NONLINEAR DYNAMICS AND PREDICTION FOR SPACE WEATHER

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We consider the problem of space weather modeling and prediction. It is suggested to utilize the nonlinear noisy black-box model to predict the Dst index. We assume the presence of a weak turbulence for the magnetosphere plasma. It is suggested to consider the solar wind parameter VB_z (a product of the solar wind speed on the south magnetic field component) as the input information for a discretized black-box model. The data on the Dst index were chosen to be the output information similarly to that for the processes of magnetospheric dynamics responsible for the Dst storms. It is shown that the relationship between VB_z input and the Dst output can be represented by a nonlinear discrete model. This model is compared to the previously developed prediction model suggested by Balikhin et al. While the both models demonstrate a high level of prediction efficacy, their lack of the sensitivity to the solar wind dynamic pressure requires their further improvement.

1. Introduction

The solar energy that reach the magnetosphere and ionosphere provides a great impact on the technological processes happening in the Space and Earth surface [1, 3, 6, 7]. Such an influence happens due to the mass ejected from the corona and the energetic particle flux from the Sun.

Here, we use a nonlinear "black-box" method based on the long-term behavior of the Dst index to predict the space weather. A mathematical model is built to simulate the processes in the magnetosphere plasma. Unlike the commonly used statistical prediction approaches, we paid attention to the dynamical characteristics of the magnetosphere behavior.

2. "Black-box" Model Approach

The suggested nonlinear "black-box" system approach is modeled as a discrete process [2]. The input is a time series which is based on the magnetic field component and the solar wind speed. The model output is the Dst index. The index describes the magnetic field perturbations and is widely used for the observed data analysis [1]. The time series that characterizes the solar wind behavior consists of multidimensional components, which increases the dimensionality of the model input, complicates the model reconstruction, and significantly increases the running time of the computation algorithm.

Due to the reasons stated above, it is important to reduce the dimensionality of the input, as well as to identify the most informative quantities. This problem can be logically divided into three stages: 1) to identify the initial predictive feature, 2) to define a time delay between the model input and the output, and 3) to select the predictors from a wide range of observed features.

To predict the Dst index, we consider the magnetosphere to be a multichannel input-output system. As the input of the system, we take the time series of the magnetic field component B_z and the solar wind speed V [7].

3. Nonlinear Model Reconstruction

The novel approach was utilized in the model. We assume a nonlinearity of the processes underlying the studied phenomenon. After a discretization of the model of magnetosphere, we arrive at the following formulation used for forecasting the Dst index:

$$y(k) = F[y(k-1), ..., y(k-n_y), u(k-1), ...,$$

$$u(k-n_u), \xi(k-1), ..., \xi(k-n_\xi), \xi(k)], \qquad (1)$$

where $F[\dots]$ denotes a nonlinear function, and y, u are, respectively, the discrete-time input and output signals. The quantity ξ accounts for the possible noise and uncertainties; n_y, n_u, and n_ξ are their associated maximum lags. The nonlinear function F can be a polynomial, a rational function, a set of radial basis functions, a wavelet decomposition, or any other function. The model with a finite number of terms can represent a wider class of nonlinear systems. The frequency characteristic can be calculated analytically from the discrete series, as well as from the Volterra series [4, 5].

We suggest $F[\dots]$ to be a polynomial function with variables $y(k)$, $u(k)$, and $\xi(k)$. The simplest

Dst index predicting technique is the one-step ahead forecasting, when the previous "black-box" input and output values are used to predict the next output value. The normalized least square error $\Psi(\theta)$ is defined as

$$\Psi(\theta) = \sqrt{((\hat{y}(k) - y(k))^2/(y(k) - \bar{y}(k))^2}, \qquad (2)$$

where $\hat{y}(k)$ is the predicted output value, $y(k)$ – experimental signal measurements, $\bar{y}(k)$ – the mean value of $y(k)$. Here, only some initial values of the input and the output are used to identify the model parameters.

We can rewrite Eq. (2) as

$$y(k) = \Psi^T(k-1)\hat{\theta} + \xi(k), \qquad (3)$$

where the index T means the transposition procedure; $\Psi^T(k-1)$ includes the input and output data, possible noise components, and all possible combinations of them up to the time moment k; $\hat{\theta}$ is a vector of the unknown model parameters; and $\xi(k)$ is the error at the step k.

Let us introduce the prediction error function

$$J_1(\hat{\theta}) = (y - \Psi\hat{\theta})^T(y - \Psi\hat{\theta}), \qquad (4)$$

where y is the Dst index at the time moment k.

The procedure of model parameter identification includes solving the mathematical programming problem. In other words, we have to find such a vector $\hat{\theta}$ that minimizes the function $J(\hat{\theta})$. Let us introduce the following vector function $J(\Theta)$, which contains various parameter dependences:

$$J(\Theta) = [J_1(\Theta), ..., J_N(\Theta)]^T. \qquad (5)$$

After expressing the function $J(\Theta)$ as the prediction error function square and substituting $J_i(\Theta)$, $i = 2, ..., N$ by the corresponding sums of other possible error squares, the multicriterion optimization problem becomes:

$$\min J(\Theta)$$

$$\text{subject to } \{\Theta \in D\}, \qquad (6)$$

where D is the model parameters restriction.

We used the following mathematical model:

$$\hat{y}(k) = x_1 y(k-1) - x_2 u(k-1) -$$

$$-x_3 y(k-2)u(k-1) - x_4 y(k-4) -$$

$$-x_5 u(k-4)u(k-6) - x_6 y(k-2) +$$

$$+x_7 y(k-7) + x_8 y(k-3)u(k-1) +$$

$$+x_9 y(k-5) + x_{10} u(k-1)u(k-7) +$$

$$+x_{11} u(k-2) + x_{12} y(k-3)u(k-2)$$

$$+x_{13} u(k-2)u(k-5) - x_{14} u(k-5) +$$

$$+x_{15} u(k-7)u(k-12). \qquad (7)$$

4. Results

We have compared several different models including a neural network model [8], a linear regression model, and a model based on the principal component analysis. As a result of the comparison, a nonlinear discrete model describing a relationship between input and output data has been selected.

We have developed two programs that use the described parameter reconstruction approach. In the first case, we minimize the prediction error to identify the informative parameters. In the second case, we search for optimal parameters using a genetic algorithm. We used satellite data with the discretization step equal to one hour.

The implementation of the above-presened algorithms provided us with the following results. The suggested model was found to give a sufficiently accurate prediction for several steps ahead and thus can be used for the practical forecasting of the Dst index. The prediction accuracy can be increased by using the adaptation algorithm for real-time model parameters. We used the quantity VB_z (Fig. 1) as an input parameter and the Dst index (Fig. 2) as an output parameter in the modeling of the magnetospheric dynamics responsible for the Dst storms.

The comparision of the real Dst index and the predicted values is shown in Figs. 3 and 4 over the 300-h time interval.

As a result of the identification procedure, the following mathematical model has been obtained:

$$\hat{y}(k) = 1,36y(k-1) - 4,98u(k-1) -$$

$$-0,18y(k-2)u(k-1) - 0,56y(k-4) -$$

$$-1,42u(k-4)u(k-6) - 0,76y(k-2) +$$

$$+0,53y(k-7) + 0,11y(k-3)u(k-1) +$$

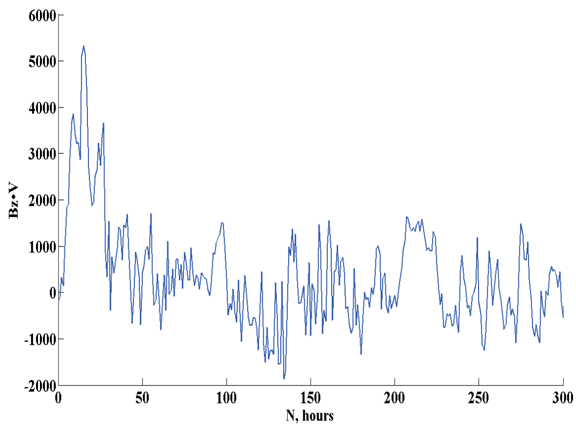

Fig. 1. Product of the magnetic field component B_z by the solar wind speed V (Space Physics Data Facility (SPDF), National Space Science Data Center (NSSDC), (2006))

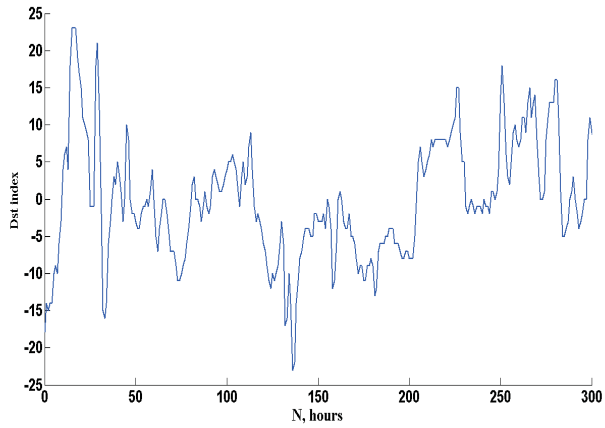

Fig. 2. Time dependence of the Dst index (Space Physics Data Facility (SPDF), National Space Science Data Center (NSSDC), (2006))

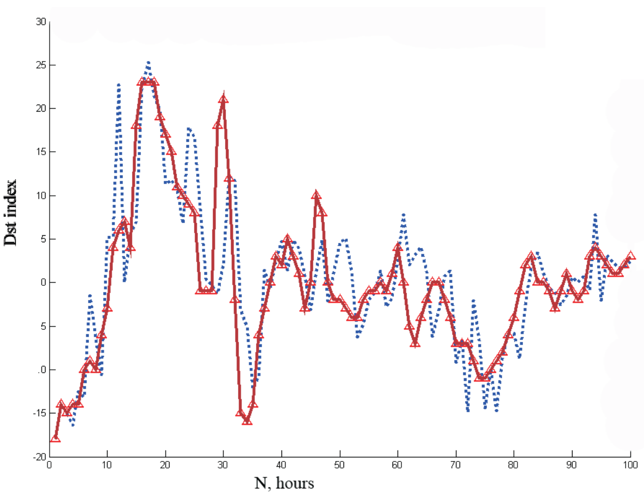

Fig. 3. Comparison of the predicted Dst index values with the experimental data (line is experimental data, △ line is the one-step ahead prediction, – line is the two-step ahead prediction (Space Physics Data Facility (SPDF), National Space Science Data Center (NSSDC), (2006)))

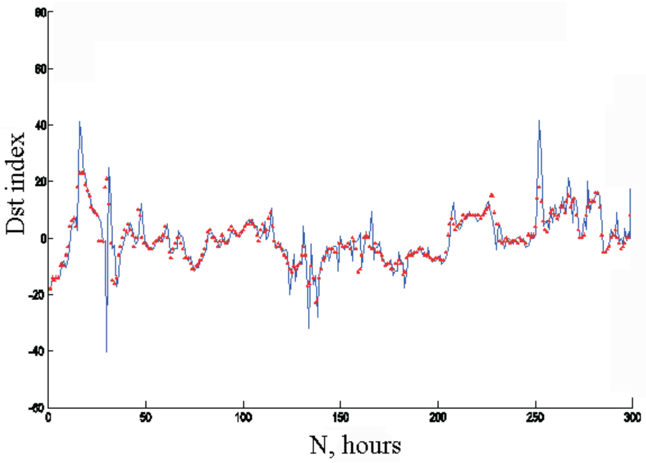

Fig. 4. Comparison of the predicted Dst index values with experimental data (curve is experimental data, △ is the one-step ahead prediction (Space Physics Data Facility (SPDF), National Space Science Data Center (NSSDC), (2006)))

$$0,36y(k-5)+0,91u(k-1)u(k-7)+$$

$$2,72u(k-2)+0,08y(k-3)u(k-2)$$

$$0,79u(k-2)u(k-5)-0,92u(k-5)+$$

$$0,24u(k-7)u(k-12), \tag{8}$$

where u_k – the product of the magnetic field component B_z by the solar wind speed V at the time moment k, y_k – the Dst index value at the time moment k.

In spite of the fact that the model identification procedure was performed in the time domain, the better spectral characteristics of the system can be achieved if the frequency domain analysis is utilized [2]. Figure 5 shows the modulus of H_i for the model identified. Obviously, the H_1 maxima correspond to the intrinsic frequency of the system.

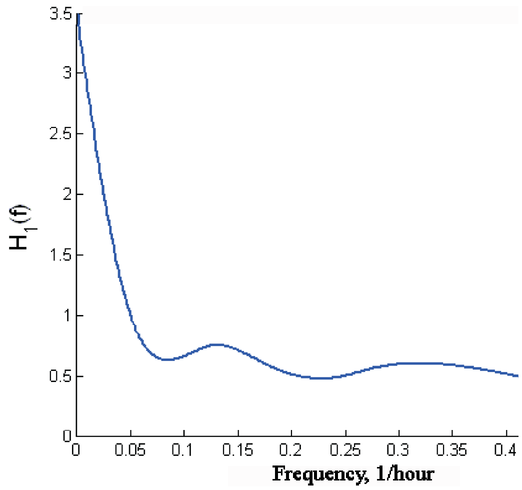

Fig. 5. Frequency dependence of the $|H_1|$ amplitude. The local $|H_1|$ maxima correspond to the system resonances

5. Summary

This paper presents an application of the multicriteria optimization method to the prediction of the Dst index, by using a nonlinear discrete dynamical model. Two novel algorithms of the identification of discrete input-output models have been developed. They allow predicting the Dst index with a high level of precision and confidence. Using the data on the solar wind VB_z as the model input and the Dst index data as the output allows us to model the magnetospheric dynamics, which is responsible for the Dst storms. To verify the algorithms, several testing approaches have been used.

The possibility of the nine-step ahead prediction of the main local Lyapunov exponent using a discrete dynamical model for systems with chaotic behavior has been shown. The result of predictions within the one-step ahead approach has demonstrated a relatively high correlation (0.97) with the real outcome. The predicion error, however, still decreases for larger time scales.

We have developed a model that predicts geomagnetic storms which are characterized by the Dst index. The model is able to forecast for 100 hours ahead, using the solar wind data. The prediction model uses a nonlinear discrete equation. The obtained prognosis is highly correlated with the actual Dst index. The correlation coefficient is in a range of 0.9–0.95. We have also compared our model with the well-known space weather prediction model developed at the University of Sheffield [3]. The accuracy of our model can be slightly improved if the real-time estimation algorithms for parameters are used.

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НЕЛІНІЙНА ДИНАМІКА
І ПРОГНОЗУВАННЯ КОСМІЧНОЇ ПОГОДИ

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Р е з ю м е

Розглядається проблема моделювання й прогнозування космічної погоди. Використовуючи припущення про слабку турбулентність магнітосферної плазми, ми пропонуємо модель нелінійного чорного ящика з шумом для прогнозування Dst-індексу. Для побудови дискретної моделі магнітосферної динаміки, що відповідає за Dst-збурення, був використаний параметр VB_z сонячного вітру у ролі входу і Dst-індекс у ролі виходу чорного ящика. Було показано, що зв'язок між параметром VB_z на вході та Dst-індексом на виході під час його збурень може бути описаний за допомогою нелінійної дискретної динамічної моделі стану магнітосферної плазми.