DIELECTRIC CHARACTERISTICS OF A MIRROR-TRAPPED PLASMA WITH ANISOTROPIC TEMPERATURE

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Analytical expressions for the wave permittivity tensor are derived for a two-dimensional axisymmetric mirror-trapped plasma model with anisotropic temperature. The dielectric characteristics are found by solving the Vlasov equations for trapped particles in the zero-order of a magnetization parameter taking the cyclotron and bounce resonances into account. The dispersion equations are derived for left-hand and right-hand polarized field-aligned cyclotron waves in the mirror-trapped plasmas.

1. Introduction

The additional plasma heating experiments in mirror-trapped plasmas by using the high frequency electromagnetic waves in the range of the electron-cyclotron [1] and ion-cyclotron [2] resonances demonstrated that the HF heating is accompanied by the increased plasma transport to the first wall under the conditions when the parallel, $T_{\|}$, and perpendicular, $T_{\perp}$, plasma temperatures are different. As is well known, the presence of particles with $T_{\|} \neq T_{\perp}$ can cause the cyclotron wave instabilities and affect consequently the transport processes.

To study the heating and stability problems in the two-dimensional (2D) plasma models, it is necessary to use the corresponding dielectric tensor components. The simplest expressions of the parallel and transverse dielectric permittivities can be used for the cold plasmas, as was done in the first 2D numerical calculations of the structure of electromagnetic fields [3–5] in the laboratory open traps. In those computations, using the so-called two-fluid MHD dielectric tensor ε_{ik}, the wave dissipation was simulated by means of artificial collisions related to enhanced electron-ion collisions. However, such an approach does not allow a study of collisionless wave dissipation in hot plasmas. The kinetic effects (using the kinetic dielectric tensor in the straight magnetic field adopted for the 2D gas dynamic open trap) were considered in work [5] analyzing the ion-cyclotron minority heating by the resonant excitation of long-wavelength fast magnetosonic waves. However, in this case, the wave-particle resonance conditions did not take the bounce resonances into account. The main feature of a 2D mirror-trapped plasma is the fact that the stationary magnetic field is axisymmetric and has one minimum in the central part of the trap. As a result, all plasma particles in the mirror trap should be considered as the trapped particles bounce-oscillating between the magnetic mirrors. As was noted in [6], the wave-particle resonance conditions in the 2D mirror-trapped plasma models should involve both the cyclotron and bounce resonances. Moreover, it was proposed that the ion-cyclotron plasma heating in mirror experiments [7, 8] can be explained by the bounce-resonance dissipation of electrons with electromagnetic waves. The contributions of the trapped particles to the longitudinal (parallel) and transverse dielectric tensor components for small-amplitude waves in the equilibrium 2D mirror-trapped plasma were derived and analyzed in work [9] accounting for the cyclotron and bounce resonances. In the present paper, we evaluate the dielectric characteristics and derive the dispersion relations for field-aligned electromagnetic waves in the non-equilibrium cylindrical magnetic mirror plasma including the particles with anisotropic temperature. The linearized Vlasov equation for the perturbed distribution functions is solved as a boundary-value problem in the zero-order in the magnetization parameter. The steady-state distribution functions of trapped particles are modeled by the bi-Maxwellian distributions.

2. Plasma Model and Vlasov Equation

Let us consider the simplest 2D collisionless mirror-trapped plasma model suitable for cylindrical axisymmetric gas-dynamic open traps [10]. In this model, the structure of the confinement magnetic field, $\mathbf{H}_0(\rho, z) = \{H_{0\rho}, 0, H_{0z}\}$, is defined by external currents and has the following $\mathbf{H}_0$-field components in the cylindrical coordinates (ρ, θ, z):

$$H_{0\rho}(\rho,z)=\frac{-\rho z\delta H_{0\min}}{L_0^2\left(1-\delta z^2/L_0^2\right)^2},\quad H_{0\theta}(\rho,z)=0,$$

$$H_{0z}(\rho,z)=\frac{H_{0\min}}{1-\delta z^2/L_0^2},\qquad(1)$$

where L_0 is the half length of a mirror trap, so that $|z|\le L_0$; $\delta=(R_m-1)/R_m$, $R_m=H_{0\max}/H_{0\min}$ is the so-called mirror ratio; $H_{0\min}$ is the minimal value of the magnetic field at the center of the trap, i.e. under $\rho=z=0$; $H_{0\max}$ is the maximum of H_{0z} at $z=\pm L_0$. The stationary magnetic field in Eqs. (1) satisfies the equation $\nabla\cdot\mathbf{H}_0=0$ and is a good approximation for devices with a large mirror ratio: $R_m\gg 1$.

To describe the oscillating currents induced by radio-frequency fields in such plasma configuration, we should solve the linearized Vlasov equation for the perturbed distribution functions of charged particles, $f(t,\mathbf{r},\mathbf{v})=f(t,\rho,\theta,z,v_\parallel,\mathrm{v}_\perp,\sigma)$, using the normal ($A_n\equiv A_1$), binormal ($A_b\equiv A_2$) and parallel ($A_\parallel\equiv A_3$) projections of the vectors $\mathbf{A}=\{\mathbf{E},\mathbf{H},\mathbf{v},\mathbf{j}\}$ which are connected with the corresponding cylindrical projections as

$$A_n=A_\rho h_z-A_z h_\rho,\quad A_b=A_\theta,\quad A_\parallel=A_\rho h_\rho+A_z h_z,\qquad(2)$$

where $\mathbf{h}=\mathbf{H}_0/H_0=(h_\rho,0,h_z)$ is the unit vector along $\mathbf{H}_0$; $\mathbf{E},\mathbf{H}$ are the perturbed electric and magnetic fields, respectively; and $\mathbf{j}$ is the perturbed current density. In the velocity space, we introduce the polar coordinates: $v_1=v_\perp\cos\sigma$, $v_2=v_\perp\sin\sigma$, and $v_3=v_\parallel$. The further solution of the kinetic equation for the perturbed distribution functions can be found by the Fourier transformation with respect to the polar angle σ in the velocity space,

$$f(t,\mathbf{r},\mathbf{v})=\sum_l^{\pm\infty}f_l(\rho,z,v_\parallel,\mathrm{v}_\perp)\exp(-i\omega t+im\theta-il\sigma),\qquad(3)$$

taking into account that the problem is homogeneous in the time t and the azimuthal coordinate θ; where m and l are the integers. To estimate the main contribution of plasma particles to the perturbed current density components, it is necessary to find the harmonics f_l with the low numbers $l=0,\pm1$. Substituting the Fourier transform (3) into the Vlasov equation, we obtain (in the zero-order in the magnetization parameter) the following equations for f_0, f_1 and f_{-1}:

$$-i\left(\omega-l\Omega_c\right)f_l+v_\parallel h_\rho\frac{\partial f_l}{\partial\rho}+v_\parallel h_z\frac{\partial f_l}{\partial z}+$$

$$+\frac{v_\perp}{2}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho h_\rho+\frac{\partial h_z}{\partial z}\right)\left(v_\perp\frac{\partial f_l}{\partial v_\parallel}-v_\parallel\frac{\partial f_l}{\partial v_\perp}\right)=Q_l,\qquad(4)$$

where $l=0,\pm1$,

$$Q_{\pm1}=\frac{ev_\perp}{Mv_{T\parallel}^2}\left[\frac{T_\parallel}{T_\perp}(E_n\pm iE_b)+\right.$$

$$\left.+\frac{v_\parallel}{c}(H_b\mp iH_n)\left(1-\frac{T_\parallel}{T_\perp}\right)\right]F,\qquad(5)$$

$$Q_0=\frac{2ev_\parallel}{Mv_{T\parallel}^2}E_\parallel F.\qquad(6)$$

Here, F is the steady-state bi-Maxwellian distribution function:

$$F=\frac{N}{\pi^{1.5}v_{T\parallel}v_{T\perp}^2}\exp\left\{-\frac{v^2}{v_{T\parallel}^2}\left[1-\mu\left(1-\frac{T_\parallel}{T_\perp}\right)\right]\right\},$$

$$v_{T\parallel}=\sqrt{\frac{2T_\parallel}{M}},\quad v_{T\perp}=\sqrt{\frac{2T_\perp}{M}}.\qquad(7)$$

The cyclotron frequency of plasma particles $\Omega_c\equiv eH_0/Mc$ in Eq. (4) is calculated by the modulus of the stationary magnetic field $H_0(\rho,z)=\sqrt{H_{0z}^2+H_{0\rho}^2}$, where, as in Eqs. (4)–(7) for f and F, the index of particle species is omitted.

Three partial derivatives in Eq. (4) can be excluded, by introducing three new variables connected with the conservation laws of energy and magnetic moment and equation for $\mathbf{H}_0$-field lines, i.e., respectively,

$$v_\parallel^2+v_\perp^2=\mathrm{const},\quad\frac{v_\perp^2}{H_0(\rho,z)}=\mathrm{const},$$

$$\frac{\rho^2}{1-\delta z^2/L_0^2}=\mathrm{const}.\qquad(8)$$

Using these invariants, the new variables v(velocity modulus), μ (dimensionless magnetic moment) and r(radial coordinate of magnetic field lines) are introduced instead of $v_\parallel$, $v_\perp$, and ρ as

$$v=\sqrt{v_\parallel^2+v_\perp^2},\quad\mu=\frac{v_\perp^2}{v_\parallel^2+v_\perp^2}\left(1-\delta\frac{z^2}{L_0^2}\right),$$

$$r = \frac{\rho}{\sqrt{1-\delta z^2/L_0^2}}. \quad (9)$$

To simplify the calculations, we assume that the radial magnetic field component is much less of the longitudinal one, i.e. $H_{0\rho} \ll H_{0z}$, and, consequently, $H_0 = \sqrt{H_{0z}^2 + H_{0\rho}^2} \approx H_{0z}$, which are valid for long mirror traps under the condition $a/L_0 \ll 1/(R_m - 1)$, where a is the plasma radius at the equatorial plane of a trap ($z = 0$). Further, we suppose that the plasma density is distributed parabolically over a radius, $N = N_0(1 - r^2/a^2)$, where N_0 is the particle density at the point $(r = 0, z = 0)$, and the plasma boundary radius along the mirror trap $r_0(z)$ follows the law: $r_0(z) = a\sqrt{1-\delta z^2/L_0^2}$.

In other words, the equidensity lines coincide in this plasma model with the magnetic field lines. In this case, using the variables r, $\varsigma = z/L_0$, v and $\kappa = \sqrt{1-\mu}$ (instead of $\rho, z, v_\parallel, \mathrm{v}_\perp$), Eqs. (4) for the principal harmonics of the perturbed distribution function,

$$f_l(\rho, z, v_\parallel, v_\perp) = \sum_{s=\pm1} f_l^{(s)}(r, \varsigma, v, \kappa), \quad l = 0, \pm 1, \quad (10)$$

can be reduced to the first-order linear differential equations for a single variable ς:

$$\sqrt{\frac{\kappa^2-\delta\varsigma^2}{\delta(1-\delta\varsigma^2)}}\frac{\partial f_l^{(s)}}{\partial\varsigma} - i\frac{sL_0}{v\sqrt{\delta}}\left(\omega - \frac{l\Omega_{c0}}{1-\delta\varsigma^2}\right)f_l^{(s)} = Q_l^{(s)},$$

$$s = \pm1, \quad l = \pm1, \quad (11)$$

where

$$Q_0^{(s)} = \frac{2eL_0E_\parallel}{\sqrt{\delta}Mv_{T\parallel}^2}\sqrt{\frac{\kappa^2-\delta\varsigma^2}{1-\delta\varsigma^2}}F, \quad \Omega_{c0} = \frac{eH_{0\,\min}}{Mc}, \quad (12)$$

$$Q_{\pm1}^{(s)} = \frac{seL_0\sqrt{1-\kappa^2}}{\sqrt{\delta}Mv_{T\perp}^2\sqrt{1-\delta\varsigma^2}}\times$$

$$\times\left[E_{\pm1} + i\frac{sv}{L_0\omega}\left(1-\frac{T_\perp}{T_\parallel}\right)\sqrt{\frac{\kappa^2-\delta\varsigma^2}{1-\delta\varsigma^2}}\frac{\partial E_{\pm1}}{\partial\varsigma}\right]F. \quad (13)$$

Other variables r, v, and $\kappa = \sqrt{1-\mu}$ appear as parameters in these equations. Here, the signs of the parallel velocity relatively to the $\mathbf{H}_0$-field direction have been taken into account by the indices $s = \pm1$:

$$v_\parallel = sv\sqrt{\frac{\kappa^2-\delta\varsigma^2}{1-\delta\varsigma^2}}. \quad (14)$$

The combinations $E_l = E_n + ilE_b$ are convenient to describe the circular left-hand (if $l = 1$) and right-hand (if $l = -1$) polarizations having the electric field rotation as ions and electrons in the strong magnetic field, respectively.

3. Trapped Particles and Current Density Components

Since the mirror-trapped magnetic field configuration is axisymmetric and has one minimum at the equatorial plane for a considered magnetic field line, the plasma particles should be separated on two groups of the so-called trapped and untrapped (passing) particles. In our case, such a separation can be done by the parameter μ or κ analyzing the conditions when $v_{||}(\kappa, \varsigma) = 0$. The domain of the functions $f_l^{(s)}$ for the trapped particles is given by the inequalities

$$0 \le \kappa \le \sqrt{\delta}, \quad -\varsigma_t(\kappa) \le \varsigma \le \varsigma_t(\kappa), \quad (15)$$

where $\pm\varsigma_t = \pm\kappa/\sqrt{\delta}$ are the stop points (or reflection points) of trapped particles. As for untrapped particles, the corresponding domain of their distribution functions is defined by the inequalities

$$\sqrt{\delta} \le \kappa \le 1, \quad -1 \le \varsigma \le 1. \quad (16)$$

However, as was shown in [9], the influence of untrapped particles on the wave processes in mirror-trapped plasmas is not substantial, since they are lost in one transit time, and the number of those passing is small compared with the number of those trapped for $R_m \gg 1$. Further, we neglect the contribution of untrapped particles to the current density components and, respectively, to the dielectric characteristics.

As a result, the transverse, $j_{(l)}$, and parallel, $j_\parallel$, perturbed current density components in the new variables can be found by the double integration of the perturbed distribution function of trapped particles in the velocity space:

$$j_{(\pm1)}(r,\varsigma) = \frac{\pi e}{1-\delta\varsigma^2}\sum_{s=\pm1}\int_0^\infty v^3dv\int_0^\delta \frac{f_{\pm1}^{(s)}\kappa\sqrt{1-\kappa^2}d\kappa}{\sqrt{\kappa^2-\delta\varsigma^2}}, \quad (17)$$

$$j_\parallel(r,\varsigma) = \frac{2\pi e}{1-\delta\varsigma^2}\sum_{s=\pm1}s\int_0^\infty v^3dv\int_0^\delta f_0^{(s)}\kappa d\kappa. \quad (18)$$

Solving Eq. (11) to describe the bounce motion of trapped particles, it is convenient to introduce the new time-like variable $\tau(\varsigma)$, instead of ς:

$$\tau(\varsigma) = \int\limits_0^{\arcsin(\sqrt{\delta}\varsigma/\kappa)} \sqrt{1-\kappa^2\sin^2\phi}\,d\phi =$$

$$= E(\arcsin(\sqrt{\delta}\varsigma/\kappa), \kappa), \quad (19)$$

where $E(\alpha, \kappa)$ is the non-complete elliptic integral of the second kind. In this case, the bounce-period of trapped particles in the mirror trap become proportional to $\tau_b = 4E(\kappa)$, where $E(\kappa)$ is the complete elliptic integral of the second kind, and the solution of Eq. (11) can be found as

$$f_l^{(s)} = \sum_p^{\pm\infty} f_{l,p}^{(s)} \exp\left[i\left(2\pi p\frac{\tau}{\tau_b} - \frac{slL_0\Omega_{c0}}{v\sqrt{\delta}}\times\right.\right.$$

$$\times \int\limits_0^{\tau} \frac{d\tau}{1-\kappa^2\sin^2\phi(\tau)} + \frac{4slL_0\Omega_{c0}}{v\sqrt{\delta}\tau_b}\tau\times$$

$$\left.\left.\times \int\limits_0^{E(\kappa)} \frac{d\tau}{1-\kappa^2\sin^2\phi(\tau)}\right)\right]. \quad (20)$$

Here, $l = 0, \pm1$, and the coupling of the variables $\phi(\tau)$ is given in the non-explicit form by Eq. (19) and $\phi(\varsigma) = \arcsin(\sqrt{\delta}\varsigma/\kappa)$. Averaging over the bounce-periods, we can find the bounce-harmonics $f_{l,p}^{(s)}$ and calculate the 2D current density components using Eqs. (17) and (18).

4. Dielectric Permittivity Tensor Components

To evaluate the high frequency conductivity and the corresponding dielectric tensor components, it is necessary to apply the Fourier transformation of the perturbed electric field and current density components with respect to the variable ς:

$$(1-\delta\varsigma^2)\mathbf{j}(r,\varsigma) = \sum_n^{\pm\infty} \mathbf{j}^{(n)}(r)\exp(i\pi n\varsigma), \quad (21)$$

$$\mathbf{E}(r,\varsigma) = \sum_{n'}^{\pm\infty} \mathbf{E}^{(n')}(r)\exp(i\pi n'\varsigma). \quad (22)$$

This procedure converts the integral operator representing the dielectric permittivity tensor into a matrix, whose elements can be calculated in a manner which does not depend upon the solutions of the Maxwell equations. The corresponding expressions for the Fourier-harmonics of the transverse and parallel currents are

$$\frac{4\pi i}{\omega} j_{(l)}^{(n)} = \sum_n^{\pm\infty} \varepsilon_l^{n,n'}\left[E_n^{(n')} + ilE_b^{(n')}\right], \quad l = \pm 1, \quad (23)$$

$$\frac{4\pi i}{\omega} j_{\parallel}^{(n)} = \sum_n^{\pm\infty} \varepsilon_{\parallel}^{n,n'} E_{\parallel}^{(n')}, \quad (24)$$

where the contribution of trapped particles to the transverse $\varepsilon_l^{n,n'}$ and parallel $\varepsilon_{\parallel}^{n,n'}$ elements of the dielectric permittivity tensor is defined by the summation of bounce resonance terms including the double integration in the velocity space, resonant denominators, and the phase coefficients:

$$\varepsilon_l^{n,n'} = \frac{\Omega_{p0}^2\left(1-\frac{r^2}{a^2}\right)L_0}{\omega\delta\pi^{1.5}v_{T\parallel}T_\perp^2/T_\parallel^2} \sum_{p=1}^{\infty} \int\limits_{-\infty}^{\infty} u^4 du\times$$

$$\times \int\limits_0^{\sqrt{\delta}} A_{l,p}^{n'}(\kappa, v)B_{l,p}^{n}(\kappa, v)\times$$

$$\times \frac{\exp\left\{-u^2\left[\kappa^2+(1-\kappa^2)\frac{T_\parallel}{T_\perp}\right]\right\}}{pu - \frac{\omega E(\kappa) - l\Omega_{c0}K(\kappa)}{\omega_b E(\kappa)}}\kappa(1-\kappa^2)d\kappa, \quad (25)$$

$$\varepsilon_{\parallel}^{n,n'} = \frac{4\Omega_{p0}^2\left(1-\frac{r^2}{a^2}\right)L_0}{\omega\delta\pi^{1.5}v_{T\parallel}T_\perp/T_\parallel} \sum_{p=1}^{\infty} \int\limits_0^{\sqrt{\delta}} A_p^{n'}(\kappa)B_p^{n}(\kappa)\kappa^3 d\kappa\times$$

$$\times \int\limits_{-\infty}^{\infty} \frac{\exp\left\{-u^2\left[\kappa^2+(1-\kappa^2)\frac{T_\parallel}{T_\perp}\right]\right\}}{pu - \omega/\omega_b} u^4 du. \quad (26)$$

Here, $\Omega_{p0}^2 = \frac{4\pi N_0 e^2}{M}$, $K(\kappa) = \int\limits_0^{\pi/2} \frac{d\varphi}{\sqrt{1-\kappa^2\sin^2\varphi}}$ and $F(\phi, \kappa) = \int\limits_0^{\phi} \frac{d\varphi}{\sqrt{1-\kappa^2\sin^2\varphi}}$ are the complete and non-complete elliptic integrals of the first kind, $\omega_b = \frac{2\pi\sqrt{\delta}v_{T\parallel}}{L_0\tau_b}$

is the bounce-frequency of trapped particles with a given parameter κ and the longitudinal temperature $T_\parallel$ which is inversely proportional to the bounce-period of trapped particles between the stop points. In Eqs. (25) and (26), we have used the following definitions for the phase coefficients:

$$A^n_{l,p}(\kappa, v) = \int\limits_0^{\pi/2} \left[1 - \frac{\pi n u v_{T\parallel}}{L_0\omega}\left(1 - \frac{T_\perp}{T_\parallel}\right)\times\right.$$

$$\left.\times\frac{\kappa\cos\phi}{\sqrt{1-\kappa^2\sin^2\phi}}\right]\cos\left[n\pi\frac{\kappa}{\sqrt{\delta}}\sin\phi - p\pi\frac{E(\phi,\kappa)}{2E(\kappa)}+\right.$$

$$\left.+\frac{lL_0\Omega_{c0}\left\langle F(\phi,\kappa)E(\kappa) - E(\phi,\kappa)K(\kappa)\right\rangle}{uv_{T\parallel}\sqrt{\delta}E(\kappa)}\right]d\phi+$$

$$+(-1)^p\int\limits_0^{\pi/2}\left[1 - \frac{\pi n u v_{T\parallel}\kappa\cos\phi}{L_0\omega\sqrt{1-\kappa^2\sin^2\phi}}\left(1-\frac{T_\perp}{T_\parallel}\right)\right]\times$$

$$\times\cos\left[n\pi\frac{\kappa}{\sqrt{\delta}}\sin\phi + p\pi\frac{E(\phi,\kappa)}{2E(\kappa)}-\right.$$

$$\left.-\frac{lL_0\Omega_{c0}\left\langle F(\phi,\kappa)E(\kappa) - E(\phi,\kappa)K(\kappa)\right\rangle}{uv_{T\parallel}\sqrt{\delta}E(\kappa)}\right]d\phi, \qquad (27)$$

$$B^n_{l,p}(\kappa, v) = \int\limits_0^{\pi/2}\cos\left[n\pi\frac{\kappa}{\sqrt{\delta}}\sin\phi - p\pi\frac{E(\phi,\kappa)}{2E(\kappa)}+\right.$$

$$\left.+\frac{lL_0\Omega_{c0}\left\langle F(\phi,\kappa)E(\kappa) - E(\phi,\kappa)K(\kappa)\right\rangle}{uv_{T\parallel}\sqrt{\delta}E(\kappa)}\right]d\phi+$$

$$+(-1)^p\int\limits_0^{\pi/2}\cos\left[n\pi\frac{\kappa}{\sqrt{\delta}}\sin\phi + p\pi\frac{E(\phi,\kappa)}{2E(\kappa)}-\right.$$

$$\left.-\frac{lL_0\Omega_{c0}\left\langle F(\phi,\kappa)E(\kappa) - E(\phi,\kappa)K(\kappa)\right\rangle}{uv_{T\parallel}\sqrt{\delta}E(\kappa)}\right]d\phi, \qquad (28)$$

$$B^n_p(\kappa) = \int\limits_0^{\pi/2}\cos\left[n\pi\frac{\kappa}{\sqrt{\delta}}\sin\phi - p\pi\frac{E(\phi,\kappa)}{2E(\kappa)}\right]\cos\phi d\phi+$$

$$+(-1)^{p-1}\int\limits_0^{\pi/2}\cos\left[n\pi\frac{\kappa}{\sqrt{\delta}}\sin\phi + p\pi\frac{E(\phi,\kappa)}{2E(\kappa)}\right]\cos\phi d\phi. \qquad (29)$$

As usual, when the denominators in Eqs. (25) and (26) are equal to zero, we obtain the conditions of the resonant interactions of waves with the trapped particles in a mirror trap,

$$\omega - l\Omega_{c0}\frac{K(\kappa)}{E(\kappa)} = \frac{p\pi v\sqrt{\delta}}{2L_0E(\kappa)}, \quad p = 0, \pm1, \pm2, ... \quad ,$$

$$l = 0, \pm1, \qquad (30)$$

which involve the wave frequency, the bounce-averaged cyclotron frequency, and the bounce frequency, where the integer p is the number of possible bounce-resonances. The trapped particles with the corresponding v and κ are named as resonant and responsible for the damping or growth rates of the wave amplitudes in the equilibrium and non-equilibrium plasmas. Moving along the stationary magnetic field lines, the trapped particles bounce-oscillate between the stop points and are able to interact many times with the wave in the two cyclotron resonance zones, which are symmetric relatively to the plane $z = 0$ in the space between the mirror-points. As an important feature of the transverse and parallel current density components in the 2D axisymmetric mirror-trapped plasma, Eqs. (23) and (24), is the fact that the whole spectrum of the perturbed electric field (by $\Sigma^{\pm\infty}_{n'}$ over the longitudinal wave numbers) contributes to the given n-th harmonic of the current density.

As was mentioned above, expressions (25) and (26) describe the contribution of trapped particles of the unspecified kind to the dielectric tensor elements. The corresponding expressions for plasma electrons and ions can be obtained from Eqs. (25) and (26) replacing the temperatures $T_\parallel$ and $T_\perp$, density N_0 , mass M, and charge of particles by the electron $T_{\parallel e}$, $T_{\perp e}$, N_{0e}, M_e, e_e and ion $T_{\parallel i}$, $T_{\perp i}$, N_{0i}, M_i, e_i parameters, respectively. It should be noted that, in the case of the isotropic temperature ($T_\parallel = T_\perp = T$), expressions (25)–(29) can be simplified to the corresponding expressions for dielectric permittivity elements in the equilibrium plasma model with the Maxwellian distribution functions.

5. Dispersion Relations for the Field-aligned Cyclotron Waves

Since the cyclotron wave instabilities can affect the transport processes in the mirror-trapped plasma, it is possible to develop a 2D numerical code to describe these instabilities using the new dielectric tensor components, accounting for the bounce-resonant effects. To have some analogy with linear theory of cyclotron waves in the straight magnetic field, let us assume that the $n-$th harmonic of the electric field gives the main contribution to the n-th harmonic of the current density (*one-mode approximation*). In this case, for the field-aligned electromagnetic cyclotron waves (when m =0, $\partial/\partial r = 0$, $E_\parallel = 0$, $H_\parallel = 0$), we get the following dispersion equation from the Maxwell's equations excluding the $E_l^{(n)}$-harmonics by Eqs. (23):

$$\left(\frac{\pi nc}{L_0\omega}\right)^2 = 1 + 2\sum_{\alpha}^{e,i_1,i_2,\ldots} \varepsilon_{l,(\alpha)}^{n,n'}(r), \tag{31}$$

where α denotes the particle species (electron, proton, heavy ions). This equation is suitable to analyze the instability of the right-hand polarized waves if $l = -1$, and the left-hand polarized waves if l =1. Note that, in our notation, the parallel wave vector is defined as $k_\parallel = n\pi/L_0$, so that $n\pi c/(L_0\omega)$ is the dimensionless parallel refractive index.

Further, Eq. (31) should be solved numerically for the real and imaginary parts of the wave frequency, $\omega = \mathrm{Re}\omega + i\mathrm{Im}\omega$, to define the conditions of the wave instabilities in the mirror-trapped plasmas with anisotropic temperature. As usual, the growth (damping) rate of the electromagnetic cyclotron waves, $\mathrm{Im}\,\omega$, is defined by the contribution of the resonant particles to the imaginary part of the transverse permittivity elements, $\mathrm{Im}\,\varepsilon_{l,(\alpha)}^{n,n}$, that can be readily derived from Eqs. (25) using the well known residue (or the Landau rule) method: $\mathrm{Im}\,\varepsilon_{l,(\alpha)}^{n,n} = \sum\limits_{p=1}^{\infty} \mathrm{Im}\,\varepsilon_{l,p,\alpha}^{n,n}$, where

$$\mathrm{Im}\,\varepsilon_{l,p,\alpha}^{n,n} = \frac{\Omega_{p\alpha}^2\left(1-\frac{r^2}{a^2}\right)L_0 T_{\parallel\alpha}^2}{\omega\delta\sqrt{\pi}v_{T\parallel\alpha}T_{\perp\alpha}^2}\frac{1}{p^5}\times$$

$$\times\int\limits_0^{\sqrt{\delta}} A_{l,p}^n\left(\kappa,\frac{Z_{l,\alpha}}{p}\right)B_{1,p}^n\left(\kappa,\frac{Z_{l,\alpha}}{p}\right)\times$$

$$\times Z_{l,\alpha}^4\exp\left\{-\frac{Z_{l,\alpha}^2}{p^2}\left[\kappa^2+(1-\kappa^2)\frac{T_{\parallel\alpha}}{T_{\perp\alpha}}\right]\right\}\kappa(1-\kappa^2)d\kappa \tag{32}$$

is the separate contribution of the bounce resonance terms to $\mathrm{Im}\,\varepsilon_{l,(\sigma)}^{n,n}$,

$$Z_{l,\alpha} = \frac{\omega E(\kappa) - l\Omega_{c0,\alpha}K(\kappa)}{\omega_{b,\alpha}E(\kappa)}. \tag{33}$$

Note that the instabilities of the ion-cyclotron and electron-cyclotron waves ($\mathrm{Im}\,\omega > 0$) in mirror traps can be realized in the frequency ranges below of the minimal cyclotron frequencies of ions, $\mathrm{Re}\,\omega < \Omega_{c0,i}$, and electrons, $\mathrm{Re}\,\omega < |\Omega_{c0,e}|$, respectively.

6. Conclusion

In conclusion, let us summarized the main results of the present paper. The transverse and parallel current density components in the 2D mirror-trapped plasma with anisotropic temperature ($T_\parallel \neq T_\perp$) are evaluated by solving the linearized Vlasov equation for the perturbed distribution functions of trapped particles in the zero order in the magnetization parameter using the standard method of switching to new variables associated with the conservation integrals of energy and magnetic moment and the equation for magnetic field lines. The new time-like variable is introduced (instead of the longitudinal z-variable) to describe the bounce-periodic motion of trapped particles along the 2D stationary magnetic field, Eq. (20).

To evaluate the transverse and longitudinal dielectric permittivity elements, the perturbed electric field and the current density components are Fourier-transformed with respect to the z-variable. The new dielectric characteristics are expressed by the summation of the bounce-resonant terms including the double integration in the velocity space, the resonant denominators, and the corresponding phase coefficients, Eqs. (26) and (27). Due to the 2D magnetic field nonuniformity, the bounce-resonance conditions for trapped particles in open traps are different from ones in the straight magnetic field, Eqs. (30); the whole spectrum of the electric field is present in the given current density harmonic, Eqs. (24) and (25); the left-hand and right-hand polarized waves are coupled in the general case.

In this paper, we have derived the dispersion relations for field-aligned cyclotron waves in the mirror-trapped plasma, Eq. (31), assuming that the energetic particles have the bi-Maxwellian distribution functions in the velocity space. To have some analogy with the linear theory of cyclotron waves in the straight magnetic field, we assumed that the $n-$th harmonic of the electric field gives the main contribution to the $n-$th harmonic of the current density, and the connection of the left-hand and right-hand waves is small. In this case, the dispersion equations for field aligned cyclotron waves have the simplest form and are suitable to analyze the instabilities of both the electron-cyclotron and ion-cyclotron waves accounting for the cyclotron and bounce resonances. Our dispersion relations can be used to analyze the eigenfrequencies and the temporal growth/damping rates of both the left-hand and right-hand circularly polarized cyclotron waves in the 2D open traps. Like the case of a uniform plasma confined in the straight magnetic field, the growth/damping rates of the cyclotron waves are defined by the contribution of the resonant particles to the imaginary part of the transverse dielectric permittivity elements.

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ДІЕЛЕКТРИЧНІ ХАРАКТЕРИСТИКИ АКСІАЛЬНО-СИМЕТРИЧНОГО ПЛАЗМОВОГО ПРОБКОТРОНУ З АНІЗОТРОПНОЮ ТЕМПЕРАТУРОЮ

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Р е з ю м е

Отримано аналітичні вирази компонентів тензора діелектричної проникності для високочастотних полів в двовимірно-неоднорідному аксіально-симетричному плазмовому пробкотроні з анізотропною температурою. Діелектричні характеристики отримано на основі розв'язку рівняння Власова для запертих частинок в нульовому наближенні за параметром замагніченості, враховуючи циклотронні та баунс-резонанси. Отримано дисперсійні співвідношення для ліво- і правополяризованих циклотронних хвиль, які поширюються вздовж стаціонарного магнітного поля у плазмовому пробкотроні.