IMPURITY ION DYNAMICS NEAR MAGNETIC ISLANDS IN THE DRIFT OPTIMIZED STELLARATOR CONFIGURATION OF WENDELSTEIN 7-X

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The radial transport of tungsten ions in a fusion plasma of a Wendelstein 7-X stellarator with five magnetic field periods is studied. The numerical code solves the guiding center equations for tungsten ions using the sixth-order Runge–Kutta method. The Coulomb scattering of tungsten ions is simulated by means of the Monte Carlo collision operator. The establishment of statistical properties of the ensemble of test particles is based on the calculation of the mean square displacement and the diffusion coefficient. It is shown that the existence of magnetic islands at the plasma periphery leads to the enhancement of the impurity transport.

1. Introduction

At the contemporary stage of the fusion development, the question of the particle transport is very important. One of the main problems for the operation of a fusion reactor is the penetration of impurity ions which are sputtered from the divertor plates or a vacuum vessel, into the plasma core. According to the present state of the art, tungsten is considered as an almost optimal material. However, even a small fraction of heavy ions in plasma leads to a significant increase of bremsstrahlung radiation losses, thus increasing the plasma temperature needed for the ignition threshold. In the present paper, the radial transport of tungsten ions is studied for two magnetic field configurations of a Wendelstein 7-X (W7-X) stellarator being constructed at Max-Planck-Institut für Plasmaphysik, Greifswald, Germany. The W7-X coil system consists of 50 modular and 20 planar non-circular coils combined in $M = 5$ magnetic periods [1]. There are also 10 control coils used to control the magnetic island position. The coil system of W7-X is developed in such a way that the drift losses are minimized in this configuration. The main parameters of W7-X are the following: the major plasma radius $R_0 = 5.5$ m, minor plasma radius $a = 0.52$ m, the average magnetic field on the magnetic axis $B_0 = 2.5$ T.

It is known that, due to the lack of axial symmetry in stellarators, some group of particles is trapped in local magnetic wells. These particles show a fast bouncing motion superposed onto a slow uncompensated radial drift. The existence of such a group of particles leads to the large neoclassical losses in stellarators as compared with the tokamak case. In a W7-X stellarator, the significant reduction of neoclassical losses is expected due to the optimized 3-D magnetic field structure.

Due to collisions, a passing particle can change its magnetic moment, which results in trapping in the magnetic well and the subsequent loss from the system. In the present work, the Coulomb scattering of tungsten ions is modeled via the Monte Carlo collision operator [2]. The statistical properties of the ensemble of tungsten ions are studied by calculating the mean square displacement and the diffusion coefficient. It is shown that the existence of magnetic islands at the plasma periphery leads to the enhancement of the particle transport.

2. Magnetic Field Model

We use the Boozer coordinates (ψ, θ, and ϕ), where ψ is the label of the magnetic surface, θ and ϕ are the poloidal and toroidal angles, respectively. The strength of an equilibrium magnetic field which has closed nested magnetic surfaces can be expanded in the Fourier series

$$B = B_0(1 + \sum_{m,n} b_{mn}(r)\cos(\mathrm{m}\theta - \mathrm{nM}\phi)). \quad (1)$$

We restrict ourselves to considering a model magnetic field, where only the diamagnetic $b_{00}(r)$, mirror $b_{01}(r)$, toroidal $b_{10}(r)$, and helical $b_{11}(r)$ harmonics are non-zero. In W7-X configurations, these harmonics are much larger than the rest of harmonics [1, 3]. The Fourier coefficients as functions of the normalized radius r are

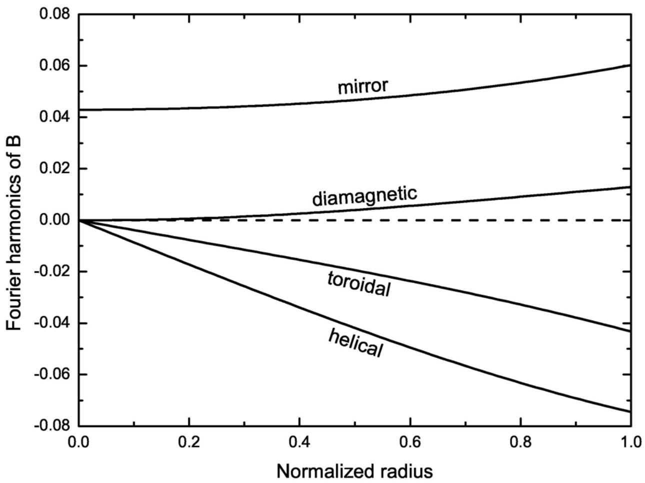

Fig. 1. Fourier harmonics of B for the standard configuration

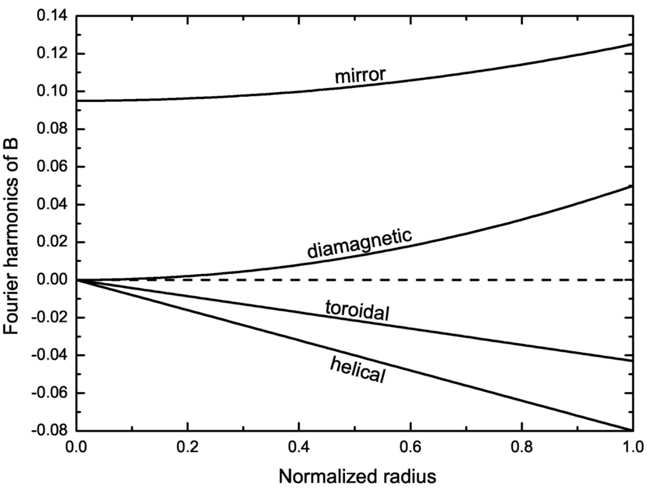

Fig. 2. Fourier harmonics of B for the high-mirror configuration

presented in Fig. 1 for the vacuum standard configuration [1] and in Fig. 2 for the finite beta high-mirror configuration [3].

The perturbation of the equilibrium magnetic field can be presented in the form [4]

$$\delta\overrightarrow{B} = \nabla \times (\alpha\overrightarrow{B_0}), \tag{2}$$

where the function α, which has the unit of length, presents a structure of the destroyed magnetic field. The perturbation function is chosen in the following form:

$$\alpha = \alpha_0 r^m \sin(\mathrm{n}\phi - \mathrm{m}\theta). \tag{3}$$

The total magnetic field can be presented as

$$\overrightarrow{B} = \overrightarrow{B_0} + \delta\overrightarrow{B}. \tag{4}$$

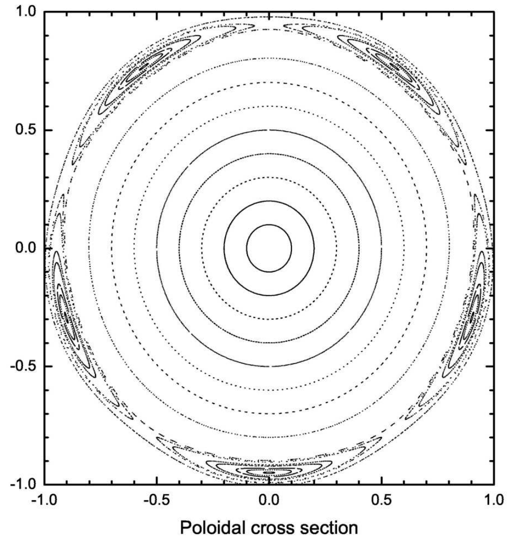

Fig. 3. Poincaré plot of magnetic surfaces for $\phi = 0$

For the W7-X configuration with 5 magnetic periods along the torus, we have studied the perturbation with the wave numbers $n = 5$, $m = 5$. This perturbation creates a chain of 5 magnetic islands (Fig. 3) near the rational magnetic surface ($r_{\mathrm{res}} = 0.95$), where the rotational transform is $\iota = 1$. Our numerical calculations confirm the radical dependence of the radial island width Δ on the perturbation amplitude α_0 [5]. In the case under consideration, the radial island width was $\Delta \approx 4.9$ cm. We note that the shape of magnetic surfaces in W7-X changes from a bean to a triangle depending on the toroidal angle ϕ. In Fig. 3, the magnetic surfaces are circular, because we operate in the Boozer magnetic coordinates, where the magnetic lines are straight in the $(\theta,\ \phi)$ plane.

3. Basic Equations

The guiding center equations written in the Hamiltonian form [4, 6, 7]

$$\left\{ \begin{array}{l} \dot{\psi} = (g\dot{P}_\theta - I\dot{P}_\phi)/\gamma, \\ \dot{\theta} = [(\mu + \rho_{||}^2 B)\frac{\partial B}{\partial \psi} g + \rho_{||} B^2(\iota - \rho_c g') - \\ -\rho_{||} B^2 \frac{\partial \alpha}{\partial \psi} g]/\gamma, \\ \dot{\phi} = [-(\mu + \rho_{||}^2 B)\frac{\partial B}{\partial \psi} I + \rho_{||} B^2(1 + \rho_c I') + \\ +\rho_{||} B^2 \frac{\partial \alpha}{\partial \psi} I]/\gamma, \\ \dot{\rho_{||}} = [(1 + \rho_c I')\dot{P}_\phi + (\iota - \rho_c g')\dot{P}_\theta]/\gamma - \\ -(\frac{\partial \alpha}{\partial \psi}\dot{\psi} + \frac{\partial \alpha}{\partial \theta}\dot{\theta} + \frac{\partial \alpha}{\partial \phi}\dot{\phi}), \\ \dot{P}_\theta = -(\mu + \rho_{||}^2 B)\frac{\partial B}{\partial \theta} + \rho_{||} B^2 \frac{\partial \alpha}{\partial \theta}, \\ \dot{P}_\phi = -(\mu + \rho_{||}^2 B)\frac{\partial B}{\partial \phi} + \rho_{||} B^2 \frac{\partial \alpha}{\partial \phi} \end{array} \right. \tag{5}$$

are solved numerically to study the particle dynamics. Here, $\rho_c = \rho_{||} + \alpha$ and $\gamma = g + \iota I + \rho_c(gI' - Ig')$. These equations are written in the normalized form. All lengths are normalized to the major plasma radius R_0, the time is given in the units of $1/\omega_0$, where $\omega_0 = \frac{ZeB_0}{mc}$ is the gyrofrequency on the axis, the energy is normalized to $m\omega_0^2 R_0^2$, and the magnetic field strength is normalized to its average value on the magnetic axis B_0. The functions I and g are proportional to the toroidal and poloidal currents inside and outside the flux surface, respectively. The rotational transform profile $\iota(r)$ is taken in the form presented in Fig. 4.

4. Collisional Operator

To model a collisional kick for a test particle in the pitch angle space, a collision operator is applied after each integration time step. The Lorentz collision operator has the form [2]

$$\lambda(t_n) = \lambda(t_{n-1})(1 - \nu_d \triangle t) \pm \sqrt{(1 - \lambda^2(t_{n-1}))\nu_d \triangle t}, \quad (6)$$

where the time step $\triangle t$ is chosen to satisfy the condition $\nu_d \triangle t \ll 1$ and the energy conservation in the non-collisional case. The sign in (6) should be chosen randomly but with equal probabilities for plus and minus. The deflection collision frequency is calculated in terms of the plasma parameters as [2,8,9]

$$\nu_d^{\alpha/\beta} = \frac{4\pi e_\alpha^2 e_\beta^2 n_\beta \Lambda_{\alpha\beta}}{m_\alpha^2 v_\alpha^3} \left(\Phi(x^{\alpha/\beta}) - \Psi(x^{\alpha/\beta}) \right). \quad (7)$$

In this formula, e_α, m_α, and v_α are the charge, mass, and velocity of a test particle, e_β and n_β are the charge and the density of background particles, $\Lambda_{\alpha\beta}$ is the Coulomb logarithm, and $x^{\alpha/\beta} = v^\alpha / \sqrt{\frac{2T_\beta}{m_\beta}}$ is the ratio of the velocity of the test particle to the thermal velocity of background particles. The functions $\Phi(x)$ and $\Psi(x)$ are given by

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt, \quad (8)$$

$$\Psi(x) = (\Phi(x) - x\Phi'(x))/2x^2. \quad (9)$$

The collisions of test impurity ions are primarily determined by the collisions with background plasma ions. In this paper, the normalized deflection frequency is chosen to be $\tilde{\nu}_d = 1.0 \times 10^{-4}$.

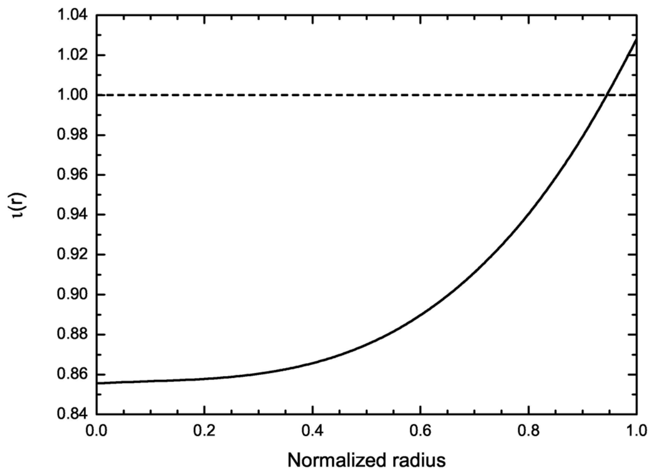

Fig. 4. Profile of rotational transform. The chain of 5 magnetic islands is created at the rational magnetic surface $\iota = 1$ ($r = 0.95$), when perturbation (3) is applied

5. Numerical Model

The monoenergetic ensemble of $N = 1000$ impurity ions with the energy $W = 3$ keV with randomly distributed initial pitch angles $\lambda = v_{||}/v$ is started from the same rational flux surface $r_0 = 0.95$ ($\iota = 1$). The initial poloidal and toroidal angles of particles are uniformly randomly distributed around the magnetic surface. We study tungsten ions with the atomic mass number $A = 184$ and the charge state $Z = 30$. Each particle evolves independently of the others, and its motion is described by Eqs. (5). The guiding center equations are solved using the sixth-order Runge–Kutta method with the constant time step (RK6). The pitch angle of a particle is changed at each time step using the Monte Carlo collision operator (6).

As the measure of statistical properties of the ensemble, the mean square displacement is calculated: $C_2 = \langle(\delta r(t) - \langle\delta r(t)\rangle)^2\rangle$, where $\delta r(t) = r(t) - r_0$ is the particle radial displacement. Brackets mean the ensemble average: $\langle X \rangle = \frac{1}{N}\sum_{i=1}^{N} X_i$, where N is the number of particles of the ensemble. The linear increase in time of $C_2(t)$ corresponds to the normal diffusive process with the diffusion coefficient defined as $D(t) = \frac{dC_2(t)}{2dt}$ [2]. In Figs. 5 and 6, the temporal dependence of the diffusion coefficient $D(t)$ is shown for two magnetic configurations (standard configuration [1] and high-mirror configuration [3]) of a W7-X for the cases with and without magnetic islands at the plasma periphery. The total integration time is chosen to be equal to 4 collisional times $t_E = 4/\nu_d$.

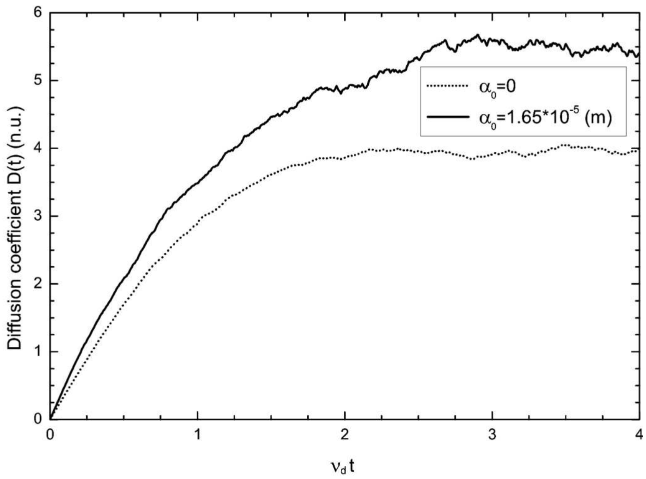

Fig. 5. The time dependence of $D(t)$ for the standard configuration with magnetic islands (solid line) and without them (dotted line)

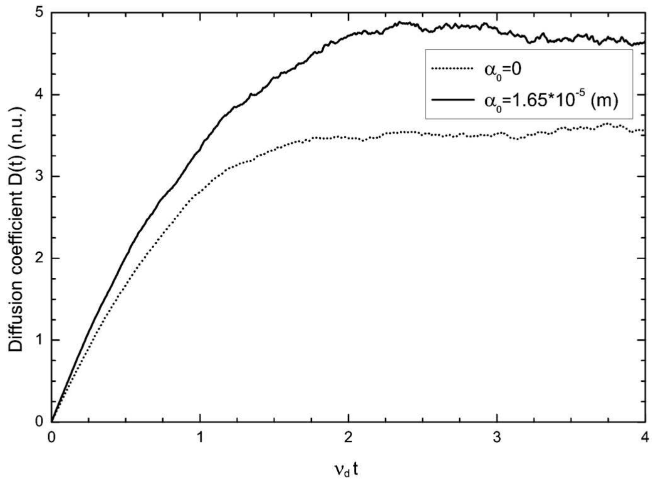

Fig. 6. The time dependence of $D(t)$ for the high-mirror configuration with magnetic islands (solid line) and without them (dotted line)

The saturation of the diffusion coefficients of impurity tungsten ions is shown in Figs. 5 and 6. The diffusion coefficients are given in the normalized units (1 n.u. $= 1.2 \times 10^{-3}$ m^2/s). We have also studied the ensembles of hydrogen and deuterium ions of the same energy and for the same normalized collision rate. The diffusion coefficients for the H and D ensembles are much higher. For the standard configuration without magnetic islands, they are of the order of 0.3 and 0.5 m^2/s, respectively. These values agree with the diffusion coefficients predicted by the neoclassical theory for the plateau regime [10].

Though the tungsten ions do not form the drift islands in the considered collision regime, the diffusion coefficients are more than 30% higher in comparison with those in the case without islands. The regimes with a smaller collisionality need a more detailed description with the extended computer power. The effect of the magnetic islands on the impurity transport in the low-collisionality regimes will be the subject of the future research.

6. Summary

The statistical properties of the particle radial diffusion in the magnetic configuration of a stellarator W7-X with the chain of 5 magnetic islands are investigated. The diffusion coefficients of impurity tungsten ions are calculated numerically. It is shown that the existence of the magnetic field perturbation leads to the enhancement of the impurity transport.

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ДИНАМІКА ІОНІВ ДОМІШКИ ПОБЛИЗУ МАГНІТНИХ ОСТРОВІВ У ДРЕЙФОВО-ОПТИМІЗОВАНІЙ КОНФІГУРАЦІЇ СТЕЛАРАТОРА "ВЕНДЕЛЬШТАЙН 7-Х"

Ж.С. Кононенко, О.О. Шишкін

Р е з ю м е

Вивчено радіальний перенос іонів вольфраму у термоядерній плазмі стеларатора "Вендельштайн 7-Х" з п'ятьма магнітними періодами. Для розв'язання рівнянь ведучого центра для іонів вольфраму числовий код використовує метод Рунге–Кутта 6-го порядку. Кулонівські зіткнення іонів вольфраму моделюються за допомогою дискретного оператора зіткнень Монте-Карло. Вивчення статистичних властивостей ансамблю тестових частинок ґрунтується на обчисленні середньоквадратичного відхилення. Показано, що наявність магнітних островів на периферії плазми приводить до збільшення переносу домішок.