NONLINEAR SHIELDING OF AN ELLIPSOIDAL CONDUCTING DUST PARTICLE IN A PLASMA

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An iteration method is applied to solve nonlinear integral equations for the field of the potentials produced by a conducting sphere and an ellipsoid in a plasma. The results are illustrated by computations for a plasma with the Boltzmann distribution of particles.

1. Introduction

The properties of colloidal and dusty plasmas essentially depend on the screening of strongly charged macroparticles (dust grains). Hence, the study of screening effects becomes a problem of special interest in complex-plasma physics [1]. In many papers, the dust grains are assumed to be spherical. Recent experiments, however, concern a variety of phenomena in plasmas with highly nonspherical macroparticles [2], rod-like grains in particular [3]. In order to comprehend the behavior of such media, we have to study how the screening depends on the macroparticle shape [4].

The stationary distribution of the grain potential is given by the solution of the Poisson equation

$$\triangle U = -4\pi f(U), \qquad (1)$$

where $f(U)$ is the distribution function for plasma particles that may be equilibrium ((1) reduces to the Poisson–Boltzmann equation) [5] or nonequilibrium [6].

The traditional way to find the potential distribution is to solve Eq. (1) with relevant boundary conditions by means of the finite-difference method. This approach is reasonable in the case of a potential depending on a single variable and becomes too involved even when the potential depends on two variables. In this paper, an alternative approach is proposed. The case of a conducting body is treated in terms of nonlinear differential equations which are reduced to the corresponding nonlinear integral equations. This makes it possible to obtain a solution in quadratures in the linear case and to apply iteration methods in the nonlinear case (under certain conditions for the nonlinearity and geometrical parameters). The structure of the Green function derived in the present paper is suitable for the treatment of problems with nonlinear terms entering both the right-hand parts of the differential equations and the boundary conditions. The realizability of this approach for the problems concerning the conducting bodies surrounded by plasmas is confirmed by the numerical calculations for a sphere and a prolate ellipsoid.

2. General Approach

Suppose a conducting macroparticle of an arbitrary shape is bounded by the surface S_1 that satisfies Lyapunov's conditions for smoothness. We consider a nonlinear problem for the Poission equation in the interior of a layer Ω bounded by the inner surface S_1 and the outer surface S_2, i.e.,

$$\triangle U = -4\pi f(U(P)), \quad P \in \Omega. \qquad (2)$$

The boundary conditions on the surfaces S_1 and S_2 are as follows. In the cases where the potential is known, we have

$$U(P)_{P\in S_1} = U_1, \qquad U(P)_{P\in S_2} = U_2. \qquad (3)$$

If the charge Q of the macroparticle is known, then

$$\oint\limits_{S_1} \left.\frac{\partial U}{\partial n}\right|_{P\in S_1} ds = -Q,$$

$$U(P)_{P\in S_1} = U_1 = \text{const}, \qquad U(P)_{P\in S_2} = U_2. \qquad (4)$$

In order to obtain the nonlinear integral equations equivalent to problems (2), (3) and (2), (4) we have to find relevant Green functions.

The Green function $G(P,P_0)$ of two variable points $(P,P_0)\in\Omega$ is determined by the equation

$$\triangle G = -4\pi\delta(P-P_0). \tag{5}$$

With respect to the variable P, the function $G(P,P_0)$ should satisfy the boundary conditions

$$G(P,P_0)_{P\in S_1}=0, \qquad G(P,P_0)_{P\in S_2}=0 \tag{6}$$

for the problem (2), (3), and

$$\frac{\partial G(P,P_0}{\partial n})\Big|_{P\in S_1}=0, \qquad G(P,P_0)_{P\in S_2}=0 \tag{7}$$

for the problem (2), (4).

We multiply Eqs. (2) and (5) by G and U, respectively, subtract the first equation from the second one, and carry out the integration within the layer Ω. Thus we have

$$U(P_0)=\int\limits_{\Omega} Gf(U)d\Omega+\frac{1}{4\pi}\int\limits_{\Omega}(G\triangle U-U\triangle G)\,d\Omega.$$

It follows from Green's formula that

$$U(P_0)=\int\limits_{\Omega} Gf(U)d\Omega+\frac{1}{4\pi}\oint\limits_{S}\left[G\frac{\partial U}{\partial n}-U\frac{\partial G}{\partial n}\right]_S d\Omega.$$

Here, $S=S_1\cup S_2$ is the surface of the domain Ω, $\frac{\partial}{\partial n}$ is the derivative along the direction of the outer normal to S. With regard for the boundary conditions (3), (6) for the first problem, we have

$$U(P_0)=\int\limits_{\Omega} Gf(U)d\Omega-$$

$$-\frac{1}{4\pi}\oint\limits_{S_1}\left[U\frac{\partial G}{\partial n}\right]_{S_1}ds-\frac{1}{4\pi}\oint\limits_{S_2}\left[U\frac{\partial G}{\partial n}\right]_{S_2}ds. \tag{8}$$

In a similar manner, with the boundary conditions for the second problem being given by (4), (7), we have

$$U(P_0)=\int\limits_{\Omega} Gf(U)d\Omega-$$

$$-\frac{1}{4\pi}\oint\limits_{S_1}\left[G\frac{\partial U}{\partial n}\right]_{S_1}ds-\frac{1}{4\pi}\oint\limits_{S_2}\left[U\frac{\partial G}{\partial n}\right]_{S_2}ds. \tag{9}$$

Formulas (8) and (9) thus obtained give the nonlinear integral equations equivalent to problems (2), (3) and (2), (4), respectively.

With the exception of the point P_0, the Green function is determined by the solutions of the homogeneous equations corresponding to (5). The constant coefficients should be found from the boundary conditions, the continuity condition, and the condition imposed on the jump of a derivative at the point P_0. The latter condition may be derived from the equation

$$\int\limits_{D^R_{P_0}}\triangle G\,d\Omega=-4\pi\int\limits_{D^R_{P_0}}\delta(P-P_0)\,d\Omega=-4\pi. \tag{10}$$

Here, $D^R_{P_0}$ is a sphere of radius R with the center at point P_0.

In what follows, we assume that the macroparticle surface is a coordinate surface. In this case, the integrands in Eqs. (8) and (9) are given by

$$\left[U\frac{\partial G}{\partial n}\right]_S ds=\left[U\frac{\partial G}{\partial x_1}\frac{H_2H_3}{H_1}\right]_S dx_2\,dx_3, \tag{11}$$

where H_n is the Lamé coefficient for the coordinate x_n.

3. Spherical Layer

If the macroparticle is spherical, problem (2), (3) can be treated in spherical coordinates. Its potential does not depend on the angular coordinates ϑ and ϕ, and thus we obtain the equations

$$\triangle U=\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial U}{\partial r}=-4\pi f(U), \quad r\in[r_1,r_2], \tag{12}$$

$$U(r)\big|_{r=r_1}=U_1, \quad U(r)\big|_{r=r_2}=U_2,$$

$$\triangle G=\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial U}{\partial r}=-4\pi\delta(r-\xi),\ r\in[r_1,r_2], \tag{13}$$

$$G(r,\xi)\big|_{r=r_1}=0, \quad G(r,\xi)\big|_{r=r_2}=0$$

with the Lamé coefficients $H_r=1$, $H_\vartheta=r$, and $H_\varphi=r\sin\vartheta$.

Then Eq. (8) reduces to

$$U(\xi)=4\pi\int\limits_{r_1}^{r_2}G(r,\xi)f(U(r))r^2dr+$$

$$+U_1r_1^2\left[\frac{dG(r,\xi)}{dr}\right]_{r=r_1}-U_2r_2^2\left[\frac{dG(r,\xi)}{dr}\right]_{r=r_2}. \tag{14}$$

As follows from (10), the jump of the derivative $G(r,\xi)$ at the point $r=\xi$ is given by

$$\frac{\partial G}{\partial r}\Big|_{r=\xi-\varepsilon}^{r=\xi+\varepsilon}=-\frac{1}{\xi^2},$$

and then the Green function is given by

$$G(r,\xi) = \frac{1}{\xi r(r_2 - r_1)} \begin{matrix} (r-r_1)(r-\xi), & r_1 \le r \le \xi, \\ (r_2-r)(\xi-r_1), & \xi \le r \le r_2. \end{matrix} \quad (15)$$

The integral equation reduces to

$$U(\xi) = \left\{ U_1 \frac{r_1(r_2-\xi)}{\xi(r_2-r_1)} + U_2 \frac{r_2(\xi-r_1)}{\xi(r_2-r_1)} \right\} +$$

$$+\frac{4\pi(r_2-\xi)}{\xi(r_2-r_1)} \int\limits_{r_1}^{\xi} (r-r_1) f(U(r)) r dr +$$

$$+\frac{4\pi(\xi-r_1)}{\xi(r_2-r_1)} \int\limits_{\xi}^{r_2} (r_2-r) f(U(r)) r dr. \quad (16)$$

Inasmuch as the domain contains neither internal sources nor sinks, only the expression within the curly braces does not vanish in the right-hand part of Eq. (16), and we obtain an expression for the potential that reproduces the potential of a spherical capacitor [9].

If r_2 tends to infinity and $U_2 = 0$, we obtain an integral equation for an infinite layer, i.e.,

$$U(\xi) = U_1 \frac{r_1}{\xi} + \frac{4\pi}{\xi} \int\limits_{r_1}^{\xi} (r-r_1) f(U(r)) r dr +$$

$$+\frac{4\pi(\xi-r_1)}{\xi} \int\limits_{\xi}^{\infty} f(U(r)) r dr.$$

In the case of a conducting sphere with charge Q, the boundary conditions of problem (4) are given by

$$\left.\frac{\partial U(r)}{\partial r}\right|_{r=r_1} = \frac{-Q}{4\pi r_1^2}, \quad U(r)\Big|_{r=r_2} = U_2.$$

The relevant Green function is described by the expression

$$G(r,\xi) = \begin{matrix} \frac{(r_2-\xi)}{\xi r_2}, & r_1 \le r \le \xi, \\ \frac{(r_2-r)}{r r_2}, & \xi \le r \le r_2. \end{matrix} \quad (17)$$

Then the integral equation looks as

$$U(\xi) = U_2 + \frac{r_2-\xi}{\xi r_2} \frac{Q}{4\pi} + 4\pi \int\limits_{r_1}^{r_2} G(r,\xi) f(U(r)) r^2 dr. \quad (18)$$

4. A Prolate Ellipsoid of Revolution

The rod-like bodies are modelled by a large-eccentricity prolate ellipsoid. In the coordinate system of a prolate ellipsoid ($0 \le \alpha \le \infty$, $0 \le \beta \le \pi$, $-\pi \le \varphi \le \pi$) and in view of the asimuthal symmetry (the potential does not depend on the coordinate φ), Eq. (2) is given by

$$\triangle U = \frac{1}{H_\alpha H_\beta} \Big[\frac{1}{\text{sh}\alpha} \frac{\partial}{\partial \alpha} \Big(\text{sh}\alpha \frac{\partial U}{\partial \alpha} \Big) +$$

$$+\frac{1}{\sin\beta} \frac{\partial}{\partial \beta} \Big(\sin\beta \frac{\partial U}{\partial \beta} \Big) \Big] = -4\pi f(U), \quad (19)$$

and Eq. (5) reduces to

$$\triangle G = \frac{1}{H_\alpha H_\beta} \Big[\frac{1}{\text{sh}\alpha} \frac{\partial}{\partial \alpha} \Big(\text{sh}\alpha \frac{\partial G}{\partial \alpha} \Big) +$$

$$+\frac{1}{\sin\beta} \frac{\partial}{\partial \beta} \Big(\sin\beta \frac{\partial G}{\partial \beta} \Big) \Big] = -4\pi \frac{\delta(\alpha-\xi)\delta(\beta-\zeta)}{H_\alpha H_\beta H_\varphi}. \quad (20)$$

Here, the Lamé coefficients are given by the expressions

$$H_\alpha = H_\beta = c\sqrt{\text{sh}^2\alpha + \sin^2\beta}, \quad H_\varphi = c\ \text{sh}\alpha \sin\beta,$$

$c = \sqrt{a^2 - b^2}$ is the scaling factor, a and b are the major and minor semiaxes of the ellipsoid, respectively.

The eigenfunctions of the Laplace operator with respect to the variable β are described by the Legendre polynomials $P_n(\cos\beta)$; those with respect to the variable α are given by the first- and second-kind Legendre functions, i.e., $P_n(\text{ch}\alpha)$ and $Q_n(\text{ch}\alpha)$, respectively [7,8].

The Green function $G(\alpha,\beta;\xi,\zeta)$ can be written as

$$G(\alpha,\beta;\xi,\zeta) = \sum_{n=0}^{\infty} G_n(\alpha,\xi) G_n(\beta,\zeta), \quad (21)$$

where

$$G_n(\beta,\zeta) = P_n(\cos\beta) P_n(\cos\zeta).$$

4.1. The first problem

The Green function (20) should satisfy (6); the boundary conditions (3) reduce to

$$U(\alpha)_{\alpha=\alpha_1} = U_1, \quad U(\alpha)_{\alpha=\alpha_2} = U_2. \tag{22}$$

Thus, we find that $G_n(\alpha,\xi)$ is given by

$$G_n(\alpha,\xi) = A \begin{array}{ll} G_n^{(1)}(\alpha,\xi), & \alpha_1 \le \alpha \le \xi, \\ G_n^{(2)}(\alpha,\xi), & \xi \le \alpha \le \alpha_2, \end{array} \tag{23}$$

where

$$A = \frac{1}{c[\gamma_n(\,\mathrm{ch}\alpha_1) - \gamma_n(\,\mathrm{ch}\alpha_2)]}, \quad \gamma_n(x) = \frac{P_n(x)}{Q_n(x)},$$

$$G_n^{(1)}(\alpha,\xi) = B(\,\mathrm{ch}\alpha,\,\mathrm{ch}\alpha_1)B(\,\mathrm{ch}\xi,\,\mathrm{ch}\alpha_2),$$

$$G_n^{(2)}(\alpha,\xi) = B(\,\mathrm{ch}\alpha,\,\mathrm{ch}\alpha_2)B(\,\mathrm{ch}\xi,\,\mathrm{ch}\alpha_1),$$

$$B(x,y) = P_n(x) - \gamma_n(y)Q_n(x).$$

With regard for the boundary conditions for $G(\alpha,\beta;\xi,\zeta)$ (the integration path is shown in Fig. 1), Eq. (8) yields an integral equation equivalent to problem (19), (22). It is given by

$$U(\xi,\zeta) = 2\pi c^3 \int_{\alpha_1}^{\alpha_2}\int_0^{\pi} G f(U) H_\alpha^2\,\mathrm{sh}\alpha\,\sin\beta\, d\alpha\, d\beta +$$

$$+c\frac{\mathrm{sh}\alpha_1}{2}U_1 \int_0^{\pi}\left[\frac{dG}{d\alpha}\right]_{\alpha=\alpha_1}\sin\beta\, d\beta -$$

$$-c\frac{\mathrm{sh}\alpha_2}{2}U_2 \int_0^{\pi}\left[\frac{dG}{d\alpha}\right]_{\alpha=\alpha_2}\sin\beta\, d\beta. \tag{24}$$

4.2. The second problem

In the case of an ellipsoid with some (known) charge, the solution of (19) should satisfy the boundary conditions (6), and G should satisfy the boundary conditions (7). Since $U(\alpha,\,\beta)_{\alpha=\alpha_1} = U(\alpha_1) = \mathrm{const}$, we have

$$\left.\frac{\partial U(\alpha)}{\partial n}\right|_{\alpha=\alpha_1} = -\frac{Q}{4\pi\,\mathrm{sh}\alpha_1 c}, \quad U(\alpha)_{\alpha=\alpha_2} = U_2, \tag{25}$$

$$\left.\frac{\partial G}{\partial n}\right|_{\alpha=\alpha_1} = 0, \quad G(P,P_0)_{P\in S_2} = 0. \tag{26}$$

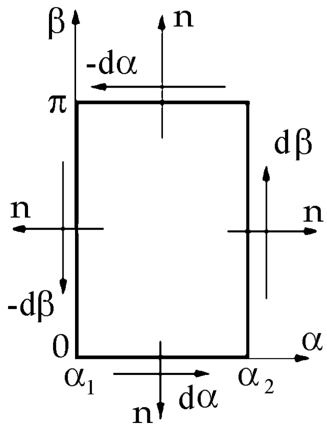


Fig. 1. Integration path

Under such conditions and in terms of the above notation, $G_n(\alpha,\xi)$ takes the form

$$G_n(\alpha,\xi) = A \begin{array}{ll} G_n^{(1)}(\alpha,\xi), & \alpha_1 \le \alpha \le \xi, \\ G_n^{(2)}(\alpha,\xi), & \xi \le \alpha \le \alpha_2, \end{array} \tag{27}$$

where

$$A = \frac{1}{c[\gamma_n{}'(\,\mathrm{ch}\alpha_1) - \gamma_n(\,\mathrm{ch}\alpha_2)]}, \quad \gamma_n'(x) = \frac{P_n'(x)}{Q_n'(x)},$$

$$G_n^{(1)}(\alpha,\xi) = B'(\,\mathrm{ch}\alpha,\,\mathrm{ch}\alpha_1)B(\,\mathrm{ch}\xi,\,\mathrm{ch}\alpha_2),$$

$$G_n^{(2)}(\alpha,\xi) = B(\,\mathrm{ch}\alpha,\,\mathrm{ch}\alpha_2)B'(\,\mathrm{ch}\xi,\,\mathrm{ch}\alpha_1),$$

$$B'(x,y) = P_n(x) - \gamma_n'(y)Q_n(x),$$

and the integral equation (9) reduces to

$$U(\xi,\zeta) = U_2 + \frac{Q}{\pi c}\Big(L(\xi) - L(\alpha_2)\Big) +$$

$$+2\pi c^3 \int_{\alpha_1}^{\alpha_2}\int_0^{\pi} G(\alpha,\beta;\xi,\zeta) f(U(\alpha,\beta)) H_\alpha H_\beta H_\varphi d\alpha\,\beta, \tag{28}$$

where

$$L(x) = \frac{\mathrm{ch}x + 1}{\mathrm{ch}x - 1}\ .$$

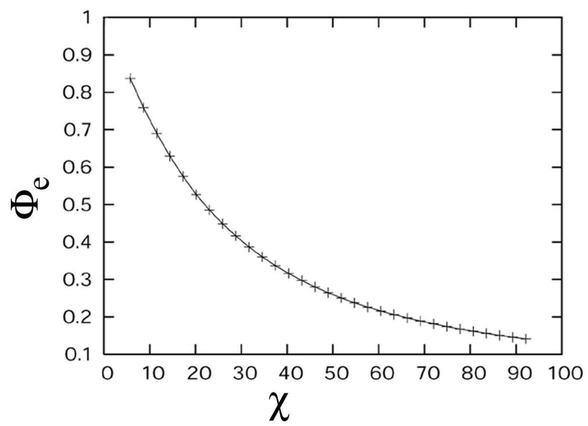


Fig. 2. χ-dependence of Φ_e for a spherical grain

4.3. Limiting transition to a sphere

Let us put $r = c\,\mathrm{ch}\alpha$, $d\alpha = dr/c\,\mathrm{sh}\alpha$ and tend $c \to 0$. Then we obtain the Green function and the integral equation similar to those for the spherical coordinate system, i.e., (15), (16) and (17), (18), respectively.

4.4. Quasi-one-dimensional ellipsoid

If c is sufficiently small, we may assume that the dependence of the potential on β is negligibly small. Thus, having averaged $U(\alpha, \beta)$ over β,

$$\bar{U}(\alpha) = \frac{c}{\pi} \int\limits_0^\pi U(\alpha, \beta) H_\beta \sin\beta d\beta, \qquad (29)$$

we find the Green function $G(\alpha, \xi)$ (in terms of the notation of (28)) given by

$$G(\alpha, \xi) = -\frac{1}{2cK(\alpha_1, \alpha_2)} \begin{matrix} G^1(\xi, \alpha), & \alpha_1 \le \alpha \le \xi, \\ G^2(\alpha, \xi), & \xi \le \alpha \le \alpha_2, \end{matrix}$$

where

$$G^1(\xi, \alpha) = K(\xi, \alpha_2) K(\alpha, \alpha_1),$$

$$G^2(\alpha, \xi) = K(\alpha, \alpha_2) K(\xi, \alpha_1),$$

$$K(x, y) = (L(x) - L(y)).$$

The integral equation reduces to

$$U(\xi) = U_1 G_1(\xi) + U_2 G_2(\xi) -$$

$$G_1(\xi) c^2 \int\limits_{\alpha_1}^{\xi} K(\alpha, \alpha_1) f(U) (\mathrm{sh}^2\alpha + \frac{2}{3}) \mathrm{sh}\alpha d\alpha +$$

$$+ G_2(\xi) c^2 \int\limits_{\xi}^{\alpha_2} K(\alpha, \alpha_1) f(U) (\mathrm{sh}^2\alpha + \frac{2}{3}) \mathrm{sh}\alpha d\alpha, \qquad (30)$$

where

$$G_1(\xi) = \frac{K(\xi, \alpha_2)}{K(\alpha_1, \alpha_2)}, \quad G_2(\xi) = \frac{K(\alpha_1, \xi)}{K(\alpha_1, \alpha_2)}.$$

The integral equations (16), (18), (24), (28), and (30) can be solved by means of successive approximations. The structure of the equations obtained and the corresponding Green functions makes it possible to appropriately choose the distances between the surfaces S_1 and S_2 and thus to improve both the convergence and the accuracy of the iteration process.

5. Application of the Method

Let us consider a conducting charged dust grain with the charge $Q = Ze_e$ that is surrounded by a quasineutral electron-ion plasma with T_e, $|e_e|$ and T_i, $Z_i = e_i/|e_e|$ being, respectively, the electron and ion temperatures and charges. We restrict the consideration to the equilibrium case. The potential distribution function is given by the solution of the Poisson–Boltzmann equation

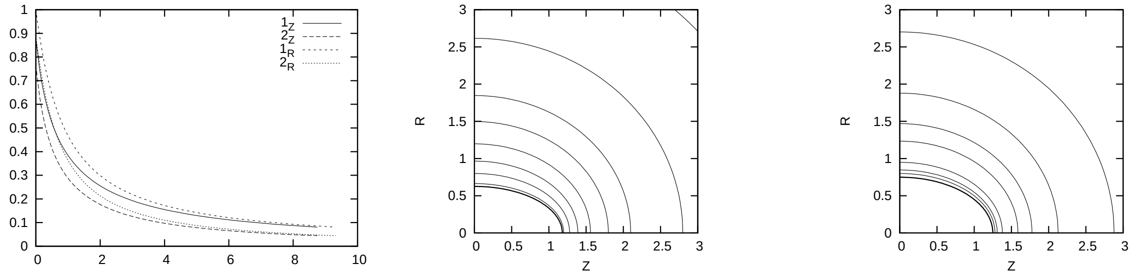
$$\triangle\Phi = -\frac{c^2}{\lambda_{\mathrm{D}}^2} \left\{ e^{-\Phi/t} - e^{\Phi} \right\}, \qquad (31)$$

where $\Phi = |e_e|\varphi/T_e$ is the dimensionless potential, λ_{D}^2 is the Debye length, and $t = T_i/Z_i T_e$.

A decrease of the ratio of the major (a) to minor (b) semiaxes or, in other words, a decrease of the eccentricity of a prolate ellipsoid is accompanied by the transformation of its coordinate surfaces from a rod ($a \gg b$) of length c to a sphere ($a = b$). With the linear dimensions being normalized to the focal radius ($c = 1$), the coordinate surface (i.e., the grain surface) is specified by the value of $\mathrm{ch}\alpha_1 = 1/\varepsilon$. If this expression yields $\alpha_1 \simeq 3$ or more, then the grain may be treated as a spherical one since the eccentricity of the coordinate surface is $\varepsilon \le 0.1$, i.e., the difference between the minor and major semiaxes does not exceed 0.5 per cent ($b/a = 0.995$). Under such conditions, the surface distribution of the charge density σ can be treated as constant.

5.1. Spherical grain

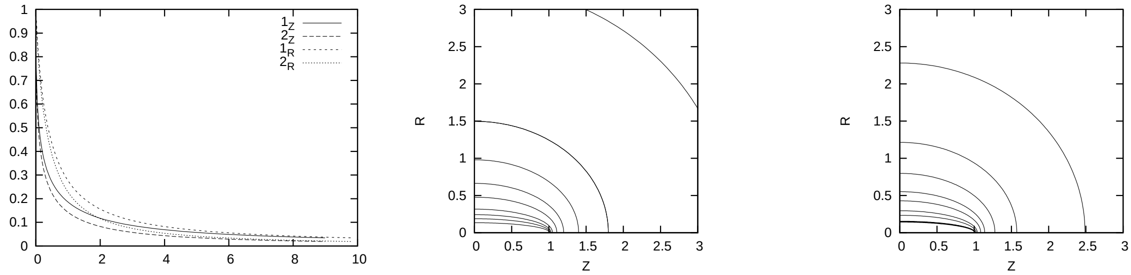
In this case, the calculations confirm the results obtained in [5]. For large distances (greater than the Debye length) from the grain surface, the potential reproduces the Yukawa potential, and the ratio Φ/Φ_D reduces to a constant that is actually equal to the value of the effective potential Φ_e. For $\chi = Ze^2/kT_e a > 1$, the potential decrease near the grain surface becomes steeper as χ increases and deviates from the Yukawa shape. At the same time, the effective charge decreases with the increase in χ and tends to zero for $\chi \to \infty$ (see Fig. 2).



(a) Potential distribution along the Z- and R-axes (the reference point corresponds to the surface)

(b) Equipotential lines in vacuum

(c) Equipotential lines in the plasma



(d) Potential distribution along the Z- and R-axes (the reference point corresponds to the surface)

(e) Equipotential lines in vacuum

(f) Equipotential lines in the plasma

Fig. 3. Stationary distribution of the potential as a function of the distance from the surface of a prolate ellipsoid with the eccentricity $\varepsilon = 0.8$ (*a, b, c*) and $\varepsilon = 0.99$ (*d, e, f*). In *a* and *d*, curves *1* correspond to the potential in vacuum, and curves *2* correspond to the potential in a plasma

5.2. Prolate-ellipsoid-shaped grain

For $\alpha_1 \leq 3$, the calculation should be performed taking into account that the surface charge density σ is proportional to the surface curvature [10], i.e.,

$$\sigma = \frac{Q}{c^2 \,\mathrm{sh}^2\alpha_1 \sqrt{\mathrm{sh}^2\alpha_1 + \sin^2\beta}} = \frac{Q}{ab^2\sqrt{\frac{r^2}{b^4} + \frac{z^2}{a^4}}}, \qquad (32)$$

where Q is the total charge of an ellipsoidal grain. Hence, we see that the ratio of the charge-density values at the ellipsoid vertices σ_z (on the Z-axis) to σ_r (on the R-axis) is equal to the semiaxes ratio, i.e.,

$$\frac{\sigma_z}{\sigma_r} = \frac{a}{b} = \frac{1}{\sqrt{1-\varepsilon^2}}. \qquad (33)$$

Thus, the charge density σ_z can be very large for large eccentricities, while the value of σ_r is moderate. At the same time, the surface potential of a conducting grain is constant. Figure 3,*b, e* show the thickening of the equipotential surfaces near the narrow vertex of the ellipse as a function of the growing eccentricity of the ellipse in vacuum. In a plasma, the effect is stimulated by screening (Figs. 3,*c,f*).

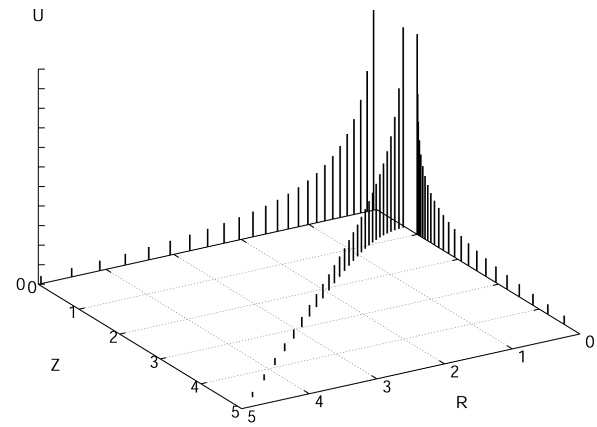


Fig. 4. Screening potential as a function of chα for a prolate ellipsoid with the eccentricity $\varepsilon = 0.99$ for three constant values of β: $\beta = 0$ (Z-axis), $\beta = \pi/4$, $\beta = \pi/2$ (R-axis)

The two-dimensional calculations with the use of Eqs. (24) and (28) show that, in the case of the Boltzmann distribution, the dependence on the coordinate β lies within the computation error, and the result is similar to that obtained for the quasi-one-dimensional problem (30). That is why Figs. 3,*a, d* show only the α-dependence of the potential. Figures 3–4 are plotted for similar conditions but different eccentricities. The potential is normalized to the wall potential. The distances are given in terms of the ellipsoid focal radius c. The Debye screening length $\lambda_D \simeq 5$. The spatial distribution of the screening potential for three constant values of β is shown in Fig. 4. In the case of a very thin ellipsoid, the screening can be nearly linear at the flat vertex of the ellipsoid and strongly nonlinear at the narrow vertex, so that the value of the effective potential near the latter becomes small.

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НЕЛІНІЙНЕ ЕКРАНУВАННЯ ЕЛІПСОЇДНОЇ ПРОВІДНОЇ ПОРОШИНКИ В ПЛАЗМІ

В. Кубайчук, Є. Мейнарович

Р е з ю м е

Ітераційним методом отримано розв'язки нелінійних інтегральних рівнянь, що описують поле потенціалів в околі провідної кулі та еліпсоїда в плазмі. Результати проілюстровано розрахунками для плазми з больцмановим розподілом частинок.