DYNAMICS OF A MODULATED ELECTRON BEAM IN HOMOGENEOUS PLASMA: 2D SIMULATION

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The interaction of a thin modulated electron beam with homogeneous plasma in a 2D planar geometry is investigated by the PIC method. The cases of subcritical and supercritical plasmas and the resonance interaction ($\omega_0 = \omega_p$) are considered separately. Effects of beam focusing and defocusing both in the longitudinal and transversal directions are observed. It is shown that a Langmuir wave is excited in the system due to the Cherenkov resonance. The influence of this wave on the beam dynamics is analyzed. The rapid excitation of an electrostatic wave with large amplitude and the nonlinear bounce oscillations of beam electrons in the potential minima of this wave are observed in the case of resonance interaction.

1. Introduction

The interaction of a modulated electron beam with plasma has been an object of investigation of fundamental and applied science during last decades, starting from the pioneer works [1, 2]. Waves' generation in the ionospheric plasma by injection of modulated electron beams [3] is in the field of interest. Wake fields excited in plasma by modulated beams can be used for particles' acceleration [4].

The interaction of a modulated electron beam with plasma in a 1D geometry was investigated analytically [1], numerically [5], and within the PIC method [6]. It was shown that the modulation depth of a beam is increased or decreased when it moves in supercritical or subcritical plasma, respectively. This effect was characterized by different signs of the dielectric permittivity for supercritical and subcritical plasmas.

In real experiments, beams are always finite in the transversal direction. Therefore, 2D and 3D models should be used for a correct description of such experiments. The question about the motion of beam electrons in the transversal direction is important for such models. In [7], the possibility of the transversal focusing of a modulated electron beam moving in plasma was firstly indicated. The problem of transversal stability of a modulated electron beam in plasma was treated analytically in [8–10]. The possibility of an equilibrium between the modulated beam and its electromagnetic field was predicted as well.

The self-focusing of a modulated ribbon relativistic beam in dense plasma was observed in the simulation [11] for the quasiresonant beam-plasma interaction (the relative detuning between plasma and modulation frequencies was $\delta = 0.1$). The 2D3V electromagnetic PIC code was used in [12] for the investigation of the ion channel formation by a sequence of relativistic electron bunches.

The excitation of wake fields in plasma by a single bunch of electrons was investigated in 1D and 2D cases in [13]. It was shown that the use of bunches of specific shapes will result in an increase of the wake field amplitude. The appearance of the focusing/defocusing wake field behind the injected bunch and the self-focusing of the injected bunch due to the Weibel instability were pointed out.

This work is aimed at the investigation of the transversal and longitudinal dynamics of a non-relativistic modulated electron beam in plasmas using the PIC method. The beam is injected into homogeneous plasma. The cases of subcritical and supercritical plasmas and the resonance interaction are considered separately.

2. Physical and Mathematical Models

The 2D electrostatic particle-in-cell code [14] was used for the simulation of the beam-plasma interaction. A rectangular area (0.6×0.1 m^2) filled with non-isothermal plasma was investigated. Plasma density was varied in the range $(1.6 \div 4.8) \times 10^{14}$ m^{-3}, electron temperature $T_e = 2.0$ eV, and ion temperature $T_i = 0.3$ eV.

A thin ($L_b = 0.1 \times L_y$) modulated electron beam (the average density $n_b = 1.0 \times 10^{12}$ m^{-3}, modulation frequency $\omega_0 = 1.0 \times 10^9$ s^{-1}, and density modulation depth $m = 100\%$) was injected into plasma from the left electrode. The resonance condition $\omega_p = \omega_0$ is fulfilled at the plasma density $n_0 = 3.2 \times 10^{14}$ m^{-3}.

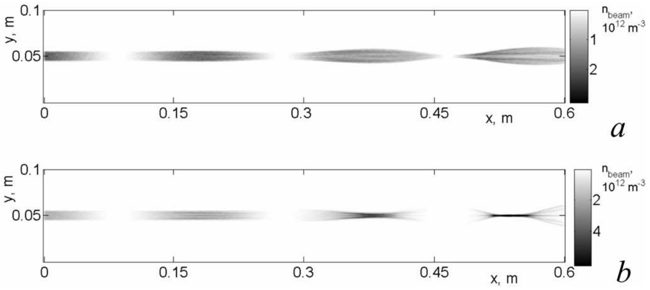

Fig. 1. Density of beam electrons at the time moment $t = 40T_{\text{mod}}$ for subcritical (a) and supercritical (b) plasmas

A difference Poisson equation was solved on the rectangular grid of 2048×256 cells. Plasma consisted of the large number of electrons ($N_e = 2 \times 10^6$) and ions ($N_i = 2 \times 10^6$).

The Dirichlet boundary conditions were applied on the left and right borders of the simulated region, and the periodic boundary conditions were applied on the top and bottom borders:

$$\phi(x = 0) = \phi(x = L_x) = 0,$$

$$\phi(y = 0) = \phi(y = L_y).$$

3. Non-resonant Thin Modulated Beam Interaction with Plasma

3.1. Dynamics of beam electrons

Here, we discuss the results of simulations of the interaction of a modulated beam with homogeneous plasma with a Langmuir frequency different from the beam modulation frequency.

The periodic change of the beam width is observed when the beam moves through homogeneous plasma. The spatial distributions of the density of beam electrons at the time moment $t = 40\ T_{\text{mod}}$ for subcritical ($n_p = 0.5 \times n_0$) and supercritical ($n_p = 1.5 \times n_0$) plasma are shown in Fig. 1. One can see that the beam dynamics in these two cases are essentially different.

In subcritical plasma, beam bunches become wider in central parts and thinner in peripheral parts. The beam dynamics in supercritical plasma is quite different: bunches becomes thinner in central parts and wider in front and rear parts.

The time dependences of the beam current on the right electrode (collector) for subcritical (a) and supercritical (b) plasma are shown in Fig. 2. One can see that the wave of a beam current injected at left electrode with constant frequency ω_0 and maximal current $I_{0,\max} = 9.6$ mA has different dynamics in the longitudinal direction for subcritical and supercritical plasma. Beam bunches in subcritical plasma become less dense because the maximal current at a collector $I_{\text{sub,max}}$ is less (Fig. 2,a) than the initial maximal current, $I_{\text{sub,max}} < I_{0,\max}$. Beam bunches in supercritical plasma become more dense, because the maximal current on a collector $I_{\text{super,max}}$ exceeds $I_{0,\max}$ (Fig. 2b). One can also see that, in both cases, the beam current on a collector is not monochromatic, but it has beating dependence. The beating periodicity is different for subcritical and supercritical plasmas.

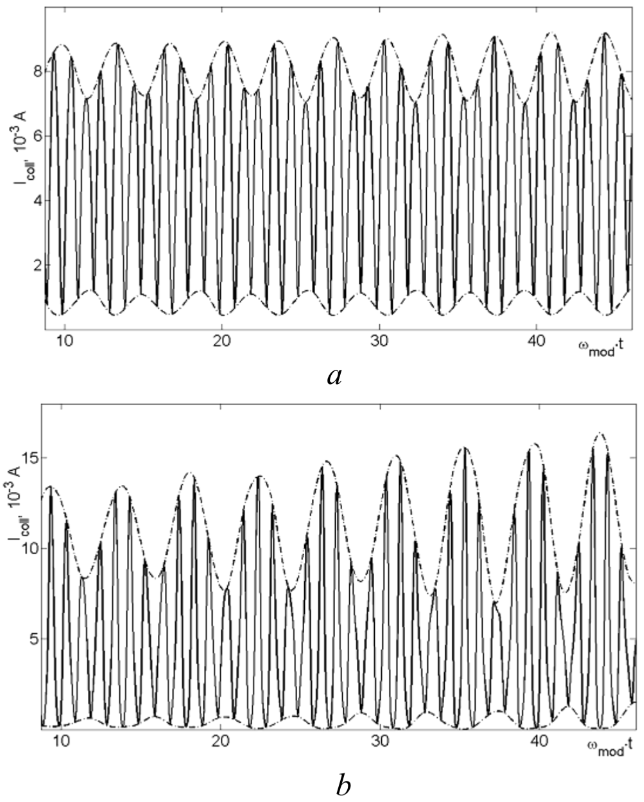

Fig. 2. Time dependence of the beam current for subcritical (a) and supercritical (b) plasmas

In both cases, the dynamics of two subsequent bunches are different.

3.2. Potential waves excited in the system

The dynamics of beam electrons described in Section 3.1 can be understood if we analyze the beam interaction with waves of electric potential excited in the system [8–11]. These waves are a forced one with the wavelength of the modulated beam and a Langmuir wave excited by

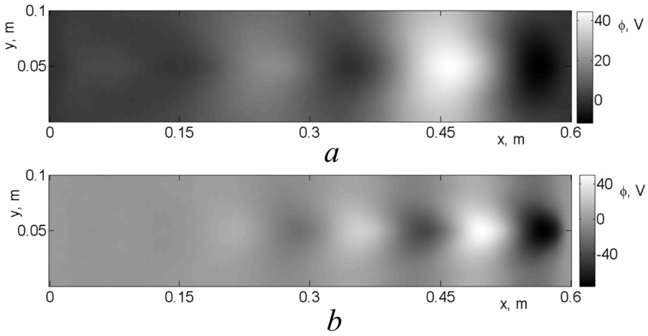

Fig. 3. Spatial distributions of the potential in the system at time $t = 40\ T_{\rm mod}$ for subcritical (a) and supercritical (b) plasmas

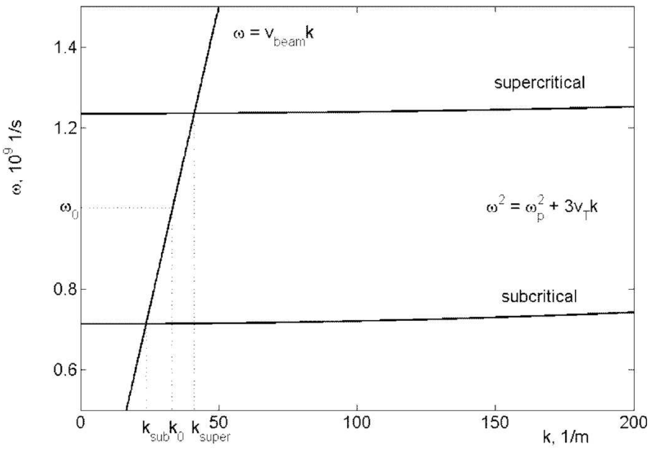

Fig. 4. Langmuir waves' dispersion curves for subcritical and supercritical plasmas. Intersections with the straight line corresponding to the modulated beam define the frequencies and the wave vectors of Langmuir waves excited in the system

the Cherenkov mechanism (for relativistic beams considered in [8–11], these waves coincide). The forced wave is excited due to the plasma electrons' response on the electric field of the oscillating beam current. Its phase relatively to this current in the first-order approximation is defined by the sign of the cold plasma dielectric permittivity $\varepsilon = 1 - \omega_p^2/\omega^2$.

The spatial distribution of a potential in the system at time $t = 40\ T_{\rm mod}$ for subcritical (a) and supercritical (b) plasmas is shown in Fig. 3. The potential patterns in both cases are formed as a superposition of forced and Langmuir waves moving from an injector to a collector with a phase velocity equal to the beam velocity $v_{\rm ph} = v_{\rm beam}$. The potential distribution is moveless in the reference frame associated with beam electrons. Therefore, they move accordingly to the potential surface, i.e. they gather around maxima and slip off from minima. One can see from the comparison of Fig. 1 and Fig. 3 that the spatial distributions of potential and beam electrons density are correlated.

A disturbance of the potential at large distances from the beam is small (Fig. 3). The potential gradient indicates the presence of a transversal electric field that causes the motion of beam electrons in the y direction. Therefore, focusing and defocusing of beam bunches in the longitudinal direction takes place along with the same processes in the transversal direction. This explains the effects described in Section 3.1.

The forced wave of the potential corresponds to the point (ω_0, k_0) at the dispersion plane (Fig. 4) and has the same wave number and frequency as the modulated electron beam. Dielectric permittivity has different signs in subcritical ($\omega > \omega_p$) and supercritical ($\omega < \omega_p$) plasmas. This means that the forced wave of the potential and the wave of the beam current will be in phase in the first case and in antiphase in the second one. Therefore, the electric field of the forced wave focuses beam bunches in supercritical plasma and defocuses them in subcritical one.

The intersections between the straight line with the slope angle tangent equal to the beam velocity $v_{\rm beam}$ and the dispersion curves for Langmuir waves in warm plasma correspond to Langmuir waves excited in the system by the Cherenkov mechanism. These waves are excited both in subcritical and supercritical plasmas. The wave numbers $k_{\rm sub}$, $k_{\rm super}$ and the frequencies $\omega_{\rm sub}$, $\omega_{\rm super}$ of these waves differ from the correspondent parameters ω_0, k_0 of the modulated beam. Numerical values of $k_{\rm sub}$, $k_{\rm super}$ and $\omega_{\rm sub}$, $\omega_{\rm super}$ found in numerical experiments are in good agreement with the values calculated from Fig. 4.

The Langmuir wave amplitude turns out to be significantly greater than the amplitude of the field induced by the beam current at large distances from a collector. Therefore, the spatial periods of the potential wave and the beam current wave are different in this region. As a result, subsequent electron bunches move with different phases relatively to a wave of potential and demonstrate different dynamics. This is the explanation of the beating dependence of the beam current on a collector shown in Fig. 2.

The fact that the electric field of a Langmuir wave excited by the Cherenkov mechanism is greater than that of a forced wave far from an injector can be directly observed on Fig. 3,b: the regions of maximum and minimum of the potential are symmetric near an injector and have an arrow shape near a collector. Arrow tips have the same direction as the beam velocity.

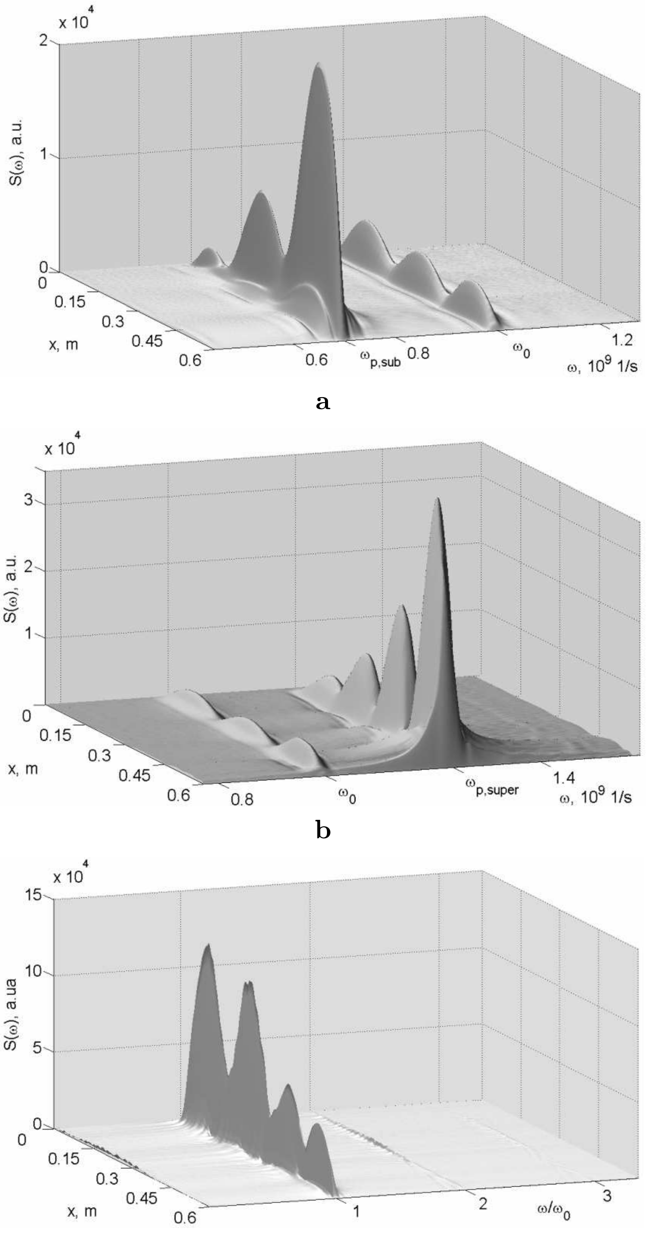

Fig. 5. Spatial dependences of excited waves' amplitudes on the axis of the system in subcritical (a) and supercritical (b) plasmas and for the resonance interaction (c)

The spectra of time oscillations of the electric potential on the axis of the system are shown in Fig. 5,a–c. They are obtained as the Fourier transform of the potential dependences on time at each point of the axis $y = L_y/2$. One can see that there are two frequencies in the spectrum (the beam modulation frequency ω_0 and the frequency of the excited Langmuir wave $\omega_{\rm sub}$ or $\omega_{\rm super}$) both for subcrititcal and supercritical plasmas.

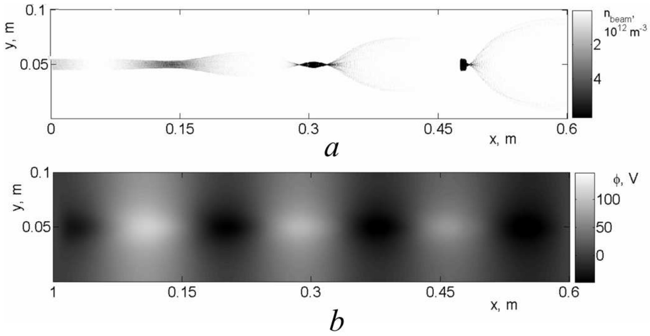

Fig. 6. Spatial distributions of the beam electrons' density (a) and the potential (b) in the case of the resonant interaction at the time moment $t = 10T_{\rm mod}$

The spatial dependence of the amplitude for both spectral lines has a standing-wave type, and the amplitude of the Langmuir wave grows from an injector to a collector. The standing wave formation for the Langmuir wave can be explained by the reflection of this wave from metallic borders. The growth of its amplitude can be explained by the amplification of the forward wave due to the interaction with the beam and the spatial damping of a reflected wave.

4. Resonance Interaction of a Thin Modulated Electron Beam with Plasma

In the resonant case, the synchronism condition between the excited Langmuir wave and the beam current wave ($\omega_L = \omega_0$, $k_L = k_0$) is fulfilled. As a result, the electric field of a big amplitude (compare Fig. 3,a,b and Fig. 6) is excited in plasma during a relatively short period of time ($t \sim 10T_{\rm mod}$). The strong electric field causes a substantial deformation of the initial profile of beam density.

The spatial distributions of the beam electrons' density (a) and the potential (b) in the case of the resonant interaction at the time moment $t = 10T_{\rm mod}$ are shown in Fig. 6. One can see that beam electrons while moving in plasma oscillate in the transversal and longitudinal directions around the bunch center. These oscillations are similar to well-known bounce oscillations on the nonlinear stage of the beam-plasma interaction. Electrons move in the potential minima of an excited wave, and the centers of beam bunches coincide with the maxima of the excited wave for the resonant interaction. Oscillations take place both in the transversal and longitudinal directions due to a finite beam width.

The excited wave amplitude grows rapidly with time because the resonance condition is satisfied. As a result, bounce oscillations of beam electrons become nonlinear. The anisochronism of the transversal oscillation can be observed in Fig. 6*a*: beam electrons in the central part of a beam bunch at $x = 0.3$ m has already passed the equilibrium point, while electrons on the periphery have not reached it yet.

The spectrum of potential oscillations on the axis of the system as a function of the x coordinate is shown in Fig. 5,*c*. In contrast to the cases of subcritical and supercritical plasmas, only the Langmuir frequency and its higher harmonics are presented in the spectrum for the resonant interaction. The excited wave of the electric field exists in the system as a standing wave similarly to the above-mentioned cases. The dynamics of two subsequent beam bunches are identical due to the coincidence of the spatial periods of the beam current wave and the excited Langmuir wave

In contrast to the non-resonant case where the amplitude of an excited Langmuir wave grows from an injector to a collector, this amplitude is now maximal near an injector. In the case of the resonant interaction, the Langmuir wave takes the beam energy on a shorter length. The further beam-plasma interaction becomes essentially nonlinear, and beam bunches undergo a strong deformation.

5. Conclusion

1. Longitudinal and transversal dynamics of a non-relativistic modulated electron beam in homogeneous plasma are defined by its interaction with the excited potential electric field. This field consists of a forced wave stimulated by the alternative beam current and a Langmuir wave excited via the Cherenkov mechanism. For a non-resonant interaction at large distances from an injector, the Langmuir wave amplitude is bigger than that of the forced wave, and this leads to different dynamics of two subsequent beam bunches.

2. Transversal and longitudinal focusings of beam bunches are observed in supercritical plasma. This effect is maximal in the case of the resonant interaction when the beam modulation frequency is equal to the plasma frequency. Similar results were obtained in [11] for relativistic beams.

3. Simulations were performed for a planar geometry with periodic boundary conditions in the transversal direction. We think that the obtained results are qualitatively correct for beams with finite width in unbounded plasma and for thin beams in a plasma filling a wave-guide with conductive walls, as well as for axially symmetric models.

1. Ya.B. Fainberg and V.D. Shapiro, Atomn. Energ. **19**, 336 (1965).
2. A.K. Berezin, Ya.B. Fainberg, and I.A. Bezyazychnyi, Pis'ma Zh. Eks. Teor. Fiz. **7**, 156 (1968).
3. J. Lavergnat and T. Lehner, IEEE Transactions **AP-32**, 177 (1984).
4. I.M. Onishchenko, Ukr. J. Phys. **53**, 314 (2008).
5. A.I. Rogashkova, Elektronn. Tekhn. Ser. 1. Elektr. SVCh **3**, 143 (1970).
6. I.O. Anisimov, O.I. Kelnyk, S.V. Soroka, Probl. At. Sci. and Technol. Ser. Plasma Physics **10**, 125 (2005).
7. A.I. Akhieser, G.Ya. Lyubarskiy, and Ya.B. Fainberg, *Theory and Calculation of Linear Accelerators* (Gosatomizdat, Moscow, 1962) (in Russian).
8. V.B. Krasovitsky, Pis'ma Zh. Eks. Teor. Fiz. **9**, 679 (1969).
9. V.P. Kovalenko and N.P. Yushmanov, Fiz. Plasm. **3**, 1284 (1977).
10. V.G. Dorofeenko and V.B. Krasovitsky, Izv. Vuzov. Radiofiz. **27**, 1043 (1984).
11. O.V. Batishchev, V.B. Krasovitsky, O.Yu. Naguchev, and Yu.S. Sigov, Fiz. Plasm. **19**, 738 (1993).
12. V.I. Karas', I.V. Karas', V.D. Levchenko *et al.*, Fiz. Plasm. **23**, 311 (1997).
13. T. Katsouleas, Phys. Rev. A **33**, 2056 (1986).
14. I.O.Anisimov and T.Eu. Litoshenko, Probl. At. Sci. and Technol. Ser. Plasma Physics **12**, 175 (2006).

ДИНАМІКА МОДУЛЬОВАНОГО ЕЛЕКТРОННОГО ПУЧКА В ОДНОРІДНІЙ ПЛАЗМІ: ДВОВИМІРНЕ МОДЕЛЮВАННЯ

І.О. Анісімов, Т.Є. Літошенко

Р е з ю м е

Методом крупних частинок (PIC) досліджується взаємодія тонкого модульованого електронного пучка із однорідною плазмою в двовимірній геометрії. Окремо розглядаються випадки докритичної, закритичної плазми та резонансної взаємодії ($\omega_0 = \omega_p$). Показано, що разом із ефектами стиснення-розширення пучка у поздовжньому напрямку такі ж ефекти наявні і у поперечному напрямку. Виявлено, що в досліджуваній системі за черенковським механізмом збуджується ленгмюрівська хвиля, а також проаналізовано її вплив на динаміку електронів пучка. У випадку взаємодії тонкого модульованого пучка із резонансною плазмою спостерігалося швидке збудження електростатичної хвилі великої амплітуди та нелінійні баунс-коливання електронів пучка в околах мінімумів потенціалу цієї хвилі.