TEMPERATURE RELAXATION PROCESSES IN A MAGNETIZED PLASMA

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By means of the kinetic fluctuation theory, the relaxation process between the electron and ion temperatures in a magnetized homogeneous plasma is considered. The cases where the external upper-hybrid pump wave excites modified convective cells, ion-acoustic waves and ion-cyclotron oscillations with ion temperature anysotropy are analyzed. The inverse relaxation time in the regime, where turbulent fluctuations are developed, is calculated for these cases, and its dependence on the pump wave and plasma parameters is deduced.

Investigations of the temperature relaxation process between electrons and ions in plasmas are important for plasma diagnostics, measurements of the efficiency of high-frequency pump power dissipation, and for plasma heating.

The theory of temperature relaxation was developed in [1–3] for an isotropic magnetized plasma and also for a plasma subjected to external electromagnetic radiation.

The theory for fluctuations in a plasma with a high-frequency pump was developed in [4–7]. On the basis of this theory, the relaxation processes in plasmas were studied in [8–10].

It has been found [8] that a high-frequency electric field close to the lower-hybrid resonant frequency has a significant influence on the relaxation rate between the electron and ion temperatures in a magnetized uniform plasma. It was shown that, due to the pump-wave field, the inverse relaxation time contains an additional term which increases anomalously when the pump-wave amplitude approaches the threshold value.

The relaxation process in a magnetized inhomogeneous plasma is considered in [9] when the external lower-hybrid pump wave decays into a daughter wave and an electron drift one. In the region above the instability threshold, the inverse relaxation time is calculated, and its dependence on the density gradient and the pump-wave intensity and frequency is obtained.

In [10], the relaxation between the electron and ion temperatures is investigated when the ion-cyclotron wave is excited parametrically by the lower-hybrid pump wave in a plasma with ion temperature anisotropy. The dependence of the relaxation time on the pump-wave amplitude and the anisotropy of the ion temperature is calculated.

In the present paper, on the basis of the kinetic fluctuation theory, the relaxation process between the electron and ion temperatures in a magnetized homogeneous plasma with a high-frequency pump is considered. The case where the external upper-hybrid pump wave decays into daughter and ion-acoustic waves in a plasma is considered. The situation when the upper-hybrid pump wave parametrically excites modified convective cells and ion-cyclotron oscillations with ion temperature anisotropy is also analyzed. The inverse relaxation time in the regime, when the turbulent fluctuations are developed, is calculated for these cases, and its dependence on the pump wave and the plasma parameters is obtained.

Consider the electron-ion plasma in an external magnetic field $Bo\vec{z}$. Furthermore, the plasma is subjected to an HF pump field, whose electric field is directed perpendicularly to the external magnetic field. For a long-wavelength ($k_0 = 0$) pump wave, we can write $\vec{E}(t) = E_0\vec{y}\cos\omega_0 t$. First, we consider the case where the pump wave frequency is close to the upper-hybrid frequency,

$$\omega_{\mathrm{UH1}} \approx (\omega_{pe}^2 + \omega_{ce}^2)^{1/2}, \tag{1}$$

where ω_{pe} is the electron plasma frequency and ω_{ce} is the electron gyrofrequency. Here, $\omega_{pe} > \omega_{ce}$, i.e. we have the case of a weakly magnetized plasma.

The important role of parametric instabilities in the region of upper-hybrid resonance was pointed out in [11–13].

We consider the decay of the pump wave into an upper-hybrid wave and modified convective cells:

$$\omega_0 = \omega_{\mathrm{UH1}} + \omega_c. \tag{2}$$

Here, $\omega_c = (m_i/m_e)^{1/2}\cos\Theta\omega_{ci}$ is the real part of the frequency of a modified convective cell, $\mathrm{Im}\omega = \gamma_c \approx \frac{1}{2}\nu_{ei}$, where ν_{ei} is the electron-ion collision frequency, Θ is the angle between $\vec{k}$ and $\vec{B}_0$, and ω_{ci} is the ion gyrofrequency. It should be noted that convective modes arise in the magnetized plasma with a small ratio of the plasma pressure to the magnetic pressure and can also occur in the ionospheric plasma [14, 15].

The dipole approximation is assumed for the pump wave, because typical ionospheric plasma parameters satisfy the condition $k_0/k_{0\perp} \ll 1$. Here, $k_0 = \omega_0/c$ (with $\omega_0 \approx \omega_{pe}$ for the upper-hybrid wave) is the wave vector of the pump upper-hybrid wave, and the wave number $k_{0\perp}$ satisfies the decay condition (2) (we assume that $k_{0\perp} \leq 1/\rho e$, where ρ is the Larmor radius). Thus, we have

$$\frac{k_0}{k_{0\perp}} \approx \frac{\omega_{pe} V_{Te}}{\omega_{ce}\, c} \ll 1,$$

where V_{Te} is the electron thermal velocity.

It is well known [8] that the connection between the inverse relaxation time and the power density for the plasma ion component is defined by the formula

$$\frac{1}{\tau_{ei}} \approx \frac{2W_i}{3n_e T_e}, \tag{3}$$

where the power density is

$$W_i = n_i \int \frac{p^2}{2m_i} I_i d\vec{p}. \tag{4}$$

In (4), I_α is the collision integral which can be represented in the Fokker–Planck-form

$$I_\alpha(p) = \sum_n d\vec{k}(L_{\alpha n} D_{\alpha n} L_{\alpha n} + L_{\alpha n} A_{\alpha n}) f(p). \tag{5}$$

The quantities $D_{\alpha n}$ and $A_{\alpha n}$ are, respectively, the diffusion coefficient in the velocity space and the dynamic friction coefficient, and the notation $L_{\alpha n}$ means

$$L_{\alpha n} = k_\| \frac{\partial}{\partial p_\|} + \frac{n\omega_{c\alpha}}{\mathrm{v}_\perp} \frac{\partial}{\partial p_\perp}.$$

Taking into account that the main contribution to the collision integral (5) for a parametrically unstable plasma is made by the diffusion coefficient (in comparison with the dynamic friction coefficient), we can present the power density in the form [8, 16]

$$W_i = \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{\langle \delta\vec{E}\delta\vec{E}\rangle_{\omega,\vec{k}}}{4\pi} \omega \,\mathrm{Im}\, \chi_i^0, \tag{6}$$

where $\langle \delta\vec{E}\delta\vec{E}\,\rangle_{\omega,\vec{k}}$ is the spectral density of turbulent fluctuations of the electric field in the region above the instability threshold. The correlator $\langle \delta\vec{E}\delta\vec{E}\,\rangle_{\omega,\vec{k}}$ near the natural plasma frequencies is obtained from the well-known expression [8], in which the plasma eigenmodes are $\omega_j = \omega_j + i\nu_{\text{eff}}$ ($j = UH, C$ for the upper-hybrid wave and a modified convective cell, respectively). Here, for the saturation of the parametric instability, we have introduced the effective collision frequency $\nu_{\text{eff}\,1} \approx \nu_{ei} E_0^2/E_{\text{th}\,1}^2$ [17] which defines the additional wave damping due to the scattering of charged particles by turbulent electric field fluctuations. Here,

$$E_{\text{th1}}^2 = 16 \frac{\omega_0^2 B_0^2}{k^2 c^2} \frac{m_e \nu_{ei}^2 \omega_{\text{UH1}}}{m_i \omega_{ce}^2 \omega_c} \tag{7}$$

is the threshold electric field for the decay instability (2).

Substituting the redefined expression for the field fluctuation spectral density into (6) and integrating with respect to ω and $\vec{k}$, we obtain the formula that determines the power density absorbed in the plasma

$$W_i \approx \frac{1}{2\pi} \left(\frac{\omega_{pe}}{\omega_0}\right)^2 \nu_{ei} \frac{E_0^4}{E_{\text{th1}}^2(k_1^*)}, \tag{8}$$

where k_1^* is the wavenumber satisfying the decay condition (2).

Substituting expression (8) into (3), we have

$$\frac{1}{\tau_{ei1}} \approx \frac{1}{12} \frac{e^2}{m_e T_e} \frac{m_i}{m_e} \frac{\omega_{ce}^2 \omega_c (kc)^2 E_0^4}{\omega_0^4 \nu_{ei} \omega_{\text{UH1}} B_0^2}. \tag{9}$$

It can be seen from (9) that the inverse relaxation time has a sharp dependence on the pump frequency and is proportional to the pump wave intensity.

As our second example, we consider the decay of the pump wave into an upper-hybrid and ion-acoustic wave

$$\omega_0 = \omega_{\text{UH2}} + \omega_s, \tag{10}$$

where $\omega_s = |\vec{k}\,|\mathrm{v}_s$ and $\mathrm{v}_s = (T_e/m_i)^{1/2}$ is the ion sound velocity. Consider an upper-hybrid wave satisfying the dispersion relation

$$\omega_{\text{UH2}} = \omega_{ce} \left(1 + \frac{\omega_{pe}^2 \sin^2\Theta}{2\omega_{ce}^2}\right). \tag{11}$$

It should be noted that expression (11) is valid in a strongly magnetized plasma for the case $\omega_{pe} \ll \omega_{ce}$. We assume that the damping rate of the upper-hybrid wave $\gamma_{\text{UH}} \approx \nu_{ei}$. We note also that the pump-wave frequency ω_0 must be slightly above ω_{UH2}, because $\omega_s \ll \omega_0, \omega_{\text{UH2}}$.

Near the eigen-frequencies of the plasma, we now express the spectral density in terms of the fluctuation intensities at the ion-acoustic and upper-hybrid frequencies [15]. Thus,

$$\langle \delta\vec{E}\delta\vec{E}\,\rangle_{\omega,\vec{k}} = \pi\Big[I_{\vec{k}}^s\, \delta(\omega - \omega_s) + I_{\vec{k}}^u\, \delta(\omega - \omega_U)\Big], \tag{12a}$$

where

$$I^s_{\vec{k}} = \frac{1}{\operatorname{Im}\varepsilon \partial \operatorname{Re}\varepsilon/\partial\omega}\left[\langle\delta\vec{E}^{\,2}\rangle^0_{\omega,\vec{k}} + \right.$$

$$\left. + \frac{\pi\mu^2}{4}(\chi^0_e)^2 \frac{\langle\delta\vec{E}^{\,2}\rangle^0_{\omega-\omega_0,\vec{k}}\,\delta(\omega-\omega_0+\omega_U)}{I_m\varepsilon_{-1}\partial R_e\varepsilon_{-1}/\partial(\omega-\omega_0)}\right]. \quad (12b)$$

Note that we do not need the expression for $I^u_{\vec{k}}$ since the instability saturation mechanism is determined mainly by low-frequency ion-acoustic turbulent fluctuations.

Taking into account that $\nu_{\mathrm{eff}\,2} \approx (E_0^2/E^2_{\mathrm{th}\,2})(\gamma_s\nu_{ei})^{1/2}$ when $\nu_{\mathrm{eff}\,2} \gg \nu_{ei}, \gamma_s$ [12], we substitute expression (12) into (6) and integrate with respect to ω and $\vec{k}$. Then using formula (3) and performing lengthy but straightforward calculations, we obtain the following expression for the inverse relaxation time:

$$\frac{1}{\tau_{ei2}} \approx \frac{1}{3}\frac{e^2E_0^2}{m_eT_e\omega_0^2}\frac{E_0^2}{E^2_{th2}}(\gamma_s\nu_{ei})^{1/2}. \quad (13)$$

It should be noted that the threshold value of the parametric decay (10) is governed by [15]

$$E^2_{\mathrm{th}\,2} \approx \frac{2\pi}{2}\left(\frac{5\pi m_e}{2m_i}\right)^{1/2}\frac{\mathrm{v}^2_{Te}}{c^2}B_0^2\frac{\omega_0^2(\omega_0^2-\omega_{ce}^2)^2}{\omega_{pe}^4\omega_{ce}^3}\nu_{ei}. \quad (14)$$

By carrying out the integration above, we have assumed that $\delta(\omega_s - \omega_0 + \omega_U) = \delta[(k-k_0)\mathrm{v}_s]$, where $k_0 = (\omega_0 - \omega_U)/\mathrm{v}_s$ is the wave number which satisfies the decay condition (10).

Thirdly, we consider the situation where the ion velocity distribution function is supposed to be anisotropic, i.e. it is characterized by different temperatures along the directions parallel and perpendicular to $\vec{B}_0$. The unperturbed particle distribution functions are therefore written in the form [18]:

$$f_{o\alpha} = n_{0\alpha}\left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2}\left(\frac{T_{\perp\alpha}}{T_{''\alpha}}\right)^{1/2}\times$$

$$\times\exp\left(-\frac{m_\alpha v^2_{\perp\alpha}}{2T_{\perp\alpha}} - \frac{m_\alpha v^2_{''\alpha}}{2T_{''\alpha}}\right), \quad (15)$$

where $n_{0\alpha}$ is the equilibrium density and m_α is the particle mass. The temperature $T_{''\alpha}$ is, in the general case, supposed to be different from $T_{\perp\alpha}$.

We limit our anlysis to electrostatic, weakly damped oscillations close to harmonics of the ion cyclotron frequency ($\omega \approx n\omega_{ci}$) which can exist if the wave propagation direction is nearly perpendicular to $\vec{B}_0$, i.e. the angle between the wave vector $\vec{k}$ and $\vec{B}_0$ is close to $\pi/2$. In order to describe these waves, we adopt the approximation $(\omega - n\omega_{c\alpha})\,/\,k_{''}v_{''\alpha} \gg 1$, $k\rho_e \ll 1$, $v_{''i} \ll \omega\,/\,k_{''} \ll v_e$. Then, for $n = 1$, we obtain the expression for the frequency and the damping rate of such oscillations [19]

$$\operatorname{Re}\omega = \omega^{(i)} = \omega_{ci}\,(1' + A_1(\beta_{\perp i}))\,, \quad (16)$$

$$\operatorname{Im}\omega = \gamma_e^{(1)} + \gamma_i^{(1)} \approx \gamma_i^{(1)} =$$

$$= (\pi/2)^{1/2}\frac{A_1^3\omega^2_{ci}}{k_{''}v_{''i}}\frac{T_{\perp i}}{T_{''i}}\exp\left(-\frac{A_1^2\omega^2_{ci}}{2k^2_{''}v^2_{''i}}\right). \quad (17)$$

In (16) and (17), $A_n(\beta_{\perp i}) = I_n(\beta_{\perp i})\,e^{-\beta_{\perp i}}$, where I_n is the modified Bessel function, $\beta_{\perp i} = (k\rho_{\perp i})^2 > 1$. Moreover, relations (16) and (17) were obtained in the case which is interesting in applications, namely, when $T_{''i}/T_{\perp i} < A_1 \ll 1$.

Note that the anisotropic distribution of ions over velocities is typical of the plasma held in adiabatic traps. In this case, all the ion anisotropic instabilities possess the frequencies close to the ion-cyclotron one and its harmonics with the increments and the conditions for the emergence of those instabilities essentially depending on a degree of ion anisotropy.

We consider the decay of the pump wave into a daughter upper hybrid wave and ion-cyclotron oscillations in a plasma with ion temperature anisotropy:

$$\omega_0 = \omega_{\mathrm{UH}\,2} + \omega^{(1)}. \quad (18)$$

The parametric instability threshold for this decay is [11]

$$E^2_{\mathrm{th}\,3} \approx \frac{32\pi}{3\sin^4\theta}\frac{v_e^2B_0^2}{c^2}\frac{\nu_{ei}\gamma_i^{(1)}}{\omega_{ct}\omega_{ci}}. \quad (19)$$

Taking into account that $\nu_{\mathrm{eff}} \approx \frac{E_0^2}{E^2_{\mathrm{th}\,3}}\nu_{ei}$ when $\nu_{ei} < \nu_{\mathrm{eff}} < \gamma_i^{(1)}$ [12] and using the analogous procedure as that which has been described earlier), we find, as a result, the relaxation time dependence on the ion temperature anisotropy

$$\frac{1}{\tau_{ei3}} \sim T^2_{\perp i}\left(\frac{T_{''i}}{T_{\perp i}}\right)^{3/2}\exp\left(\frac{C}{T_{''i}T_{\perp i}}\right). \quad (20)$$

Here, C is a constant which does not depend on the electron and ion temperatures.

In the present paper, the relaxation processes between the electron and ion temperatures in the magnetized plasma with an upper-hybrid pump are investigated. The expressions for the inverse relaxation time in the cases where the upper-hybrid pump parametrically excites modified convective cells, ion-acoustic waves, and ion-cyclotron oscillations in a plasma with ion temperature anisotropy are obtained.

These results can be of interest for plasma diagnostics and for considering the plasma heating efficiency.

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ПРОЦЕСИ ТЕПЛОВОЇ РЕЛАКСАЦІЇ
У ЗАМАГНІЧЕНІЙ ПЛАЗМІ

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Р е з ю м е

На основі кінетичної теорії флуктуацій розглянуто процеси релаксації між електронною і іонною температурами в магнітоактивній однорідній плазмі. Проаналізовано випадки, коли зовнішня верхньогібридна хвиля накачки збуджує конвективні комірки, іонно-звукові хвилі та іонно-циклотронні осциляції з анізотропією іонної температури. Обчислено обернений час релаксації в режимі турбулентних флуктуацій для цих випадків та отримано його залежність від параметрів плазми і хвилі накачки.