DIFFUSION IN TURBULENT PLASMA WITH ION TEMPERATURE ANISOTROPY

V.N. PAVLENKO, V.G. PANCHENKO

Institute for Nuclear Research, Nat. Acad. Sci. of Ukraine
(47, Prosp. Nauky, Kyiv 03680, Ukraine)

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On the base of the kinetic fluctuation theory, we consider the parametric interaction of lower hybrid and upper hybrid waves with ion-cyclotron oscillations in a plasma, where the ion velocity distribution function is anisotropic. The turbulent diffusion coefficient is found to depend significantly on the ratio between the parallel and perpendicular ion temperatures. Our results can be of interest in the investigation of anomalous transport processes in space and laboratory plasmas with ion temperature anisotropy.

Turbulent transport driven by low frequency electrostatic fluctuations is recognized to be of significant importance and may account for the major of the cross-field transport occuring in plasma.

Investigations of diffusion processes in a hot magnetized plasma and in a dusty plasma are widely conducted last years [1–5].

Anomalous diffusion and parametric excitation of electrostatic fluctuations in a helicon-produced plasma were studied in [2].

It is shown in [3] that the anomalous transport in fusion devices arises from the turbulent diffusion caused by electromagnetic fluctuations in the edge plasma region.

The appropriateness of the Fokker–Planck equation for grains in a dusty plasma is discussed in [4]. A more general description of the velocity dependent friction and diffusion coefficients in a dusty plasma is formulated.

The existence of the regions with reduced levels of diffusion attributed to zonal flows in the toroidal ion temperature gradient mode turbulence with the polarization drift effects is indicated in [5].

It is of interest to consider diffusion in the presence of enhanced fluctuations due to a parametric instability. Thus, such problems as the anomalous transport in the presence of turbulent fluctuations and the enhanced diffusion due to ion-cyclotron waves are important. As for the latter problem, a more detailed study which would treat the real diffusion with realistic ion temperature anisotropy is needed.

We use the fluctuation theory for a magnetized plasma in the presence of a pump wave [6–10] to calculate the diffusion coefficient of the turbulent plasma.

In [11], the anomalous diffusion in a magnetoactive inhomogeneous plasma in the presence of the hybrid pump wave parametric instability is investigated. For the region above the instability threshold, it is shown that the turbulent diffusion coefficient grows with increase in the density gradient and the intensity of a pump wave.

In this paper, we have considered the parametric excitation of short wavelength ion-cyclotron waves in the plasma with ion temperature anisotropy by the lower hybrid and upper hybrid pump waves. The parametric interaction of lower hybrid waves with such low-frequency modes can lead to the three-wave decay instability [12,13]. Under such parametric instability conditions, the electric field intensity of superthermal fluctuations is much higher than the level of the thermal noise.

The expression for the diffusion coefficient is calculated, and its dependence on the ion temperature anisotropy is obtained. It is found that, for typical laboratory plasma parameters, the magnitude of the diffusion coefficient obtained in this article is essentially greater than the corresponding one for a stable plasma in the absence of a pump wave.

We consider the uniform electron-ion plasma in a constant external magnetic field $\vec{B}_0 = B_0\vec{z}$. The ion velocity distribution function is supposed to be anisotropic, i.e. it is characterized by different temperatures in parallel and perpendicularly to $\vec{B}_0$. The unperturbed particle distribution functions can be therefore written in the form [12]

$$f_{o\alpha} = n_{0\alpha}(\frac{m_\alpha}{2\pi T_\alpha})^{3/2}(\frac{T_{\perp\alpha}}{T_{\parallel\alpha}})^{1/2}\exp(-\frac{m_\alpha v_{\perp\alpha}^2}{2T_{\perp\alpha}} - \frac{m_\alpha v_{\parallel\alpha}^2}{2T_{\parallel\alpha}}), \quad (1)$$

where $n_{0\alpha}$ is the equilibrium density and m_α is the particle mass. The temperature $T_{\parallel\alpha}$ is supposed, in general, to be different from $T_{\perp\alpha}$.

We limit our anlysis to electrostatic weakly damped oscillations close to harmonics of the ion cyclotron frequency ($\omega \approx n\Omega_i$) which can exist if the wave propagation direction is nearly perpendicular to $\vec{B}_0$, i.e. the angle between the wave vector $\vec{k}$ and $\vec{B}_0$ is close to $\pi/2$. In order to describe these waves, we adopt the approximation $(\omega - n\Omega_\alpha)/k_\parallel v_{\parallel\alpha} \gg 1$, $k\rho_e \ll 1$, $v_{\parallel i} \ll \omega/k_\parallel \ll v_e$. Then, for $n = 1$, we obtain the following expressions for the frequency and the damping rate of such oscillations [13]:

$$\mathrm{Re}\omega = \omega^{(i)} = \Omega_i\,(1' + A_1(\beta_{\perp i}))\,, \tag{2}$$

$$\mathrm{Im}\omega = \gamma_e^{(1)} + \gamma_i^{(1)} \approx \gamma_i^{(1)} =$$

$$= (\pi/2)^{1/2}\frac{A_1^3\Omega_i^2}{k_\parallel v_{\parallel i}}\frac{T_{\perp i}}{T_{\parallel i}}\exp(-\frac{A_1^2\Omega_i^2}{2k_\parallel^2 v_{\parallel i}^2}.) \tag{3}$$

In (2) and (3), $A_n(\beta_{\perp i}) = I_n(\beta_{\perp i})e^{-\beta_{\perp i}}$, where I_n is the modified Bessel function, and $\beta_{\perp i} = (k\rho_{\perp i})^2 > 1$. Moreover, (2) and (3) were obtained in the case which is interesting in applications, namely when $T_{\parallel i}/T_{\perp i} < A_1 \ll 1$.

Note that the anisotropic distribution of the ions over velocities is typical of the plasma held in adiabatic traps. In this case, all the ion anisotropic instabilities possess the frequencies close to the ion-cyclotron one and its harmonics, and the increments and the conditions for the emergence of those instabilities depend significantly on a degree of ion anisotropy.

It is known that macroscopic plasma properties are governed by particle collisions. So their calculations calls for the solution of a kinetic equation with collision integral. Thus, the determination of the collision integral and, consequently, the associated diffusion of particles are the actual problems of plasma theory.

The collision integral of charged particles in plasma can be presented in the form [6]

$$I_\alpha(\vec{p}) = -\frac{e_\alpha}{n_\alpha}\mathrm{Re}\int\frac{d\omega}{2\pi}\int\frac{d\vec{k}}{(2\pi)^3}\frac{d}{d\vec{p}}\times$$

$$\times\langle\delta f_\alpha(\omega,\vec{k},\vec{p})\delta\vec{E}(\omega,\vec{k})\rangle. \tag{4}$$

In (4), $\delta f_\alpha(\omega,\vec{k},\vec{p})$ is a fluctuation of the distribution function, and $\delta\vec{E}$ is the fluctuating electric field. We will study fluctuations in an electron-ion magnetized plasma under the influence of a radio-frequency pump wave field $\vec{E}_0(t) = E_0\vec{y}\cos\omega_0 t$ which is taken in the dipole approximation.

1) Firstly, we consider the case where the frequency of a pump wave ω_0 lies in the lower hybrid (LH) frequency region $\Omega_i \ll \omega_0 \sim \omega_{\mathrm{LH}} \ll \Omega_e$, where $\omega_{\mathrm{LH}} \approx \omega_{pi}(1 + \omega_{pe}^2/\Omega_e^2)^{-1/2}$ is the LH resonance frequency and $\omega_{p\alpha}$ is the plasma frequency of s particle of the α species $(\alpha = e, i)$. The damping rate of an LH wave is given by

$$\gamma_{\mathrm{LH}} \approx \Big(\frac{\pi}{8}\Big)^{1/2}\frac{m_i}{m_e}\frac{\omega_{\mathrm{LH}}^4}{k^3 v_e^3\cos\theta}\exp\Big(-\frac{\omega_{\mathrm{LH}}^2}{2k_\parallel^2 v_e^2}\Big). \tag{5}$$

Consider the parametric decay of a pump wave into a daugter lower hybrid wave and ion-cyclotron oscillations (2):

$$\omega_0 = \omega_{\mathrm{LH}} + \omega^{(1)}. \tag{6}$$

We study the turbulent plasma mode, when the amplitude of a pump wave exceeds the field threshold value [14]:

$$E_{\mathrm{th}1}^2 = \frac{p}{T_{\parallel i}}\Big(\frac{T_{\perp i}}{T_{\parallel i}}\Big)^{1/2}\exp\Big(-\frac{q}{T_{\parallel i}T_{\perp i}}\Big), \tag{7}$$

where the coefficients p and q are determined by the expressions

$$p = \frac{4\omega_0^2 B_0^2 r_{De}^2\gamma_{\mathrm{LH}}}{k^2c^2\cos\theta\,\omega_{\mathrm{LH}}}m_i\Omega_i^2, \quad q = \frac{\Omega_i^4 m_i^2}{4\pi k^4\cos^2\theta}.$$

The expression for the threshold field can be used in the region $k_\parallel\rho_{\parallel i}k_\perp\rho_{\perp i} < T_{\parallel i}/T_{\perp i} < k_\perp\rho_{\perp i}$. In reality, the parameter $T_{\parallel i}/T_{\perp i}$ is expected to be less than unity, and the threshold electric field will thus generally increase with increase in the temperature anisotropy. We note that the real (anisotropic) plasma is more stable than the usual model plasma, where $T_{\parallel i} = T_{\perp i}$.

Under those conditions, the fluctuations of the electric field intensively develop, by considerably exceeding the thermal noise level. We assume that the instability saturates due to a stabilization mechanism associated with the scattering of charged particles by turbulent fluctuations of the electric field. We will characterize this process by the effective collision frequency ν_{eff} which determines the rate of plasma heating by LH waves, and it is connected with the coefficient of turbulent diffusion $D_\perp$ by the expression [15]

$$\nu_{\mathrm{eff}} \approx k_\perp^2 D_\perp. \tag{8}$$

To calculate ν_{eff}, we use the energy balance equation for the plasma [16]:

$$\frac{1}{2}\frac{\nu_{\mathrm{eff}}e^2n_e}{m_e\omega_0^2}E_0^2 = \sum n_\alpha\int d\vec{p}\frac{p^2}{2m_\alpha}I_\alpha(\vec{p}), \tag{9}$$

where $I_\alpha(\vec{p})$ is the collision integral determined by (4) and depending on ν_{eff}. Thus, expression (9) is a nonlinear integral equation for ν_{eff}. Solving this equation and using

(8) in the region above the instability threshold, we find the turbulent diffusion coefficient in the form

$$D_{\perp 1} \approx \frac{1}{k_{0\perp}^2} \frac{E_0^2}{E_{\rm th}^2(k_0)} [\gamma_i^{(1)}(k_0)\gamma_{\rm LH}(k_0)]^{1/2}, \tag{10}$$

where k_0 is the wave number to be determined from the decay condition (6):

$$k_0 = \quad \Omega_i/(2\pi)^{1/2}\rho_{\perp i}(\omega_0 - \omega_{\rm LH} - \Omega_i). \tag{11}$$

Note that the fluctuations due to the pump field give the dominant contribution to the diffusion coefficient governed by (10).

It can be seen from (10) that the diffusion coefficient is sensitive to the ion temperature anisotropy:

$$D_{\perp 1} = B_1 T_{\perp i} \left(\frac{T_{\| i}}{T_{\perp i}}\right)^{3/4} \exp\left(\frac{C}{T_{\| i}T_{\perp i}}\right), \tag{12}$$

where B_1 and C are constants which do not depend on the electron and ion temperatures. In the derivation of (12), it was assumed that $A_1(\beta_{\perp i}) \approx 1/(2\pi)^{1/2} k\rho_{\perp i}$ at $k\rho_{\perp i} > 1$.

2) Now we are intersted in the case where the frequency of a pump wave lies in the upper hybrid frequency region,

$$\omega_{\rm UH1} \approx \quad \Omega_e(1 + \omega_{pe}^2 \sin^2\theta/2\Omega_e^2), \tag{13}$$

where θ is the angle between the wave vector and the external magnetic field.

Here, we have the case of strongly magnetized plasma, i.e. $\omega_{pe} \ll \Omega_e$, and we assume that the damping rate of the upper hybrid wave γ_U is approximately equal to the electron-ion collision frequency ν_{ei} [9], i.e. $\gamma_U \approx \nu_{ei}$.

It should be emphasized that the condition $\omega_{pe} \ll \Omega_e$ for a strongly magnetized plasma is, in fact, not fulfilled for typical fusion plasma parameters, for which ω_{pe} and Ω_e are of the same order, by referring to the central part of the plasma. Further out, towards the boundary of the fusion plasma, the assumption $\omega_{pe} \ll \Omega_e$ is more appropriate.

As our second example, we consider the decay of a pump wave into the daughter upper hybrid wave (UH1) and ion-cyclotron oscillations in a plasma with ion temperature anisotropy:

$$\omega_0 = \omega_{\rm UH1} + \omega^{(1)}. \tag{14}$$

The parametric instability threshold for this decay is [17]

$$E_{\rm th2}^2 \approx \frac{32\pi}{3\sin^4\theta} \frac{\omega_o^2 B_0^2}{k^2c^2} \frac{\nu_{ei}\gamma_i^{(1)}}{\Omega_e\Omega_i}. \tag{15}$$

Taking into account that $\nu_{\rm eff} \approx \frac{E_0^2}{E_{\rm th2}^2}\nu_{ei}$ when $\nu_{ei} < \nu_{\rm eff} < \gamma_i^{(1)}$ [9] and using the procedure analogous to that which has been described in Section 1, we find the diffusion coefficient dependence on the ion temperature anisotropy for the parametric decay (14):

$$D_{\perp 2} = B_2 T_{\perp i}^2 \left(\frac{T_{\| i}}{T_{\perp i}}\right)^{3/2} \exp\left(\frac{2C}{T_{\| i}T_{\perp i}}\right). \tag{16}$$

Here, B_2 is the constant which does not depend on the electron and ion temperatures.

3) Finally, we investigate the case where the frequency of a pump wave lies in another part of the upper-hybrid frequency region,

$$\omega_0 \sim \omega_{\rm UH2} \approx (\omega_{pe}^2 + \Omega_e^2)^{1/2}, \tag{17}$$

i.e. we have the case of weakly magnetized plasma, when $\omega_{pe} > \Omega_e$.

Consider the pump wave decay into the upper hybrid wave (UH2) and ion-cyclotron oscillations with ion temperature anisotropy:

$$\omega_0 = \omega_{\rm UH2} + \omega^{(1)}. \tag{18}$$

Note that such a decay can occur in the ionospheric plasma.

Using the procedure as that which has been described above, we obtain

$$D_{\perp 3} \sim \exp\left(\frac{\Omega_i^4 m_i^2}{4\pi k^4 \cos^2\theta\, T_{\| i}T_{\perp i}}\right). \tag{19}$$

It is worth noting that the diffusion coefficient in a turbulent plasma (in the assumption that the turbulent damping dominates the collisional one) in the case of short-wavelength oscillations ($k_\perp\rho_{\perp i} \gg 1$) is found to be [15]

$$D_\perp^0 = \frac{1}{2\pi} \frac{v_i}{(nL_\|)} \ln\left(\frac{\rho_i}{r_{De}}\right)^{1/2}, \tag{20}$$

where $L_\|$ is a finite size of the plasma system along the magnetic field.

Note that expression (20) can be also obtained when the external pump wave is absent, i.e. in the case of stable plasma.

Comparing the diffusion coefficients (10) and (16) with the correspoding expression (20), we can see that, for typical parameters of a hot plasma ($n = 10^{20}$ m^{-3}, $T_e = 10$ keV, $\nu_i = 10^4$ s^{-1}, $\omega_{\rm LH} = 5 \times 10^9$ s^{-1},

$\Omega_e = 1.5 \times 10^{11}$ s^{-1}, $B_0 = 50$ kGs, $k_\perp r_{De} \approx 10^{-1}$), the diffusion coefficients $D_{\perp 1}$ and $D_{\perp 2}$ are much greater than $D_\perp^0$ (more than two orders of magnitude).

On the base of the theory of kinetic fluctuations, we have considered the parametric interaction of lower hybrid and upper hybrid waves with ion-cyclotron oscillations in a plasma with anisotropic ion velocity distribution function. The turbulent diffusion coefficient is found to depend on the ratio between the parallel and perpendicular ion temperatures. The results obtained in this article allow one to clarify the influence of the ion temperature anisotropy on the diffusion processes in space and laboratory plasmas.

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ДИФУЗІЯ У ТУРБУЛЕНТНІЙ ПЛАЗМІ З АНІЗОТРОПІЄЮ ТЕМПЕРАТУРИ ІОНІВ

В.Н. Павленко, В.Г. Панченко

Р е з ю м е

На основі кінетичної теорії флуктуацій розглянуто параметричну взаємодію нижньогібридної і верхньогібридної хвиль із іонно-циклотронними осциляціями в плазмі з анізотропною функцією розподілу за швидкостями. Знайдено, що коефіцієнт турбулетної дифузії суттєво залежить від співвідношення між іонними температурами вздовж і поперек магнітного поля. Результати роботи є цікавими при дослідженні процесів аномального транспорту в космічній і лабораторній плазмі з іонною анізотропією.