NUMERICAL SIMULATION OF THE DEVELOPMENT OF AN ION CYCLOTRON PARAMETRIC INSTABILITY UNDER SELECTIVE SEPARATION OF ISOTOPES BY ICR METHOD

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The development of a parametric instability of ion Bernstein modes under selective separation of Li^6 and Li^7 isotopes by the ICR method is considered, which is important for the controlled fusion problem. It is shown that, under the separation of these isotopes, the oscillations of ions of different species relative to each other across the magnetic field under the action of forced Alfven oscillations can result in the excitation of parametrically unstable shortwave electrostatic ion cyclotron oscillations, when the ion cyclotron resonance condition is met. The numerical simulation of the evolution of this instability by means of a macroparticle technique under the separation of lithium isotopes by the ICR method has shown that its development leads to the turbulent heating of both resonant and nonresonant isotopes and, as a consequence, to the selection efficiency deterioration of the resonant isotope from plasma.

1. Introduction

The earlier research revealed [1] that the oscillations of isotopes of different species relative to each other across the magnetic field in the process of selective separation of heavy isotopes, such as gadolinium ones, under the conditions of the ion cyclotron resonance result in the excitation of parametrically unstable short-wave electrostatic ion cyclotron fluctuations. The modeling of this instability by the method of macroparticles (for the data [2] on the separation of gadolinium isotopes by the ICR method) has shown that the development of this instability leads to the turbulent heating of both resonant and nonresonant isotopes and to the efficiency deterioration of the selection of resonant isotopes from plasma.

The present work considers the possibility of the development of this instability under the separation of Li^6 and Li^7 isotopes by this method, which is important for the problem of thermonuclear fusion. Using the data of experiments [3], we show that, in this case, the development of the parametric instability of ion Bernstein modes and the turbulent heating of ions of both species are possible too.

The mechanism of selective separation of isotopes by the ICR method consists in the following. In the plasma which comes out with the thermal velocity from a source of ions into the heating area under an antenna, the resonant ions under selection obtain the transverse energy from the RF field under conditions of the ion cyclotron resonance, and they are heated up on the average. Due to the Doppler effect, the nonresonant ions obtain a smaller energy, and their heating can be neglected, if the generator frequency $\omega_0 \approx \omega_{c\,\mathrm{res}}$ and the magnetic field are chosen such that the condition $\left|\omega_0 - \omega_{\mathrm{cres}} - k_{\|\,0} V_{\|}\right| \lesssim k_{\|\,0} V_{Ti}$ is met, where $V_{\|}$ is the velocity of particles along the magnetic field, V_{Ti} is their thermal velocity, $k_{\|\,0} = 2\pi/L_A$, and L_A is the antenna length. For nonresonant particles, the cyclotron frequency differs from the cyclotron frequency of the resonant ones by the value $\Delta\omega = \omega_0\,(\Delta A/A)$, where A is the isotope atomic number, ΔA is the difference between the atomic numbers of different isotopes for the given element, and the condition of the resonant absorption is not met for them if $\left|\omega_0 - \omega_{c\,\mathrm{nores}} - k_{\|\,0}\bar{V}_{\|}\right| \omega_{ci}\Delta A/A \gg \sqrt{2} k_{\|\,0} V_{Ti}$. These conditions restrict the antenna length. It should exceed a certain critical length considerably, viz.

$$L_A \gg L_{\mathrm{cr}} = \sqrt{8}\pi\,(V_{Ti}/\omega_0)\,(A/|\Delta A|)\,. \tag{1}$$

In the RF field, the ions get the oscillation velocity perpendicular to the magnetic field $u_{\perp i} = (u_x + iu_y)$. For the resonant ions, it has the value of the order of

$$u_{\perp i} \sim \frac{e_i}{m_i} E_{\perp} \Delta t, \quad \Delta t \le \Delta t_s \sim \frac{1}{k_{\|0}\, V_{Ti}}, \tag{2}$$

where $E_{\perp}$ is the complex amplitude of the field rotating in the direction of the cyclotron rotation of ions. The velocity of the resonant ions can be, due to inequality (1), much greater than the velocity of nonresonant ones, as the oscillation velocity of nonresonant ions will have

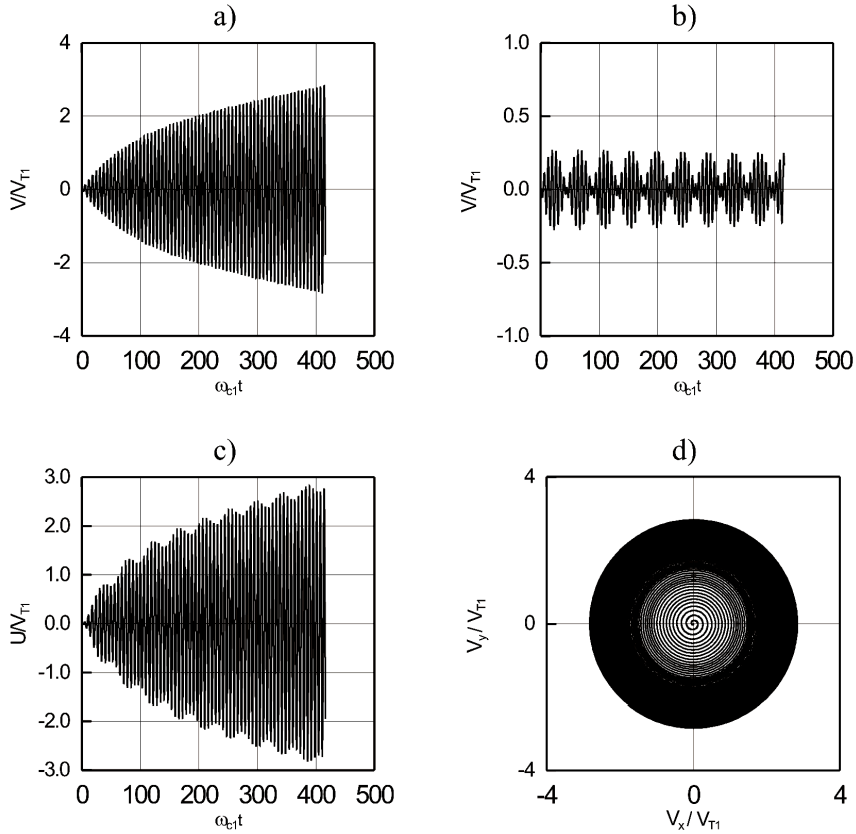

Fig. 1. The time dependence of the average transversal velocity of Li^6 isotope ions (a), the average transversal velocity of Li^7 isotope ions (b), the relative velocity of Li^6 and Li^7 ions (c), and amplitude evolution of the velocity of resonant ions in the phase (d) plane

the order

$$u_{\perp \mathrm{nores}} \sim \frac{e_i}{m_i}\frac{E_\perp}{\omega_{ci}}\frac{A}{\Delta A} \sim c\frac{E_\perp}{B_0}\frac{\Delta A}{A} \ll v_{\perp \mathrm{res}} \simeq$$
$$\simeq \frac{eE_\perp}{m_i k_{\parallel 0} V_{Ti}} = c\frac{E_\perp}{B_0}\frac{\omega_{ci}}{k_{\parallel 0}V_{Ti}}. \quad (3)$$

A more accurate expression for the oscillation velocity can be derived from the equations of motion, by neglecting the small longitudinal electric field $E_\parallel$ and by averaging the result over the initial Maxwell distribution. Then we derive the expression [1] for the averaged transverse velocity of particles

$$\vec{u}_\perp = \left\langle \vec{V}_{\perp i} \right\rangle = \sqrt{\frac{\pi}{2}}\frac{e_i}{m_i}\frac{\vec{E}_\perp \exp\left(-i\omega_{ci}t\right)}{k_{\parallel 0}V_{Ti}} \times$$
$$\times \left\{ \exp\left[-i\left(\omega_0 - \omega_{ci}\right)\ t\right] \exp\left[-\frac{\left(k_{\parallel 0}V_{Ti}t\right)^2}{2}\right] \times \right.$$
$$\left. \times W\left(\frac{\omega_0 - \omega_{ci}}{\sqrt{2}k_{\parallel 0}V_{Ti}} - i\frac{k_{\parallel 0}V_{Ti}t}{\sqrt{2}}\right) - W\left(\frac{\omega_0 - \omega_{ci}}{\sqrt{2}k_{\parallel 0}V_{Ti}}\right) \right\}, \quad (4)$$

where $W(z) = \exp\left(-z^2\right)\left(\frac{k_{\parallel 0}}{|k_{\parallel 0}|} + \frac{2i}{\sqrt{\pi}}\int\limits_0^z \exp\left(t^2\right)\, dt\right)$.

If the relative velocity of resonant and nonresonant ions becomes of the order of their initial thermal velocity, the parametric instability of potential ion cyclotron fluctuations will arise in such a plasma due to the relative ion oscillations [4].

2. Computer Modeling Results

The computer modeling of the nonlinear evolution of the ion cyclotron parametric instability by the macroparticle method has been carried out with the code [5] for ions of the lithium isotopes with the parameters [3]: the initial temperature of ions is 4 eV, plasma density is $\bar{n}_0 = 10^{12}$ cm^{-3}, magnetic field is B=0.25 T, ion density of the resonant isotope Li^6 $n_{Li^6} = 0.1\bar{n}_0$, intensity of the left-hand polarized electric pumping field is 0.5 V/cm, RF coil has the length ("period") along the magnetic field $L_A = 80$ cm, and the pumping field frequency

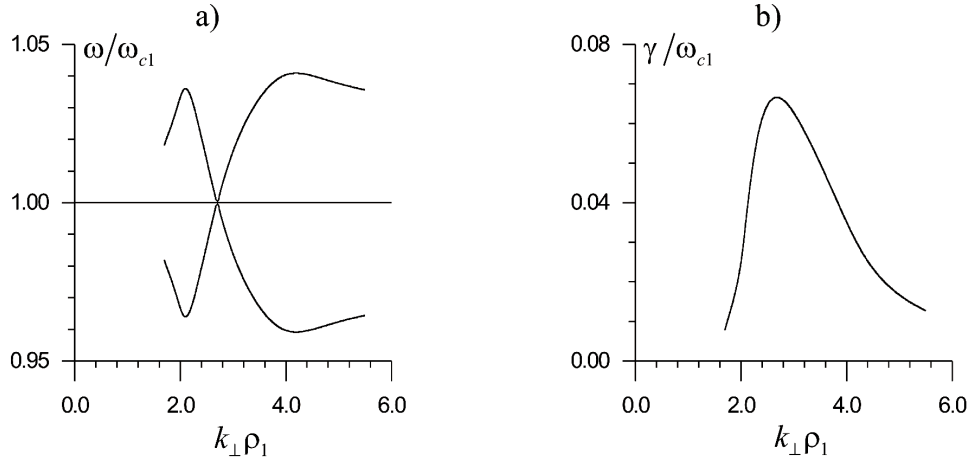

Fig. 2. Frequency (a) and the growth rate (b) of parametrically unstable fluctuations against their transverse wave number in the case where the transverse velocity is $u_1/V_{T1} = 2.5$ for Li^6 ions and $u_2/V_{T1} = 0.2$ for Li^7 ions ($k_\parallel \rho_1 = 0.01$). For the fluctuations with the greater values of $k_\parallel$ ($k_\parallel \rho_i \geq 0.02$), the growth rate is 2–4 times less than its maximum

$\omega_0 = \omega_{c\,\mathrm{res}} = 3.99 \times 10^6$ s^{-1}. With these parameters, the time of flight of lithium ions under the coil is $\Delta t_{\mathrm{f}\,1} \simeq L_A/V_{Ti} \approx 400/\omega_{ci} \approx 10^{-4}$ s, and the value of the steady-state time is $\Delta t_s = 1/\sqrt{2}k_\parallel V_{Ti} = L_A/\sqrt{2}2\pi V_{Ti} \sim 0.1\Delta t_{\mathrm{f}\,1}$. In this case, the amplitude of the oscillation velocity of Li^6 ions increases, by reaching the initial thermal velocity $V_{Ti} \approx 0.8 \times 10^6$ cm/s at $t\omega_{ci} \sim 50$, and it is up to $2.8V_{Ti}$ at the end of the flight under an antenna (Fig. 1,a). At the same time, the amplitude of the ion velocity of Li^7 nonresonant isotope does not exceed $0.25V_{Ti}$. It does not increase, but changes periodically, and is modulated with the frequency $\Delta\omega = \omega_0\,(\Delta A/A) = 5.7 \times 10^5$ s^{-1} (Fig. 1,b). The relative velocity of Li^6 and Li^7 isotope ions is shown in Fig. 1,c. It is actually equal to the velocity of Li^6 resonant ions with some modulation by the same frequency $\Delta\omega$. Figure 1,d depicts the trajectory of the average transverse velocity of Li^6 resonant ions in the phase plane. On the phase plane, the trajectory of the average velocity of resonant ions represents a non-uniform spiral. The density of spiral coils increases in time. The final scope or the diameter of the spiral reaches almost six initial values of the thermal velocity of Li^6 ions.

At such ion oscillation velocity values of different elements relative to one another, the parametric instability appears in plasma in the region of the ion cyclotron frequency. The frequency and the growth rate obtained from the solution of a linear parametric dispersion equation indicate it. A dispersion equation has the form

$$\det |a_{mn}|_{-\infty}^{\infty} = 0, \tag{5}$$

$$a_{mn} = \delta_{mn} + \frac{1}{1 + \delta\varepsilon_e\,(k\omega + m\omega_0)} \times$$

$$\times \sum_\alpha \sum_{p=-\infty}^{\infty} J_{p+m}\,(a_a)\,J_{p+n}\,(a_a)\,\delta\varepsilon_\alpha\,(\omega + p\omega_0),$$

where a_α is the displacement of α species ions relative to electrons in the pumping wave field, $J_p\,(a_\alpha)$ are Bessel functions of the first kind, and $\delta\varepsilon_\alpha$ is the contribution of α species particles to the permittivity.

The dispersion curves obtained via solving Eq. (5) for Li^6 and Li^7 isotopes are shown in Fig. 2. Here and further on, we assume the pumping field frequency ω_0 to be equal to the ion cyclotron frequency of Li^6 isotope. It is clear from Fig. 2,a, that the frequency of parametric unstable fluctuations differs from the cyclotron frequency of Li^6 ions (and also, due to periodicity, from the multiples of the ion cyclotron frequency) by the amount less than $0.05\omega_{c1}$. Then the maximum growth rate, as is shown in Fig. 2,b, has the value $\gamma/\omega_{c1} \approx 0.06$. It is reached at $k_\parallel\rho_1 = 0.01$, $k_\perp\rho_1 = 2.8$.

Some modes of the electric field in a wide enough range of transverse wave numbers, $2 \lesssim k_\perp\rho_1 \lesssim 6$, become unstable and begin to increase exponentially, when the relative velocity of different species ions becomes greater than their thermal velocity. In Fig. 3,a the time behavior of the most unstable mode ($k_\perp\rho_1 = 2.8$, $k_\parallel\rho_1 = 0.01$) is shown. Its amplitude begins to increase intensively at $\omega_{c1}t \gtrsim 300$, when the value of the relative velocity of Li^6 and Li^7 ions exceeds twice

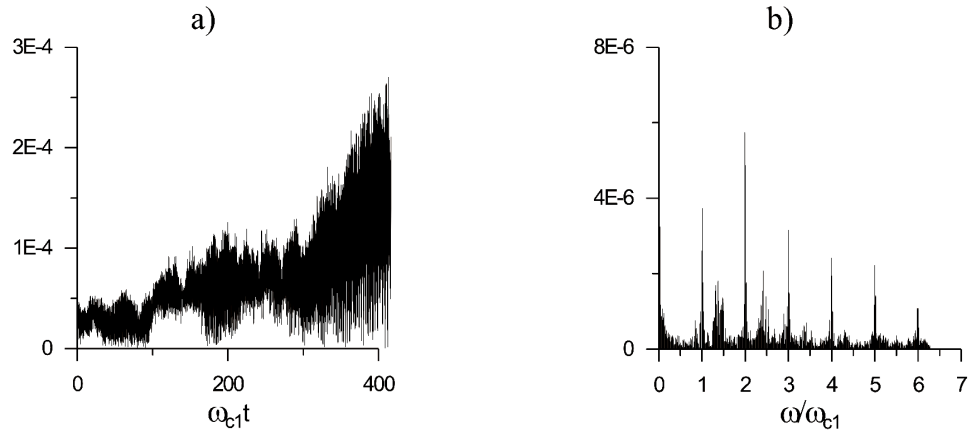

Fig. 3. *a* – time dependence of the most unstable mode ($k_\perp \rho_1 = 2.8$, $k_\| \rho_1 = 0.01$); *b* – frequency spectrum of this mode

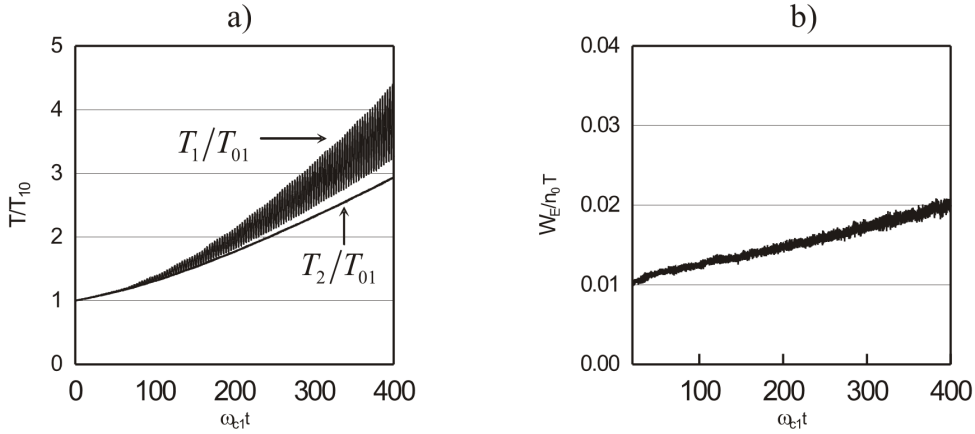

Fig. 4. Time variation of temperatures of both species ions related to the initial temperature (T_1/T_{01} is the temperature of Li^6 ions; T_2/T_{01} is the temperature of Li^7 ions) (*a*); energy density of the self-consistent electric field $W_E = E^2\ 8\pi$ related to the density of the initial thermal energy of ions $n_0 T_{01}$ (*b*)

the initial thermal velocity. The frequency spectrum of this mode is shown in Fig. 3,*b*. In this spectrum, two groups of harmonics stand out against others.

The first group of harmonics, with the frequencies very close to the multiple values of the cyclotron frequency of Li^6 ions is unstable. The big peaks shown in Fig. 3,*b* correspond to it. The greatest peak is located near the second cyclotron harmonic of Li^6 ions. The second group of harmonics has smaller amplitudes than the first one. These frequencies are located between the multiples of the cyclotron frequency of Li^6 ions. The frequencies of these harmonics are close to the multiple values of the cyclotron frequency of Li^7 ions.

With the rise of the relative velocity there is the increase of the temperatures of Li^6 and Li^7 ions (Fig. 4,*a*) and the growth of the energy density of the self-consistent electric field $W_E = E^2/8\pi$ (Fig. 4,*b*). The temperature of Li^6 ions increases by 4.5 times during the modeling, and their thermal energy appears nearby 18 eV at $t = 400\omega_{c1}^{-1}$. At the same time, the energy of the directed motion of Li^6 ions makes 36 eV, i.e. the energy of the directed motion exceeds twice their thermal energy. The total kinetic energy of Li^6 ions, thus, makes 54 eV, that corresponds to the values obtained in the experimental study [3]. Heating the nonresonant Li^7 ions occurs more slowly, and their final temperature is only three times greater than the initial one. Therefore, their thermal energy is less than the thermal energy of Li^6 ions by a factor of 1.5, and it is equal to 12 eV. The energy of the directed motion of Li^7 ions is insignificant in comparison with their thermal energy.

At the nonlinear stage, the noise level $\langle E^2 \rangle / 8\pi n_0 T_0$ increases in time linearly from 10^{-2} to 2×10^{-2}, the transverse temperature of Li^6 ions reaches the value

$4.8T_0 = 18$ eV, and, for Li^7 ions, it is up to $3T_0 = 12$ eV. In the experiments [3], the saturation of the current of accelerated Li^6 ions was observed at a collector with increase in the RF current amplitude in the antenna. This evidences the efficiency deterioration of this isotope separation method. In [3], it is associated with the possible appearance of a plasma instability.

3. Conclusions

1. By means of the macroparticle technique, we have carried out the computer modeling of the ICR isotope separation with the parameters of plasma, magnetic field, and alternating electric field typical of the experiments [3] under conditions of the ion cyclotron resonance for the mixture of Li^6 and Li^7 isotopes. The computations have shown that the small-scale potential ion cyclotron fluctuations arise in plasma due to the parametric instability, when the relative velocity of different isotopes reaches the value of the order of the ion thermal velocity (or even a little smaller). Their characteristic transverse wavelength is of the order of the ion Larmor radius, and the longitudinal wavelength exceeds the ion Larmor radius considerably.

2. The frequency spectrum analysis of unstable fluctuations has revealed the presence of the peaks at the frequencies $\omega \approx n\omega_{ci}$ (ω_{ci} is the cyclotron frequency of the resonant isotope), where $n = 1,2...,6$, and also the presence of the additional peaks with somewhat smaller amplitudes at $n - 1 < \omega/\omega_{ci} < n$, where $n = 1,2 \ldots$.

3. The growth rate of these fluctuations reaches the value $\gamma/\omega_{ci} \lesssim 0.06 \div 0.08$. The instability development results in an increase of the transverse "temperature" of both resonant and nonresonant ions

$$n_0 T_{\perp i} = \frac{m_i}{n_0} \int (v_\perp - \langle v \rangle_i)^2 f_0 \left(v_\perp, v_{||}\right) dv_\perp dv_{||},$$

where $\langle v \rangle_i = (1/n_0) \int v_\perp f_0 dv_\perp dv_\parallel$ is the average velocity of ions across the magnetic field. In the case of the modeling of the selective separation of the lithium isotopes with weights m_i/m_p equal to 6 and 7, the ion temperature increases approximately up to 3–4 values of the initial temperature $T_0 = 4$ eV during the passage of the area under an antenna. The heating of nonresonant components and the deterioration of the separation efficiency were noted in work [3]. In authors' opinion, it is caused by a plasma instability. The modeling of the instability in this system shows that the plasma instability is a parametric one which is excited by the relative fluctuations of different isotopes in the pumping field.

For a mixture of the gadolinium isotopes which are much heavier than the lithium ones, this instability appears more dangerous, as is shown in [1]. Already during the time of the order of $\Delta t = 200/\omega_{ci}$ which takes only 1/8 of the flight time in the RF field region, the temperature of both resonant and nonresonant isotopes reaches 60 eV (the initial temperature is 10 eV). It is clear that, during all time of the flight under an antenna, the temperatures of both nonresonant and resonant ions will increase considerably, and the efficiency of this method will be depreciated substantially.

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ЧИСЛОВЕ МОДЕЛЮВАННЯ РОЗВИТКУ ІОННОЇ ЦИКЛОТРОННОЇ ПАРАМЕТРИЧНОЇ НЕСТІЙКОСТІ ПРИ СЕЛЕКТИВНОМУ РОЗДІЛЕННІ ІЗОТОПІВ МЕТОДОМ ІЦР

В.В. Ольшанський, К.М. Степанов

Р е з ю м е

Розглянуто розвиток параметричної нестійкості на іонних модах Бернштейна при селективному розділенні ізотопів літію (Li^6 та Li^7) методом ІЦР, важливому для проблеми керованого термоядерного синтезу. Показано, що осциляції ізотопів різних сортів відносно один одного впоперек магнітного поля при розділенні цих ізотопів в умовах циклотронного резонансу під впливом вимушених альфвенівських коливань можуть приводити до збудження параметрично нестійких короткохвильових електростатичних іонних циклотронних коливань. Моделювання методом макрочастинок еволюції цієї нестійкості при розділенні ізотопів літію ІЦР-методом показало, що її розвиток приводить до турбулентного нагрівання як резонансних, так і нерезонансних ізотопів і, як наслідок, до погіршення ефективності відбору з плазми резонансного ізотопу.