ON A MODIFICATION OF THE PHYSICAL THEORY FORMALISM DUE TO THE ESTABLISHMENT OF THE TRANSFORMATION SYMMETRY POSTULATE AND THE PROBLEM OF SECOND-ORDER EFFECTS IN THE OPTICS OF MOVING BODIES

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The postulate of a symmetry of formulas used at the derivation of Lorentz transformations has been used to check other transformations. To reach a better adequacy between the experiment and the theory, the method of symmetrization of classical transformations has been used, and necessary modifications to the theory dealing with certain problems have been introduced in such a way. The Lorentz transformations have been obtained without imposing any confinements upon the magnitude of velocities of the reference frames and signals. A modified "a posteriori" theory of the Michelson experiment has been constructed, and its schematic generalization onto the case of mechanical signals has been made. The result of this experiment has been substantiated by means of symmetric transformations and on the basis of the Fermat principle. It has been demonstrated that the classical theory, owing to the application of nonsymmetric transformations, had mistakenly predicted the existence of the second-order effect, which the experiment had been designed to seek for.

1. Introduction

In work [1], while deriving the Lorentz transformations, the requirement of their symmetry has been formulated as a postulate. The latter demands that the direct transformations, which describe a transition from the reference frame K to the reference frame K', and the inverse ones describing the inverse transition $K' \to K$ be equivalent. Hence, its realization is a mathematical background of the kinematic relativity principle. There are two ways to invert the transformation. The first one consists in swapping the primed and nonprimed variables and changing the sign of the transformation parameter. The second way is a direct solution of the transformation equations to obtain either primed or nonprimed variables. The transformations are symmetric, if the results of both ways of their inversion are identical.

The postulate of transformation symmetry is based on the statement that the application of nonsymmetric transformations does not provide the sufficient correctness to the theoretical results obtained. Whence, there emerges an idea that some modifications have to be made to the existing theory, because the neglecting of this postulate brought about a disagreement between the theory and the experiment [2, 3]. To put this idea into action, the method of transformation symmetrization is applied; the essence of the method is that nonsymmetric transformations of an "old" theory are used to construct new, symmetric ones. In such a way, we change over to a "new", modified theory.

In works [4–8], an attempt was made to demonstrate how the problems of electrodynamics and optics of moving bodies can be considered in a different way, by modifying the formalism of the theory on the basis of the symmetry postulate. In this work, we have the same goal in view, but present a more profound substantiation of the drawn conclusions.

2. Symmetrization of Transformations

1. Let the points O and O' be the origins of the reference frames K and K', respectively. Let the positions of an arbitrary point M in those reference frames be given by the vectors $\mathbf{r} = \overline{OM}$ and $\mathbf{r}' = \overline{O'M}$, respectively. The corresponding geometric transformation is

$$\mathbf{r}' = \mathbf{r} - \mathbf{s}, \quad (\mathbf{s} = \overline{OO'}) \tag{1}$$

where $\mathbf{s}$ is the vector $\overline{OO'}$. Similarly to the case where a transition to the kinematics by means of the equation $\mathbf{s} = \mathbf{v}t$ introduces a new variable t into consideration (this variable is to be transformed in order that the transformations comprise a complete system of equations), the use of a new parameter $\boldsymbol{\beta} = \mathbf{s}/r$ instead of $\mathbf{s}$ is accompanied by the introduction of an extra variable r, the lengths of the vector $\mathbf{r}$. Then, we obtain a complete system of vector-longitudinal transformations:

$$\mathbf{r}' = \mathbf{r} - \boldsymbol{\beta} r, \quad r' = \sqrt{(1+\beta^2)r^2 - 2(\boldsymbol{\beta}\mathbf{r})r}. \tag{2}$$

Let us symmetrize these transformations in the case where the vector $\mathbf{r}'$ and its norm are expressed by the formulas

$$\mathbf{r}' = kr(\mathbf{n} - \boldsymbol{\beta}), \quad r'^2 = k^2 r^2 (\mathbf{n} - \boldsymbol{\beta})^2, \tag{3}$$

where $\mathbf{n} = \mathbf{r}/r$, and we introduced a new quantity k which has the meaning of the similarity coefficient and, simultaneously, is the factor of transformation symmetrization, after the latter has been determined on the basis of the symmetry postulate. Sequentially inverting the norm of the vector by two ways indicated above, we obtain $r'^2 = r^2/[k^2(\mathbf{n} + \boldsymbol{\beta})^2]$. Demanding that the two latter expressions should be equal, which is required by the symmetry postulate, we obtain $k = \Gamma$, where $\Gamma = 1/\sqrt{\pm(1-\beta^2)}$ is an analog of the Lorentz factor, and the sign plus or minus corresponds to the case $\beta < 1$ or $\beta > 1$, respectively.

Let us write down the symmetric transformation for the length of vector (3) in the polar (spherical) coordinates (r, θ), by assuming that the polar axis is directed along the vector $\mathbf{s}$:

$$r'(\theta) = r\Gamma f_p(-\beta, \theta),$$

$$\cos\theta' = \left[(1+\beta^2)\cos\theta - 2\beta\right] f_p^{-2}(-\beta, \theta), \tag{4}$$

where

$$f_p(-\beta, \theta) = \sqrt{1 - 2\beta\cos\theta + \beta^2},$$

and θ is the angle between the vectors $\mathbf{s}$ and $\mathbf{r}$. Angular dependence (4) is known from the problem of light reflection from a moving mirror (Einstein, 1905). Here, θ' can be regarded as a quantity defined by the given dependence.

Let us find transformations which would be more convenient than Eqs. (4). Making use of both an expression for r', which can be derived from Eq. (3) in form (2), and the formula $r'\cos\theta' = r\Gamma(\cos\theta - \beta)$, we obtain the transformation

$$r'(\theta') = \pm r\Gamma f_q(\mp\beta, \theta'), \quad \theta = \theta', \tag{5}$$

where

$$f_q(\beta, \theta') = \sqrt{1 - \beta^2 \sin^2\theta'} + \beta\cos\theta',$$

and θ' is the angle between the vectors $\mathbf{s}$ and $\mathbf{r}'$.

It is worth noting the equivalence between lengths (4) and (5), as well as the condition of its validity,

$$r'(\theta) = r'(\theta'), \quad \cos\theta' = (\cos\theta - \beta) f_p^{-1}(-\beta, \theta). \tag{6}$$

2. Now, consider the issue concerning the symmetrization of transformations expressed in rectangular coordinates. Let the axes x, y, z (the reference frame K) and x', y', z' (the reference frame K') be relatively oriented in such a manner, that the transformations of transverse coordinates ($y = y'$, $z = z'$) are symmetric. By projecting Eqs. (2) onto the axes of the reference frames, we obtain a complete system of coordinate-longitudinal transformations. Into the nonsymmetric expressions for x' and r', we introduce the symmetrization factor (we continue to designated it, as it was above, by the symbol k); afterwards, we find the corresponding inverse expressions. The result of inversion made following the first way is evident. The inversion by the second method brings about the equations

$$x = \pm\Gamma^2(x' \pm \beta r^*)/k, \quad r = \Gamma^2(r^* \pm \beta x')/k. \tag{7}$$

Here, there spontaneously appeared an analog of the Lorentz factor Γ which was introduced above, and the length

$$r^* = \sqrt{r'^2 - \beta^2(r'^2 - x'^2)}. \tag{8}$$

The latter, in comparison with the length r' of the vector $\mathbf{r}'$, is an incompletely defined quantity in that sense that the r^*-magnitude does not represent the length of a definite Euclidean vector. Therefore, we did not manage here to find the symmetrization factor k, having the inverse transformations obtained by two ways.

Difficulties associated with the determination of the symmetrization factor in the case where the transformations are expressed by means of rectangular coordinate systems, can be overcome, if we confine ourselves to the approximation of affine transformations. Being satisfied with the variable r^* instead of r', we can symmetrize system (7) in the case $\beta < 1$ and obtain a geometric analog of Lorentz transformations. Now, we will deduce these transformations for the cases $\beta < 1$ and $\beta > 1$, making use of formula (8) written down in the form

$$r^{*2} = k^2[(x - \beta r)^2 \pm \Gamma^{-2}(y^2 + z^2)]. \tag{9}$$

We take advantage of the equation $r^{*2} = x'^2 \pm (y'^2 + z'^2)$, which – from the viewpoint of affine geometry – describes either an ellipsoid or a two-sheet hyperboloid of revolution in the K'-system, depending on the selected plus or minus sign, respectively. The transformation of the latter expression to Eq. (9) is ensured by the affine coordinate transformations

$$x' = k(x - \beta r), \quad y' = k\Gamma^{-1}y, \quad z' = k\Gamma^{-1}z. \tag{10}$$

The longitudinal transformation, according to Eq. (9), looks like

$$r^* = \pm k(x - \beta r). \tag{11}$$

By symmetrizing, in Eqs. (10) and (11), the transformations of transverse coordinates and, separately, the system of two remained equations, we obtain $k = \Gamma$. While writing down the transformations, we use the notation k for the symmetrization factor in order to involve the cases where these transformations belong to either the classical ("old") theory ($k = 1$) or the modified one ($k = \Gamma$).

3. Now, let us deduce kinematic transformations from geometric ones. In the case of transformations (1), we put $\mathbf{s} = \mathbf{v}t$. We have the spatial Galilean transformations. Together with the equality $t' = t$, they constitute the complete system of transformations. In the cases of transformations with the parameter $\boldsymbol{\beta}$, besides the equation of motion of the system K', we use the equation $r = ct$ and either the equation $r' = ct'$ or $r^* = ct'$. The first equation is a formal definition of a certain velocity on the basis of the length r and the time. This equation is used to obtain the kinematic parameter $\beta = v/c$. The second equation defines the transformation of the variable t on the basis of the transformed length. If c is the speed of light in vacuum, the kinematic transformations obtained from Eqs. (10) and (11) in the case $\beta < 1$ are the Voigt space-time transformations (Voigt, 1887) if $k = 1$, and the Lorentz transformations if $k = \Gamma$.

3. Generalized Model and the Theory of Michelson Experiment

1. The scheme of experiment carried out by Michelson can be generalized, in particular, owing to the existence of contemporary acoustic interferometers. Let we have a system consisting of an optical or sound locator and a specular reflector of locator signals connected with the former by a rod. The locator moves in the system K with a velocity $\mathbf{v}$, and the rod of the length l steadily rotates around it. In so doing, the total time of the signal motion to the reflector and back again is determined at various orientations of the rod with respect to the vector $\mathbf{v}$. Let us calculate this time in the framework of two methods. The first is based on the reciprocal relation between the relative velocity and the time of the signal motion, the second on the principle of motion independence.

First, consider the case where the rod is oriented normally to the velocity $\mathbf{v}$. In the first method, the velocity of the signal motion V_τ along the rod is determined from the relation $\mathbf{V} = \mathbf{V}_\tau + \mathbf{v}$, where $|\mathbf{V}| = c$ is the velocity of the signal in the system K. Therefore, $V_\tau = c\sqrt{1 - \beta^2}$, and the time of the signal motion forward is

$$t_\tau = \gamma t, \tag{12}$$

where $t = l/c$ and, for $\beta < 1$, $\gamma = \Gamma$. In the second method, the signal itself at $v = 0$ and together with the locator at $c = 0$ shifts independently by the vectors $\mathbf{l} = \mathbf{V}t$ and $\mathbf{s} = \mathbf{v}t$, respectively, in the time interval t; so that the total shift is $\mathbf{r}_\tau = \mathbf{l} + \mathbf{s}$ and the corresponding time $t_\tau = t\sqrt{1 - \beta^2}$. The latter, with an accuracy to the $O(\beta^2)$-terms, coincides with quantity (12). In this case (the transverse rod orientation), the use of the Pythagorean theorem evidently provides practically identical results in both approaches. The total time of the signal motion, in accordance with Eq. (12), can be expressed by the formula

$$T_\tau = 2\gamma l/c. \tag{13}$$

Now, let the rod be oriented along the direction of locator motion. In the first method, if the signal moves to the reflector, the relative velocity V' and the time t' are determined by the expressions

$$V't' = l, \quad V' = kc(1 - \beta), \tag{14}$$

where the symmetrization factor k was introduced. The time of the signal motion to the mirror is

$$t' = t/k(1 - \beta). \tag{15}$$

In the second method, when the signal, freely and together with the locator, has independent shifts $l = ct$ and $s = vt$, respectively, during the time interval t, the total shift of the signal moving to the reflector equals $r' = kl(1 + \beta)$. Whence, we obtain

$$t' = kt(1 + \beta). \quad (16)$$

In the first and second approaches, the time of the signal motion backward is determined from Eqs. (15) and (16) by substituting $\beta \to -\beta$. The total durations of the motion in these approaches are, respectively,

$$T' = 2\gamma^2 l/(kc), \quad T' = 2kl/c. \quad (17)$$

Hence, in the case of the longitudinal orientation of the rod, the classical theory ($k = 1$) produces different results. Thus, it does not satisfy the requirements of unambiguity of the result obtained and correctness of the conclusions drawn.

2. Let us demonstrate now that the ambiguity in the theory of the Michelson experiment originates from the asymmetry of the transformations engaged. We are going to find a transformation that is inverse to expression (15) following the first way and, afterwards, following the second way, to invert the result obtained. As a result, instead of transformation (15), which would have been the final result if transformation (15) had been symmetric, we obtain expression (16). This fact, first, substantiates the second method of finding t' and, second, proves the asymmetry of transformations (15) and (16). The symmetry postulate demands that the symmetrization of transformations be carried out. For this purpose, by equating those expressions, we find the condition of their symmetry: $k = \gamma$. Provided this condition, formula (14) and the expression used for r' are symmetric, and transformations (15) and (16), which were obtained with their help, become identical and symmetric as well. Expressions (17) now coincide with each other and with expression (13). The former coincidence proves the equality of both approaches for the determination of T', and the latter proves the absence of the second-order effect predicted by the classical theory but not revealed in the Michelson experiment. The negative result of this experiment revealed the failure of its classical theory and demonstrated the necessity of making modifications to the formalism of this theory.

3. The Michelson experiment has proved that the total time, $T' = T_\tau = T$, and the path, $L = cT$, of the signal motion are constant for all rod orientations. From the viewpoint of geometric optics, it is the conditions of tautochronism and stationarity of optical paths of light in the absolute reference frame K that are obeyed in the experiment discussed, when light moves between the start and the return points. This means that the Fermat principle is realized. As is known, the condition of path stationarity is satisfied for light that comes to an end point after having been reflected from a concave specular surface, the shape of which is an ellipsoid of revolution. The start and end points for light in the system K are the foci O_1 and O' of this ellipsoid. While traveling from one focus to another, the signal becomes reflected at a point M^* of a plane mirror in the Michelson device, which is tangent to the ellipsoid surface at this point. Light that reaches the point O' follows the paths $O_1M^* = r_1^*$ and $M^*O' = r^*$. They are the focal radii of the given ellipsoid; hence, for any position of the point M^* on the ellipsoid surface, the total path $L = r_1^* + r^*$ and the total time T are constant.

The equations for the Fermat–Michelson ellipsoid described above can be obtained on the basis of expression (11) for the length taken from the Lorentz transformations at $\beta < 1$ and $k = \gamma$. In terms of (r, θ)- and (r, θ')-variables, we obtain the following equivalent formulas for the right focal radius:

$$r^*(\theta) = kr(1 - \beta\cos\theta), \quad r^*(\theta') = kr\gamma^{-2}(1 + \beta\cos\theta')^{-1}, \quad (18)$$

which are known for $k = 1$. The equivalency of those expressions gives a formula for signal aberration (cf. Eq. (6))

$$\cos\theta' = (\cos\theta - \beta)/(1 - \beta\cos\theta),$$

which demonstrates that light in the Michelson experiment deviates from the axis of the device shoulder in the same manner as from the telescope axis, when stars are observed.

While describing the scenario of light propagation in the Michelson experiment, the discovery of the electromagnetic microwave background radiation (A.A. Penzias and R.W. Wilson, 1965; the 1978 Nobel Prize in Physics) is of great importance. This radiation was found to propagate isotropically in a fixed reference frame that is unique for the whole Universe. The discovery of an absolute system proves that every laboratory on the Earth's surface is an optically anisotropic system. The picture of the signal propagation in the Michelson experiment, which was described above on the basis of the Fermat principle, agrees with the conclusion about the existence of a fixed system K, in which light propagates irrespective of its source motion.

4. Now, we are going to demonstrate that the Michelson experiment points out the necessity of making modifications to the Euclidean metric geometry. We shall take advantage of formulas for distances, which can be obtained from transformations (4) and (5), provided $\beta < 1$. In the case $\Gamma = k$, we have the expressions

$$r'(\theta) = krf_p(-\beta, \theta), \quad r'(\theta') = krf_q(-\beta, \theta'). \tag{19}$$

Let an ellipsoid of revolution be inscribed into the sphere $(0, kr)$. The focal radii of this ellipsoid can be found in a regular geometric manner. We impose a restriction

$$r'(\theta) + r_1(\theta) = 2kr, \tag{20}$$

where the length $r_1(\theta)$ is determined by the expression for $r'(\theta)$ with an accuracy to the substitution $\beta \to -\beta$. Using expression (19) for $r'(\theta)$ and condition (20), we obtain the formulas for the focal radii $r^*(\theta)$ and $r_1^*(\theta)$ of the ellipsoid of revolution inscribed into the given sphere. The radius $r^*(\theta)$ is determined by the first equation in (18). The Fermat-Michelson ellipsoid is described by this formula by putting $k = \gamma$.

We have shown above that, while calculating the total signal path in the Michelson experiment for the case of the transverse rod orientation, one can use the cosine theorem, which coincides, in this special case, with the Pythagorean theorem. In the case of the longitudinal rod orientation, the symmetry postulate demands that the factor γ should be introduced into the cosine theorem, in order to provide a concordance between the theory and the experiment. Owing to the availability of such a factor in the expression for $r'(\theta)$, formulas (18) are obtained, in which γ is the similarity factor. The symmetrization of the classical expressions has, in this case, the meaning of a similarity transformation.

Let us calculate now the lengths $r'(\theta')$ and $r_1(\theta')$ by the second formula (19). The arithmetic mean value of those lengths at $\theta' = \theta$ (see Eq. (5)) is the function $r_0^* = kr(1 - \beta^2 \sin^2\theta)^{1/2}$, which, at $k = \gamma$ and $r = \text{const}$, describes the Fermat–Michelson ellipsoid. The expressions for lengths, which were used here, are mutually conjugated in the sense that their product is a rational expression $r'(\theta')r_1(\theta') = k^2\gamma^{-2}r^2$. At $k = 1$, this formula illustrates the theorem about the product of the segments of a chord, which is drawn in the sphere (O, r) through the point O' and is divided at this point by the sphere's diameter into the given segments. At $k = \gamma$, we obtain an invariant, which has no illustrative correspondence in geometry, but agrees with the transformation symmetry postulate.

The geometry applied here, which was modified making use of the transformation symmetry postulate, combines the Euclidean metric geometry and the geometry of similarity [7].

4. Conclusions

In this work, two – in our opinion, very important – discoveries were used to consider, from the contemporary viewpoint, and estimate the known phenomena in the optics of moving bodies. These are, first, the postulate about the exclusive meaning of symmetric transformations and, second, the experimental discovery of the exclusive reference frame in the Universe. Those discoveries made it possible to explain the results of the Michelson experiment, as well as the results of other experiments of the second order (see works [3–8]), similarly to the Lorentz's explanation (1895) of the negative results of first-order experiments in the optics of moving bodies. Lorentz put the theory in agreement with the experiment, by having proved that effects, which are not observed experimentally, should not manifest themselves from the theoretical point of view as well. This statement is known as the Lorentz's theorem. Owing to the establishment of the transformation symmetry postulate and the postulate-induced modifications made to the formalism of the theory, it became possible to extend the scope of the Lorentz's theorem applicability upon the case of second-order effects.

From the established fact of illusiveness of the second-order effects, it follows that illusory are also those hypotheses and postulates which have been invented for compensating these, erroneous in their essence, effects.

We saw above that, in the theory, together with completely defined distances r', there appear incompletely defined ones r^*. It turns out that the formal incorrectness arises even if symmetric transformations are applied, but the distances defined completely and incompletely are made use of, without taking into account the features of the phenomena to be described. In particular, in the theory of the motion of celestial bodies, the distances r^* are used in order to describe – kinematically – the orbits of these bodies, but it is the completely defined distances that are used in dynamics for the determination of gravitational interactions. Similarly, the Lorentz transformations, which have been derived on the basis of an incompletely defined distance, provide an opportunity to obtain correct results in the optics of moving bodies. But, in electrodynamics (for instance, in the theory of the field created by a moving point charge), they, as well as the classical theory, bring about the results, the incorrectness of which has been revealed by the Trouton–Noble experiment (1903). A disagreement between the theory and the experiment can be eliminated in this case, if the Liénard–Wiechert potentials are calculated making use of the completely defined distance [4, 5, 8]. This fact testifies that the Lorentz transformations are not universal.

Concerning the insertion of modifications into the electrodynamics of moving bodies by using completely defined distances and making allowance for the transformation symmetry postulate, we are going to consider this issue in a separate work.

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ПРО ВНЕСЕННЯ ЗМІН У ФОРМАЛІЗМ ФІЗИЧНОЇ ТЕОРІЇ У ЗВ 'ЯЗКУ З ВСТАНОВЛЕННЯМ ПОСТУЛАТУ СИМЕТРІЇ ПЕРЕТВОРЕНЬ ТА ПРОБЛЕМОЮ ЕФЕКТІВ ДРУГОГО ПОРЯДКУ В ОПТИЦІ РУХОМИХ ТІЛ

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Р е з ю м е

Постулат симетричності формул, використаний при виведенні перетворень Лоренца, поширено на випадки знаходження інших перетворень. З метою досягнення більшої відповідності між експериментом і формалізмом теорії застосовано метод симетризації класичних перетворень і таким способом внесено необхідні зміни в теорію деяких задач. Одержано перетворення Лоренца без накладання обмежень на величини швидкостей систем відліку і сигналів. Побудовано змінену апостеріорі теорію досліду Майкельсона при схематичному узагальненні його на випадки використання механічних сигналів. Результат цього досліду обґрунтовано за допомогою симетричних перетворень та на основі принципу Ферма. Доведено, що класична теорія передбачила існування ефекту другого порядку, що був шуканий на досліді, помилково внаслідок вживання нею несиметричних перетворень.