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## POLARIZATION EFFECTS IN THE PHOTON-INDUCED PROCESS OF ELECTRON-POSITRON PAIR CREATION IN A MAGNETIC FIELD, STUDIED IN THE ULTRA-QUANTUM-MECHANICAL APPROXIMATION

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The photon-induced process of electron-positron pair creation in a strong homogeneous magnetic field, provided that the polarization of particles is arbitrary, has been considered. The polarization of a photon is described in terms of the well-known Stokes parameters, and the relevant probabilities of the process turn out to have simple analytical expressions, which allows us to analyze the polarization and spin effects. A substantial influence of the linear polarization of a photon on the spin orientations of electrons and positrons has been demonstrated.

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### 1. Introduction

Quantum-mechanical electrodynamic processes, which involve photons and electrons in strong external electromagnetic fields, do not lose their urgency for both experimental and theoretical studies, despite a good number of available literature sources.

The process of creation of an electron-positron pair by a single photon in a magnetic field has been considered for the first time by Klepikov [1] in the approximation of ultra-relativistic motion of particles. The operator method for the consideration of this problem was applied by Baier and Katkov in the quasiclassic ultra-relativistic case [2,3]. In recent years, there has appeared the work of these authors [4], where the operator method was used to study the process of electron-positron pair creation by a photon, with the particles being located at low-energy Landau levels. We

also mention work [5], where the process of photon-induced creation with the participation of polarized particles has been considered for arbitrary Landau levels and magnetic field values. Nevertheless, the complexity of general formulas prohibited the authors from carrying out their detailed analysis.

A relatively little attention has been drawn to studying the polarization effects in the given process in the case of nonrelativistic electrons in weakly excited states, because the description of such a process requires the application of the ultra-quantum-mechanical approximation, which, in its turn, demands the presence of a rather strong magnetic field approaching the critical Schwinger one  $H_c = m^2 c^3 / e \hbar = 4.41 \times 10^{13}$  Gs.

This work aims at calculating the probability of the photon-induced creation of an electron-positron pair in a strong magnetic field and at elucidating the roles of polarization and spin effects, as well as the correlations between them. We found expressions for the probability of the process in a general quantum-mechanical relativistic case and without putting any additional confinements onto the values of such parameters as the momenta, energies, field magnitude, and so on. The researches of similar expressions was carried out by other authors [5], but the complexity of expressions prohibited them from revealing the regularities associated with the polarization of the photon and the spins of particles.

Owing to the specific choice of the approximation – low Landau levels and magnetic fields with the magnitude below the critical one – we managed to derive simple analytical expressions for the probability concerned which explicitly depend on the Stokes parameters that characterize the polarization of the photon.

## 2. Probability of the Process

For the wave functions of an electron and a positron, we used the following expressions (hereafter, we use the system of units, where  $\hbar = c = 1$ ) [6]:

$$\begin{aligned} \Psi^- &= \frac{1}{\sqrt{S}} e^{-i(E^- t - p_y^- y - p_z^- z)} A_{l^-} \left[ i \sqrt{\tilde{m}^- - \mu^- m} U_{l^-}(\zeta^-) + \right. \\ &\left. + \mu^- \sqrt{\tilde{m}^- + \mu^- m} U_{l^- - 1}(\zeta^-) \gamma^1 \right] u_{l^-}^-, \\ \Psi^+ &= \frac{1}{\sqrt{S}} e^{i(E^+ t - p_y^+ y - p_z^+ z)} A_{l^+} \left[ i \sqrt{\tilde{m}^+ + \mu^+ m} U_{l^+}(\zeta^+) - \right. \\ &\left. - \mu^+ \sqrt{\tilde{m}^+ - \mu^+ m} U_{l^+ - 1}(\zeta^+) \gamma^1 \right] u_{l^+}^+. \end{aligned} \quad (1)$$

Here,  $S$  is the normalization area,  $A_{l^\pm}$  are the normalization constants,  $p_y^\pm$  are the  $y$ -components of the electron and positron momenta,  $(\tilde{m}^\pm)^2 = m^2 + 2l^\pm eH$ ,

$$\zeta^\pm = \sqrt{eH} \left( x - \frac{p_y^\pm}{eH} \right), \quad (2)$$

$U_l(\zeta) = \frac{1}{\sqrt{\pi^{1/2} 2^l l!}} \exp(-\zeta^2/2) H_l(\zeta)$  is the Hermite function,  $u_{l^\pm}$  are constant bispinors, and  $\mu^-$  and  $\mu^+$  are the polarizations of the electron and the positron, respectively.

For the wave function of the initial photon, we used the standard expression [7]

$$A = \frac{\sqrt{4\pi}}{\sqrt{2\omega V}} \gamma^\mu e_\mu e^{-ikx}, \quad (3)$$

where  $V$  is the normalization volume,  $\gamma^\mu$  are the Dirac matrices, and  $e_\mu$  is the vector of photon polarization.

Let the vector of a uniform magnetic field be directed along the axis  $z$ . Then, the wave function (1) is connected with the following gauge of the electromagnetic potential:

$$A_0 = 0, \quad \vec{A} = (0, xH, 0). \quad (4)$$

In this case, the energy spectrum of the electron (positron) looks like

$$(E_l^\pm)^2 = (p_z^\pm)^2 + (\tilde{m}^\pm)^2 = (p_z^\pm)^2 + m^2 + 2l^\pm eH, \quad (5)$$

where  $l^\pm = 0, 1, 2, \dots$  is the principal quantum number (the Landau level number) of the electron or positron.

Let us introduce the parameter  $h = eH/m^2$  which is the ratio between the magnetic field  $H$  and the critical field  $H_c$ . Thus,

$$\tilde{m}^\pm = m \sqrt{1 + 2hl^\pm}. \quad (6)$$

The following conservation laws are valid for the process under consideration:

$$\begin{cases} p_z^\pm + p_z^{\pm l} = k_z = \omega \cos(\theta), \\ E^- + E^+ = \omega, \end{cases} \quad (7)$$

where  $\theta$  is the angle between the photon propagation direction and the magnetic field. We are going to determine the threshold values for the energy and the pair momentum. Let us introduce the function

$$f(p) = \omega - \sqrt{(\tilde{m}^-)^2 + p^2} - \sqrt{(\tilde{m}^+)^2 + (\omega u - p)^2}, \quad (8)$$

where the notation  $u = \cos(\theta)$  is used and – for convenience – the subscripts for the  $z$ -components of the electron momentum are omitted. The conservation laws are fulfilled, provided the condition  $f(p) = 0$  is true. The threshold of the process is determined by the maximum point of the given function. After differentiating, we find

$$f(p_m) = \frac{\omega(p_m - uE_m)}{p_m}. \quad (9)$$

The subscript  $m$  denotes the threshold values. It is obvious that the process is impossible, if the photon propagates along the field, since,  $u = 1$  in this case, so that  $E > p$  at any time (Fig. 1, *a*). Figure 1, *b* demonstrates the plot of the function  $f(p)$  for  $\theta = \pi/2$ .

By putting Eq. (9) equal to zero, we determine the required threshold values:

$$\omega_m = \frac{(\tilde{m}^+ + \tilde{m}^-)}{\sqrt{1 - u^2}}, \quad (10)$$

$$E_m^- = \frac{\tilde{m}^-}{\sqrt{1 - u^2}} = \frac{\tilde{m}^-}{\tilde{m}^- + \tilde{m}^+} \omega_m, \quad (11)$$

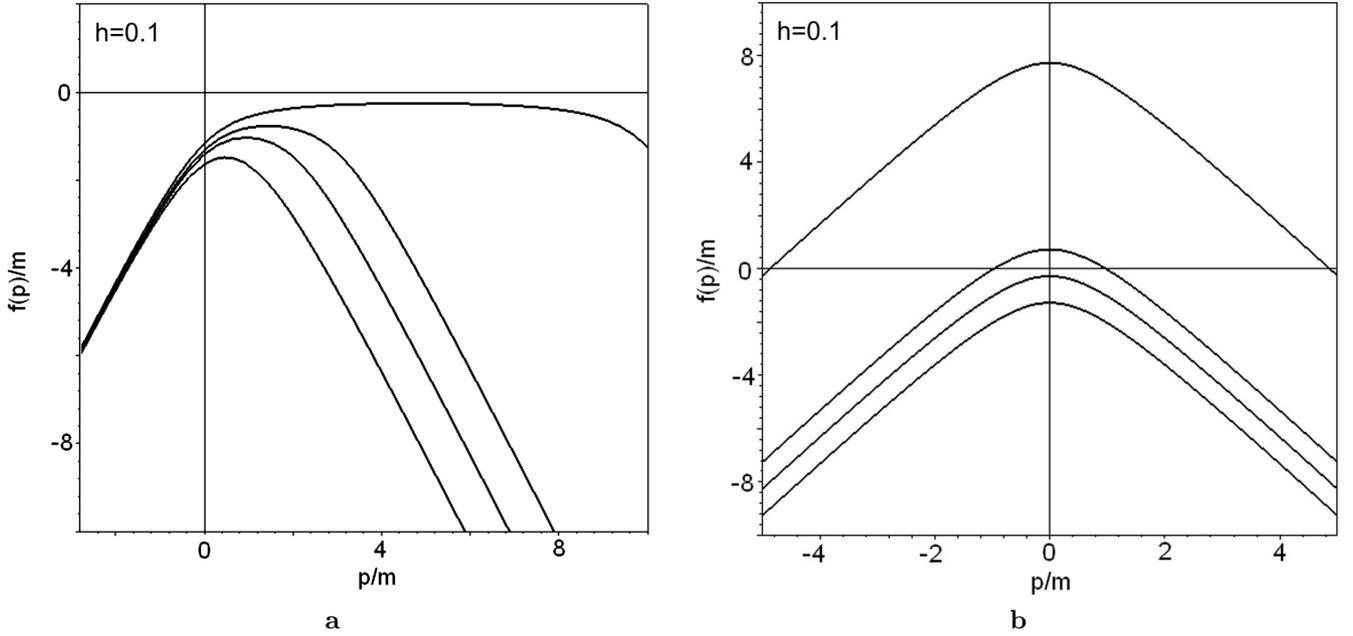


Fig. 1. Plots of the function  $f(p)$  at various frequencies of 1, 2, 3, and 10 (in terms of electron's mass units) and for the values  $\theta = 0$  (a) and  $\pi/2$  (b)

$$p_m^- = uE_m^-, \quad (12)$$

$$E_m^+ = \frac{\tilde{m}^+}{\sqrt{1-u^2}} = \frac{\tilde{m}^+}{\tilde{m}^- + \tilde{m}^+} \omega_m, \quad (13)$$

$$p_m^+ = uE_m^+. \quad (14)$$

To elucidate the physical meaning of the obtained expressions (10)–(14), we pass to a reference frame, where  $(p_m^-)' = 0$ . For this purpose, let us write down the Lorentz transformation for the energy and the momentum of the particle:

$$(p_m^-)' = \gamma(p_m^- - VE_m^-), \quad (15)$$

$$(E_m^-)' = \gamma(E_m^- - Vp_m^-), \quad (16)$$

where  $\gamma = 1/\sqrt{1-V^2}$ . Since  $(p_m^-)' = 0$ , it follows from Eqs. (11) and (16) that

$$V = u, \quad \gamma = \frac{\omega_m}{(\tilde{m}^- + \tilde{m}^+)}, \quad (17)$$

$$(E_m^-)' = \tilde{m}^-, \quad (E_m^+)' = \tilde{m}^+, \quad (18)$$

$$(p_m^+)' = 0. \quad (19)$$

Taking Eq. (7) into account, we find that

$$\omega'_m = \tilde{m}^- + \tilde{m}^+, \quad (20)$$

$$u' = 0. \quad (21)$$

Thus, the threshold value for the photon frequency corresponds to the photon frequency in the reference frame, where  $\mathbf{k} \perp \mathbf{H}$ , the frequency is equal to the sum of the effective masses of electron and positron, and the longitudinal momenta of particles are zero.

In the general case, the solution of Eq. (7) gives

$$p_{1,2}^- = \frac{au \pm b}{2\omega(1-u^2)}, \quad (22)$$

$$p_{1,2}^+ = \frac{a^+u \mp b}{2\omega(1-u^2)}, \quad (23)$$

$$E_{1,2}^- = \frac{a \pm bu}{2\omega(1-u^2)}, \quad (24)$$

$$E_{1,2}^+ = \frac{a^+ \mp bu}{2\omega(1-u^2)}, \quad (25)$$

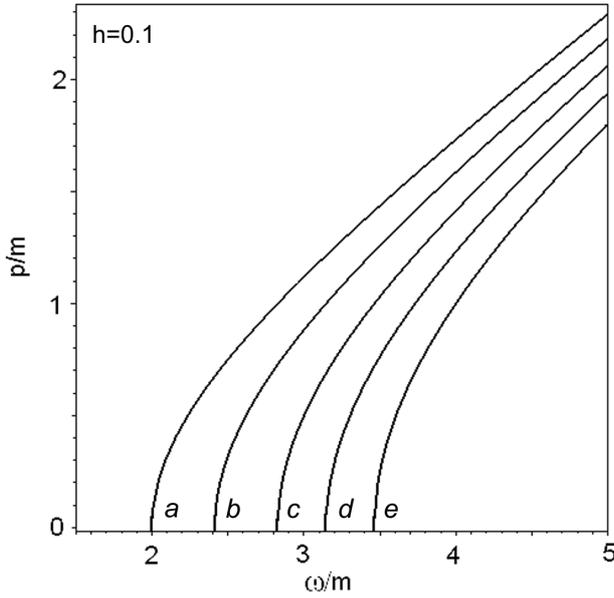


Fig. 2. Dependences of the electron momentum on the frequency of the initial photon (in terms of electron's mass units) for the following Landau levels of the electron and the positron: (a)  $l^- = 0$  and  $l^+ = 0$ , (b)  $l^- = 0$  and  $l^+ = 5$ , (c)  $l^- = 5$  and  $l^+ = 5$ , (d)  $l^- = 5$  and  $l^+ = 10$ , and (e)  $l^- = 10$  and  $l^+ = 10$

where

$$a = \omega^2(1 - u^2) + (\tilde{m}^-)^2 - (\tilde{m}^+)^2, \quad (26)$$

$$a^+ = \omega^2(1 - u^2) - (\tilde{m}^-)^2 + (\tilde{m}^+)^2, \quad (27)$$

$$b^2 = a^2 - 4(\tilde{m}^-)^2 \omega^2(1 - u^2) =$$

$$= (a^+)^2 - 4(\tilde{m}^+)^2 \omega^2(1 - u^2). \quad (28)$$

Without loss of generality, we may assume that

$$u = \cos \theta = 0, \quad (29)$$

because it is always possible to pass to the corresponding reference frame, which moves along the magnetic field and where equality (29) is obeyed; the configuration of the external field remains untouched at that. For condition (29), the following expressions for the momenta can be obtained:

$$p^- = \pm \sqrt{\frac{1}{4}\omega^2 - m^2 - (l^- + l^+)hm^2 + (l^- - l^+)^2 \frac{m^4 h^2}{\omega^2}}, \quad (30)$$

$$p^+ = -p^-. \quad (31)$$

Formula (30) is plotted in Fig. 2.

The values of momenta of the created pair of particles substantially depend on the frequency of the initial photon. In the case  $\omega = \omega_m = \tilde{m}^- + \tilde{m}^+$ , the electron and the positron are created accurately at the Landau levels, and  $p_z^\pm = 0$ . This gives rise, as is known, to the emergence of divergences in the process amplitude.

The probability of the process, according to the well-known rules of quantum electrodynamics [7], looks like

$$dW = |S_{fi}|^2 dN^- dN^+, \quad (32)$$

where  $S_{fi}$  is the process amplitude;

$$dN^\pm = \frac{d^2 p^\pm S}{(2\pi)^2} \quad (33)$$

are the intervals of final states of the electron and the positron, respectively; and  $S$  is the normalization area. According to general rules, the amplitude of the process is determined – in the first order of perturbation theory – by the formula

$$S_{fi} = -ie \int \bar{\Psi}^- A^\mu \gamma_\mu \Psi^+ d^4 x. \quad (34)$$

Considering the expressions for the wave functions of the electron and the positron (see Eq. (1)), as well as Eq. (3), and carrying out the necessary integration procedure, we obtain

$$S_{fi} = \frac{\Phi (2\pi)^3 \sqrt{2\pi}}{4S\sqrt{\omega V} \sqrt{E^- E^+ \tilde{m}^- \tilde{m}^+}} A_{fi} \delta^3(p^- + p^+ - k), \quad (35)$$

where

$$\begin{aligned} A_{fi} = & \sqrt{\tilde{m}^- - \mu^- m} \sqrt{\tilde{m}^+ + \mu^+ m} R_+ \sin(\theta) \cos(\alpha) \times \\ & \times J(l^+, l^-) + (-\mu^+) \sqrt{\tilde{m}^- - \mu^- m} \sqrt{\tilde{m}^+ - \mu^+ m} R_- T_+ \times \\ & \times J(l^+ - 1, l^-) + \mu^- \sqrt{\tilde{m}^- + \mu^- m} \sqrt{\tilde{m}^+ + \mu^+ m} R_- T_- \times \\ & \times J(l^+, l^- - 1) + (-\mu^- \mu^+) \sqrt{\tilde{m}^- + \mu^- m} \times \\ & \times \sqrt{\tilde{m}^+ - \mu^+ m} R_+ \sin(\theta) \cos(\alpha) J(l^+ - 1, l^- - 1), \end{aligned} \quad (36)$$

and  $\Phi$  is the phase factor. In formula (36), the following notations were introduced:

$$R_+ = \frac{1}{R_m^+ R_m^-} \left( (R_m^+)^2 p^- + (R_m^-)^2 p^+ \right), \quad (37)$$

$$R_- = \frac{1}{R_m^+ R_m^-} \left( (R_m^+)^2 (R_m^-)^2 - p^+ p^- \right), \quad (38)$$

$$T_{\pm} = \cos(\theta) \cos(\alpha) \pm i e^{i\beta} \sin(\alpha), \quad (39)$$

$$R_m^{\pm} = \sqrt{E^{\pm} - \mu \tilde{m}^{\pm}}, \quad (40)$$

$$\tilde{m}^{\pm} = m \sqrt{1 + 2l^{\pm} \hbar}. \quad (41)$$

The special functions  $J(l^+, l^-)$  arise owing to the integration of the amplitude over the coordinate  $x$ , along which the motion of a particle in the magnetic field is quantized. The explicit expressions for the special functions were taken from work [1]:

(i) for  $l^- > l^+$ ,

$$J(l^+, l^-) = e^{-\frac{\eta}{2}} \eta^{\frac{l^- + l^+}{2}} \sqrt{\frac{l^-!}{l^+! (l^- - l^+)!}} \times \\ \times F(-l^+, l^- - l^+ + 1, \eta); \quad (42)$$

(ii) for  $l^- < l^+$ ,

$$J(l^+, l^-) = e^{-\frac{\eta}{2}} \eta^{\frac{l^- + l^+}{2}} (-1)^{l^-} \sqrt{\frac{l^+!}{l^-! (l^+ - l^-)!}} \times \\ \times F(-l^-, l^+ - l^- + 1, \eta), \quad (43)$$

where  $F(n, m, \eta)$  is the degenerate hypergeometric function. Its argument looks like

$$\eta = \frac{\omega^2}{2m^2 \hbar}. \quad (44)$$

We seek for the amplitude in the form

$$S_{fi} = \frac{1}{S\sqrt{V}} A \delta(p_z^- + p_z^+ - k_z) \times \\ \times \delta(p_y^- + p_y^+ - k_y) \delta(E^- + E^+ - \omega), \quad (45)$$

where

$$A = \frac{\Phi (2\pi)^3 \sqrt{2\pi}}{4\sqrt{\omega} \sqrt{E^- E^+ \tilde{m}^- \tilde{m}^+}} A_{fi}. \quad (46)$$

According to formula (32), the probability within the time unit is

$$dW = \frac{|A|^2}{(2\pi)^7} \frac{S}{V} \delta(p_z^- + p_z^+ - k_z) \times \\ \times \delta(p_y^- + p_y^+ - k_y) \delta(E^- + E^+ - \omega) d^2 p^- d^2 p^+. \quad (47)$$

Here, we used the following properties of the  $\delta$ -function:

$$(\delta(E^- + E^+ - \omega))^2 = \frac{T}{2\pi} \delta(E^- + E^+ - \omega) \quad (48)$$

and

$$(\delta(p_i^- + p_i^+ - k_i))^2 = \frac{L_i}{2\pi} \delta(p_i^- + p_i^+ - k_i), \quad (49)$$

where the subscript  $i$  designates the corresponding coordinate ( $x$  or  $y$ ), and  $L_i$  is the normalization distance.

After integrating over  $d^2 p^+$  and  $dp_y^-$ , and taking into account that the integrand does not depend on  $p_y$ , we find that

$$dW = \frac{|A|^2}{(2\pi)^7} \frac{S p_y^-}{V} \delta(E^- + E^+ - \omega) dp_z^-. \quad (50)$$

As is seen, the factor  $S p_y^- / V$  appeared. To determine its form, consider the argument of the wave function (2). Identifying the quantity  $p_y^- / eH$  as a characteristic distance of quantization  $L_x$ , which in terms of classical physics has the meaning of the  $x$ -coordinate of the Larmor orbit center, we obtain the relation

$$\frac{S p_y^-}{V} = \frac{p_y^-}{L_x} = \hbar m^2. \quad (51)$$

Formula (50) is the general expression for the probability of the creation of an electron-positron pair with the participation of polarized particles and without any restriction put on the energies of those particles and the magnitude of the magnetic field.

In what follows, we assume that  $\hbar \ll 1$ . Moreover, we use the so-called ultra-quantum-mechanical, or LLL- (Low Landau Levels) approximation:

$$l^{\pm} \sim 1. \quad (52)$$

We should emphasize that the ultra-quantum-mechanical case demands that a strong enough magnetic field should be present. Really, even for an insignificant transverse energy of an electron  $E$  of about 10 keV, the

experimentally obtainable fields give  $l^\pm$  of the order of  $10^5$ . For conditions (52) to be fulfilled, one needs such a magnetic field which approaches the critical one by its order of magnitude. Such fields are observed near pulsars.

Let the condition

$$hl^\pm \ll 1 \tag{53}$$

be fulfilled. In this approximation, we can determine the approximate values of the momentum for the electron. Depending on the difference between the frequency  $\omega$  and the critical frequency  $\omega_m$ , expression (30) has the following forms in the ultra-quantum-mechanical approximation:

(i) in the case where the additive to the critical frequency is of the order of  $h^2$ , i.e.  $\omega = \omega_m + amh^2$ , where  $a$  is about unity, the expression for the momentum looks like

$$p_z^- = \pm m\sqrt{ah}; \tag{54}$$

(ii) if the condition  $\omega = \omega_m + amh$  is satisfied, the momentum will be equal to

$$p_z^- = \pm m\sqrt{ah}; \tag{55}$$

(iii) at last, if the photon frequency considerably exceeds the critical one for the selected Landau levels, i.e.  $\omega = \omega_m + am$ , we have

$$p_z^- = \pm \frac{m}{2} \sqrt{a(a+4)}. \tag{56}$$

Taking the aforesaid into consideration, we can expand Eq. (36) in a power series in  $h$ . Let us write down the expressions for the probability of the pair creation process at the Landau level  $l^-, l^+$  in the first approximation. After integrating over  $dp_z^-$ , we obtain (the superscripts denote the polarization of the electron and the positron, respectively)

$$W^{++} = \frac{1}{4} \frac{\alpha m^4 h^2}{\omega E |p_z^-|} Bl^- (1 - \xi_3), \tag{57}$$

$$W^{--} = \frac{1}{4} \frac{\alpha m^4 h^2}{\omega E |p_z^-|} Bl^+ (1 - \xi_3), \tag{58}$$

$$W^{-+} = \frac{1}{2} \frac{\alpha m^4 h}{\omega E |p_z^-|} B (1 + \xi_3), \tag{59}$$

where  $\alpha$  is the fine-structure constant,  $\xi_3$  the Stokes parameter,

$$\eta = \frac{\omega^2}{2m^2h}, \tag{60}$$

$$B = \frac{e^{-\eta} \eta^{l^- + l^+}}{l^- ! l^+ !}. \tag{61}$$

In the energetically unfavorable case  $\mu^- = 1$  and  $\mu^+ = -1$ , the form of expression depends on the difference between the values of the frequencies  $\omega$  and  $\omega_m$ :

(i) in the case  $\omega = \omega_m + amh^2$ ,

$$W^{+-} = \frac{\alpha m^3 h^5}{32\omega |p_z^-|} Bl^- l^+ \left( (1 + \xi_3) + \frac{16 (p_z^-)^2}{m^2 h^2} (1 - \xi_3) \right); \tag{62}$$

(ii) if  $\omega = \omega_m + amh$ ,

$$W^{+-} = \frac{\alpha m h^3}{2\omega} |p_z^-| Bl^- l^+ (1 - \xi_3); \tag{63}$$

(iii) and for  $\omega = \omega_m + am$ ,

$$W^{+-} = \frac{\alpha m^4 h^3}{4\omega E^3} |p_z^-| Bl^- l^+ \left( 2(1 - \xi_3) + \frac{(p_z^-)^2 (1 + \xi_3)}{2E^2} \right). \tag{64}$$

The total probability of the pair creation at arbitrary Landau levels is given by the formula

$$W_t^{\mu^+ \mu^-} = \sum_{l^+, l^-} W^{\mu^+ \mu^-}. \tag{65}$$

One can see that, depending on the polarization of created particles, the value of the process probability can be characterized by different orders of magnitude, which are determined by the degree of the small parameter  $h$ . Highest is the probability of the process of pair creation in an energetically profitable state, where  $\mu^+ = 1$  and  $\mu^- = -1$ . Provided that  $h = 0.1$ , it is by an order of magnitude larger than the others; therefore, the total probability  $W$  of the photon-induced creation of a pair with arbitrary values of particle spin projections is governed just by this summand:

$$W = \frac{1}{2} \frac{\alpha m^4 h}{\omega E |p_z^-|} B (1 + \xi_3). \tag{66}$$

Let us average this expression over the polarizations of the initial photon. The photon polarization is described by the quantum number  $\lambda$  which accepts two values

$\lambda = \pm 1$ . The change of the sign corresponds to the sign change of the Stokes parameters. Thus, we obtain

$$\langle W \rangle = \frac{1}{2} (W(+\xi_3) + W(-\xi_3)) = \frac{\alpha m^4 h}{2\omega E |p_z^-|} B. \quad (67)$$

One can see that the final formula is extremely simple. The plot of dependence (67) is depicted in Fig. 3. The parameter  $r$  looks like

$$r = \frac{\omega^2}{4m^2}. \quad (68)$$

It should be noted that our result obtained for small  $l^+, l^-$  is in accordance with those obtained in works [4,5]. As the numbers of Landau levels grow, a deviation of our results from those of work [4] is observed, which is associated with the violation of condition (52) accepted by us.

Now, let us elucidate the origin of the resonance lines which are observed in Fig. 3. According to Eq. (50), the general formula includes the multiplier  $\delta(E^- + E^+ - \omega) dp$ . To integrate over  $dp$ , we must pass to the  $\delta$ -function depending on momenta. As is known,  $\delta(f(p)) = \sum \delta(p - p_i) / |df/dp|$ , where  $p_i$  are the roots of the function  $f(p)$ . In our case,  $f(p) = E^- + E^+ - \omega$ . After the corresponding differentiation, the function  $\delta(E^- + E^+ - \omega)$  looks like

$$\delta(E^- + E^+ - \omega) = \sum \frac{E^- E^+}{|E^- p^+ + E^+ p^-|} \delta(p^- - p_i^-). \quad (69)$$

The denominator under the sum sign is zero, if  $p^- = p^+ = 0$ , i.e. if the pair becomes created accurately at the Landau level and with zero longitudinal momenta.

From the physical point of view, the presence of singularities in the probability of the pair creation is associated with the neglected radiation emission of soft photons, which always accompanies quantum-mechanical electrodynamic processes. This phenomenon is similar to the so-called ‘‘infra-red catastrophe’’ that occurs in the course of the bremsstrahlung process at the scattering by a Coulomb center [8].

The process of electron scattering by a Coulomb center followed by the emission of a photon is known to possess a similar divergence (Fig. 4, *a*). The cross-section of such a process  $d\sigma \sim 1/\omega'$ , where  $\omega'$  is the final-photon frequency. At  $\omega' \rightarrow 0$ , we have  $d\sigma \rightarrow \infty$ , which was named as the ‘‘infra-red catastrophe’’. An analogous

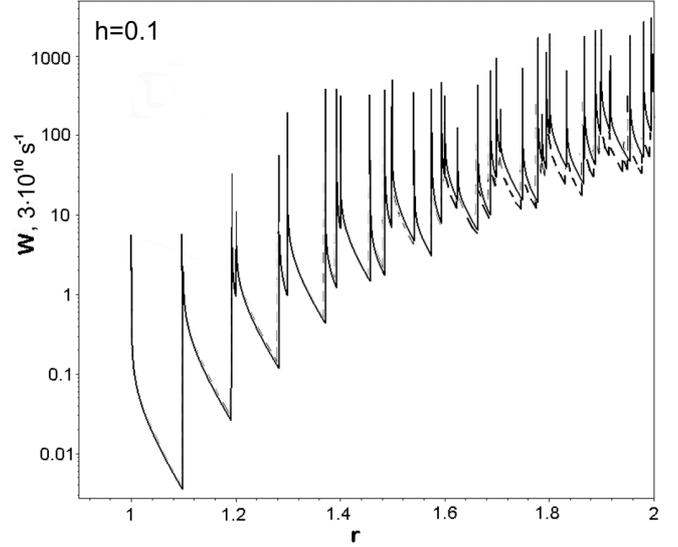


Fig. 3. Dependence of probability (67) summarized over spins and averaged over photon polarization on the parameter  $r = \omega^2/4m^2$ . The dashed curve corresponds to the results of work [4]

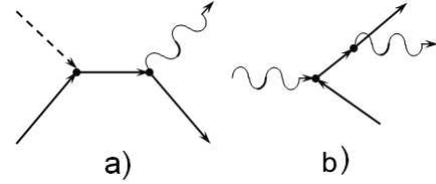


Fig. 4. Feynman diagrams of (a) bremsstrahlung and (b) photon-induced pair creation with emission of a final photon

situation also takes place at the photon-induced creation of the pair (Fig. 4, *b*). Divergences arise, because the perturbation theory becomes incorrect in the case of soft photon emission.

### 3. Polarization Effects

The circular polarization  $\xi_2$  and the total linear polarization  $\sqrt{\xi_1^2 + \xi_3^2}$  are known to be invariant with respect to the choice of a coordinate system, whereas the parameters  $\xi_1$  and  $\xi_3$  to be not. It is convenient to determine the parameters in that frame system, where their values are the same as those for magnetic bremsstrahlung. Let the  $x$ - and  $y$ -axes lie in a plane which is perpendicular to the wave vector  $\mathbf{k}$ ; the  $x$ -axis being oriented normally and the  $y$ -axis in parallel to the magnetic field  $\mathbf{H}$ . In this case, the parameter  $\xi_3$  characterizes the polarization of the photon along the  $x$ -direction, and, provided the condition  $\xi_3 = -1$ , the photon becomes completely polarized perpendicularly to the magnetic field. The parameter  $\xi_1$  determines the

polarization along those directions which are oriented at an angle of  $45^\circ$  to the magnetic field.

It should be emphasized that the probability of the photon-induced pair creation depends on the parameter  $\xi_3$  only, i.e. on the linear polarization of the photon. This can be understood on the basis of simple geometrical considerations. Owing to our choice of the reference frame, where the condition  $u = \cos\theta = 0$  is satisfied, the vector of the electric field of the photon lies in the same plane with the external field  $\mathbf{H}$ . Only the component of the electric field that is normal to  $\mathbf{H}$  and lies in the plane of the classical orbit of the electron plays an important role in the process. Therefore, only the linear polarization can be included into the expressions for pair creation probability. The probability cannot depend on  $\xi_1$ , because, owing to the symmetry of the problem, no direction that forms a  $45^\circ$ -angle with  $\mathbf{H}$  is singled out.

Expressions (57)–(64) readily demonstrate that there is a clear correlation between the polarization of the photon and the spin projections of the created pair. The processes with  $(\mu^+ = 1, \mu^- = 1)$  and  $(\mu^+ = -1, \mu^- = -1)$  differ from the most probable one by the sign of the initial photon polarization; the energetically unprofitable case is characterized by a complicated dependence on the photon polarization, which changes its shape with the variation of the ratio between the transverse and forward energies of the created particles.

In probability (59), we can select any value for the polarization of the initial photon, because, actually, it is determined by the experimental setup. In the case  $\xi_3 = -1$ , formula (59) brings about the zero probability of the process. Therefore, for a correct comparison of probability values to be made in such a case, it is necessary to determine the next correction with respect to  $h$  in Eq. (59). The corresponding calculation gave rise to the expression

$$W^{-+} = \frac{\alpha m^2 B}{2\omega} \frac{h}{g\varepsilon_0^5} (1 + \xi_3) \times \left[ \varepsilon_0^4 + \frac{1}{2} h (3\varepsilon_0^4 (l^- + l^+) - 2l^- l^+ \varepsilon_0^2) \right], \quad (70)$$

where the notations  $g = p_z^-/m$  and  $\varepsilon_0^2 = 1 + g^2$  were introduced. One can see that the character of the probability dependence on the polarization is preserved, and, in the case  $\xi_3 = -1$ , the processes of creation of pairs with spin projections  $(\mu^+ = 1, \mu^- = 1)$  and  $(\mu^+ = -1, \mu^- = -1)$  really dominate.

Thus, by selecting the polarization of the initial photon, one can influence the spin polarization of new

particles. As an example, we find the polarization degree for electrons. By definition, it looks like

$$P_{e^-} = \frac{W^+ - W^-}{W^+ + W^-}. \quad (71)$$

In our case,  $W^+ = W^{++} + W^{+-}$  and  $W^- = W^{-+} + W^{--}$ . Taking into account the fact that the probability  $W^{+-}$  is by one order of magnitude lower than the other ones, we can write

$$P_{e^-} = \frac{W^{++} - W^{-+} - W^{--}}{W^{++} + W^{-+} + W^{--}}. \quad (72)$$

Depending on the  $\xi_3$ -value, polarization (71) is different. If  $\xi_3 \neq -1$ , the contribution  $W^{-+}$  exceeds all other terms, so that, neglecting  $W^{++}$  and  $W^{--}$ , we obtain

$$P_{e^-} \approx -1. \quad (73)$$

Hence, if the condition  $\xi_3 \neq -1$  is fulfilled, the beam of created electrons will be almost completely polarized against the field direction.

Making use of Eqs. (57), (58), and (70), one can find a correction to  $P_{e^-}$ . Let us substitute the indicated expressions into Eq. (72), expand the quantity  $P_{e^-}$  in a power series in the small parameter  $h$ , and confine ourselves to the first order of the expansion. After simple calculations, we obtain

$$P_{e^-} = -1 + hl^- \frac{1 - \xi_3}{1 + \xi_3}. \quad (74)$$

But if  $\xi_3 \rightarrow -1$ , then the quantity  $W^{-+}$  in expression (72) can be neglected according to Eq. (70), and we obtain

$$P_- = \frac{l^- - l^+}{l^- + l^+}. \quad (75)$$

Therefore, if the condition  $\xi_3 \rightarrow -1$  is obeyed, the polarization degree of electron spins depends only on the numbers of Landau levels for the electron and the positron. In the specific case  $l^+ = l^- = l$ , the degree of particle polarization is equal to zero. In such a case, the quantity  $P_{e^-}$  is determined by terms that have higher order with respect to the smallness parameter  $h$ ; and the probability of the process with  $\mu^+ = -1$  and  $\mu^- = 1$  should be taken into account in the calculation procedure. As is clear from Eqs. (62)–(64), this probability has a simple form, namely

$$W^{+-} = \frac{\alpha m^2}{\omega} \frac{e^{-\eta} \eta^{l+l}}{(l!)^2} l^2 h^3 \frac{g}{\varepsilon_0^3}, \quad (76)$$

under the conditions specified.

From Eqs. (77)–(79) in the Appendix, we also obtain that  $W^{+-} = W^{-+}$  and  $W^{++} = W^{--}$ ; therefore, the numerator in Eq. (71) is equal to zero as well. Hence, if  $\xi_3 \rightarrow -1$  and  $l^+ = l^-$ , the polarization degree of electrons vanishes to within  $h^3$ .

Expressions (74) and (75) determine the polarization degree of electrons in the case of their creation at the fixed Landau levels  $l^+$  and  $l^-$ . In order to find it in the case of the photon-induced creation at arbitrary levels, the summation of probabilities over  $l^+$  and  $l^-$  has to be carried out in relevant expressions.

#### 4. Conclusions

In this work, the probability for the process of photon-induced creation of an electron-positron pair in a strong magnetic field has been found, taking into account the polarization of the initial photon and the values of spin projections of created particles. The total probability averaged over the photon polarization (Eq. (67)) has a sawtooth dependence (Fig. 3) and is in agreement with the results of other authors [4,5]. The derived expressions depend on the parameter of photon polarization through the well-known Stokes parameters and have a simple analytical form, which allowed us to carry out the analysis of the polarization and spin effects.

A neat correlation was observed between the polarization of the photon and the probability of the creation of particles with preset spin projections. The corresponding probabilities depend on the linear polarization  $\xi_3$  only; provided  $\xi_3 \neq -1$ , the complete polarization of particles' spins is observed. But if  $\xi_3 \rightarrow -1$ , the degree of spin polarization depends only on the numbers of Landau levels and is equal to zero in the specific case  $l^+ = l^-$ .

#### APPENDIX

For a more detailed analysis of polarization effects, it is useful to know the expressions for the probability with a higher accuracy with respect to the small parameter  $h$ . Below, the results of calculations are given:

$$\begin{aligned}
 W^{-+} = & \frac{\alpha m^2 B}{2\omega} \frac{h}{g\varepsilon_0^5} (1 + \xi_3) \left[ \varepsilon_0^4 + \frac{1}{2} h \times \right. \\
 & \times 3\varepsilon_0^4 l^- + l^+ - 2l^- l^+ \varepsilon_0^2 - \frac{1}{4} h^2 \varepsilon_0^4 l^2 + l'^2 + \\
 & + ll' l^- + l^+ 2\varepsilon_0^2 + 1 - 2l^- l^+ - 8 - 12g^2 - 5g^4 \left. + \right. \\
 & \left. + \frac{\alpha m^2 B}{4\omega} \frac{h^3 g}{\varepsilon_0^3} l^- + l^+ \right]^2, \quad (77)
 \end{aligned}$$

$$\begin{aligned}
 W^{++} = & \frac{\alpha m^2 B}{4\omega} \frac{h^2}{g\varepsilon_0^5} (1 - \xi_3) l^- \times \\
 & \times \left[ \varepsilon_0^4 + \frac{1}{2} h l^- k_1 + l^+ k_2 - 2l^- l^+ \varepsilon_0^2 \right], \quad (78)
 \end{aligned}$$

$$\begin{aligned}
 W^{--} = & \frac{\alpha m^2 B}{4\omega} \frac{h^2}{g\varepsilon_0^5} (1 - \xi_3) l^+ \times \\
 & \times \left[ \varepsilon_0^4 + \frac{1}{2} h l^- k_2 + l^+ k_1 - 2l^- l^+ \varepsilon_0^2 \right], \quad (79)
 \end{aligned}$$

where

$$k_1 = 3 - 2\varepsilon_0^3 + 4g^2 + g^4, \quad (80)$$

$$k_2 = 3 - 2\varepsilon_0^3 + 2g^2 - g^4. \quad (81)$$

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#### ПОЛЯРИЗАЦІЙНІ ЕФЕКТИ В ПРОЦЕСІ ФОТОНАРОДЖЕННЯ ЕЛЕКТРОН-ПОЗИТРОННОЇ ПАРИ В МАГНІТНОМУ ПОЛІ В УЛЬТРАКВАНТОВОМУ НАБЛИЖЕННІ

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Резюме

Розглянуто процес фотонародження електрон-позитронної пари в сильному однорідному магнітному полі з довільною поляризацією частинок. Поляризація фотона характеризується загальновідомими параметрами Стокса, а вирази для ймовірностей процесу мають просту аналітичну форму, що дозволило провести аналіз поляризаційних та спінових ефектів. Показано суттєвий вплив лінійної поляризації фотона на орієнтації спінів електронів та позитронів.