EXCITED ROTATIONAL-VIBRATIONAL STATES OF EVEN-EVEN NUCLEI WITH QUADRUPOLE AND OCTUPOLE DEFORMATIONS

SH. SHARIPOV, M.S. NADIRBEKOV, G.A. YULDASHEVA

Institute of Nuclear Physics, Academy of Sciences of Republic Uzbekistan
(Ulugbek, Tashkent 100214, Uzbekistan)

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The energies of excited rotational-vibrational states of deformable, axially symmetric even-even nuclei with quadrupole and octupole deformations have been studied. The variation of the nucleus deformation magnitude with the growth of the spin and the energy of a level has been taken into account. The ratios between the energies of excited levels with positive and negative parities and the energy of the first excited level with spin 2^+ in the basic band of the nuclei concerned have been calculated.

1. Introduction

Nuclei with quadrupole and octupole deformations are known to possess two rotational bands with opposite parities [1–6]. Between the levels belonging to bands with opposite parities, there occur strong dipole transitions which are caused by the presence of polarization-induced electric dipole moment in those nuclei [2–6]. The parameters of quadrupole, β_2, and octupole, β_3, deformations in these nuclei are of the same order of magnitude [1–7].

Atomic nuclei with octupole deformation possess two minima of the potential energy which correspond to opposite values of the parameter of octupole deformation β_3 and are characterized by the two-fold level degeneration which is eliminated due to a tunnel transition through the potential barrier separating the β_3- and $-\beta_3$-configurations [2, 3, 8]. In this case, nuclear states with positive parity are described by a symmetric $|+\rangle$-combination of wave functions in the $\pm\beta_3$-potential wells, while nuclear states with negative parity by an antisymmetric combination $|-\rangle$ of those functions [2, 3, 8]. Besides the main vibrational band 0^+, 2^+, 4^+, and so on, the band 1^-, 3^-, 5^-, ... with the spin projection $K = 0$ onto the nucleus symmetry axis was also observed for all nuclei with quadrupole and octupole deformations [2, 3, 8–11].

The properties of nuclei with octupole and quadrupole deformations have been widely studied in the framework of geometric, algebraic, and microscopic approximations (see review [7] and the references therein). In the algebraic model [12], the corresponding wave functions of the main and excited bands consist of components which, in addition to quadrupole bosons, include or do not include dipole bosons and are related with mononuclear or cluster components, respectively. In works [13, 14], the algebraic model, which included octupole bosons in addition to dipole ones, was considered, and the low-lying states of actinides with negative parity were described. In those models, the relative distance between the centers of mass of clusters under mass asymmetry is a key collective coordinate for the description of bands with alternating parity.

In the framework of the cluster model, the energies of levels belonging to the bands with alternating parity have been considered in work [15] for low and intermediate values of angular momenta and in the algebraic approximation. But, in this work, the maximal spin values for levels with alternative parity were equal to 12^+ for isotopes ^{232}U, ^{234}U, ^{236}U, and ^{238}U; 17^- for isotopes ^{220}Ra, ^{222}Ra, ^{224}Ra, and ^{226}Ra; and 20^+ for isotopes ^{220}Pu, ^{222}Pu, ^{224}Pu, and ^{226}Pu. At the same time, the experimental data are available for levels with alternative parity and higher spin values which cannot be described by the given model.

A large body of experimental data has been accumulated for nuclei with quadrupole and octupole deformations [2,3]. Hence, the problem of the description of energy level locations in the even and odd parity bands, which would take into account the shape modification of a nucleus at its excitation, including states with high spins, seems challenging.

The excited high-spin states of atomic nuclei arise owing to either the collective rotation or the arrangement of the angular momenta of individual nucleons along the nuclear symmetry axis [8, 16, 17]. The availability of high-spin states due to rotation can originate from the coupling between multipole deformations of the nuclear surface and its rotation [8]. In works [18–23], the models of forced rotation and varying inertia moment were used to study the energy levels of atomic nuclei with high spins. In those models, only rotational states with high spins were examined. But, as the rotation frequency increases, the nuclear deformation increases due to the action of centrifugal effects, similarly to the behavior of the vibrational-rotational interaction in molecules [8, 24, 25].

In this connection, the researches of high-spin states of atomic nuclei, which take into account – in the geometric approximation – the coupling between the rotational motion of a nucleus and nuclear surface vibrations characterized by different multipolarities seem topical.

2. Energy Levels and Wave Functions

Consider how the phenomenological nonadiabatic collective theory describes energy levels in even and odd bands [26–28]. Let an axially symmetric even-even nucleus with quadrupole and octupole deformations execute β_2- and β_3-vibrations and rotate around an axis perpendicular to the symmetry one.

The Schrödinger equation describing excited rotational-vibrational states can be written down as follows [27]:

$$-\sum_{\lambda=2,3}\frac{\hbar^2}{2B_\lambda}\frac{1}{\beta_\lambda^2}\frac{\partial}{\partial\beta_\lambda}\left(\beta_\lambda^2\frac{\partial\Psi_I^\pm(\beta_\lambda,\theta)}{\partial\beta_\lambda}\right)+$$

$$+\frac{\hbar^2 I(I+1)\Psi_I^\pm(\beta_\lambda,\theta)}{6(B_2\beta_2^2+B_3\beta_3^2)}+$$

$$+V(\beta_2,\beta_3)\Psi_I^\pm(\beta_\lambda,\theta)=E_I\Psi_I^\pm(\beta_\lambda,\theta), \qquad (1)$$

where B_λ are the mass parameters, I is the nuclear spin, $V(\beta_2,\beta_3)$ is the potential energy of quadrupole and octupole vibrations of the nuclear surface; and $\vartheta=\vartheta_1,\vartheta_2,\vartheta_3$ are the Euler angles that describe a spatial orientation of the nucleus. We seek the solutions of the Schrödinger equation (1) in the form

$$\Psi_I^\pm(\beta_2,\beta_3,\theta)=(\beta_2\beta_3)^{-1}\Phi_I^\pm(\beta_2,\beta_3)|IM0,\pm\rangle, \qquad (2)$$

where the function $|IM0,+\rangle$ describes the rotation of an axially symmetric even-even nucleus with the spin projection M onto the axis Z, and $K=0$. In the general case, the function $|IMK,\pm\rangle$ looks like [3, 10]

$$|IMK,\pm\rangle=\sqrt{\frac{2I+1}{16\pi^2(1+\delta_{K0})}}\times$$

$$\times\left[D_{MK}^I(\theta)\pm(-1)^I D_{M,-K}^I(\theta)\right], \qquad (3)$$

where δ_{K0} is the Kronecker symbol, and $D_{MK}^I(\theta)$ is the Wigner spherical function.

From Eq. (3), one can see that $|IM0,+\rangle\neq 0$ if $I=0,2,4,\ldots$, and $|IM0,-\rangle\neq 0$ for $I=1,3,5,\ldots$. Equation (1) gives rise to the equation for the wave functions $\Phi_I^\pm(\beta_2,\beta_3)$:

$$-\frac{\hbar^2}{2B_2}\frac{d^2\Phi_I^\pm(\beta_2,\beta_3)}{d\beta_2^2}-\frac{\hbar^2}{2B_3}\frac{d^2\Phi_I^\pm(\beta_2,\beta_3)}{d\beta_3^2}+$$

$$+\left[\frac{\hbar^2 I(I+1)}{6(B_2\beta_2^2+B_3\beta_3^2)}+V(\beta_2,\beta_3)-E_I\right]\Phi_I^\pm(\beta_2,\beta_3)=0. \qquad (4)$$

Here, it is convenient to pass to the polar coordinates σ and ε [2, 3]:

$$\beta_2=\sqrt{B/B_2}\sigma\cos\varepsilon,\quad \beta_3=\sqrt{B/B_3}\sigma\sin\varepsilon,$$

$$B=\frac{B_2+B_3}{2},\quad 0\le\sigma<\infty,\quad -\frac{\pi}{2}\le\varepsilon\le\frac{\pi}{2}. \qquad (5)$$

Then, Eq. (4) reads

$$-\frac{\hbar^2}{2B}\left[\frac{d^2}{d\sigma^2}+\frac{1}{\sigma}\frac{d}{d\sigma}+\frac{1}{\sigma^2}\frac{d^2}{\varepsilon^2}\right]\Phi_I^\pm(\sigma,\varepsilon)+$$

$$+\left[\frac{\hbar^2 I(I+1)}{6B\sigma^2}+V(\sigma,\varepsilon)-E_I^\pm\right]\Phi_I^\pm(\sigma,\varepsilon)=0. \qquad (6)$$

In nuclei with octupole deformation, there are two minima of the potential energy which are located at (β_2^0,β_3^0) and $(\beta_2^0,-\beta_3^0)$ [or, equivalently, at (σ_0,ε_0) and $(\sigma_0,-\varepsilon_0)$]. Expanding the potential energy V in a power series of a displacement of either of those equilibrium positions and neglecting the crossed terms, we obtain the following expression for V in the vicinity of the minima located at $(\sigma_0, \pm\varepsilon_0)$ [2–4]:

$$V(\sigma,\varepsilon) = V(\sigma) + \frac{C_\varepsilon}{2\sigma^2}(\varepsilon \mp \varepsilon_0)^2. \quad (7)$$

In this case, the variables in Eq. (6) can be separated:

$$\Phi_I^{\pm}(\sigma,\varepsilon) = F_I^{\pm}(\sigma)\chi_\nu(\varepsilon \mp \varepsilon_0), \quad (8)$$

where the functions $\chi_\nu(\varepsilon \pm \varepsilon_o)$ and $F_I^{\pm}(\sigma)$ satisfy the equations

$$\frac{d^2\chi_\nu(\varepsilon \mp \varepsilon_0)}{d\varepsilon^2} + \frac{2B}{\hbar^2}\left[\pm\varepsilon_\nu + \frac{C_\varepsilon}{2}(\varepsilon \mp \varepsilon_0)^2\right]\chi_\nu(\varepsilon \mp \varepsilon_0) = 0 \quad (9)$$

and

$$-\frac{\hbar^2}{2B}\left[\frac{d^2F_I^{\pm}(\sigma)}{d\sigma^2} + \frac{dF_I^{\pm}(\sigma)}{\sigma d\sigma}\right] +$$

$$\left[\frac{\hbar^2 I(I+1)}{6B\sigma} + V(\sigma) \mp \frac{\varepsilon_\nu}{\sigma^2} - E_I^{\pm}\right]F_I^{\pm}(\sigma) = 0, \quad (10)$$

respectively. The eigenvalues of Eq. (9) are

$$\pm\varepsilon_\nu = (\nu + \frac{2}{2})\hbar\omega_\varepsilon, \quad \omega_\varepsilon = \sqrt{C_\varepsilon/B}, \quad \nu = 0,1,2,.... \quad (11)$$

Note that the tunneling brings about the splitting of oscillator levels. The quantity $2\varepsilon_\nu$ is the splitting of the doubly degenerate ν-th level which occurs owing to the tunnel transition between nuclear forms with opposite values of the octupole deformation parameter [2,3].

For Eq. (10) to be solved, it is necessary to choose firstly the dependence $V(\sigma)$ for the potential energy. In the collective phenomenological theory, the choice of the potential energy is one of the most complicated problems. The expression for the potential energy is often selected so that it should reflect certain key features obtained from microscopic calculations and, at the same time, contain a small number of parameters; these parameters are determined from a comparison of the results of theoretical calculations with experimental data.

While solving Eq. (10), the potential $V(\sigma)$ can be presented in the form

$$V(\sigma) = \frac{C}{2}(\sigma - \sigma_0)^2 + \frac{\hbar^2}{8B\sigma^2}, \quad (12)$$

where the quantities C and σ_0 determine the elastic constant and the deformation parameter, respectively, of an even-even nucleus with quadrupole and octupole deformations at $I = 0$. Then, by introducing the function $\varphi_I^{\pm}(\sigma)$ which satisfies the boundary conditions $\varphi_I^{\pm}(\sigma \to 0) = 0$ and $\varphi_I^{\pm}(\sigma \to \infty) = 0$, we obtain

$$F_I^{\pm}(\sigma) = \frac{\varphi_I^{\pm}(\sigma)}{\sqrt{1\sigma}}. \quad (13)$$

In this case, Eq. (10) reads

$$\left\{-\frac{\hbar^2}{2B} + W_{I\nu}^{\pm} - E_{I\nu}^{\pm}\right\}\varphi_I^{\pm}(\sigma) = 0, \quad (14)$$

where

$$W_{I\nu}^{\pm} = \frac{C}{2}(\sigma - \sigma_o)^2 + \frac{\hbar^2 I(I+1)}{6B\sigma^2} \mp \frac{\varepsilon_\nu}{\sigma^2}. \quad (15)$$

Provided $I \neq 0$, a second term in the effective potential energy (15) differs from zero, which shifts the equilibrium position and changes the elastic constant. Expanding the function $W_{I\nu}^{\pm}(\sigma)$ into a power series in σ and confining the expansion to the linear and quadratic terms, expression (15) acquires the form

$$W_{I\nu}^{\pm}(\sigma) = W_{I\nu}^{\pm}(\sigma_{I\nu}) + \frac{C_{I\nu}^{\pm}}{2}(\sigma - \sigma_{I\nu})^2, \quad (16)$$

where

$$C_{I\nu}^{\pm} = C\left\{1 + \left(\frac{\mu}{p_{I\nu}^{\pm}}\right)^4 [I(I+1) \mp 3\varepsilon'_\nu]\right\}, \quad (17)$$

$$(p_{I\nu}^{\pm})^3(p_{I\nu}^{\pm} - 1) = \mu^4\left[\frac{I(I+1)}{3} \mp \varepsilon'_\nu\right], \quad (18)$$

$$W_{I\nu}^{\pm}(\sigma_{I\nu}) = \hbar\omega_0\left\{\left(\frac{p_{I\nu}^{\pm} - 1}{\sqrt{2}\mu}\right)^2 + \right.$$

$$\left. + \left(\frac{\mu}{p_{I\nu}^{\pm}}\right)^2\left[\frac{I(I+1)}{3} \mp \varepsilon'_\nu\right]\right\}, \quad (19)$$

$$p_{I\nu}^{\pm} = \frac{\sigma_{I\nu}^{\pm}}{\sigma_0} \geq 1, \quad \mu = \frac{1}{\sigma_0}\left[\frac{\hbar^2}{BC}\right]^{\frac{1}{4}},$$

$$\varepsilon'_\nu = \frac{2B}{\hbar^2}\varepsilon_\nu, \quad \omega_0 = \sqrt{\frac{C}{B}}. \quad (20)$$

Substituting expression (16) into Eq. (14), we find that

$$\left\{-\frac{\hbar^2}{2B}\frac{d^2}{d\sigma^2}+\frac{C^{\pm}_{I\nu}}{2}(\sigma-\sigma_0)^2+\right.$$

$$\left.+[E^{\pm}_{I\nu}-W^{\pm}_{I\nu}(\sigma_{I\nu})]\right\}\varphi^{\pm}_{I\nu}(\sigma)=0. \quad (21)$$

Let us introduce a quantity

$$\mu^{\pm}_{I\nu}=\mu\left[\frac{C}{C^{\pm}_{I\nu}}\right]^{\frac{1}{4}}=\mu\left\{1+\frac{\mu}{p^{\pm}_{I\nu}}\left[I(I+1)\mp 2\varepsilon'_{\nu}\right]\right\}^{\frac{1}{4}}, \quad (22)$$

and a new variable

$$\xi=\frac{p^{\pm}_{I\nu}(\sigma-\sigma_0)}{\mu^{\pm}_{I\nu}\sigma_{I\mu}},\quad -\frac{p^{\pm}_{I\nu}}{\mu^{\pm}_{I\nu}}\le\xi<\infty. \quad (23)$$

Let us assume that

$$\varphi^{\pm}_{I\nu}(\xi)=U^{\pm}_{I\nu}(\xi)exp\left\{-\frac{\xi^2}{2}\right\}. \quad (24)$$

Using expressions (21)–(23,) we obtain

$$\left\{\frac{d^2}{d\xi^2}-2\xi\frac{d}{d\xi}+2q^{\pm}_{\nu}\right\}U^{\pm}_{I\nu}(\xi)=0, \quad (25)$$

where

$$q^{\pm}_{\nu}=\frac{E^{\pm}_{I\nu}-W^{\pm}_{I\nu}(\sigma_{I\nu})}{\hbar\omega^{\pm}_{I\nu}}-\frac{1}{2},\qquad \omega^{\pm}_{I\nu}=\sqrt{\frac{C^{\pm}_{I\nu}}{B}}. \quad (26)$$

The function $U^{\pm}_{I\nu}(\xi)$ has to satisfy the following boundary conditions:

$$U^{\pm}_{I\nu}(\xi)\left(-\frac{p^{\pm}{}_{I\nu}}{\mu^{\pm}{}_{I\nu}}\right)_{\xi\to 0}=0,$$

$$U^{\pm}_{I\nu}(\xi)e^{-\frac{\xi^2}{2}}_{\xi\to\infty}=0. \quad (27)$$

The solutions of Eq. (25) can be selected in the form

$$U^{\pm}_{I\nu}(\xi)=N^{\pm}H^{\pm}_{q^{\pm}_{\nu}}(\xi), \quad (28)$$

where $N^{\pm}$ are the normalizing coefficients,

$$H^{\pm}_{q^{\pm}_{\nu}}(\xi)=\left[2\Gamma(-q^{\pm}_{\nu})\right]^{-1}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}(2\xi)^k\cdot\Gamma\left(\frac{k-q^{\pm}_{\nu}}{2}\right) \quad (29)$$

is the Hermite function of the first kind, and $\Gamma(x)$ is the gamma-function. The quantity $q^{\pm}$ in expression (29) is a root of the transcendental equation

$$H^{\pm}_{q^{\pm}_{\nu}}\left(-\frac{p^{\pm}{}_{I\nu}}{\mu^{\pm}{}_{I\nu}}\right)=0 \quad (30)$$

stemming from boundary condition (27). This equation has an infinite set of roots $q^{\pm}_{\nu}$ which are enumerated in the ascending order as follows: $q^{\pm}_0$, $q^{\pm}_1$, $q^{\pm}_2$, and so on [22]. The quantities $p^{\pm}_{I\nu}$ and $\mu^{\pm}_{I\nu}$ are determined from Eqs. (18) and (22) for every pair of parameters μ and ε'_{ν}. Here, $p^{\pm}_{I\nu}$ determines an increase of the equilibrium deformation σ_o of the ground state of an even-even nucleus with quadrupole and octupole deformations, which accompanies the nucleus transition into the $I\nu$-states, so that $\sigma^{\pm}_{I\nu}=p^{\pm}_{I\nu}\sigma_o$. The energies of excited rotational-vibrational $I\nu$-states are determined – according to formulas (17)–(19) and taking expression (26) into account – by the expression

$$E^{\pm}_{I\nu}=\hbar\omega_0\left\{\left(q^{\pm}_{\nu}+\frac{1}{2}\right)\sqrt{1+\left(\frac{\mu}{p^{\pm}_{I\nu}}\right)^4[I(I+1)\mp 3\varepsilon'_{\nu}]}+\right.$$

$$\left.+\left(\frac{\mu}{p^{\pm}_{I\nu}}\right)^2\left[\frac{I(I+1)}{6}\mp\varepsilon'_{\nu}\right]+\frac{1}{2}\left(\frac{p^{\pm}_{I\nu}-1}{\mu}\right)^2\right\}. \quad (31)$$

It is evident from this equation that the level energies depend on three parameters, namely, ω_0, μ, and ε'_{ν}. If the energies of excited states are measured in units of $\hbar\omega_0$ for a given even-even nucleus, there remain only two independent parameters, μ and ε'_{ν}:

$$\frac{E^{\pm}_{I\nu}}{\hbar\omega_0}=f^{\pm}_{I\nu}(\mu,\varepsilon'_{\nu}). \quad (32)$$

The energy of the ground state of an even-even nucleus, measured in $\hbar\omega_o$-units, is determined by the quantum numbers $(I,\nu,q^{\pm})=(0+,1,q^{\pm})$. Hence, the excitation energy for an even-even nucleus is equal to the difference

$$\Delta\varepsilon_{I\nu q^{\pm}}=f_{I\nu q^{\pm}}-f_{01q^{\pm}}. \quad (33)$$

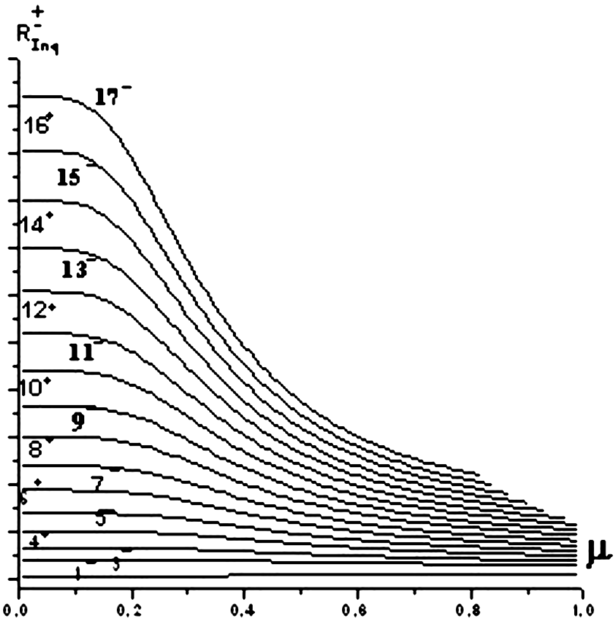
For simplicity, it is convenient to analyze the ratio between the excitation energy and the energy of the first excited level

$$R_{I\nu q^{\pm}}=\frac{\varepsilon_{I\nu q^{\pm}}}{\varepsilon_{21q^{\pm}}} \quad (34)$$

rather than the excitation energy (33) itself. It is evident that the ratios between the energies of excited levels (34) in axially symmetric even-even nuclei with quadrupole and octupole deformations are the functions of two parameters, μ and ε'_ν.

To illustrate the results obtained, the dependences of the ratio between the energies of excited levels with positive and negative parities and the energy of the first excited level in the main band with spin 2^+ on the parameter μ are depicted in the Figure for the fixed parameter $\varepsilon'_\nu = 2.04$. The Figure demonstrates that the ratios between the energies of levels with positive and negative parities diminish with increase in the parameter μ value, provided that the value of the parameter ε'_ν is fixed, which corresponds to the formation of equidistant levels similar to the levels in spherical nuclei.

The Table quotes experimental [29] and calculated values obtained for the ratio between the energy of the excited levels with positive and negative parities and the energy of the first excited level with spin 2^+, as well as the values for the parameters μ and ε'_ν, and for the roots $q_0^\pm$, $q_1^\pm$, and $q_2^\pm$ of the transcendental equation (30) for ^{150}Sm, ^{154}Dy, ^{162}Dy, ^{232}Th, ^{232}U, ^{236}U, and ^{238}U nuclei. The listed data make it evident that the root of transcendental Eq. (30) is not an integer number for ^{150}Sm and ^{154}Dy nuclei ($\mu > 1/3$), but it is an integer number for other, heavier nuclei ($\mu < 1/3$) [30]. For deformable nonaxial even-even nuclei with quadrupole deformation, the roots of the transcendental equation (30) were calculated earlier in works [26, 30]. For odd nuclei, the energies of the excited levels with positive and negative parities were calculated in work [31], but the roots of the transcendental equation (30) were supposed integer there.



Dependences of the ratio between the energy of excited levels with positive and negative parities and the energy of the first excited level of the main band with spin 2^+ on the parameter μ for a fixed value of the parameter $\varepsilon'_v = 2.04$

Experimental and theoretical values of the ratio between the energies of excited levels and the energy of the first excited level

Nuclei and their parameters	$q_\nu^\pm$	Spin	Energy		$q_\nu^\pm$	Spin	Energy	
			Theor.	Exper.			Theor.	Exper.
1	2	3	4	5	6	7	8	9
^{150}Sm	$q_0^+ = 1.3 \times 10^{-2}$	2^+	1	1	$q_0^- = 8.2 \times 10^{-3}$	3^-	2.58	3.21
μ =0.51	$q_0^+ = 5.7 \times 10^{-3}$	4^+	2.32	2.32	$q_0^- = 3.5 \times 10^{-3}$	5^-	3.78	4.07
$\varepsilon'_\nu = 1$	$q_0^+ = 2.3 \times 10^{-3}$	6^+	3.80	3.83	$q_0^- = 1.4 \times 10^{-3}$	7^-	5.18	5.28
[3]	$q_0^+ = 9.2 \times 10^{-4}$	8^+	5.39	5.50	$q_0^- = 5.6 \times 10^{-4}$	9^-	6.73	6.68
	$q_0^+ = 3.6 \times 10^{-4}$	10^+	7.07	7.29	$q_0^- = 2.2 \times 10^{-4}$	11^-	8.39	8.22
	$q_0^+ = 1.4 \times 10^{-4}$	12^+	8.83	9.04	$q_0^- = 9.0 \times 10^{-5}$	13^-	10.12	9.86
	$q_0^+ = 5.8 \times 10^{-5}$	14^+	10.64	11.00	$q_0^- = 3.7 \times 10^{-5}$	15^-	11.91	11.72
	$q_0^+ = 2.4 \times 10^{-5}$	16^+	12.49	12.89	$q_0^- = 1.5 \times 10^{-5}$	17^-	13.76	13.79
	$q_0^+ = 1.0 \times 10^{-4}$	18^+	14.39	14.76	$q_0^- = 6.5 \times 10^{-5}$	19^-	15.64	16.00
	$q_0^+ = 4.3 \times 10^{-6}$	20^+	16.32	16.74	$q_0^- = 2.9 \times 10^{-6}$	21^-	17.57	18.28
^{154}Dy	$q_0^+ = 1.1 \times 10^{-2}$	2^+	1	1	$q_1^+ = 1.02$	6^+	7.46	4.96
$\mu = 0.5$	$q_0^+ = 4.8 \times 10^{-3}$	4^+	2.41	2.23	$q_1^+ = 1.01$	8^+	9.34	6.47
$\varepsilon'_\nu = 1$	$q_0^+ = 1.9 \times 10^{-3}$	6^+	4.01	3.66	$q_1^+ = 1$	10^+	11.30	8.25
[26]	$q_0^+ = 7.7 \times 10^{-4}$	8^+	5.75	5.22	$q_0^- = 1.3 \times 10^{-2}$	1^-	1.80	4.24
	$q_0^+ = 3.0 \times 10^{-4}$	10^+	7.59	6.89	$q_0^- = 3.0 \times 10^{-3}$	5^-	3.99	4.62

Continue table

1	2	3	4	5	6	7	8	9
	$q_0^+ = 1.2 \times 10^{-4}$	12^+	9.51	8.65	$q_0^- = 1.2 \times 10^{-3}$	7^-	5.52	5.87
	$q_0^+ = 4.9 \times 10^{-5}$	14^+	11.49	10.4 9	$q_0^- = 4.8 \times 10^{-4}$	9^-	7.21	7.24
	$q_0^+ = 2.0 \times 10^{-5}$	16^+	13.53	12.4 7	$q_0^- = 6.9 \times 10^{-3}$	3^-	2.69	3.61
	$q_0^+ = 8.5 \times 10^{-6}$	18^+	15.61	14.5 4	$q_0^- = 1.9 \times 10^{-4}$	11^-	9.02	8.61
	$q_0^+ = 3.7 \times 10^{-6}$	20^+	17.73	16.7 0	$q_0^- = 7.6 \times 10^{-5}$	13^-	10.92	10.13
	$q_0^+ = 1.6 \times 10^{-6}$	22^+	19.89	18.9 7	$q_0^- = 3.1 \times 10^{-5}$	15^-	12.89	11.90
	$q_0^+ = 7.6 \times 10^{-7}$	24^+	22.08	21.3 9	$q_0^- = 1.3 \times 10^{-5}$	17^-	14.97	13.87
	$q_0^+ = 3.6 \times 10^{-7}$	26^+	24.30	23.9 8	$q_0^- = 5.5 \times 10^{-6}$	19^-	16.99	15.95
	$q_0^+ = 1.8 \times 10^{-7}$	28^+	26.54	26.0 6	$q_0^- = 2.4 \times 10^{-6}$	21^-	19.11	18.04
	$q_0^+ = 1.0 \times 10^{-7}$	30^+	28.80	28.8 8	$q_0^- = 1.1 \times 10^{-5}$	23^-	21.26	20.18
	$q_0^+ = 0$	32^+	31.09	31.2 0	$q_0^- = 5.2 \times 10^{-7}$	25^-	23.44	22.47
		34^+	33.39	33.8 2	$q_0^- = 2.5 \times 10^{-7}$	27^-	25.66	24.91
		36^+	35.70	36.4 9	$q_0^- = 1.3 \times 10^{-7}$	29^-	27.90	27.54
	$q_1^+ = 1.1$	0^+	2.41	1.97	$q_0^- = 1.0 \times 10^{-7}$	31^-	30.16	30.35
	$q_1^+ = 1.08$	2^+	3.93	2.71	$q_0^- = 0$	33^-	32.44	33.31
	$q_1^+ = 1.04$	4^+	5.66	3.74	$q_0^- = 0$	35^-	34.74	36.04
^{162}Dy	$q_0^+ = 0$	2^+	1	1		6^+	6.80	6.80
$\mu = 0.205$		4^+	3.29	3.29		8^+	11.40	11.42
$\varepsilon'_\nu = 9$		10^+	16.99	17.05		11^-	34.35	31.05
[26]		12^+	23.45	23.58	$q_1^+ = 1$	0^+	20.34	17.36
		14^+	30.70	30.90		2^+	21.47	18.02
		16^+	37.64	38.91		4^+	24.05	19.53
		18^+	47.21	47.56		6^+	27.95	21.91
	$q_0^- = 0$	1^-	16.61	15.82		8^+	33.02	24.62
		3^-	18.06	16.83		10^+	39.13	28.04
		5^-	20.63	18.82		12^+	46.12	32.15
		7^-	24.25	21.76		14^+	53.89	36.64
		9^-	28.85	26.05				
^{232}Th	$q_0^+ = 0$	2^+	1	1		7^-	23.71	21.11
$\mu = 0.21$		4^+	3.28	3.28		9^-	28.14	25.29
$\varepsilon'_\nu = 9$		6^+	6.74	6.74		11^-	33.41	30.33
[26]		8^+	11.26	11.27		13^-	39.44	36.12
		10^+	16.70	16.74		15^-	46.16	42.54
		12^+	22.95	23.01		17^-	53.50	49.49
		14^+	29.92	30.0		19^-	61.40	56.94
		16^+	37.51	37.61		21^-	69.80	64.85
		16^+	37.51	37.61		23^-	78.65	73.20
		18^+	45.65	45.78		25^-	87.93	81.98
		20^+	54.30	54.46		27^-	97.59	91.21
		22^+	63.39	63.62	$q_1^+ = 1$	0^+	18.09	14.78
		24^+	72.89	73.24		2^+	19.23	15.67
		26^+	82.76	83.29		4^+	21.82	17.65
		28^+	92.97	93.73		6^+	25.70	20.72
		30^+	103.50	104.5		8^+	30.71	24.73
	$q_0^- = 0$	1^-	16.32	14.46		10^+	36.68	29.69
		3^-	17.73	15.67	$q_1^- = 1$	1^-	35.52	21.15
		5^-	20.21	17.89		3^-	37.09	22.38
^{232}U	$q_0^+ = 0$	2^+	1	1		16^+	37.51	38.41
$\mu = 0.21$		4^+	3.28	3.33		18^+	45.65	46.89
$\varepsilon'_\nu = 9$		6^+	6.74	6.78		20^+	54.30	55.85
[26]		8^+	11.26	11.37	$q_1^+ = 1$	0^+	18.09	14.52
		10^+	16.70	16.93		2^+	19.23	15.43
		12^+	22.95	23.36		4^+	21.82	17.63
		14^+	29.92	30.55		6^+	25.70	20.70
		8^+	30.71	24.94		7^-	23.71	19.23
		10^+	36.68	30.14		9^-	28.14	23.76
	$q_0^- = 0$	1^-	16.32	11.83		11^-	33.41	29.33
		3^-	17.73	13.21		13^-	39.44	35.48

Continue table

1	2	3	4	5	6	7	8	9
		5^-	20.21	15.90				
^{236}U	$q_0^+=0$	2^+	1	1	$q_0^-=0$	1^-	13.34	15.21
$\mu=0.195$		4^+	3.30	3.30		3^-	14.90	16.47
$\varepsilon'_\nu=7$		6^+	6.85	6.85		5^-	17.61	18.75
[26]		8^+	11.54	11.54		7^-	21.45	22.15
		10^+	17.28	17.28		9^-	26.32	26.55
		12^+	23.98	23.98		11^-	32.18	31.96
		14^+	31.55	31.53		13^-	38.93	38.36
		16^+	39.90	39.81		15^-	46.51	45.62
		18^+	48.95	48.73		17^-	54.83	53.72
		20^+	58.65	58.19		19^-	63.84	62.48
		22^+	68.94	68.13		21^-	73.48	71.88
		24^+	79.75	78.52	$q_1^+=1$	0^+	23.48	20.34
		26^+	91.06	89.37		2^+	24.59	21.34
		28^+	102.83	100.62		4^+	27.15	23.16
		30^+	115.01	112.30				
^{238}U	$q_0^+=0$	2^+	1	1		1^-	14.97	15.13
$\mu=0.203$		4^+	3.30	3.30		3^-	16.45	16.28
$\varepsilon'_\nu=8$		6^+	6.81	6.84		5^-	19.07	18.40
[26]		8^+	11.44	11.53		7^-	22.77	21.50
		10^+	17.07	17.27		9^-	27.47	25.62
		12^+	23.60	23.97		11^-	33.09	30.70
		14^+	31.93	31.51		13^-	39.55	36.72
		16^+	38.98	39.81		15^-	46.78	43.67
		18^+	47.68	48.78		17^-	54.70	51.35
		20^+	56.96	58.30		19^-	63.24	59.87
		22^+	66.76	68.30		21^-	72.39	69.12
		24^+	77.04	78.70		23^-	82.18	78.99
		26^+	87.76	89.45	$q_1^+=1$	0^+	21.09	20.65
		28^+	98.89	100.57		2^+	22.22	21.54
		30^+	110.38	112.57		4^+	24.79	23.49

From the Table, it is also evident that the experimental and calculated values of the ratio between the energies of the excited levels with positive and negative parities are in good agreement with one another, especially for uranium isotopes, including their high-spin states.

Thus, it follows from the results of our researches reported above that the proposed variant of the model developed in works [2–4] satisfactorily describes the spectrum of energy levels with positive and negative parities in deformable even-even nuclei with quadrupole and octupole deformations.

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ОБЕРТАЛЬНО-КОЛИВАЛЬНІ ЗБУДЖЕНІ СТАНИ ПАРНО-ПАРНИХ ЯДЕР З КВАДРУПОЛЬНОЮ ТА ОКТУПОЛЬНОЮ ДЕФОРМАЦІЯМИ

Ш. Шаріпов, М.С. Надирбеков, Г.А. Юлдашева

Р е з ю м е

Досліджено енергії обертально-коливальних збуджених станів аксиально-симетричних парно-парних ядер з квадрупольною та октупольною деформаціями. Враховано зміну величини деформації ядра зі збільшенням спіну та енергії рівнів. Обчислено відношення енергії збуджених рівнів додатної та від'ємної парності до енергії першого збудженого рівня зі спіном 2^+ основної полоси вищевказаних ядер.