FREE FIELDS QUANTIZATION IN THE “KREIN” SPACE

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It has been shown that the presence of negative-norm states or negative-energy solutions (unphysical states) is indispensable for a fully covariant quantization (Krein space quantization) of the minimally coupled free scalar field in the de Sitter spacetime [1,2]. This method of quantization is extended here to free fields in the Minkowski spacetime. Contrary to the scalar field [3], some unphysical (negative-energy) states of spinor and vector fields have positive norms. This departure from the usual Krein space led to the defined “Krein” space. The presence of unphysical negative-frequency states plays the role of an automatic renormalization tool for the free quantum field theory (QFT).

1. Introduction

It has been shown that negative-norm states necessarily appear in a covariant quantization of the free minimally coupled scalar field in the de Sitter spacetime [1,2] which plays an important role in the inflationary model, as well as in the linear quantum gravity in the de Sitter spacetime [4, 5]. Negative-norm states were considered by Dirac in 1942 [6]. In 1950, Gupta [7] and Bleuler [8] applied this idea to the Lorentz covariance quantization of a massless vector field. The gauge theory and the Lagrangian with higher derivatives also lead to ghosts, states with negative norms. For a minimally coupled scalar field in the de Sitter spacetime [2], it has been proved mathematically that the use of the two sets of solutions (positive- and negative-frequency states) is unavoidable for the preservation of causality (locality) and covariance [2]. In this case, the ultraviolet and infrared divergences have been automatically eliminated [9].

We generalized this method to a scalar field in the Minkowski space [3]. In this approach, the auxiliary negative-frequency states have been utilized, the modes of which do not interact with the physical states or the real physical world. We obtained that the vacuum energy is automatically renormalized, and the normal ordering procedure is rendered useless [2]. Applying the method to various problems, we obtained that the presence of “unphysical"(negative-energy) states plays the role of an automatic renormalization tool for the theory [10–14].

In this paper, we present the quantization of free vector and spinor fields within this method. Their vacuum energies and the associated two-point functions are calculated. Contrary to the scalar field [3], some unphysical (negative-energy) states of spinor and vector fields have the positive norms. This departure from the usual Krein space led to the defined “Krein” space. Again, it is seen that the presence of unphysical states plays the role of an automatic renormalization tool for QFT.

2. Quantization of Free Scalar Fields in the Krein Space

We briefly recall the scalar field quantization in the Krein space. The scalar field satisfies the Klein–Gordon equation

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu + m^2)\phi(x) = 0,$$

$$\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1). \tag{1}$$

The two sets of solutions of (1) are given by

$$u_p(k, x) = \frac{e^{-ikx}}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}} = \frac{e^{-i(\omega_{\mathbf{k}}t-\mathbf{k}.\mathbf{x})}}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}},$$

$$u_n(k, x) = \frac{e^{ik.x}}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}} = \frac{e^{i(\omega_{\mathbf{k}}t-\mathbf{k}.\mathbf{x})}}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}},$$

where $\omega_{\mathbf{k}} = k^0 = \sqrt{\mathbf{k}.\mathbf{k} + m^2} \geq 0$. The modes u_p (u_n) represent positive (negative) norm states. The quantization operator of a free scalar field in the Krein space is [3]

$$\phi(x) = \frac{1}{\sqrt{2}}\ [\phi_p(x) + \phi_n(x)], \tag{2}$$

where

$$\phi_p(x) = \int d^3\mathbf{k}\ [a_{\mathbf{k}} u_p(k,x) + a^\dagger_{\mathbf{k}} u^*_p(k,x)],$$

$$\phi_n(x) = \int d^3\mathbf{k}\ [b_{\mathbf{k}} u_n(k,x) + b^\dagger_{\mathbf{k}} u^*_n(k,x)].$$

The two independent operators $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ are constrained to obey the commutation rules

$$[a_{\mathbf{k}}, a_{\mathbf{k}'}{}^\dagger] = +\delta^3(\mathbf{k}-\mathbf{k}'),$$

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}{}^\dagger] = -\delta^3(\mathbf{k}-\mathbf{k}'), \quad [a_{\mathbf{k}}, b_{\mathbf{k}'}{}^\dagger] = 0.$$

The vacuum state $|0\rangle$ is defined as a state that is destroyed by all annihilation operators,

$$a_{\mathbf{k}}|0\rangle = 0, \quad b_{\mathbf{k}}|0\rangle = 0, \quad \forall \mathbf{k}.$$

One-physical and unphysical particle states are defined by

$$a^\dagger_{\mathbf{k}}|0\rangle = |1_{\mathbf{k}}\rangle, \quad b^\dagger_{\mathbf{k}}|0\rangle = |\bar{1}_{\mathbf{k}}\rangle, \quad \forall \mathbf{k}.$$

Calculating the energy operator in terms of Fourier modes gives

$$H = \int d^3\mathbf{k}\ \omega_{\mathbf{k}}\ (a_{\mathbf{k}}{}^\dagger a_{\mathbf{k}} + b_{\mathbf{k}}{}^\dagger b_{\mathbf{k}} + a_{\mathbf{k}}{}^\dagger b_{\mathbf{k}}{}^\dagger + a_{\mathbf{k}} b_{\mathbf{k}}),$$

where the energy of the vacuum state is zero, and the imposition of the normal ordering prescription is not required.

The time-ordered propagator for a scalar field in the Krein space is [3, 16]

$$G_T(x,x') = -i\langle 0|T\phi(x)\phi(x')|0\rangle = \text{Re}\ G_F(x,x') =$$

$$= -\frac{1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}, \tag{3}$$

where $2\sigma_0 = \eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu) \geq 0$. This function is singular only on the light cone. However, it has been shown that the incorporation of quantum metric fluctuations removes the singularities of Green's functions on the light cone [15]. In a previous work, it has been established that the combination of QFT in the Krein space together with the consideration of quantum metric fluctuations results in QFT without any divergence [16]:

$$\langle G_T(x-x')\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}\exp(-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}) +$$

$$+\frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}, \tag{4}$$

where $\langle\ \rangle$ is the average over metric fluctuations. The quantity $\langle\sigma_1^2\rangle$ is related to the density of gravitons [15]. When $\sigma_0 = 0$, due to the metric quantum fluctuations $\langle\sigma_1^2\rangle \neq 0$, we have

$$\langle G_T(0)\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}} + \frac{m^2}{16\pi}.$$

It should be noted that the auxiliary negative-frequency states cannot propagate in the physical world, and they only play the role of an automatic renormalization tool for the theory. This method can be easily used for charged scalar fields.

3. Spinor Field in the "Krein" Space Quantization

A free spinor field satisfies the Dirac equation

$$(i\ \partial\!\!\!/ - m)\psi(x) = 0.$$

The two sets of solutions are [17, 18]

$$\mathcal{U}^s(k,x) = \sqrt{\frac{m}{(2\pi)^3\omega_{\mathbf{k}}}}\ u^s(\mathbf{k})e^{-ikx} \quad \text{(positive energy)}, \tag{5}$$

$$\mathcal{V}^s(k,x) = \sqrt{\frac{m}{(2\pi)^3\omega_{\mathbf{k}}}}\ v^s(\mathbf{k})e^{ikx} \quad \text{(negative energy)}, \tag{6}$$

with $s = 1, 2$. These solutions are not complex conjugate of each other, but they satisfy the relations [17, 18]

$$u^s(\mathbf{k}) = \gamma^5 v^s(\mathbf{k}), \quad v^s(\mathbf{k}) = \gamma^5 u^s(\mathbf{k}), \tag{7}$$

$$\bar{u}^s(\mathbf{k}) = \bar{v}^s(\mathbf{k})\gamma^5, \quad \bar{v}^s(\mathbf{k}) = \bar{u}^s(\mathbf{k})\gamma^5, \tag{8}$$

where $\bar{u} = u^\dagger\gamma^0$ is the Dirac adjoint. The time-ordered spinor propagator can be written in terms of the scalar counterpart

$$S_F(x-x') \equiv (i\not\partial + m)G_F(x,x'). \quad (9)$$

By using this equation, the time-ordered product of the spinor field in the new method of field quantization can be written as

$$-i\langle 0 \mid T\bar{\psi}(x)\psi(x') \mid 0\rangle =$$

$$= S_T(x-x') \equiv (i\not\partial + m)G_T(x,x'). \quad (10)$$

By using Eqs. (2), (3), and (9), the spinor field operator is defined by

$$\psi(x) = \frac{1}{\sqrt{2}}\ [\psi_p(x) + \psi_n(x)], \quad (11)$$

where

$$\psi_p(x) = \int d^3\mathbf{k} \sum_{s=1,2} [b_{\mathbf{k}s}\mathcal{U}^s(k,x) + d^\dagger_{\mathbf{k}s}\mathcal{V}^s(k,x)]$$

positive energy mode

$$\psi_n(x) = \int d^3\mathbf{k} \sum_{s=1,2} [a_{\mathbf{k}s}\mathcal{V}^s(k,x) + c^\dagger_{\mathbf{k}s}\mathcal{U}^s(k,x)]$$

negative energy mode.

Similarly to the standard QFT, one can define the nonzero (equal-time) anticommutation relation

$$\{\psi_\alpha(x), \psi^\dagger_\beta(x')\}_{x^0=x'^0} = \delta_{\alpha\beta}\ \delta^3(\mathbf{x}-\mathbf{x}'),$$

$$(\alpha,\beta = 1,2,3,4).$$

The creation and annihilation operators are also constrained to obey the anticommutation rules

$$\{b_{\mathbf{k}s}, b^\dagger_{\mathbf{k}'s'}\} = \{d_{\mathbf{k}s}, d^\dagger_{\mathbf{k}'s'}\} = \delta_{ss'}\ \delta^3(\mathbf{k}-\mathbf{k}'),$$

$$\{a_{\mathbf{k}s}, a^\dagger_{\mathbf{k}'s'}\} = \{c_{\mathbf{k}s}, c^\dagger_{\mathbf{k}'s'}\} = \delta_{ss'}\ \delta^3(\mathbf{k}-\mathbf{k}').$$

All other anticommutation relations are equal to zero. The vacuum state $\mid 0\rangle$ is defined as a state destroyed by all annihilation operators

$$b_{\mathbf{k}s} \mid 0\rangle = 0, \quad d_{\mathbf{k}s} \mid 0\rangle = 0 \quad \forall\mathbf{k} \in R^3,$$

$$a_{\mathbf{k}s} \mid 0\rangle = 0, \quad c_{\mathbf{k}s} \mid 0\rangle = 0 \quad \forall\mathbf{k} \in R^3.$$

It should be noted that the spinor field operators defined by this method are constructed by modes which have all positive norms. This departure from the usual Krein space quantization (which encompasses negative norms as well) led to the generalized "Krein" space quantization method. The physical states (consisting of particle and antiparticle states associated with the $b_{\mathbf{k}s}$ and $d_{\mathbf{k}s}$ operators) and unphysical states (consisting of two modes with positive norms but negative energy associated with $a_{\mathbf{k}s}$ and $c_{\mathbf{k}s}$ operators) are defined by

$$b^\dagger_{\mathbf{k}s} \mid 0\rangle = \mid 1^b_{\mathbf{k}s}\rangle = \mid \text{one particle state}\rangle,$$

$$c^\dagger_{\mathbf{k}s} \mid 0\rangle = \mid \bar{1}^c_{\mathbf{k}s}\rangle = \mid \text{one unphysical state}\rangle,$$

$$d^\dagger_{\mathbf{k}s} \mid 0\rangle = \mid 1^d_{\mathbf{k}s}\rangle = \mid \text{one antiparticle state}\rangle,$$

$$a^\dagger_{\mathbf{k}s} \mid 0\rangle = \mid \bar{1}^a_{\mathbf{k}s}\rangle = \mid \text{one conjugate unphysical state}\rangle.$$

The commutation relations together with the normalization of vacuum, $\langle 0 \mid 0\rangle = 1$, lead to positive norms for both physical and unphysical parts:

$$\langle 1^b_{\mathbf{k}s} \mid 1^d_{\mathbf{k}'s}\rangle = \delta_{bd}\ \delta^3(\mathbf{k}-\mathbf{k}'),$$

$$\langle \bar{1}^a_{\mathbf{k}s} \mid \bar{1}^c_{\mathbf{k}'s}\rangle = \delta_{ac}\ \delta^3(\mathbf{k}-\mathbf{k}').$$

The Hamiltonian operator of the spinor field in terms of Fourier modes is

$$H = \int d^3\mathbf{k}\ \omega_{\mathbf{k}} \sum_{s=1,2} [b^\dagger_{\mathbf{k}s}b_{\mathbf{k}s} + b^\dagger_{\mathbf{k}s}c^\dagger_{\mathbf{k}s} + c_{\mathbf{k}s}b_{\mathbf{k}s} +$$

$$+ c_{\mathbf{k}s}c^\dagger_{\mathbf{k}s} - d_{\mathbf{k}s}d^\dagger_{\mathbf{k}s} - d_{\mathbf{k}s}a_{\mathbf{k}s} - a^\dagger_{\mathbf{k}s}d^\dagger_{\mathbf{k}s} - a^\dagger_{\mathbf{k}s}a_{\mathbf{k}s}].$$

It is immediately seen that the energy of the vacuum state is automatically zero, $\langle 0 \mid H \mid 0\rangle = 0$, and the imposition of the normal ordering prescription is not required.

The time-ordered propagator of the spinor field in the "Krein" space is defined by

$$S_T(x,x') = (i\not\partial + m)\frac{1}{2}[G^p_F(x,x') +$$

$$+ G^{p*}_F(x,x')] = (i\not\partial + m)G_T(x,x') =$$

$$= \frac{1}{2}\left(S^p_F(x,x') + \gamma^5\gamma^0[S^p_F(x,x')]^\dagger\gamma^0\gamma^5\right). \qquad (12)$$

For the new propagator in the momentum space, we obtain [17, 19]

$$\tilde{S}_T(k) = \tilde{S}^p_F(k) - \gamma^0[\tilde{S}^p_F(k)]^\dagger\gamma^0.$$

Inserting (2) into (12) gives the time-ordered propagator of the spinor field in a generalized "Krein" space

$$S_T(x,x') = \frac{1}{8\pi}\, i\gamma^\mu(x_\mu - x'_\mu)\times$$

$$\times\left\{\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}e^{-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}}\left[\frac{\sigma_0}{\langle\sigma_1^2\rangle} + m^2\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}\right]+\right.$$

$$\left.+\frac{m}{2\sqrt{2}}\theta(\sigma_0)\left[\sqrt{2m^2\sigma_0}J_0(\sqrt{2m^2\sigma_0}) - 2J_1(\sqrt{2m^2\sigma_0})\right]\right\}+$$

$$+\frac{m}{8\pi}\left[-\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}e^{-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}} + m^2\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}\right]$$

which is free of any divergence.

4. Quantization of Free Massless Vector Fields

Let us briefly recall the Gupta–Bleuler quantization for the electromagnetic field. The electromagnetic field $A_\mu(x)$ defined on the Minkowski spacetime satisfies the Maxwell equation [17, 18]

$$\partial^2 A_\mu(x) - \partial_\mu(\partial^\nu A_\nu(x)) = 0 \qquad \text{with} \qquad \partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$$

and $\quad x = (t, \mathbf{x})$.

Now the four-vector potential $A_\mu(x)$ is defined up to a gauge transformation

$$A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu\Lambda(x).$$

This gauge symmetry couples the components of the vector potential and reduces the effective degrees of freedom from 4 to 2. It is left free to quantize only these two independent (physical) components of the field (Coulomb gauge). Since the vector potential is described covariantly by a four-vector, we assure to manifest covariance using the Lorentz gauge characterized by the condition

$$\partial_\mu A^\mu(x) = 0.$$

We now analyze this indecomposable structure in terms of the (vector) plane waves with components

$$\phi^\lambda_\mu(x) = \epsilon^\lambda_\mu(k)\frac{e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_{\mathbf{k}}t)}}{\sqrt{2\omega_{\mathbf{k}}}} = \epsilon^\lambda_\mu(k)u_p(k,x)$$

with $\quad k_0 = \omega_{\mathbf{k}} = \sqrt{|\mathbf{k}|^2}$,

where the polarization vectors are

$$\epsilon^0(k) = (\epsilon^0_\mu(k)) = \begin{pmatrix} 1 \\ \mathbf{0}\end{pmatrix}, \qquad \epsilon^i(k) = \begin{pmatrix} 0 \\ \boldsymbol{\epsilon}_i\end{pmatrix},$$

with $\quad \mathbf{k}\cdot\boldsymbol{\epsilon}_i(k) = 0, \quad \boldsymbol{\epsilon}_i(k)\cdot\boldsymbol{\epsilon}_j(k) = \delta_{ij} \quad \text{for} \quad i = 1, 2,$

$$\epsilon^3(k) = \begin{pmatrix} 0 \\ \mathbf{k}/\omega_{\mathbf{k}}\end{pmatrix}.$$

These vectors obey the identity

$$\epsilon^\lambda_\mu(k)\,\epsilon^{\lambda'}_\nu(k)\,\eta^{\mu\nu} = \eta^{\lambda\lambda'}.$$

As usual, the polarization vectors $\epsilon^i(k)$ with i=1,2 are the physical transverse polarizations. The plane waves are normalized (in the Bohr integral sense) to

$$||\phi^\lambda||^2 = +1 \quad \text{for} \quad \lambda = 1,2,3 \quad \text{and} \quad ||\phi^0||^2 = -1.$$

It is important to note that the negative norm is due to the polarization vector $\epsilon^3(k)$, but their solution has positive energy.

In the Feynman gauge, the time-ordered vector propagator can be written in terms of the scalar counterpart

$$D_(\mu\nu)^F(x - x') \equiv -\eta_{\mu\nu}G_F(x,x'). \qquad (13)$$

By using this equation, the time-ordered product of the spinor field in the new method of field quantization can be written as

$$-i\langle 0 \mid TA_\mu(x)A_\nu(x') \mid 0\rangle =$$

$$= S^T_{\mu\nu}(x - x') \equiv -\eta_{\mu\nu}G_T(x,x'). \qquad (14)$$

By using Eqs. (2), (3), and (9), the vector field operator is defined by

$$A_\mu(x) = \frac{1}{\sqrt{2}}[A^p_\mu(x) + A^n_\mu(x)], \qquad (15)$$

where

$$A^p_\mu(x)=\int d^3\mathbf{k}\sum_{\lambda=0}^{3}\epsilon^\lambda_\mu(\mathbf{k})\ [a^\lambda_\mathbf{k}u_p(k,x)+a^{\lambda\dagger}_\mathbf{k}u_n(k,x)]$$

(positive energy mode),

$$A^n_\mu(x)=\int d^3\mathbf{k}\sum_{\lambda=0}^{3}\epsilon^\lambda_\mu(\mathbf{k})\ [b^{\lambda\dagger}_\mathbf{k}u_p(k,x)+b^{\lambda}_\mathbf{k}u_n(k,x)]$$

(negative energy mode).
By using the nonzero (equal-time) commutation relation

$$[A_\mu(x),\pi_\nu(x')]_{x^0=x'^0}=i\eta_{\mu\nu}\delta^3(\mathbf{x}-\mathbf{x}'),$$

the nonzero commutation relations for the creation and annihilation operators read

$$[a^\lambda_\mathbf{k},a^{\lambda'\dagger}_{\mathbf{k}'}]=-\eta^{\lambda\lambda'}\delta^3(\mathbf{k}-\mathbf{k}'),$$

$$[b^\lambda_\mathbf{k},b^{\lambda'\dagger}_{\mathbf{k}'}]=+\eta^{\lambda\lambda'}\delta^3(\mathbf{k}-\mathbf{k}'). \quad (16)$$

In the Gupta–Bleuler quantization, the unphysical states with positive and negative norms are introduced in order to preserve the Lorentz invariance. The negative-norm states appear due to the sign of the Minkowski metric. However, in the Krein space quantization, the appearance of additional negative-norm states owes itself to the negative-frequency solutions. It could be easily seen that, of the four added unphysical (negative energy) states, only one has positive norm. Vacuum is defined by

$$a^\lambda_\mathbf{k}|0\rangle=0,\quad b^\lambda_\mathbf{k}|0\rangle=0,\quad \forall\mathbf{k}.$$

By the standard choice of the polarization vectors with respect to a fixed but arbitrary frame, a one-physical-particle state is obtained by the action of $a^{\lambda\dagger}_\mathbf{k}$ on the vacuum state only for $\lambda=1,2$. We have

$$|1^\lambda_{\mathbf{k},a}\rangle\propto a^{\lambda\dagger}_\mathbf{k}|0\rangle,\quad \lambda=1,2.$$

These are related to the transverse polarizations of a photon. For the one-unphysical-particle states,

$$|\bar{1}^\lambda_{\mathbf{k},a}\rangle\propto a^{\lambda\dagger}_\mathbf{k}|0\rangle,\quad \lambda=0,3,$$

$$|\bar{1}^\lambda_{\mathbf{k},b}\rangle\propto b^{\lambda\dagger}_\mathbf{k}|0\rangle,\quad \lambda=0,1,2,3.$$

The norms of these states follow from (16) in the standard manner. Of the unphysical states, only $|\bar{1}^3_{\mathbf{k},a}\rangle$ and $|\bar{1}^0_{\mathbf{k},b}\rangle$ have positive norm, but they have no transverse polarizations. The Hamiltonian of the vector field is given by

$$H=\frac{1}{2}\int d^3\mathbf{x}(\pi^\mu\pi_\mu+\nabla A^\mu.\nabla A_\mu),$$

yielding

$$H=\int d^3\mathbf{k}\ \omega_\mathbf{k}\sum_{\lambda=0}^{3}(-\eta_{\lambda\lambda})\times$$

$$\times(a^{\lambda\dagger}_\mathbf{k}a^\lambda_\mathbf{k}+b^{\lambda\dagger}_\mathbf{k}b^\lambda_\mathbf{k}+a^{\lambda\dagger}_\mathbf{k}b^{\lambda\dagger}_\mathbf{k}+a^\lambda_\mathbf{k}b^\lambda_\mathbf{k}).$$

Therefore, $\langle 0|H|0\rangle=0$ without resort to the normal ordering procedure. We recall again that the Fock space of states, on which these new field operators act, has indefinite metric. This is, of course, due to the presence of auxiliary unphysical states which do not contribute to the observable quantities.

5. Conclusion

In quantum field theory, to eliminate the divergences that appear in the physical quantities, the normal ordering (renormalization) procedure has been adopted for the free (interacting) fields. However, the divergences seem to disappear, once the requirement of the positivity of norm and energy is relaxed. The addition of the new unphysical states, thus, leads us to the Krein space quantization. In this paper, the quantization of free boson and spinor fields is reformulated in the "Krein" space. Once again it is found that the theory is automatically renormalized. In the forthcoming papers, the interaction quantum fields will be studied within the framework of this method [20].

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КВАНТУВАННЯ ВІЛЬНИХ ПОЛІВ
У “ПРОСТОРІ КРЕЙНА”

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Р е з ю м е

Показано, що наявність станів з від'ємною нормою (або розв'язків з від'ємною енергією) є обов'язковим для повністю коваріантного квантування (квантування у просторі Крейна) вільного скалярного поля з мінімальним зв'язком у просторі–часі де Сіттера. Цей метод поширено для вільних полів у просторі–часі Мінковського. Всупереч випадку скалярного поля, деякі нефізичні (з від'ємною енергією) стани спінорного та векторного полів мають позитивну норму. Ця різниця із випадком звичайного простору Крейна веде до визначеного “простору Крейна”. Наявність нефізичних станів з від'ємною частотою відіграє роль автоматичного знаряддя ренормалізації в теорії вільного квантового поля.