ELASTIC AND INELASTIC SCATTERING FOR ^{12}C–^{12}C REACTIONS

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The double folding potential proposed by Wilson and the M3Y potential are used for the calculation of elastic and inelastic differential cross-sections. These calculations are performed for the ^{12}C–^{12}C reactions at the energies $E_{\rm lab} = 1016$, 1440, and 2400 MeV with regard for the Pauli correlation effect. A satisfactory agreement with the experimental data is obtained. The agreement is better, as the energy is increased.

1. Introduction

Experimental data on the elastic and inelastic scatterings for heavy ions are available and stimulate many theoretical calculations. The cross-sections of elastic and inelastic scatterings for the ^{12}C–^{12}C system were measured [1, 2] at $E_{\rm lab} = 360$, 1016, 1440, and 2400 MeV. The diffraction theory of excitations of collective states of nuclei by scattered nucleons was first considered in [3–5], where the adiabatic approximation for the amplitude of a process was used. The cross-sections of elastic and inelastic scatterings were calculated with the use of the Glauber model in [6]. These calculations cover a wide range of the projectile energy ($E/A_p \approx$ 30–350 MeV), the mass number of colliding nuclei ($4 \leq A_p \leq 4$, $12 \leq A_T \leq 208$), and the multipolarities of the excited states. The only input of the model is the densities and transition densities of the nuclei and the elementary nucleon-nucleon scattering amplitude. Within the context of diffractive collision theory [7], the elastic and inelastic scatterings of high-energy particles by permanently deformed, axially symmetric nuclei were investigated. In those calculations, the nuclear matter density and the optical potential are expanded into multipoles of the order $L = 0, 2, 4, \ldots$. In the high-energy approach, Lukyanov obtained [8] analytical expressions for the elastic and inelastic scattering amplitudes, where all the orders in the deformation parameters were included. In these calculations, the phenomenological optical potential was generalized to include the Coulomb and nuclear interactions caused by the dynamical deformation of the surface of a nucleus. The results of calculations of inelastic cross-sections for the scattering of ^{17}O ions on different nuclei at about tens of MeV/nucleon or higher are compared with experimental data, and the important role of the Coulomb excitation was established. A semimicroscopic double folding nucleus-nucleus optical potential was used to consider the inelastic scattering with the excitation of collective nuclear states by using the adiabatic approach and the elastic scattering amplitude in the high-energy approximation [9]. The analytical expression for the inelastic scattering amplitude was obtained keeping the first-order terms in the deformation parameter of a potential.

In the present work, the double folding potential proposed by Wilson is used to calculate the cross-sections of the elastic and inelastic scatterings for ^{12}C–^{12}C reactions at the energies equal to 1016, 1440, and 2400 MeV. The adiabatic approach and the elastic scattering amplitude in the high-energy approximation are used to calculate the inelastic scattering cross-section.

The formalism is presented in Section II. Section III includes the results and discussion. The conclusion is given in Section IV.

2. Formalism

2.1. Optical potential

The nucleus–nucleus optical potential derived by Wilson has the form [10]

$$V(r) = A_P A_T \int d^3 r_T \rho_T(r_T) \times$$

$$\times \int d^3 y \rho_P(r + y + r_T) t(e, y)[1 - C(y)], \quad (1)$$

where $A_i (i = P, T)$ are the mass numbers of the projectile and the target, ρ_i are the ground-state single particle nuclear densities for the colliding nuclei, $t(e, y)$ is the constituent – averaged energy – dependent two-body transition amplitude [11],

$$t(e, y) = -(e/m)^{1/2} \sigma(e)[\alpha(e) + i][2\pi B(e)]^{-3/2} \times$$

$$\times \exp[-y^2/2B(e)], \quad (2)$$

where e is the nucleon energy in the two-body center-of-mass frame, y is the two-nucleon separation distance, m is the nucleon mass, $\sigma(e)$ is the average nucleon-nucleon (NN) total cross-section, $\alpha(e)$ is the average of the ratio of the real to the imaginary parts of the NN forward scattering amplitude, $B(e)$ is the slope parameter of the NN elastic scattering differential cross-section, and $C(y)$ is the Pauli correlation function in the Fermi gas model given by

$$C(y) \approx \frac{1}{4} \exp(-k_F^2 y^2/10), \quad \text{and} \quad k_F = 1.36 \text{ fm}^{-1}. \quad (3)$$

The six-dimensional integral (1) is calculated using the momentum space method developed by Greiner [12]. If the Fourier transform of a function $f(\bar{x})$ is denoted by $f(\bar{k})$, the folded potential is given by

$$V(r) = (2\pi)^{-3} \int d^3 k \exp(-i\bar{k}r) \rho_p(+\bar{k}) \bar{\rho}_T(-k) \bar{t}'(e, \bar{k}). \quad (4)$$

Here,

$$t'(e, y) = t(e, y)[1 - C(y)],$$

i.e. the Fourier-transformed integrand reduces to a product of the Fourier transforms of the two densities and the transition nucleon-nucleon scattering amplitude.

In Eq. (1), $t'(e, y)$ can be replaced by any effective two-body interaction potential. In the present work, we also used the standard M3Y potential

$$V(r) = \left(7999.0 \frac{\exp(-4r)}{4r} - 2134.25 \frac{\exp(-2.5r)}{2.5r}\right) +$$

$$+ J(E)\delta(r). \quad (5)$$

The first term represents the direct part, and the second term does the exchange part of the interaction potential. The exchange part can be written to good approximation in the form

$$J(E) = -276(1 - 0.005 \frac{E}{A}), \quad (6)$$

where E is the energy of the incident particle in the center-of-mass system and A is the mass number of the projectile.

2.2. Elastic scattering differential cross-section

The elastic scattering amplitude considering the Coulomb effect is given by [13]

$$F(q) = f_{pc}(q) + ik \int_0^\infty db b J_0(qb) e^{i\phi_{pc}} (1 - e^{i\phi_N + i\delta\phi_{uc}}), \quad (7)$$

where

$$f_{pc}(q) = -\frac{2k\eta}{q^2} e^{-2i\eta \ln(q/2k) + 2i\sigma_0},$$

$\sigma_0 = \arg\Gamma(1 + i\eta)$ is the Coulomb phase,

$$\delta\phi_{uc} = \phi_{uc} - \phi_{pc},$$

$$\phi_{uc} = \begin{cases} 2\eta\{\ln(kR_c + \ln(1 + \sqrt{1 + \frac{b^2}{R_c^2}} - \\ -\frac{1}{3}\sqrt{1 + \frac{b^2}{R_c^2}} \left(4 - \frac{b^2}{R_c^2}\right), \quad b \le R_c, \\ 2\eta \ln(kb), \quad b > R_c, \end{cases}$$

$$\phi_{pc} = 2\eta ln(kb),$$

$$\eta = Z_1 Z_2 e^2/\hbar v,$$

and the nuclear eikonal phase

$$\phi_N = -\frac{i}{\hbar v} \int_{-\infty}^{\infty} U(\sqrt{b^2 + z^2}) dz. \quad (8)$$

2.3. Inelastic scattering differential cross-section

In the case of the excitation of a low-lying rotational or vibrational state |IM⟩ of even-even nuclei having the ground state spin and its projection |00⟩, the amplitude of the process is [8]

$$f_{\mathrm{IM}}(q) = \langle IM|f(q,\{\alpha_{\lambda\mu}\})|00\rangle, \tag{9}$$

where $|\mathrm{IM}\rangle = \sqrt{\frac{2I+1}{8\pi^2}}D^I_{M0}(\theta_i)$, $\alpha_{2\mu} = \beta_2 D^{(2)*}_{\mu 0}(\theta_i)$, β is the deformation parameter, θ_i is the rotational angles aroung intrinsic axes, $q = 2k\sin(\theta/2)$ is the transfer momentum, k is the relative momentum, and θ is the angle of scattering. Here, $f(q,\{\alpha_{\lambda\mu}\})$ is the amplitude of the elastic scattering on a system with "frozen" coordinates of collective motion $\{\alpha_{\lambda\mu}\}$. In order to introduce the dependence on the internal collective variables $\alpha_{\lambda\mu}$, we make a respective change of the spatial coordinates [9]

$$r \to r + \delta r, \quad \delta r = -r\sum_{\lambda\mu}\alpha_{\lambda\mu}Y_{\lambda\mu}(\theta,\phi). \tag{10}$$

Then, by expanding the potential in δr, we obtain the generalized optical potential consisting of two terms, the spherically symmetric and deformed one:

$$U^N(r,\{\alpha_{\lambda\mu}\}) = U^N(r) + U^N_{\mathrm{int}}(r,\{\alpha_{\lambda\mu}\}), \tag{11}$$

where the transition potential (its nuclear part) is as follows:

$$U^N_{\mathrm{int}} = -r\frac{d}{dr}U(r)\sum_{\mu}\alpha_{2\mu}Y_{2\mu}(\theta,\phi). \tag{12}$$

The quadrupole part of the generalized Coulomb potential $U^C\{r(\alpha_{\lambda\mu})\}$ looks as

$$U^C_{\mathrm{int}} = \frac{3}{5}U_B\left[\left(\frac{r}{R_C}\right)^2\Theta(R-r)+\right.$$

$$\left.+\left(\frac{R_C}{r}\right)^3\Theta(r-R)\right]\sum_{\mu}\alpha_{2\mu}Y_{2\mu}(\theta,\phi), \tag{13}$$

where

$$R_c = r_c(A_1^{1/3} + A_2^{1/3})$$

and

$$U_B = Z_1Z_2e^2/R_C.$$

Then we use the expression for the high-energy amplitude of scattering

$$f(q) = i\frac{k}{2\pi}\int b\,db\,d\phi\,e^{iqb\cos\phi}[1-e^{i\Phi}]. \tag{14}$$

Here, the integration is performed over the impact parameter b and the azimuthal angle ϕ, and the eikonal phase is determined by the nucleus-nucleus potential

$$\Phi = -\frac{1}{\hbar v}\int_{-\infty}^{\infty}U(r+\delta r)dz, \quad r = \sqrt{b^2+z^2}, \tag{15}$$

where v is the relative velocity of the colliding nuclei. Substituting the total potential having the central and transition terms, we can write

$$\Phi = \Phi_0(b) + \Phi_{\mathrm{int}}(b,\{\alpha_{\lambda\mu}\},\phi), \tag{16}$$

$$\Phi_{\mathrm{int}} = \beta_2\sum_{\mu=0,\pm2}G_\mu(b)D^{(2)*}_{\mu 0}(\theta_i)e^{i\mu\phi}, \tag{17}$$

$$G_\mu(b) = -\frac{2}{\hbar v}\int_0^\infty dz Y_{2\mu}(\arccos(z,r),0)\left[-r\frac{dU(r)}{dr}+\right.$$

$$\left.+\frac{3}{5}U_B[\left(\frac{r}{R_C}\right)^2\Theta(R-r)+\left(\frac{R_C}{r}\right)^3\Theta(r-R)\right]. \tag{18}$$

Substituting (16) in (14) and (7) and expanding the exponential function $\exp(I\phi_{\mathrm{int}})$, we retain only a term of the first order in β_2. The inelastic scattering amplitude $f_{\lambda\mu}(q)$ and the differential cross-section are as follows:

$$f_{20}(q) = \frac{k}{\sqrt{5}}\beta_2\int_0^\infty bdbJ_0(qb)G_0(b)e^{i\Phi_0(b)}, \tag{19}$$

$$f_{22}(q) = \frac{-k}{\sqrt{5}}\beta_2\int_0^\infty bdbJ_2(qb)G_2(b)e^{i\Phi_0(b)}, \tag{20}$$

$$\frac{d\sigma_{\mathrm{in}}}{d\Omega} = |f_{20}(q)|^2 + 2|f_{22}(q)|^2. \tag{21}$$

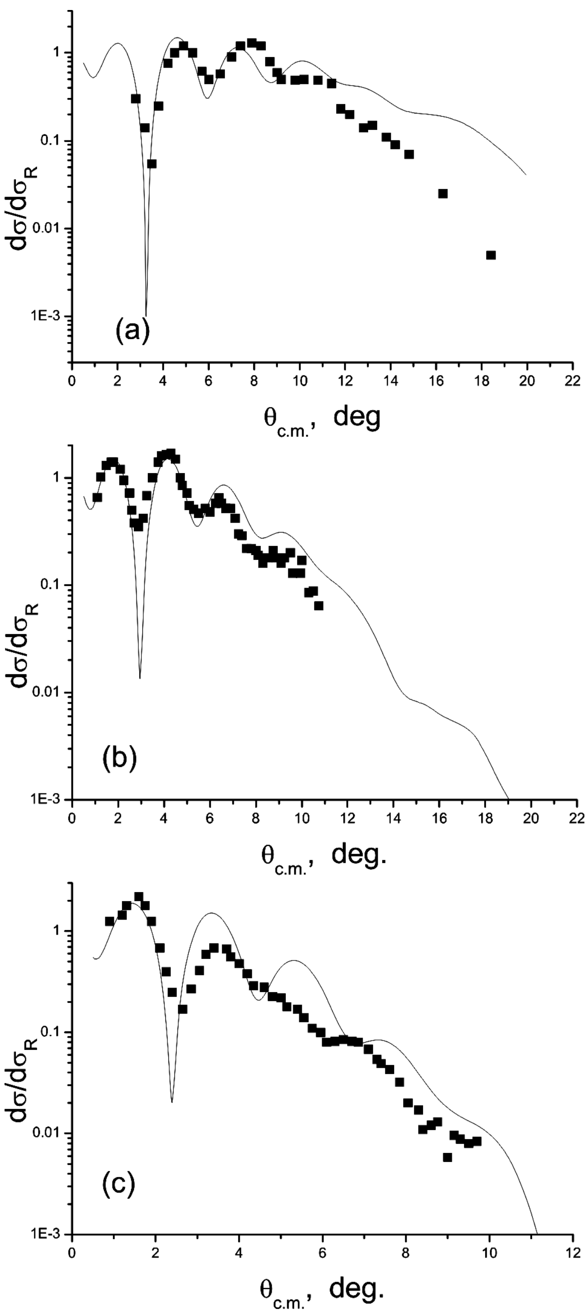

Fig. 1. Elastic scattering differential cross-sections for the ^{12}C–^{12}C reaction at energies of (*a*) 1016, (*b*) 1440, and (*c*) 2400 MeV. Dots are the experimental data [1]

3. Results and Discussion

The elastic scattering differential cross-section has been calculated for the interactions of ^{12}C–^{12}C at energies of 1016, 1440, and 2400 MeV. The density used for ^{12}C is taken from the modified Fermi model, and the Pauli correlation effect has been considered in our calculations. Figure 1 shows the elastic scattering differential cross-section for the ^{12}C–^{12}C reaction at energies of (*a*) 1016 MeV, (*b*) 1440 MeV, and (*c*) 2400 MeV. One can see from Fig. 1,*a* that the theoretical calculations agree with the experimental data up to $\theta = 10°$. At larger scattering angles, the theoretical calculations give higher values than the experimental data. From Fig. 1,*b*, one can see that the theoretical calculation give satisfactory agreement with the experimental data up to $\theta = 6°$. But it gives larger values than the experimental data for larger scattering angles. One can see from Fig. 1,*c* that the results of calculations agree satisfactorily with the experimental data at small scattering angles and give larger values at large scattering angles.

Figure 2 shows the inelastic scattering differential cross-section for the ^{12}C–^{12}C reaction at energies of (*a*) 1016 MeV, (*b*) 1440 MeV, and (*c*) 2400 MeV. In our calculations, the deformation parameter β_2^n for the nuclear potential is a free parameter and is considered to be 0.8, 0.6, and 0.5 for the ^{12}C–^{12}C reaction at energies of 1016, 1440, and 2400 MeV, respectively. The deformation parameter β_2^c for the Coulomb potential is taken from [14] using the known reduced electric transition probabilities $B(E2\uparrow)$. In our calculation, $B(E2\uparrow)$ is considered to be 42 e^2 fm^4 [15] for ^{12}C nucleus.

Thus, the results of theoretical calculations presented in Fig. 2 give a satisfactory agreement with the experimental data.

One can see from Fig. 1,*a* that the theoretical calculations with the use of the Wilson potential give larger values than the experimental data at large scattering angles. To obtain a good agreement at large scattering angles, we used the effective M3Y interaction for the ^{12}C–^{12}C system at an energy of 1016 MeV. Figure 3 shows the elastic scattering differential cross-section for the ^{12}C–^{12}C system at an energy of 1016 MeV for the M3Y effective interaction. One can see that the agreement between theoretical calculations and experimental data is improved at large scattering angles. In these calculations, we used $V(r) = N_R\ V_R(r) + N_{\rm Im}V_{\rm Im}(r)$, where N_R and $N_{\rm Im}$ are the normalization constants. They are taken to be $N_R = 0.6$ and $N_{\rm Im} = 1$ for the ^{12}C–^{12}C system at 1016 MeV. Figure 4 shows the inelastic scattering differential cross-section for the ^{12}C–^{12}C system at an energy of 1016 MeV. The deformation parameter β_2^n is chosen to be 0.7 to obtain a good fit with the experimental data. It is seen from Figs. 2 and 4 that the M3Y interaction improves the agreement with smaller deformation parameter.

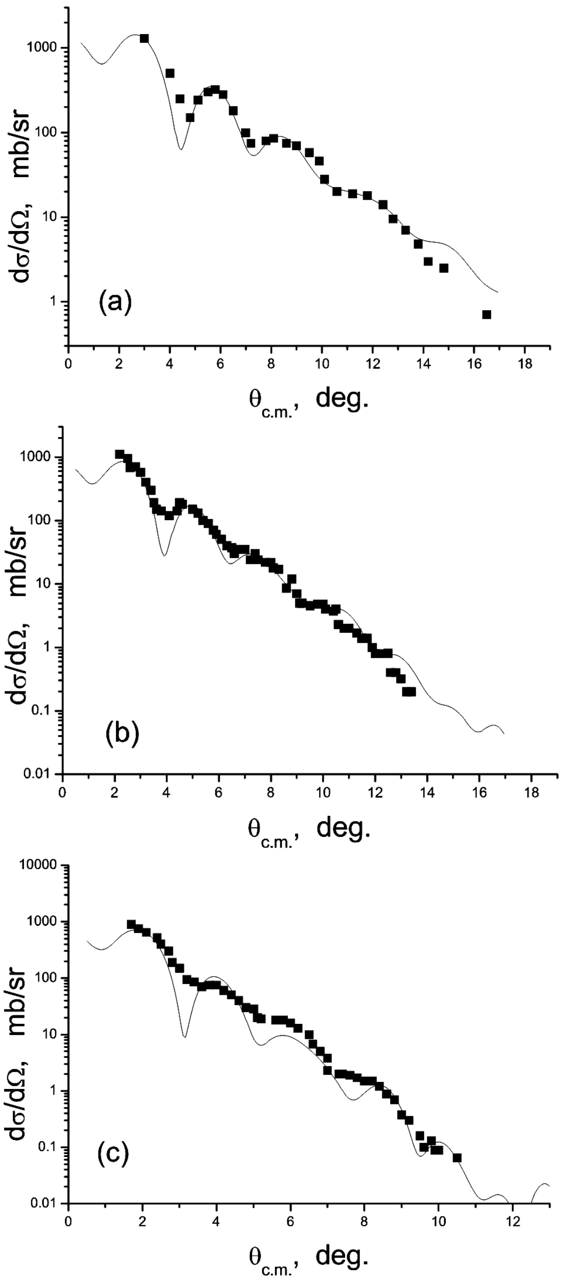

Fig. 2. Inelastic scattering differential cross-sections for the ^{12}C–^{12}C reaction at energies of (*a*) 1016, (*b*) 1440, and (*c*) 2400 MeV. Dots is the experimental data [6]

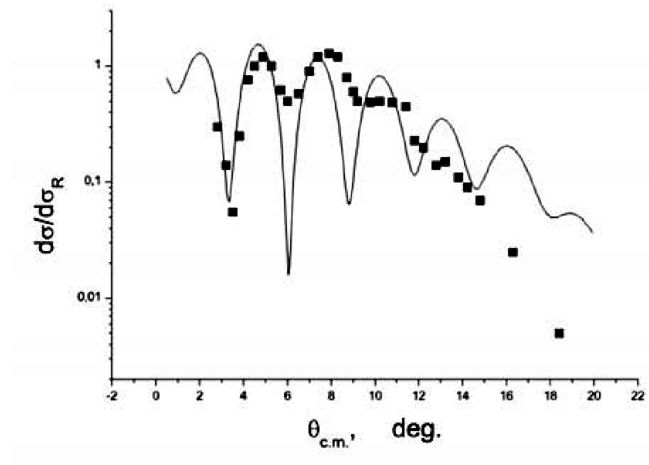

Fig. 3. Elastic scattering differential cross-section for the ^{12}C–^{12}C reaction at 1016 MeV using the M3Y interaction

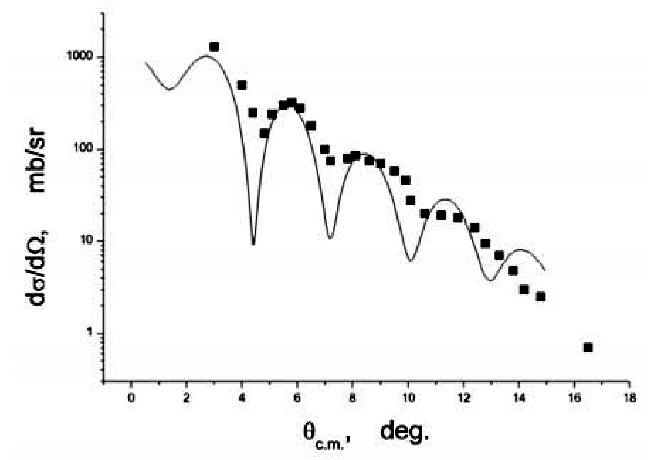

Fig. 4. Inelastic scattering differential cross-section for the ^{12}C–^{12}C reaction at 1016 MeV using the M3Y effective interaction

4. Conclusion

Here, we have calculated the elastic and inelastic scattering differential cross-sections for the ^{12}C–^{12}C reaction at energies of 1016, 1440, and 2400 MeV. For the elastic scattering, one can see that the theoretical calculations do not agree with the experimental data at large scattering angles but give a satisfactory agreement at small scattering angles. The calculations for the inelastic scattering agree with the experimental data, but the deformation parameter for the nuclear potential is large as compared with that used in [6] which was 0.45 for the ^{12}C–^{12}C inelastic scattering at 1440 and 2400 MeV.

Thus, the M3Y effective interaction improves the agreement for the elastic scattering differential cross-section at large scattering angles. In addition, it leads to a smaller deformation parameter in the calculations of the inelastic scattering differential cross-section. Hence, one may conclude that, within the present calculations, one can improve the agreement with the experimental data by changing the effective interaction and/or adjusting its parameters.

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ПРУЖНЕ ТА НЕПРУЖНЕ ^{12}C–^{12}C РОЗСІЯННЯ

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Р е з ю м е

Потенціал подвійної згортки, запропонований Вільсоном, і M3Y-потенціал використано для розрахунку диференціальних перерізів пружного та непружного ^{12}C–^{12}C розсіяння для енергій $E_{\rm lab}$ = 1016, 1440 і 2400 МеВ з врахуванням кореляційного ефекту Паулі. Отримано задовільну узгодженість з експериментальними даними, яка стає кращою з зростанням енергії.