ION DRAG FORCE ACTING ON AN ABSORBING BODY IN HIGHLY COLLISIONAL PLASMAS[1]

S.A. KHRAPAK

Max-Planck-Institut für extraterrestrische Physik
(D-85741 Garching, Germany; e-mail: skhrapak@mpe.mpg.de)

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The force acting on a small absorbing body embedded in a highly collisional plasma with drifting ions is calculated using the linear response formalism. It is shown that the ion absorption introduces new physical effects leading to a drastic reduction of the force. The physical reasons for this reduction are explained. The importance of this result is briefly discussed, mostly in the context of complex (dusty) plasmas research, but it can be relevant to many other situations, ranging from astrophysics, thunderclouds, dust in fusion devices, colloidal suspensions, biological systems, etc.

1. Introduction

The ion drag force arising due to momentum transfer from the ions drifting relative to highly charged grains is known to play an important role in complex (dusty) plasmas. Many studies were recently focused on this important topic (see, e.g., [1]). However, no comprehensive model describing all possible situations of interest has yet been constructed. Rather, there exist several approaches which can be used under certain conditions.

The focus of this paper is the effect of ion-neutral collisions on the ion drag force. In the collisional regime, available numerical simulations have demonstrated a *decrease* of the ion drag force compared to that in the collisionless situation, and even *negative values* were reported (i.e., ion drag force was directed oppositely to the ion flow) [2, 3]. On the contrary, an analytical model [4, 5] developed recently predicts an *increase* of the ion drag force with the ion collisionality. The effects associated with the plasma collection on grains were, however, neglected in these theoretical studies [4, 5].

This motivated us to perform a detailed analysis of the highly collisional limit, where the effects of plasma absorption on the grain surface can be easily accounted for, using the linear response formalism. It is demonstrated that the plasma absorption and related effects play an important role and lead to a considerable reduction of the ion drag force. The magnitude of this reduction is proportional to the value of the ion flux absorbed by a grain, and if an asymptotic expression for the ion flux in the continuum limit is used, the force can indeed assume *negative* values. The physical interpretation of this result is given.

2. Model

The problem is formulated as follows: We consider a small individual stationary grain of charge Q immersed in a highly collisional plasma. Quasineutral bulk plasma conditions are assumed, where the ions exhibit the subthermal drift while the electron component can be considered as stationary (ambipolar plasma regime). The neutral component is stationary as well. There are no plasma sources and sinks in the vicinity of a grain (except for the grain surface). Physically, this corresponds to the situation where the characteristic ionization/recombination length is considerably larger than the characteristic size of the plasma perturbation by the grain, i.e., compensation of plasma losses to the grain occurs very far from the grain. This is also relevant to those numerical simulations, in which plasma losses to the grain are compensated by the plasma injection from the boundary of a computation cell.

Within these assumptions, the collisional ion component is described by the continuity and momentum equations in the hydrodynamic approximation as

$$\nabla(n_i\mathbf{v}_i) = -J_i\delta(\mathbf{R}), \quad (1)$$

$$(\mathbf{v}_i\nabla)\mathbf{v}_i = -\frac{e}{m_i}\nabla\phi - \frac{\nabla n_i}{n_i}v_{T_i}^2 - \nu\mathbf{v}_i + \frac{\mathbf{f}}{m_i}, \quad (2)$$

where n_i, $\mathbf{v}_i$ and m_i are the ion density, velocity, and mass, J_i is the ion flux to a pointlike grain placed at the origin, $\mathbf{R} = 0$, ϕ is the electric potential, $v_{T_i} = \sqrt{T_i/m_i}$ is the ion thermal velocity, ν is the (constant) momentum transfer frequency in ion-neutral collisions,

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and $\mathbf{f}$ is an external force responsible for the ion drift. In the equilibrium, we have $\mathbf{u} = \mathbf{f}/m_i\nu$, where $\mathbf{u}$ is the ion drift velocity. In the considered case, the ion drift is caused by a weak ambipolar electric field, but the results derived below are independent of the nature of the drift-generating term. The electron density n_e satisfies the Boltzmann relation

$$n_e \simeq n_0 \exp(e\phi/T_e), \quad (3)$$

where n_0 is the unperturbed plasma density and T_e is the electron temperature. Expressions (1)–(3) are supplemented by the Poisson equation

$$\Delta\phi = -4\pi e(n_i - n_e) - 4\pi Q\delta(\mathbf{R}). \quad (4)$$

3. Results and Discussion

We assume $n_i = n_0 + \delta n_i$, $n_e = n_0 + \delta n_e$, $\mathbf{v}_i = \mathbf{u} + \delta\mathbf{v}_i$, $\phi = \delta\phi$, all the perturbations are proportional to $\exp(i\mathbf{kR})$, and linearize Eqs. (1)–(4). The inverse Fourier transformation then yields the following expression for the potential distribution around a test grain:

$$\phi(\mathbf{R}) = \frac{4\pi Q}{(2\pi)^3}\int \frac{\exp(i\mathbf{kR})d\mathbf{k}}{\chi_1(\mathbf{ku},k)} + \frac{4\pi e}{(2\pi)^3}\int \frac{\exp(i\mathbf{kR})d\mathbf{k}}{\chi_2(\mathbf{ku},k)}, \quad (5)$$

where

$$\chi_1(\mathbf{ku},k) = k^2 + k_{\mathrm{De}}^2 + k_{\mathrm{D}i}^2\left[1 + \frac{\mathbf{ku}(i\nu - \mathbf{ku})}{k^2 v_{T_i}^2}\right]^{-1}, \quad (6)$$

and

$$\chi_2(\mathbf{ku},k) = -i\frac{k^2 v_{T_i}^2(k^2 + k_{\mathrm{D}}^2)}{J_i(i\nu - \mathbf{ku})} -$$

$$-i\frac{\mathbf{ku}(i\nu - \mathbf{ku})(k^2 + k_{\mathrm{De}}^2)}{J_i(i\nu - \mathbf{ku})}. \quad (7)$$

Here, $k_{\mathrm{D}i(e)} = \lambda_{\mathrm{D}i(e)}^{-1}$ is the inverse ion (electron) Debye radius, $\lambda_{\mathrm{D}i(e)} = \sqrt{T_{i(e)}/4\pi e^2 n_0}$, and $k_{\mathrm{D}} = \sqrt{k_{\mathrm{De}}^2 + k_{\mathrm{D}i}^2}$ is the inverse linearized Debye radius. The first term in Eq. (5) is the usual expression for the potential around a pointlike non-absorbing grain in the limit of high collisionality [4]. The second term arises due to ion absorption [see Eq. (1)]. The electron absorption can similarly be accounted for; however, this would yield corrections to the force on the order of $(\ell_i/\ell_e)(v_{T_i}/v_{T_e}) \ll 1$, where $\ell_{i(e)}$ is the ion (electron) mean free path. Therefore, the electron absorption can be neglected for our purposes.

The force associated with the plasma anisotropy induced by the ion flow is $F = -Q(d\phi/dz)|_{z=0}$, where the flow is directed along the z axis. Using Eq. (5), we get

$$F = \frac{Q^2}{\pi}\int_0^\infty dk \int_{-1}^{1} \mu d\mu k^3 \mathrm{Im}\left[\chi_1^{-1}(ku\mu,k)\right] +$$

$$+\frac{eQ}{\pi}\int_0^\infty dk \int_{-1}^{1} \mu d\mu k^3 \mathrm{Im}\left[\chi_2^{-1}(ku\mu,k)\right], \quad (8)$$

where $\mu = \cos\theta$ and θ is the angle between the vectors $\mathbf{k}$ and $\mathbf{u}$. To proceed further with the calculation of the force, let us consider the limit of vanishing ion flow, $k\ell_i \gg u/v_{T_i} \equiv M_T$. The hydrodynamic approach used to derive the expressions for χ_1 and χ_2 requires $k\ell_i \ll 1$. In this regime, the dominant contributions to the imaginary parts of χ_1^{-1} and χ_2^{-2} are

$$\mathrm{Im}[\chi_1^{-1}] \simeq \frac{k_{\mathrm{D}i}^2}{(k^2 + k_{\mathrm{D}}^2)^2}\frac{\nu\mu}{kv_{T_i}}M_T \quad (9)$$

and

$$\mathrm{Im}[\chi_2^{-1}] \simeq \frac{k^2 + k_{\mathrm{De}}^2}{(k^2 + k_{\mathrm{D}}^2)^2}\frac{J_i\nu^2\mu}{k^3 v_{T_i}^3}M_T, \quad (10)$$

respectively. Integrations in Eq. (8) yield

$$F \simeq (1/6)Q^2 k_{\mathrm{D}i}^2(\ell_i k_{\mathrm{D}})^{-1}M_T +$$

$$+(1/6)Qek_{\mathrm{D}i}^2(\ell_i k_{\mathrm{D}})^{-1}(J_i/k_{\mathrm{D}i}^2 v_{T_i}\ell_i)M_T, \quad (11)$$

where the integration over k is performed from $k_{\min} = M_T/\ell_i$ to $k_{\max} = 1/\ell_i$ [where approximations (9) and (10) are valid], and it is assumed that $k_{\min}/k_{\mathrm{D}} \ll 1$ and $k_{\max}/k_{\mathrm{D}} \gg 1$. The contribution from small and large ranges of k are of minor importance [6].

Let us briefly analyze the structure of expression (11). The first term yields the force acting on a non-absorbing grain. It coincides with the expression obtained in [4] using a more general kinetic approach provided $\ell_i \ll \lambda_{\mathrm{D}}$, i.e., ions are highly collisional. The second term (absent in the previous consideration [4]) corresponds to the effect of absorption. It yields a *negative* contribution to the force, since $Q < 0$. Thus, in the highly collisional limit, the ion absorption on a grain *reduces* the absolute magnitude of the ion drag force.

To proceed further with the quantitative analysis, an expression for the ion flux J_i is required. To give an idea of how important the effect of absorption can be, let us use the well-known asymptotic expression for the ion flux on an infinitesimal grain ($a/\lambda_{\rm D} \to 0$) in the continuum limit ($\ell_i/a \to 0$) which can be written, in the present notation, as $J_i \simeq 4\pi R_{\rm C} n_0 \ell_i v_{T_i} [1-\exp(-R_{\rm C}/a)]^{-1} \simeq 4\pi R_{\rm C} n_0 \ell_i v_{T_i}$, where $R_{\rm C} = |Q|e/T_i$ is the Coulomb radius for the ions. Then, the expression for the ion drag force is

$$F \simeq -(1/6) Q^2 k_{{\rm D}i}^2 (\ell_i k_{\rm D})^{-1} [T_i/(T_e+T_i)] M_T, \qquad (12)$$

i.e., the force *reverses sign*! Thus, the analytical result, a decrease of the ion drag force and its sign reversal at high gas pressures, is compatible with the previous numerical simulations [2, 3]. Although the direct comparison between results of these simulations performed in the regime of moderate collisionality and our results derived in the highly collisional limit is not possible, our analysis identifies the ion absorption in the presence of ion-neutral collisions as the physical process responsible for the effect.

Let us briefly discuss the physical reason for the ion drag reduction. With no absorption taken into account, the ion-neutral collisions would *enhance* the ion drag force (compared with that in the collisionless case). As explained in [4], this is because of the collision-induced focusing of the ions downstream from the grain. The focusing implies a local increase in the ion density behind the grain which induces an additional electric field and a (drag) force acting in the direction of the ion drift for a negatively charged grain. With increasing collisionality, the focusing center moves closer and closer to the grain and the drag force increases. In contrast, the absorption of ions causes a rarefication of the ion density downstream from the grain. The two effects are added in a simple superposition in our linear model, and it turns out that the rarefication can cancel the effect of the focusing. This leads to a reduction of the ion drag force acting on an absorbing grain.

4. Summary

To summarize, we have shown analytically that the force acting on a small absorbing body in a collision-dominated plasma with slowly drifting ions can be decreased substantially in comparison with the force acting on a non-absorbing body and can even reverse direction. Although the range of the direct applicability of the obtained results for complex plasmas is relatively narrow, they can be instructive in explaining the results from recent numerical simulations predicting negative values of the ion drag force.

The described effect can have important consequences on the grain component behavior in plasmas. For instance, it can lead to a superfluid-like motion of the grains: A collection of absorbing grains can move freely in a highly collisional weakly ionized plasma at some critical velocity which is determined from the balance between the oppositely directed ion and neutral drag forces [7]. Apart from complex plasmas, it would be interesting to consider this effect in the context of the steady-state propagation of a spherical electrode in a leaky dielectric, as well as the operation of bacterial flagellar motors [8]. The mechanisms responsible for these phenomena can be quite similar to the one considered in the present paper.

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1. S.A. Khrapak, A.V. Ivlev, G.E. Morfill, and H.M. Thomas, Phys. Rev. E **66**, 046414 (2002); S.A. Khrapak, A.V. Ivlev, G.E. Morfill, and S.K. Zhdanov, Phys. Rev. Lett. **90**, 225002 (2003); S.A. Khrapak, A.V. Ivlev, G.E. Morfill, S.K. Zhdanov, and H.M. Thomas, IEEE Trans. on Plasma Sci. **32**, 555 (2004); V.E. Fortov, A.V. Ivlev, S.A. Khrapak, A.G. Khrapak, and G.E. Morfill, Phys. Reports **421**, 1 (2005).

2. I.V. Schweigert *et al.*, IEEE Trans. on Plasma Sci. **32**, 623 (2004).

3. S.A. Maiorov, Plasma Phys. Reports **31**, 690 (2005).

4. A.V. Ivlev, S.A. Khrapak, S.K. Zhdanov, G.E. Morfill, and G. Joyce, Phys. Rev. Lett. **92**, 205007 (2004).

5. A.V. Ivlev, S.K. Zhdanov, S.A. Khrapak, and G.E. Morfill, Phys. Rev. E **71**, 016405 (2005).

6. S.A. Khrapak, S.K. Zhdanov, A.V. Ivlev, G.E. Morfill, J. Appl. Phys. **101**, (2007) 033307.

7. S.V. Vladimirov, S.A. Khrapak, M. Chaudhuri, and G.E. Morfill, Phys. Rev. Lett. **100**, 055002(2008).

8. A.L. Yarin, Appl. Phys. Lett. **90**, 024103 (2007).

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СИЛА ГАЛЬМУВАННЯ ІОНАМИ, ЩО ДІЄ
НА ПОГЛИНАЮЧЕ ТІЛО У ПЛАЗМІ
З ЧАСТИМИ ЗІТКНЕННЯМИ

С.А. Храпак

Р е з ю м е

Використовуючи формалізм лінійного відгуку розраховано силу, діючу на невелике поглинаюче тіло у плазмі з частими зіткненнями та дрейфом іонів. Показано, що поглинання іонів приводить до нових фізичних ефектів та суттєвого зменшення сили, і дано пояснення фізичним причинам цього зменшення. Значення цього результату обговорено в контексті досліджень запорошеної плазми, хоча він має відношення до інших ситуацій, що виникають в астрофізиці, фізиці грозових хмар, домішок в приладах для синтезу, та колоїдних суспензій, у біосистемах тощо.