IMPORTANCE OF PLASMA ABSORPTION TO CHARACTERIZE THE TOTAL FORCE ACTING ON A DUST PARTICLE IN HIGHLY COLLISIONAL PLASMA SUBJECT TO A WEAK EXTERNAL ELECTRIC FIELD[1]

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The linear dielectric response formalism has been used to calculate the total force acting on a small absorbing spherical grain immersed in a highly collisional, weakly ionized plasma subject to a weak external electric field. Taking both the ion and electron absorptions on a grain into account, it is shown that the total force, which is the resultant of the electric, ion and electron drag forces, is always directed along the direction of the electric force. The "effective" charge of a grain, which can be defined as the ratio of the total force to the strength of the electric field, is comparable to the magnitude of the actual grain's charge.

1. Introduction

The "complex" or "dusty" plasma is a mixture of electrons, ions, micron-size charged particles, and a neutral gas. Often a large-scale electric field is present in complex plasmas. The direct effect of the electric field is to exert the electric force on highly charged dust particles. On the other hand, the indirect effect is to produce the ion and electron drag forces on grains which are nothing but the momentum transfer rates from drifting ions and electrons to grains. The competition between these different types of forces is responsible for different types of static and dynamic properties of the grain component, it affects wave phenomena, *etc.* [1]. Often the ion drag force dominates over the electron drag force because of a large ion-to-electron mass ratio. However, this situation can be changed, when the electrons drift much faster than the ions because of their much higher mobility. It was shown in [2] that, in the collisionless regime, the electron drag force can indeed dominate over the electric and ion drag forces provided the electron-to-ion temperature ratio is not too high.

In this work, we analyze the electric force, ion and electron drag forces in a highly collisional plasma subject to a weak electric field taking the plasma absorption on the grain surface into account. The importance of the plasma absorption on a grain has been discussed recently [3, 4]. We use the linear response formalism to derive the electric potential distribution around a grain. From this, the resulting electric field at the grain position and the corresponding total force acting on the grain are obtained. In doing so, we show that the total force, which is the sum of electric, ion drag and electron drag forces, is proportional to the electric field and always acts in the direction of the electric force. The proportionality constant represents the effective grain charge such as $\mathbf{F} = Q_{\rm eff}\mathbf{E}$. It is further found that $Q_{\rm eff} \simeq Q$, where Q is the actual charge of the particle, implying that the ion and electron drag forces are of minor importance as compared to the electric force in the considered highly collisional regime.

2. Formulation

In our model, we consider a small spherical negatively charged grain which is placed in the highly collisional,

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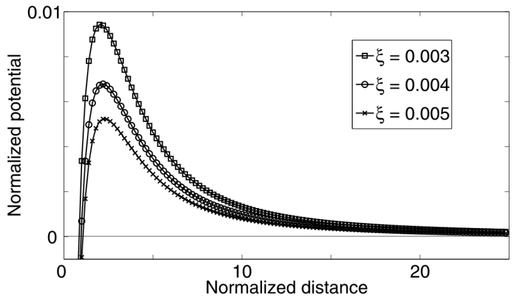

Fig. 1. Potential distribution behind a non-absorbing spherical grain for three different normalized ion mean free paths, ξ. Other plasma parameters are $z = 1$, $\tau = 2$, $a/\lambda_D = 0.01$, $M_T = 0.03$, where z is the normalized grain charge and a is the grain radius

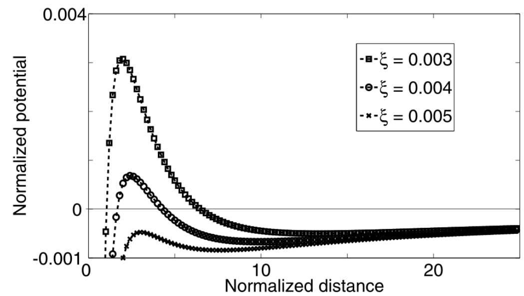

Fig. 2. Potential distribution behind an absorbing spherical grain for three different normalized ion mean free paths, ξ. Other plasma parameters are the same as those in Fig. 1

weakly ionized plasma with a constant external electric field $\mathbf{E}_0$. The grain is stationary and absorbs plasma on its surface. The ions drift in the direction of the electric field, whereas the electrons drift in the opposite direction. The electric field is sufficiently weak so that both the electron and ion drifts are subthermal: $M_{T_\alpha} = |u_\alpha|/v_{T_\alpha} < 1$, where $v_{T_\alpha} = \sqrt{T_\alpha/m_\alpha}$ is the thermal velocity and M_{T_α} is the thermal Mach number of the corresponding species. Here, $\alpha = i(e)$ for ions (electrons). There are no plasma sources or sinks except at the grain surface which is fully absorbing. In the highly collisional regime, both the ion and electron components are suitably described by the hydrodynamic equations. The corresponding continuity and momentum equations are

$$\nabla(n_\alpha \mathbf{v}_\alpha) = -J_\alpha \delta(\mathbf{r}), \quad (1)$$

$$(\mathbf{v}_\alpha \nabla)\mathbf{v}_\alpha = q_\alpha (e/m_\alpha)\mathbf{E} - (\nabla n_\alpha / n_\alpha) v_{T_\alpha}^2 - \nu_\alpha \mathbf{v}_\alpha, \quad (2)$$

where n_α and $\mathbf{v}_\alpha$ are, respectively, the density and velocity of the corresponding species, $J_{i(e)}$ denotes the ion (electron) fluxes that the grain collects from the plasma, and $\mathbf{E}$ is the electric field. In Eq. (2), $q_i = +1$ and $q_e = -1$. The above system of equations is closed with the Poisson equation

$$\Delta\phi = -4\pi e(n_i - n_e) - 4\pi Q \delta(\mathbf{r}), \quad (3)$$

where Q is the particle charge.

3. Results, Discussion, and Conclusion

The self-consistent electrostatic potential around an absorbing point-like grain is calculated using the standard linear dielectric response technique. Assuming the plasma perturbation to be proportional to $\propto \exp(i\mathbf{kr})$, we get [3]

$$\phi_{\rm P}(\mathbf{r}) = \frac{4\pi Q}{(2\pi)^3} \int \frac{\exp(i\mathbf{kr}) d\mathbf{k}}{\chi_1} +$$

$$+ \frac{4\pi e}{(2\pi)^3} \sum_{\alpha=i,e} \int \frac{\exp(i\mathbf{kr}) d\mathbf{k}}{\chi_\alpha}, \quad (4)$$

where

$$\chi_1 = k^2 \left[1 + \sum_{\alpha=i,e} \left(\frac{\omega_{p\alpha}}{\Omega_\alpha} \right)^2 \right],$$

$$\chi_\alpha = i q_\alpha \left[\frac{\chi_1 \Omega_\alpha^2}{J_\alpha (\mathbf{ku}_\alpha - i\nu_\alpha)} \right].$$

Here, $\omega_{p\alpha} = \sqrt{4\pi n_\alpha e^2/m_\alpha}$ is the plasma frequency of the corresponding species and $\Omega_\alpha^2 = k^2 v_{T_\alpha}^2 - \mathbf{ku}_\alpha(\mathbf{ku}_\alpha - i\nu_\alpha)$. The first term in Eq. (4) represents the potential of a non-absorbing point-like grain [5], while the second term represents the sum of the contributions to the potential due to ion and electron absorptions respectively [3]. Figures 1 and 2 show the potential distribution behind non-absorbing and absorbing grains, respectively. For a non-absorbing grain, there always appears a positive space charge region behind the grain, which corresponds to the ion focusing (the so-called "wake effect"). Absorption affects qualitatively the distribution of the electric potential. The magnitude of the effect is determined by three dimensionless parameters: the electron-to-ion temperature ratio (τ), ion drift velocity normalized to the ion thermal velocity (M_T), and the ratio of the ion mean-free path to the plasma screening length (ξ). Depending on these parameters, the potential downstream from an absorbing grain is either negative for all distances or can take positive values at intermediate distances [4].

From Eq. (4), we get the polarization part of the total force experienced by a test grain using the relation $F_{\rm P} = -Q\nabla\phi_{\rm P}\,|_{r=0}$. This yields

$$F_{\rm P} = \pi^{-1}\int_0^\infty k^3 dk \int_{-1}^{1} \mu d\mu \times$$

$$\left[Q^2 {\rm Im}\left\{\chi_1^{-1}(\mu,k)\right\} + Qe\sum_{\alpha=i,e} {\rm Im}\left\{\chi_\alpha^{-1}(\mu,k)\right\}\right], \qquad (5)$$

where $\mu = \cos\theta$ and θ is the angle between $\mathbf{k}$ and $\mathbf{E}_0$. In our model, we have considered the hydrodynamic approach both for ions and electrons, the applicability of which requires $k\ell_\alpha \ll 1$. Using the flux balance condition $J_i = J_e$ for a floating grain and small ion and electron drift velocities, $k\ell_\alpha \gg M_{T_\alpha}$, and integrating, we get

$$F_{\rm P} = (1/6)(Q^2/k_{\rm D})\sum_{\alpha=i,e}(q_\alpha k_{{\rm D}\alpha}^2/\ell_\alpha)M_{T_\alpha} +$$

$$+(1/6)(Qe/k_{\rm D})\sum_{\alpha=i,e}(J_\alpha M_{T_\alpha}/\ell_\alpha^2 v_{{\rm T}_\alpha})\times$$

$$\times[(2 - k_{{\rm D}\alpha}^2/k_{\rm D}^2) + (\ell_i v_{{\rm T}_i}/\ell_e v_{{\rm T}_e})^{q_\alpha}(k_{{\rm D}\alpha}/k_{\rm D})^2]. \qquad (6)$$

The first term in the above equation represents the sum of ion and electron drag forces acting on a non-absorbing grain. The expression for the ion drag force coincides with the expression derived earlier in [5] with the use of a more general kinetic approach in the considered limit of highly collisional ions ($\ell_i \ll \lambda_{\rm D}$). The result shows that the ion drag force increases with the ion collisionality. The physical reason is the ion focusing effect: behind a negatively charged grain, ions form a positive space charge region due to the focusing. As the collisionality increases, the focusing point shifts toward the grain, and the amplitude of the potential increases, which makes the ion drag force larger (see Fig. 1). The ion and electron drag forces experienced by a non-absorbing grain are directed along the drift velocity of the corresponding species, i.e. they act in the opposite directions. The ratio of their absolute magnitudes is $(T_e/T_i)^2$. This implies that, in one-temperature plasmas ($T_e = T_i$), they exactly cancel each other. In highly non-thermal plasma ($T_e \gg T_i$), the ion drag force (directed opposite to the electric force) dominates. The ratio of the ion drag force to the electric force is $(1/6)\beta$ where $\beta = z\tau(a/\lambda_{\rm D})$ is the so-called scattering parameter [6,7]. Here, $z = |Q|e/aT_e$ is the dimensionless grain charge, $\tau = T_e/T_i$ is the electron-to-ion temperature ratio, and a is the grain radius.

The second sum in Eq. (6) is the contribution to the drag forces due to ion and electron absorptions. Under most typical plasma conditions $\ell_e/\ell_i \sim 10 - 100, v_{T_e}/v_{T_i} \sim 10^2 - 10^3$, and $T_e/T_i \sim 1 - 100$, we can approximate the absorption parts of the ion and electron drag forces as

$$(F_i)_{\rm abs} \simeq (1/6)(Qe/k_{\rm D})(J_i M_{T_i}/\ell_i^2 v_{{\rm T}_i})(1 + k_{{\rm D}e}^2/k_{\rm D}^2),$$

and

$$(F_e)_{\rm abs} = (1/6)(Qe/k_{\rm D})(J_e M_{T_e}/\ell_i \ell_e v_{{\rm T}_i})(k_{{\rm D}e}/k_{\rm D})^2,$$

whose ratio is $|F_i/F_e|_{\rm abs} \approx (T_e/T_i)^2$. It is clear that the effect of ion absorption reduces the absolute magnitude of the total ion drag force for negatively charged grains, whereas the effect of electron absorption increases the total electron drag force. We note that, by neglecting the electron absorption ($J_e = 0$) and the electron drift ($u_e = 0$), we get the expression for the ion drag force obtained earlier in [3].

In the continuum regime ($\ell_\alpha \ll a$) and for infinitesimally small grains ($a \ll \lambda_{\rm D}$), we use the simple analytical asymptotic expressions for the charging fluxes, $J_e = J_i \simeq 4\pi a z\tau n_0 \ell_i v_{T_i}$ where n_0 is the unperturbed plasma density [8]. The total drag force can be then written as

$$F_{\rm P} = -(1/6)Q^2 k_{{\rm D}_e}^2 M_{T_i}(\ell_i k_{\rm D})^{-1}(k_{{\rm D}i}/k_{\rm D})^2 -$$

$$-(1/6)Q^2 k_{{\rm D}_e}^2 M_{T_e}(\ell_e k_{\rm D})^{-1}(1 + k_{{\rm D}i}^2/k_{\rm D}^2) =$$

$$= (1/6)(QE_0)(\beta/\tau), \qquad (7)$$

i.e., both the ion and electron drag forces are directed in the same direction (opposite to the electric field). This is associated with the fact that the ion absorption on a grain changes the direction of the ion drag force in highly collisional plasmas [3], whilst the electron absorption increases the magnitude of the electron drag force. The absolute ratio of the total ion and electron drag forces is $|F_i/F_e| \approx (T_e/T_i)$.

In our model, it is not possible to self-consistently estimate the forces associated with the "drift momentum" transfer from ions and electrons on the absorption by a grain. A rough estimate of these forces is $\sim J_0 m_{i(e)} u_{i(e)}$ for the ions (electrons). These contributions are smaller than the corresponding drag forces by a factor of $\sim (\tau/\beta)(\ell_i/\lambda_{\rm D})^2$ for ions and $\sim (\tau^2/\beta)(v_{T_i}/v_{T_e})(\ell_i/\lambda_{\rm D})(\ell_e/\lambda_{\rm D})$ for electrons. Thus, in highly collisional plasma, it is reasonable to neglect the above-mentioned effect. The grain effective charge, $Q_{\rm eff} = F/E_0$, in the considered parameter regime can be written as

$$Q_{\rm eff}/Q = 1 + (1/6)(\beta/\tau). \quad (8)$$

The application of the linear theory requires $\beta \lesssim 1$ [5], while τ is usually larger than unity. Thus, we have $Q_{\rm eff} \approx Q$. This implies that both the ion and electron drag forces acting on an absorbing grain are small as compared with the electric force in the considered case of the highly collisional plasma.

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ВАЖЛИВІСТЬ ПОГЛИНАННЯ ПЛАЗМОВИХ ЧАСТИНОК ДЛЯ РОЗРАХУНКУ РЕЗУЛЬТУЮЧОЇ СИЛИ, ЩО ДІЄ НА ПОРОШИНКУ У ПЛАЗМІ З ЧАСТИМИ ЗІТКНЕННЯМИ У ЗОВНІШНЬОМУ ЕЛЕКТРИЧНОМУ ПОЛІ

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Р е з ю м е

Застосовано формалізм лінійного діелектричного відгуку для розрахунку результуючої сили, що діє на невелику поглинаючу сферичну порошинку в слабко іонізованій плазмі з частими зіткненнями, яка знаходиться у слабкому зовнішньому електричному полі. З урахуванням поглинання електронів та іонів порошинкою показано, що результуюча електричної сили і сил гальмування електронами та іонами завжди має напрямок електричної сили. "Ефективний" заряд порошинки, який можна визначити як відношення результуючої сили до напруження електричного поля, є порівняним за величиною з її фактичним зарядом.