e_0-ELECTRON YIELD FROM THE SURFACE OF RADIOACTIVE SOURCE

A.I. FEOKTISTOV

Institute for Nuclear Research, Nat. Acad. of Sci. of Ukraine
(47, Nauka Ave., Kyiv 03680, Ukraine)

UDC 539.163

Supposing that e_0-electron emission is an electron transition from the surface of radioactive source into vacuum stimulated by a sudden appearance of an electric charge near the surface (shake-up), a formula for the yield of this process has been derived.

1. Introduction

The phenomenon of atomic self-ionization at β- and α-decays was considered theoretically for the first time by Feinberg [1] and Migdal [2]. It consists in that the transition of an atomic electron from its bound state into the continuous range of the spectrum might take place, giving rise to the ionization of the atom, if the nucleus spontaneously changes its charge under the action of a sudden perturbation. This phenomenon was coined "shake-up" in the literature. However, the shake-up of electron shells occurs not only at nuclear transformations, but at transitions in atomic electron shells as well. Without taking the shake-up process into account, not only the final states of the system cannot be described completely, but also the processes that give rise to the excitation of electron shells [3]. This work is devoted to the consideration of some kind of the shake-up phenomenon.

At radioactive decay, the source surface always emits electrons with a very low energy; these electrons are responsible for the emergence of a peak in the electron spectrum which is referred to as "zero-energy peak" [4]. The peak maximum is located at an energy of about 0.5 eV. Its intensity quickly diminishes with the growth of energy, so that it can be neglected at energies of about 20 eV [5]. A similar peak is observed, if targets are bombarded with charged particles. Electrons that form the "zero-energy peak" are called near-zero-energy electrons or e_0-electrons, in contrast to other electrons of the spectrum which we call fast electrons or e_f-electrons.

On the basis of experimental data and the results of our researches [6], we propose the following mechanism of e_0-electron formation. If an electric charge suddenly appears in the vicinity of the source surface, it can provoke the transition of an electron from the surface into a state of the continuous spectrum. An atom can acquire a large charge due to the emission of a good many Auger-electrons at filling the initial vacancy which has formed in one of the atomic shells. The vacancy itself can emerge in the course of β-decay, electron capture, internal conversion of γ-rays, or the process of atomic ionization near the surface, when atoms are bombarded by charged particles. This process is similar to the shake-up effect at β-decay. As follows from the analysis of numerous experimental data, the closer to the surface the charge emerges, the more e_0-electrons are shaken up, and vice versa: if the charge emerges at a distance of 5–6 atomic layers into the source depth, the shake-up phenomenon is practically absent. The yield of e_0-electrons Y is the average number of outgoing electrons which fly out owing to the appearance of a charge in atoms that are regularly arranged at different places in the near-surface layer. The electron yield from the surface of a solid is proportional to the square of the charge that has emerged in the atom, and the energy distributions of emitted e_0-electrons are close to one another for different types of radioactive decay.

In this work, an attempt was made to connect the e_0-electron yield from the source surface at the

shake-up observed in different experiments with other physical quantities that describe this process. The charge can emerge at various places of a radioactive source; accordingly, the e_0-electron can be shaken off – individually or simultaneously – from various surface sections. In addition, this work also aims at analyzing the formula for e_0-electron distribution over energy in the continuous range of the spectrum [7] and comparing it with the experimental dependence, also obtained in the present work. It was done in order to confirm the correctness of using the "shake-up" concept for the description of the e_0-electron emission from the source surface in the case where a charge suddenly emerges near a surface electron.

2. Explanation of the e_0-electron Emission Phenomenon From the Viewpoint of Contemporary Theoretical Concepts

Consider our explanation of the nature of near-zero energy electron emission from the surface of a radioactive source from the viewpoint of modern theoretical ideas. The latter are expounded in Section 41 of reference [7]. According to them, the transition probability at a sudden perturbation can be determined by the formula [8]

$$dW = \frac{\left|\int \psi_f^* \frac{\Delta Z e^2}{r} \psi_i^{(0)} dq\right|^2}{(E_{e_0} + |\varphi|)^2} d\nu = W_{if} d\nu, \tag{1}$$

where dW is the shake-up probability for a surface electron in the energy interval from E_{e_0} to $E_{e_0} + dE$, $\psi_i^{(0)}(q)$ is the wave function of a surface electron at the time moment of the charge emergence, $\psi_f^*(q)$ is the wave function of the e_0-electron in the continuous spectral range after flying out from the source surface, q are the coordinates of the wave function, ΔZe is the value of the suddenly emerged charge, r is the distance between the emerged charge and the surface electron, E_{e_0} is the e_0-electron energy, φ is the electron work function, $d\nu$ is the number of e_0-electron states within the energy interval from E_{e_0} to $E_{e_0} + dE$, and W_{if} is the probability of the electron transition from state i into state f.

Either of the wave functions is the coordinate part of the stationary wave function $\Psi = \psi(q)e^{(-iEt)/\hbar}$ for the relevant Hamiltonian, $\hat{H}_0$ or $\hat{H}$. Since the perturbation is sudden, the initial state of the system $\psi_i^{(0)}$ "has no time" to change and remains the same as it was before the perturbation was switched on. That is why this state is marked by superscript (0) [7]. However, since the initial state "had no time" to change, the sudden perturbation $V = \frac{\Delta Z e^2}{r}$ also "had no time" to penetrate into $\psi_i^{(0)}$, it acts as a whole, and there is no $r(q)$-dependence; that is, the quantity r is also sudden and constant, as a part of V. Formula (1) is applicable for the description of the shake-up phenomenon only if the perturbation time $\tau < \omega^{-1}$, where $\omega = (E_{e_0} + |\varphi|)/\hbar$. Otherwise, to determine the probability of the $(i \to f)$ transition, one has to use formula (41.4) from reference [7].

The perturbation becomes completely transferred to the final state, so that the surface electron, if it is capable, by overcoming the work function, of fulfilling the transition from the bound state into vacuum, does that. The photoeffect can serve as an analog to this phenomenon: in that case, an electron, after having instantly absorbed a photon, overcomes the work function to find itself in vacuum. In our case, after a charge having suddenly arisen, the electron also suddenly acquires the potential energy $\frac{\Delta Z e^2}{r}$ and overcomes the work function to find itself in vacuum:

$$\frac{\Delta Z e^2}{r} = E_{e_0} + |\varphi|. \tag{2}$$

The probability of the $(i \to f)$-transition can also be determined by calculating the overlap integral of the wave functions [7]:

$$W_{if} = \left|\int \psi_f^* \psi_i^{(0)} dq\right|^2. \tag{3}$$

"The wave function $\psi_i^{(0)}$"has no time" to change within the transition time interval and remains the same, as it was before the perturbation. However, it is not the characteristic function of the Hamiltonian H of the system anymore, i.e. the state $\psi_i^{(0)}$ is no more stationary. The probabilities W_{if} for the system to transit into any of the new stationary states, after all, are determined, in accordance with general rules of quantum mechanics, by the coefficients of the expansion of the function $\psi_i^{(0)}$ into a series in the characteristic functions ψ_f of the Hamiltonian H" [7].

By comparing Eqs. (1) and (3), one can see that the transition probability does not depend on the perturbation magnitude. However, in our case, owing to relationship (2), the kinetic energy of the e_0-electron is always strictly connected with the perturbation energy, and vice versa. Substituting Eq. (2) into Eq. (1), one can see that the transition probability also does not depend on the perturbation energy. If the perturbation magnitude increases, the e_0-electron energy increases as

well, whereas the probability of the $(i \to f)$ transition remains the same.

Therefore, the multiplier $\frac{\Delta Z e^2}{r}$ can be factored outside the integral sign in formula (1). Now, we can turn to the consideration of the e_0-electron energy distribution.

3. Energy Distribution of e_0-Electrons

With regard for the phase volume for surface electrons that can transit into vacuum, the density of final state levels can be expressed by the following formula:

$$\frac{d\nu}{dE} = a\sqrt{E}, \tag{4}$$

where

$$a = \frac{\sqrt{2}m^{3/2}x^3}{\pi^2\hbar^3}, \tag{5}$$

m is the electron mass, and x is the distance between two nearest surface electrons which are capable of making the transition into vacuum. To connect the quantity x with the distance between atomic layers, we can write down

$$a = \frac{\sqrt{2}m^{3/2}V\delta^3}{\pi^2\hbar^3},$$

where $x^3 = V\delta^3$, $V = A/(\rho N_A)$ is the volume occupied by an atom, A is the atomic weight of the source substance, ρ is its density, and N_A is the Avogadro constant. Hence, the parameter $\delta = x/\sqrt[3]{V}$ connects the distance between two neighbor electrons, which are capable of making the transition into vacuum, with the distance between two neighbor atoms on the source surface, and δ^2 is the ratio between the numbers of atoms and electrons on the surface. If $\delta^2 < 1$, the number of electrons is higher than the number of surface atoms; and if $\delta^2 > 1$, the number of surface atoms it larger than the number of such electrons.

The distribution of e_0-electrons over the energy is described by the expression

$$\frac{dN}{dE} = \frac{a\sqrt{E}}{(E + |\varphi|)^2}. \tag{6}$$

In Fig. 1, the distributions of e_0-electrons for several values of the work function φ are shown. They have the shape of a narrow peak with the maximum that is located at the energy $E = |\varphi|/3$ and the position of which does not depend on the magnitude of perturbation energy $\frac{\Delta Z e^2}{r}$. At an energy of about 20 eV, its intensity can be neglected. The halfwidth of the peak increases from 0.9 to 2.7 eV as the work function φ varies from -0.5 to -2 eV.

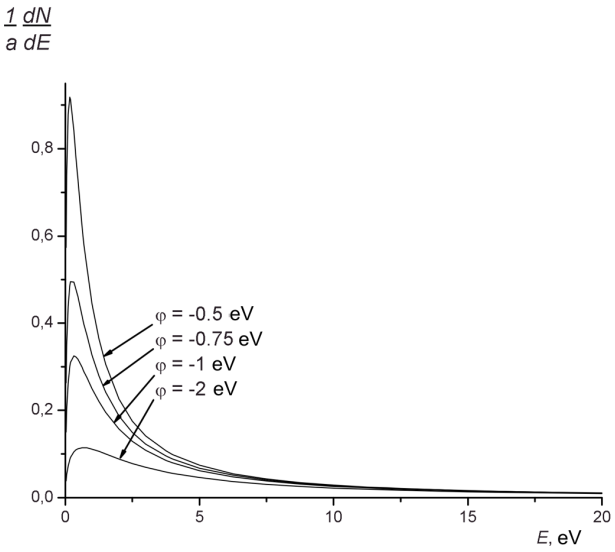

Fig. 1. Distributions of e_0-electrons over their energy for various values of the work function φ

In our measurements, the energy distribution of e_0-electrons was determined as a derivative of the dependence of the e_0-electron count number on the bias voltage applied between the source and the e_0-electron detector (two microchannel plates which composed a chevron). In a decelerating field corresponding to $+20$ V at the source, the experimental distribution is in good agreement with theoretical values, if the value of the work function φ is considered as a fitting parameter. In Fig. 2, the e_0-electron count curve and its derivative are exhibited, as well as the theoretical distribution over the energy for a ^{238}Pu source, where the value $\varphi = -0.5$ eV is adopted for the work function. The calculated value of dN/dE at $E = 1$ eV was adjusted to the experimental value obtained at this point. Near the zero value and at negative values of the source voltage, when e_0-electrons are accelerated, the count curve is distorted, because the detector attracts electrons that are emitted by the source in different directions, so that this section of distribution should not be taken into consideration. Figure 2 demonstrates that the values of dN/dE calculated by formula (6) agree well with the experimental curve for the energy distribution of electrons, thus confirming the validity of the model, where the emission of electrons from the surface is described as their perturbation

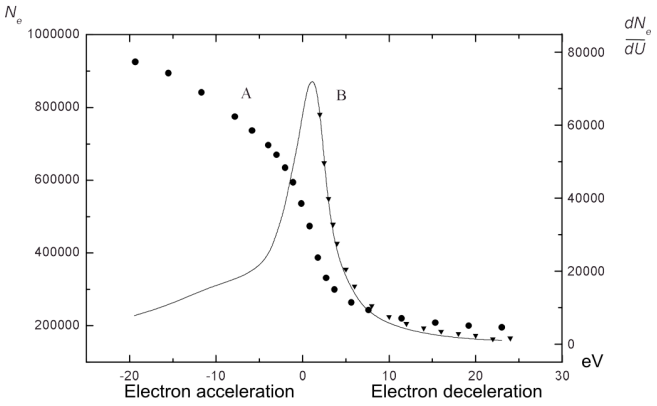

Fig. 2. Experimental determination of the e_0-electron distribution over the energy: (A) the count curve (N_e is the number of electrons registered for 160 s); (B) the energy distribution of e_0-electrons (the derivative of the count curve); (▼) the energy distribution of e_0-electrons calculated by formula (6)

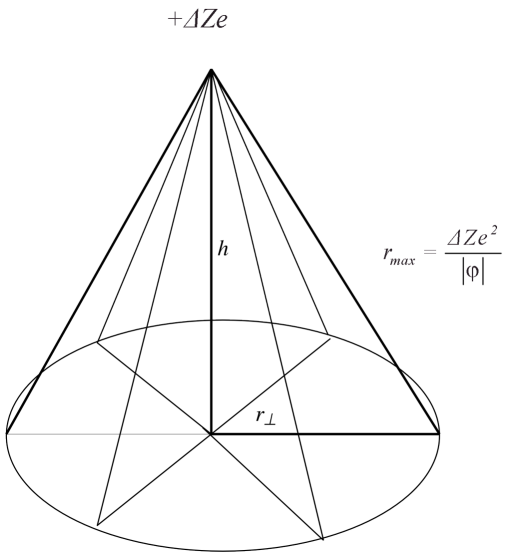

Fig. 3. Cone of interaction between a charge located at the cone vertex and surface electrons located at the cone basis

invoked by an electric charge that suddenly arises near the surface.

4. Formula for e_0-Electron Yield at Shake-up

To derive a formula that would describe the e_0-electron yield at the shake-up which is observed at a radioactive decay, consider first the perturbation of a surface electron by a suddenly arising charge which is located at the distance r from it (see Fig. 3). For this purpose, consider a cone with height h; the charge is located at the cone vertex, and the cone basis coincides with the source surface. The cone generatrix is associated with the perturbation energy $\frac{\Delta Ze^2}{r} = |\varphi|$ which is transferred from the charge to a surface electron located at the intersection between the cone generatrix and the cone basis. All the electrons that are located at the circle of the cone basis obtain an identical energy. Inside the cone, all the electrons located at concentric circles also obtain an identical energy, but this energy is higher than that obtained by electrons located at the cone generatrix. Electrons that are located along the radius $r_\perp$ obtain the energy of all possible values from $\frac{\Delta Ze^2}{r} = |\varphi|$ to $\frac{\Delta Ze^2}{h}$, but the transition probability is identical for every of them. At last, an electron that is located at the center of the circle obtains the highest energy equal to $\frac{\Delta Ze^2}{h}$, whereas those located at the cone generatrix get the lowest one $\frac{\Delta Ze^2}{r} = |\varphi|$, where φ is the work function; however, the circle, along which they are located, embraces the largest area occupied by electrons. Electrons beyond this circle obtain an energy lower that $|\varphi|$, which does not enable them to transit into vacuum. The key role in the e_0-electron emission is played by the electrons which are located at the surface in the vicinity of the cone generatrix.

The e_0-electron yield into vacuum, provided that the charge perturbs all surface electrons located along the circle radius $r_\perp$ (so that their distance to the charge varies from the maximal distance between the source and the surface electron, $r_{\max} = \frac{\Delta Ze^2}{|\varphi|}$, to the minimal distance equal to h), can be written down as follows:

$$Y_\perp(h) = \left(\frac{\Delta Ze^2}{h}\right)^2 \left|\int \psi_f^* \psi_i^{(0)} dq\right|^2 \int_0^{E_{max}} \frac{a\sqrt{E}}{(E+|\varphi|)^2} dE, \tag{7}$$

where $E_{\max} = \frac{\Delta Ze^2}{h} - |\varphi|$, E is the variable corresponding to the e_0-electron energy, and

$$\int_0^{E_{\max}} \frac{\sqrt{E}}{(E+|\varphi|)^2} dE = \frac{1}{\sqrt{|\varphi|}} \times$$

$$\times \mathrm{arctg}\sqrt{\frac{\Delta Ze^2}{h|\varphi|} - 1} - \left(\frac{\Delta Ze^2}{h}\right)^{-1} \sqrt{\frac{\Delta Ze^2}{h} - |\varphi|}. \tag{8}$$

Now, let us calculate the average e_0-electron yield for electrons located along $r_\perp$, which is produced by a

sudden charge that arises at the distance h from the surface:

$$\bar{Y}(h) = Y_{\perp}(h)\frac{x}{r_{\perp}}, \tag{9}$$

where

$$r_{\perp} = \sqrt{\left(\left(\frac{\Delta Z e^2}{\varphi}\right)^2 - h^2\right)}$$

is the circle radius, x is the average distance between neighbor surface electrons located along the radius $r_{\perp}$, and $r_{\perp}/x$ is the number of surface electrons, which are located along the circle radius $r_{\perp}$ and which can transit into vacuum. Then, the total yield of e_0-electrons $Y(h)$ produced by a sudden charge $\Delta Z e$ that arises at the distance h from the surface is determined as

$$Y(h) = \bar{Y}\frac{\pi r_{\perp}^2}{x^2} = Y_{\perp}(h)\frac{\pi r_{\perp}}{x}. \tag{10}$$

At last, if the source has the thickness of d atomic layers, and if radioactive nuclei are distributed regularly over the source thickness, the e_0-electron yield produced by a charge $+\Delta Z e$ that arises in one decay can be expressed as follows:

$$Y(d) = \frac{1}{d}\sum_{h=1b}^{h=db} Y(h), \tag{11}$$

where $b = \sqrt[3]{V}$ is the thickness of an atomic layer.

The final formula for the e_0-electron yield in the case where the point charge $+\Delta Z e$ arises with an equal probability at any point of the source that is composed of d atomic layers looks like

$$Y(d) = \frac{1}{d}\sum_{h=1b}^{h=db} \sqrt{\left(\frac{\Delta Z e^2}{\varphi}\right)^2 - h^2}\left(\frac{\Delta Z e^2}{h}\right)^2 \times$$

$$\times \left|\int \psi_f^* \psi_i^{(0)} dq\right|^2 \frac{\sqrt{2m^3}}{\pi\hbar^3}\left(\frac{A}{\rho N_A}\right)^{2/3}\delta^2 \times$$

$$\times \left[\frac{1}{\sqrt{|\varphi|}}\mathrm{arctg}\sqrt{\frac{\Delta Z e^2}{h|\varphi|} - 1} - \left(\frac{\Delta Z e^2}{h}\right)^{-1}\sqrt{\frac{\Delta Z e^2}{h} - |\varphi|}\right]. \tag{12}$$

The first multiplier in formula (12) corresponds to the radius of the circle lying in the cone basis. It changes weakly with the variation of h and weakly affect the variation of $Y(d)$, because, in our case, the cone generatrix length $\frac{\Delta Z e^2}{|\varphi|}$ is, as a rule, several times longer than the cone height h.

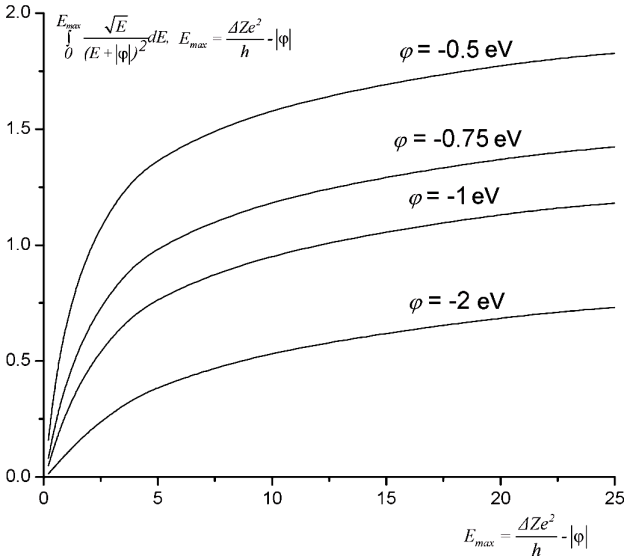

Fig. 4. Dependences of the integral $\int_0^{E_{\max}} \frac{\sqrt{E}dE}{(E+|\varphi|)^2}$ on the e_0-electron energy for various values of the work function φ

The dependence of the e_0-electron yield on the source thicknesses is mainly governed by the corresponding dependence of the quantity $\left(\frac{\Delta Z e^2}{h}\right)^2$. The yield quickly falls down as the distance h from the charge to the surface increases; and this phenomenon explains the emergence of a near-surface layer in the source. For instance, the e_0-electron yield produced by the atomic ionization in the fifth layer amounts to only 4% of that produced by the atomic ionization in the first layer of the source. The larger the perturbation $\frac{\Delta Z e^2}{h}$, the more the number of surface electrons which are able to transit into vacuum.

The dependence of the value, which acquires the expression in square brackets on the right-hand side of formula (12), on the e_0-electron energy is presented in Fig. 4. As the perturbation energy $\frac{\Delta Z e^2}{h}$ grows, this value changes; however, in general, its influence on the yield $Y(d)$ change is small, because the behavior of $Y(d)$ is governed by a few h-values which are close to one another.

5. Applications of the Formula

Let us evaluate the transition matrix element in the case of the ^{46}Sc β-decay, making use of formula (12). If the energy of β-particles is high, the contribution to

the ionization of an atom made by "direct collisions" of a β-particle with electrons beloning to own shells of the atom can be neglected. In this case, it follows from the results of our measurements [9] that $Y = 0.5e_0$ and $\Delta Ze = +1.5e$; while it follows from the count curve that $\varphi = -0.74$ eV. For these values and supposing that $\delta = 1$, we obtain that the matrix element of the $(i \to f)$-transition amounts to $\left|\int \psi_f^* \psi_i^{(0)} dq\right| = 0.08$.

1. E.L. Feinberg, J. Phys. (USSR) **4**, 424 (1941).
2. A. Migdal, J. Phys. (USSR) **4**, 449 (1941).
3. V.I. Matveev and E.S. Parilis, Usp. Fiz. Nauk **138**, 573 (1982).
4. M.S. Freedman, F.T. Porter, F.I. Wagner, and P.P. Day, Phys. Rev. **108**, 836 (1957).
5. B.V. Bobykin, V.P. Burminskii, and S.K. Lyubov, Poverkhn. Fiz. Khim. Mekh. **9**, 71 (1992).
6. V.T. Kupryashkin, L.P. Sidorenko, A.I. Feoktistov *et al.*, Izv. Ross. Akad. Nauk, Ser. Fiz. **67**, 1446 (2003).
7. L.D. Landau and E.M. Lifshitz, *Quantum Mechanics. Non-Relativistic Theory* (Pergamon Press, New York, 1965).
8. V.T. Kupryashkin, L.P. Sidorenko, A.I. Feoktistov, and I.P. Shapovalova, Izv. Ross. Akad. Nauk, Ser. Fiz. **68**, 1208 (2004).
9. V.T. Kupryashkin, L.P. Sidorenko, A.I. Feoktistov, and I.P. Shapovalova, Izv. Ross. Akad. Nauk, Ser. Fiz. **69**, 1657 (2005).

Received 10.04.08.
Translated from Ukrainian by O.I. Voitenko

ОПИС ВИХОДУ e_0-ЕЛЕКТРОНІВ З ПОВЕРХНІ ДЖЕРЕЛА ПРИ РАДІОАКТИВНОМУ РОЗПАДІ

О.І. Феоктістов

Р е з ю м е

Одержано формулу для визначення виходів e_0-електронів з поверхні радіоактивного джерела виходячи із припущення, що випромінювання e_0-електрона є переходом електрона поверхні в вакуум під дією збурення, яке створюється раптовою появою електричного заряду поблизу поверхні (струс).