PIC-SIMULATION OF WAKE FIELD EXCITATION IN NONLINEAR MODE IN A CYLINDRICAL DIELECTRIC RESONATOR

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A nonlinear mechanism giving rise to the restriction by amplitude of the wake field excited by a train of relativistic electron bunches in a cylindrical resonator partially filled with insulator has been simulated numerically. The resonance dependence of the wake field amplitude on the bunch repetition frequency in the train has been studied.

1. Introduction

While studying the acceleration of an electron beam in a wake field excited by a train of electron bunches in a dielectric resonator, the main issue is the electric field amplitude that is excited in the system. The growth of the electric field amplitude is caused by the injection of charge bunches into the accelerating resonator, and the field saturation is associated, first, with damping owing to a finite quality factor of the resonator and, second, with nonlinear processes invoked by the reverse influence of the field with high amplitude on charged bunches. In this work, the second, nonlinear mechanism of restriction of the electric field amplitude in the system concerned has been analyzed.

From the viewpoint of the particle acceleration, it is of interest to determine the largest feasible field amplitude in the system. One of the parameters that affect the amplitude of the field in the resonator is the bunch repetition frequency or, equivalently, the distance between bunches. It is natural that energy losses by train bunches and, as a consequence, the field amplitude become maximal at the repetition frequencies close to the characteristic frequencies of the resonator. Therefore, to optimize the process of wake field excitation, it is reasonable to determine, for one thing, the excitation spectrum of the resonator and, afterwards, to obtain the dependence of the field amplitude on the repetition frequency of electron bunches injected into the system. The bunch repetition frequency will be varied at that within a narrow interval around one of the resonance frequencies of the excited field.

The problem of the excitation of a slow wave by charged particle bunches in a resonator has been considered for the first time in works [1, 2]. While studying the nonlinear stage of resonator excitation by means of a density-modulated electron beam (a train of point-like electron bunches), the maximal attainable amplitude of the electric field was found. Its value turned out independent of the beam current, and the number of electron bunches, which should be injected into the resonator to achieve the field amplitude saturation, was reciprocal to the beam current.

The indicated research was carried out at a qualitative level and in the one-dimensional approximation. The problem can be studied in more details making use of the numerical simulation method. In this work, the simulation results concerning the excitation of an electromagnetic field by means of a train of relativistic charged bunches in a cylindrical resonator partially filled with an insulator are reported. The simulation was carried out making use of a specially developed 2.5-dimensional electromagnetic PIC-code [3].

2. Formulation of the Problem

The geometry of the system under investigation is shown in Fig. 1. A train of relativistic bunches *1* is injected into a dielectric resonator, which is a piece of the cylindrical waveguide of radius R partially filled with insulator *3*. The radius of drift chamber *2* is equal to R_1. The left and the right end face of the waveguide are short-circuited with the help of metal walls which are transparent for particles. The electron beam radius is r_b, and the average electron current is I_b. At the resonator input ($z = 0$), the injected beam is monochromatic, and the transverse components of electrons' velocities are equal to zero.

Owing to the axial symmetry of the problem, the system of Maxwell's equations, being written down in the cylindrical coordinate system, is split into two subsystems for field components: one for E_r (D_r), E_z (D_z), and H_φ, and the other for H_r, H_z, and E_φ (D_φ). The current density includes three components: j_r, j_z, and j_φ. The relative dielectric permittivity of the medium $\tilde{\varepsilon}(r)$ is given in the form of a step function:

$$\tilde{\varepsilon}(r) = \begin{cases} \varepsilon, & r \geq R_1; \\ 1, & r < R_1. \end{cases} \tag{1}$$

While simulating the dynamics of electromagnetic fields numerically, the latter were calculated taking advantage of the mechanism of current and charge "distribution" over the nodes of a two-dimensional spatial mesh.

In order to find the charge and current densities, one has to know the positions and velocities of macroparticles. They were determined by the equations of motion [4].

The boundary conditions for the fields are as follows: the zero-valued tangential components of the electromagnetic fields at the resonator walls,

$$E_z|_{r=R} = 0; \quad E_\varphi|_{r=R} = 0;$$

$$D_r|_{z=0} = 0; \quad D_r|_{z=L} = 0,$$

and the zero-valued field components D_r and H_φ at the resonator axis,

$$D_r|_{r=0} = 0; \quad H_\varphi|_{r=0} = 0.$$

No additional boundary conditions were given at the vacuum–insulator interface, because, according to the constructed algorithm, the dielectric permittivity function $\tilde{\varepsilon}(r)$ was given as a continuous quantity over the whole system (see Eq. (1)).

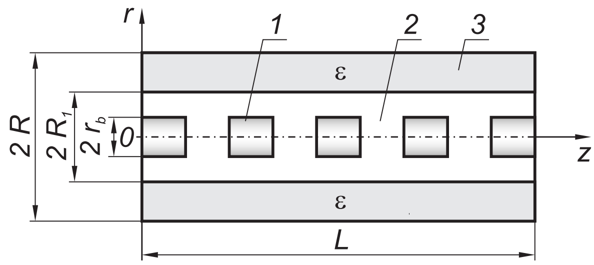

Fig. 1. Geometry of the system under investigation

3. Numerical Algorithm and Simulation Results

The solution of Maxwell's equations and the distribution of current and charge densities were determined in the framework of the finite-difference method and making use of the spatial and temporal meshes which are shifted with respect to each other.

Time quantization of the equations of motion was fulfilled making use of the predictor-corrector method. In so doing, the values of macroparticle velocities were calculated at half-integer time moments $t^{n+1/2} = (n + 1/2)\,\tau$, whereas their coordinates (z_p, r_p) at integer ones $t^n = n\tau$, where n is an integer, and τ is the time step. Hence, the field components in the equations of motion were calculated by means of a linear interpolation between two consecutive mesh nodes.

In accordance with the method selected, the solution of Maxwell's equations should be sought for twice as frequently as the solution of the equations of motion. The magnetic field component H_φ is worth calculating at the time moments $t^{n\pm1/4} = (n \pm 1/4)\,\tau$, the electric field component E_z at t^n, and the electric field component E_r at $t^{n\pm1/2}$.

At the initial time moment, all the components of the electromagnetic field in the system were supposed to be equal to zero, and there were no particles in the resonator.

To prevent the emergence of the electromagnetic field noise with a period equal to the mesh one, the procedure of averaging over nine calculated points was carried out after every ten thousand calculation steps.

The key parameters of the problem were selected to be close to those of an "Almaz" installation; namely, the radius of the drift chamber was $R = 4.3$ cm, the chamber was partially filled with the insulator with the relative dielectric permittivity $\varepsilon = 2.1$, the channel radius in the insulator was $R_1 = 1.05$ cm, the chamber length $L = 55.3$ cm, the radius of electron bunches $r_b = 0.5$ cm, their energy $eU_b = 5$ MeV, the repetition period $T_r = 0.37$ ns, and the bunch duration $T_b = 0.078$ ns. To reduce the

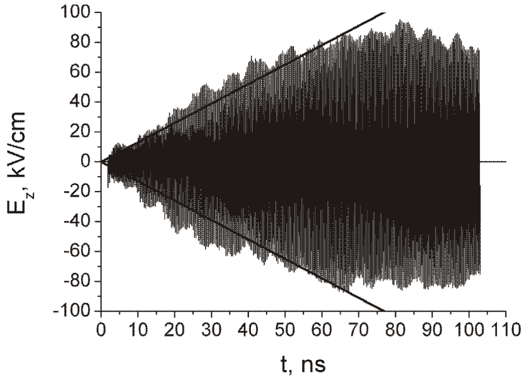

Fig. 2. Dependence of the electric field strength at the axis of the resonator at its right end face on the time

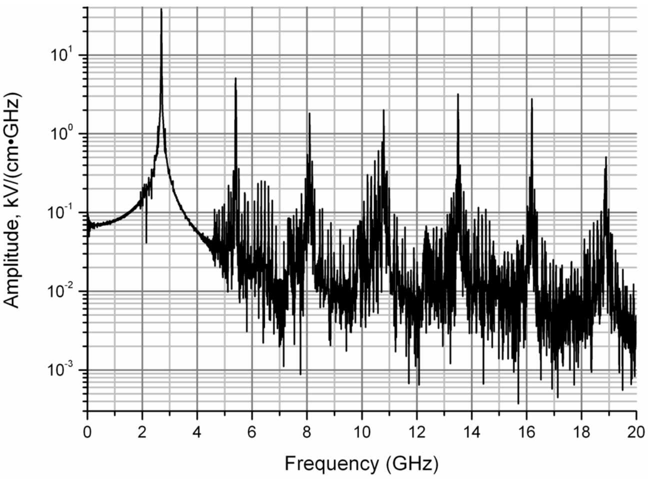

Fig. 3. Spectrum of the electric field shown in Fig. 2

calculation time and calculation errors, the amplitude of the average injected current was chosen to be 20 times larger than that attained in the installation, namely, 10 A.

The results of simulation showed that, within the first 85.5 ns, which corresponds to the injection of 230 bunches, a practically linear growth of the amplitude of the longitudinal electric field up to 95 kV/cm was observed (see Fig. 2). Note that the expected number of bunches, at which the amplitude becomes saturated, is equal to 4600 for a current of 0.5 A.

The spectrum of the electric field, which corresponds to the time dependence of its strength depicted in Fig. 2, is exhibited in Fig. 3. One can see that the main maximum in the spectrum of electric field oscillations is located at a frequency of 2.7 GHz. Other maxima in the spectrum, which are located at higher frequencies and characterized by amplitudes that are approximately 10 times smaller, extend to a frequency of 16.2 GHz.

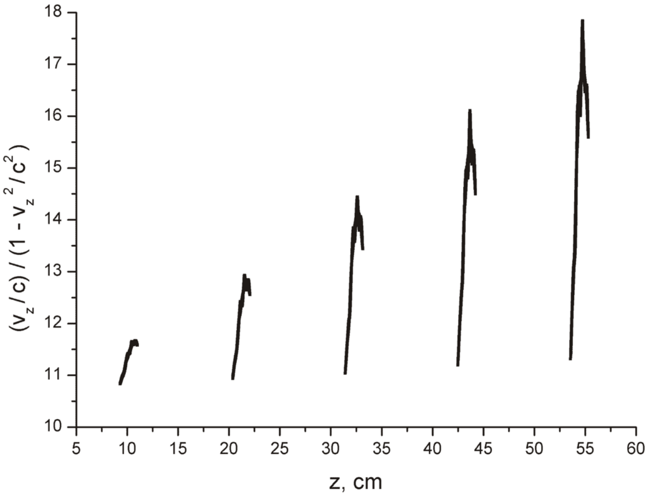

Fig. 4. Phase plane for macroelectrons: the scatter of longitudinal velocities

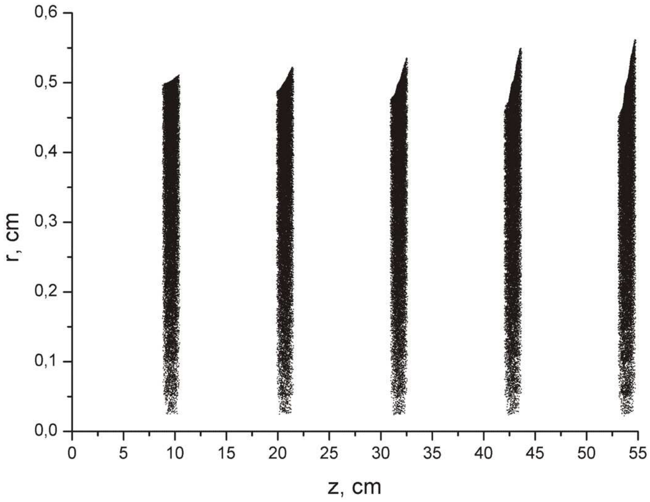

Fig. 5. Configuration space for macroelectrons

The reverse influence of the field on electron bunches, which invoked it, is implemented as a scattering of the longitudinal and transverse components of bunch velocities in the phase plane.

Despite that the velocities of macroparticles became scattered (see Fig. 4), their relativistic factor remained high. Therefore, in the configuration space, only the scatter of transverse coordinates of macroparticles was observed, whereas there was no scatter of macroparticles in the longitudinal direction (see Fig. 5).

Hence, the executed numerical simulation showed that the nonlinear capture processes invoked by the reverse influence of the field with large amplitude on charged bunches brought about a restriction of the electric field amplitude in the system. In the course of

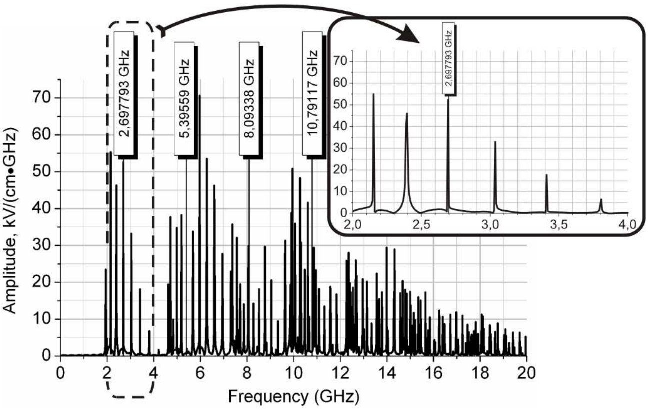

Fig. 6. Spectrum of the longitudinal electric field excited by a single bunch in the resonator. Numbers denote the resonance frequencies of the dielectric waveguide

numerical experiments, the rise time of the field amplitude and the amplitude itself were obtained for the given parameters of the system.

4. Optimization of the System Parameters

Further researches revealed that small variations of the repetition frequency – or, equivalently, the distance between bunches – are accompanied by substantial changes of the field amplitude in the resonator. It is natural to suppose that the maximum of energy losses by the bunches in the train and the corresponding maximum of the wake field amplitude would be observed at repetition frequencies close to the characteristic frequencies of the resonator. Therefore, for the sake of finding the maximal feasible field amplitude, the parameters of the system were optimized in the following way:

– the excitation spectrum of the dielectric resonator was determined;

– the dependence of the field amplitude on the repetition frequency of electron bunches injected into the system in the vicinity of the resonance frequency was obtained;

– the optimal repetition frequency, which corresponds to the maximum of the dependence obtained, was found;

– the field spectrum, which corresponds to the optimal frequency was found, and its difference from a non-optimal spectrum was demonstrated.

The excitation spectrum of the resonator (see Fig. 6) was registered in 50 ns after a single bunch had been injected into the system. In Fig. 6, the numbers indicate the characteristic frequencies of an empty dielectric waveguide (with no electron bunch) calculated theoretically for a symmetric E-wave in the course of

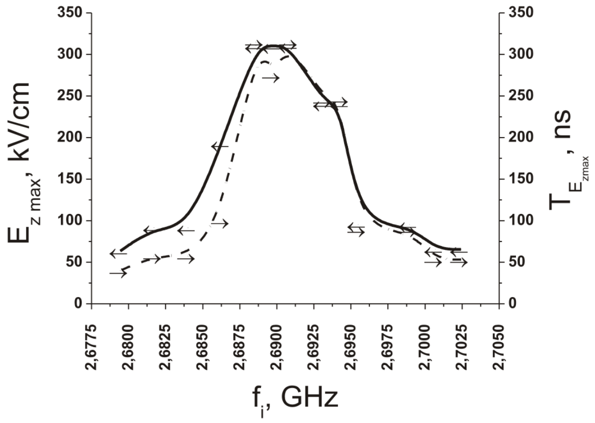

Fig. 7. Dependences of the maximal strength of the excited wake field $E_{z_{\max}}$ (solid curve, left y-axis) and the injection time $T_{E_{z_{\max}}}$ which is needed for the maximum to be attained (dotted curve, right y-axis) on the bunch repetition frequency f_i

solving numerically the dispersion equation

$$\frac{\varepsilon}{\sqrt{\beta_0^2\varepsilon-1}}\frac{J_1(k_dR_1)\,N_0(k_dR)-N_1(k_dR_1)\,J_0(k_dR)}{J_0(k_dR_1)\,N_0(k_dR)-N_0(k_dR_1)\,J_0(k_dR)}-$$

$$-\gamma_0\frac{I_1(k_vR_1)}{I_0(k_vR_1)}=0,$$

where $\beta_0 = v_0/c$, $\gamma_0 = \sqrt{1-v_0^2/c^2}$, $k_v = \omega/(v_0\gamma_0)$, $k_d = \frac{\omega}{v_0}\sqrt{\beta_0^2\varepsilon-1}$; J_j, N_j, and I_j $(j = 0,1)$ are the cylindrical functions of the j-th order, v_0 is the longitudinal velocity of electrons at the input into the drift chamber, and ω is the cyclic frequency.

We are interested in the spectral peak of mode $E_{0,1,4}$ which is located at a frequency of 2.697793 GHz. Then, an optimal value of the bunch repetition frequency, which corresponds to the maximal amplitude of the excited wake field, should be searched for in the vicinity of that frequency. With this purpose in view, a number of numerical experiments were carried out for various distances between bunches in the train at the identical energy $eU = 5$ MeV and the average current $I_b = 10$ A of the injected electron beam.

In Fig. 7, the dependences of the maximal strength of the excited wake field $E_{z_{\max}}$ (solid curve, left y-axis) and the injection time $T_{E_{z_{\max}}}$, which is needed for the maximum to be attained (dotted curve, right y-axis), on the bunch repetition frequency f_i are depicted. One can see that the maximal strength of the wake field $E_{z_{\max}}(f_i)$ and the corresponding injection time $T_{E_{z_{\max}}}(f_i)$ demonstrate a pronounced resonance dependence on the bunch repetition frequency f_i, with a maximum located at $f_i = 2.691$ GHz and possessing

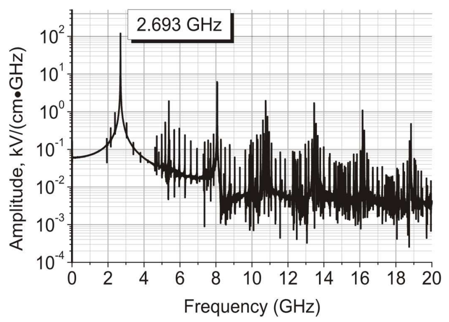

Fig. 8. Spectrum of the wake field at the optimal bunch repetition frequency $f_i = 2.691$ GHz

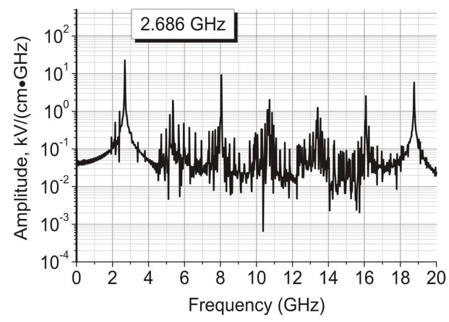

Fig. 9. Spectrum of the wake field at the non-optimal bunch repetition frequency $f_i = 2.679$ GHz

the half-width $\Delta f = 7.5$ MHz. The two dependences are mutually correlated at that, and they practically reproduce each other.

The spectrum of the wake field, which corresponds to the optimal repetition frequency $f_i = 2.691$ GHz, is exhibited in Fig. 8. In the spectrum, there is a well-pronounced line which corresponds to the exciting resonance repetition frequency. At the same time, this line is not observed in the spectrum for the non-optimal frequency $f_i = 2.679$ GHz (see Fig. 9).

5. Conclusion

The numerical simulation showed that the nonlinear capture processes invoked by the reverse influence of the field with large amplitude on charged bunches bring about a restriction of the electric field amplitude in the system. The field-amplitude rise time and the filed amplitude itself were determined. The number of electron bunches, which should be injected into the resonator for the field maximum to be attained, was estimated. The field amplitude was demonstrated to depend strongly on the accuracy of maintaining the resonance between the characteristic frequency of the resonator and the repetition frequency of bunches in the train. In the course of the optimization, it was shown that the field amplitude can be 6 times larger at the optimum repetition frequency in comparison with that in the non-optimum case.

It should be noted that there is a quantitative discrepancy between the values of maximal amplitude obtained from numerical experiments and analytical estimations fulfilled in the framework of the one-dimensional single-wave theory.

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PIC-МОДЕЛЮВАННЯ НЕЛІНІЙНОГО
РЕЖИМУ ЗБУДЖЕННЯ КІЛЬВАТЕРНОГО ПОЛЯ
В ДІЕЛЕКТРИЧНОМУ ЦИЛІНДРИЧНОМУ РЕЗОНАТОРІ

П.І. Марков

Р е з ю м е

Чисельно промодельований нелінійний механізм обмеження амплітуди кільватерного поля, збуджуваного послідовністю релятивістських електронних згустків в циліндричному резонаторі, частково заповненому діелектриком. Моделювання показало, що нелінійні процеси захоплення, викликані зворотним впливом поля великої амплітуди на заряджені згустки, приводять до обмеження амплітуди електричного поля в системі. Отримано час наростання амплітуди поля та його амплітуда. Оцінено кількість електронних згустків, які слід інжектувати у резонатор для досягнення максимуму поля. Показано, що амплітуда поля сильно залежить від точності підтримки резонансу між власною частотою резонатора та частотою надходження згустків послідовності. У ході виконаної оптимізації показано, що за оптимальної частоти повторення амплітуда поля може бути в шість разів вищою порівняно з неоптимальним випадком.