THERMALIZATION IN HEAVY-ION COLLISIONS

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We propose a model for isotropization and corresponding thermalization in a system formed after the collision of two N-particle systems (two nuclei). Two-particle collisions are taken into account. The model is based on two assumptions: (i) three collisions exerted by every particle give rise to the total randomization of its momentum and (ii) the single-particle momentum space is confined from above due to the finite total energy of the system. These features have been shown to result in a smearing of the particle momenta about their initial values and, as a consequence, in their partial isotropization. The nonequilibrium single-particle distribution function has been obtained.

1 Introduction

The problem of isotropization and thermalization in the course of collisions between heavy relativistic ions attracts much attention, because, while describing experimental data, the application of thermodynamic models is one of the basic phenomenological approaches. Recently [1], this issue was examined for quark-gluon plasma produced as a result of ultra-relativistic $A + A$ collisions in experiments at CERN (Geneva) and BNL (Upton) [2].

Nowadays, there exist a few interesting models for the explanation of the thermalization phenomenon in parton systems. Among them, there is a model, which considers, in the framework of quantum chromodynamics, the influence of external color fields on vacuum with the following creation of a thermalized system of hadrons [3]. There is also a model of thermalization in heavy ion collisions, which is considered as a consequence of the Hawking–Unruh effect [4].

Such an enhanced attention to those phenomena arose, because, along with other factors, the assumption about a local thermodynamic equilibrium is successfully applied in various domains of high-energy physics. For instance, the spectra of transverse hadrons look thermalized not only if heavy ions collide, when the creation of many-particle statistical systems is adopted as an indisputable fact [5], but also at an e^+e^- annihilation (see work [6]), when the issue concerning the creation of a many-hadron system remains open for discussion.

In this work, we try to find an explanation for the thermalization phenomenon, starting from such basic concepts as the conservation laws of energy and momentum and leaving the details which are characteristic of every specific process, aside for further researches. Later on, this approach can be considered as a basic one, when studying the mechanisms of isotropization and thermalization at a more microscopic level. We offer a model simplified to a large extent and called *the maximal isotropization model* (MIM) in what follows. This model belongs, to a certain degree, to transport ones; we consider the evolution of the system, but we parameterize this development by the number of collisions of every particle in the system, rather than by the time variable. The idea of maximal isotropization consists in that we suppose that every two-particle scattering is isotropic in the center-of-mass system.

We also suppose that, on the average, such an s-like scattering correctly simulates the process of scattering in a many-particle system. It is obvious that the isotropization in such a model is realized at a maximal rate that is allowed by the conservation laws: (i) the conservation law of the total 4-momentum and (ii) the conservation law of the 4-momentum in every individual two-particle collision. For the time being, we neglect such possible inelastic phenomena as the creation of secondary particles, the existence of resonances, the creation of strings, and so on.

The main idea of our approach consists in the following. Owing to the features of collisions between heavy ions, we assume that the initial (before the first collision) momenta of particles in N-particle system A and N-particle system B (see Fig. 1) are known exactly. More precisely speaking, the initial momentum of every particle in system A is $\boldsymbol{k}_a = \boldsymbol{k}_0 = (0, 0, k_{0z})$, while the initial momentum of every particle in system B is $\boldsymbol{k}_b = -\boldsymbol{k}_0 = (0, 0, -k_{0z})$. The energy and the momentum

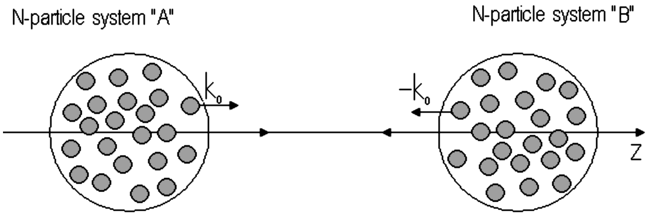

Fig. 1. Scheme of collision between two N-particle systems A and B

are conserved in every separate collision of two particles

$$\omega(\boldsymbol{k}_a) + \omega(\boldsymbol{k}_b) = \omega(\boldsymbol{p}_a) + \omega(\boldsymbol{p}_b) \quad \boldsymbol{k}_a + \boldsymbol{k}_b = \boldsymbol{p}_a + \boldsymbol{p}_b, \tag{1}$$

where $\boldsymbol{k}_a$ and $\boldsymbol{k}_b$ are the initial momenta of this particle pair, while $\boldsymbol{p}_a$ and $\boldsymbol{p}_b$ are the corresponding final momenta. We assume that the particles are on the mass shell, so that $\omega(\boldsymbol{k}) = \sqrt{m^2 + \boldsymbol{k}^2}$ and $\omega(\boldsymbol{p}) = \sqrt{m^2 + \boldsymbol{p}^2}$. We also use the system of units, where $\hbar = c = 1$.

Thus, after the first collision, we have only four equations for the determination of six unknown quantities, $\boldsymbol{p}_a$ and $\boldsymbol{p}_b$. This means that two quantities, e.g., $(\boldsymbol{p}_a)_x$ and $(\boldsymbol{p}_b)_x$, remain uncertain and can be considered as such which accept random values. After the third collision, every component of the momentum of any particle from either of systems A or B become completely uncertain, and we can consider them completely stochastic. One may assume that it is this mechanism of particle-momentum randomization within three collisions only that is the basic mechanism of momentum distribution isotropization and further thermalization of the whole $2N$-particle system.

2 Particle Distribution in Momentum Space

Consider successive variations of the momentum of the n-th particle from system A which moves along the collision axis from left to right. The model is confined to the case, where every particle experiences the identical number of collisions. Every m-th collision induces the variation of the momentum of the n-th particle by the value of $\boldsymbol{p}_n^{(m)}$, so that, after M collisions, the particle acquires the momentum $\boldsymbol{k}_n$:

$$\boldsymbol{k}_0 \;\to\; \boldsymbol{k}_0 + \boldsymbol{p}_n^{(1)} \;\to \cdots \to\; \boldsymbol{k}_0 + \sum_{m=1}^{M} \boldsymbol{p}_n^{(m)} \equiv \boldsymbol{k}_n\,,$$

where $n = 1, 2, \ldots, N$. In full analogy, one can trace the series of momentum values for the n-the particle in system B which moves from right to left; namely,

$$-\boldsymbol{k}_0 \;\to\; -\boldsymbol{k}_0 + \boldsymbol{p}_n^{(1)} \;\to \cdots \to\; -\boldsymbol{k}_0 + \sum_{m=1}^{M} \boldsymbol{p}_n^{(m)} \equiv \boldsymbol{k}_n\,,$$

where $n = N+1, N+2, \ldots, 2N$.

The main goal of this work is to determine f_{2N}, the density distribution functions in the momentum space, which describes two colliding N-particle systems after M collisions per particle. For this purpose, we use the conservation laws of total energy and momentum:

$$E_{\rm tot} = \sum_{n=1}^{2N} \epsilon_n \quad \text{and} \quad \boldsymbol{P}_{\rm tot} = \sum_{n=1}^{2N} \boldsymbol{k}_n\,, \tag{2}$$

where $\epsilon_n = \omega(\boldsymbol{k}_n) = \sqrt{m^2 + \boldsymbol{k}_n^2}$. To be more illustrative, the set of random variables $\boldsymbol{p}_n^{(m)}$ can be presented in the form of a table, where every row appears after that every of N particles from system A (the left side of the table) and B (the right side of the table) has collided. The final momentum $\boldsymbol{k}_1$ of the first particle can be determined by summing up the quantities in the first column of the table, the final momentum $\boldsymbol{k}_2$ of the second particle by summing up the quantities in the second column, and so on. Though the initial values of particle momenta in systems A and B are selected in such a way that the total momentum of the whole system is equal to zero, $\boldsymbol{P}_{\rm tot} = 0$, it is convenient to leave it as an argument. Let us write down the density distribution function in the form

$$f_{2N} = C\widetilde{f}_{2N}\,, \tag{3}$$

where C is the normalization constant. We determine the element of volume in the momentum space accessible for the n-th particle in the series of M collisions as

$$d^3P_n \equiv \frac{1}{V}\,\frac{d^3p_n^{(1)}}{V_p}\,\frac{d^3p_n^{(2)}}{V_p}\cdot\ldots\cdot\frac{d^3p_n^{(M)}}{V_p}, \tag{4}$$

V_p is the accessible volume in the one-particle momentum space, and V is the volume of the system

	1	…	N	$N+1$	…	$2N$
	$\boldsymbol{k}_0$	…	$\boldsymbol{k}_0$	$-\boldsymbol{k}_0$	…	$-\boldsymbol{k}_0$
1	$\boldsymbol{p}_1^{(1)}$	…	$\boldsymbol{p}_N^{(1)}$	$\boldsymbol{p}_{N+1}^{(1)}$	…	$\boldsymbol{p}_{2N}^{(1)}$
2	$\boldsymbol{p}_1^{(2)}$	…	$\boldsymbol{p}_N^{(2)}$	$\boldsymbol{p}_{N+1}^{(2)}$	…	$\boldsymbol{p}_{2N}^{(2)}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
M	$\boldsymbol{p}_1^{(M)}$	…	$\boldsymbol{p}_N^{(M)}$	$\boldsymbol{p}_{N+1}^{(M)}$	…	$\boldsymbol{p}_{2N}^{(M)}$

in the coordinate space. Then the nonnormalized distribution function $\widetilde{f}_{2N}$ can be defined as follows:

$$\widetilde{f}_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\dots,\boldsymbol{k}_{2N})=$$

$$=V_p\,\delta\left(E_{\rm tot}-\sum_{n=1}^{2N}\epsilon_n\right)\delta^3\left(\boldsymbol{P}_{\rm tot}-\sum_{n=1}^{2N}\boldsymbol{k}_n\right)\times$$
$$\times\int\limits_{V_p}\dots\int\limits_{V_p}d^3P_1\dots d^3P_{2N}\times$$
$$\times\prod_{n=1}^{N}\left[\delta^3\left(\boldsymbol{k}_n-\boldsymbol{k}_0-\sum_{m=1}^{M}\boldsymbol{p}_n^{(m)}\right)\right]\times$$
$$\times\prod_{n=N+1}^{2N}\left[\delta^3\left(\boldsymbol{k}_n+\boldsymbol{k}_0-\sum_{m=1}^{M}\boldsymbol{p}_n^{(m)}\right)\right]. \qquad (5)$$

Integration over the momenta $\boldsymbol{p}_n^{(m)}$ in expression (5) stems from the facts that these quantities are considered completely random and equiprobable and every two-particle scattering is spherically symmetric. The multiplier V_p appears at the beginning of the right-hand side of formula (5), because we used the function $\delta^3\left(\boldsymbol{P}_{\rm tot}-\sum_{n=1}^{2N}\boldsymbol{k}_n\right)$, which reflects the conservation of the total momentum of the system and provides the dimension of the distribution density in the microcanonical ensemble, as it is of common use in statistical mechanics.

The momentum space is confined, because, whatever large the initial total momentum of nuclei is prior to the collision, it has a fixed value in any case.

We normalize the density distribution function so that the density of states in the system is simultaneously determined:

$$\Omega_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot})=\int d\tilde{k}_1\dots d\tilde{k}_{2N}\times$$
$$\times\widetilde{f}_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\dots,\boldsymbol{k}_{2N}), \qquad (6)$$

where the element of phase volume for one particle looks like (in units of $\hbar$)

$$d\tilde{k}_n=V\frac{d^3k_n}{(2\pi)^3}. \qquad (7)$$

Hence, the normalization constant C (see Eq. (3)) is equal to

$$C=\frac{1}{\Omega_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot})}. \qquad (8)$$

Making allowance for notation (4), the nonnormalized distribution density (5), which defines a microcanonical ensemble, can be written down in the form

$$\widetilde{f}_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\dots,\boldsymbol{k}_{2N})=$$

$$=\delta\left(E_{\rm tot}-\sum_{n=1}^{2N}\epsilon_n\right)V_p\,\delta^3\left(\boldsymbol{P}_{\rm tot}-\sum_{n=1}^{2N}\boldsymbol{k}_n\right)\times$$
$$\times\prod_{n=1}^{N}\left[\frac{1}{V}\int\frac{d^3a_n}{(2\pi)^3}\,e^{-i\boldsymbol{a}_n\cdot(\boldsymbol{k}_n-\boldsymbol{k}_0)}\times\right.$$
$$\left.\times\prod_{m=1}^{M}\left(\int_{V_p}\frac{d^3p_n^{(m)}}{V_p}\,e^{i\boldsymbol{a}_n\cdot\boldsymbol{p}_n^{(m)}}\right)\right]\times$$
$$\times\prod_{n=N+1}^{2N}\left[\frac{1}{V}\int\frac{d^3b_n}{(2\pi)^3}\,e^{-i\boldsymbol{b}_n\cdot(\boldsymbol{k}_n+\boldsymbol{k}_0)}\times\right.$$
$$\left.\times\prod_{m=1}^{M}\left(\int_{V_p}\frac{d^3p_n^{(m)}}{V_p}\,e^{i\boldsymbol{b}_n\cdot\boldsymbol{p}_n^{(m)}}\right)\right]. \qquad (9)$$

Here, we presented δ-functions, which correspond to the conservation of momentum in a series of two-particle collisions, in terms of the Fourier integrals over the variables $\boldsymbol{a}_n$ and $\boldsymbol{b}_n$.

Let us define an auxiliary function

$$g(\boldsymbol{a})\equiv\int\limits_{V_p}\frac{d^3p}{V_p}\,e^{i\boldsymbol{a}\cdot\boldsymbol{p}}=\prod_{i=1}^{3}\frac{\sin\,(a_ip_{\rm max})}{a_ip_{\rm max}}, \qquad (10)$$

where the confinement of the momentum space has been taken into account: $V_p\propto p_{\rm max}^3$. Making use of this function, the nonnormalized distribution density $\widetilde{f}_{2N}$ can be rewritten in the unified form as

$$\widetilde{f}_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\dots,\boldsymbol{k}_{2N})=$$

$$=\delta\left(E_{\rm tot}-\sum_{n=1}^{2N}\epsilon_n\right)V_p\,\delta^3\left(\boldsymbol{P}_{\rm tot}-\sum_{n=1}^{2N}\boldsymbol{k}_n\right)\times$$
$$\times\prod_{n=1}^{N}\left[\frac{1}{V}\int\frac{d^3a_n}{(2\pi)^3}\,e^{-i\boldsymbol{a}_n\cdot(\boldsymbol{k}_n-\boldsymbol{k}_0)+M\ln g(\boldsymbol{a}_n)}\right]\times$$
$$\times\prod_{n=N+1}^{2N}\left[\frac{1}{V}\int\frac{d^3b_n}{(2\pi)^3}\,e^{-i\boldsymbol{b}_n\cdot(\boldsymbol{k}_n+\boldsymbol{k}_0)+M\ln g(\boldsymbol{b}_n)}\right]. \qquad (11)$$

To obtain the partition function, we should make the Laplace transformation of the density-of-states function

(6) with respect to the variable $E_{\rm tot}$ (it is evident that the function Ω_{2N} depends on the external parameters $E_{\rm tot}$ and $\boldsymbol{P}_{\rm tot}$):

$$Z_{2N}(\beta,\boldsymbol{P}_{\rm tot}) = \int\limits_{E_{\min}}^{\infty} dE_{\rm tot}\, e^{-\beta E_{\rm tot}}\, \Omega_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot}) =$$

$$= \int\limits_{E_{\min}}^{\infty} dE_{\rm tot}\, e^{-\beta E_{\rm tot}} \int d\tilde{k}_1 \ldots d\tilde{k}_{2N} \times$$

$$\times \widetilde{f}_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\ldots,\boldsymbol{k}_{2N})\,. \tag{12}$$

This procedure, after the order of integration has been changed, brings about the Laplace transformation of the nonnormalized density distribution function $\tilde{f}_{2N}$:

$$\widetilde{\mathbb{F}}_{2N}(\beta,\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\ldots,\boldsymbol{k}_{2N}) =$$

$$= \int\limits_{E_{\min}}^{\infty} dE_{\rm tot}\, e^{-\beta E_{\rm tot}}\, \widetilde{f}_{2N}(E_{\rm tot},\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\ldots,\boldsymbol{k}_{2N})\,, \tag{13}$$

Taking Eq. (11) into account, we obtain

$$\widetilde{\mathbb{F}}_{2N}(\beta,\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\ldots,\boldsymbol{k}_N;\,\boldsymbol{k}_{N+1},\ldots,\boldsymbol{k}_{2N}) =$$

$$= V_p\, \delta^3\left(\boldsymbol{P}_{\rm tot} - \sum_{n=1}^{2N}\boldsymbol{k}_n\right) \times$$

$$\times \prod_{n=1}^{N}\left[e^{-\beta\epsilon_n}\frac{1}{V}\int \frac{d^3a_n}{(2\pi)^3}\, e^{-i\boldsymbol{a}_n\cdot(\boldsymbol{k}_n-\boldsymbol{k}_0)+M\ln g(\boldsymbol{a}_n)}\right]\times$$

$$\times \prod_{n=N+1}^{2N}\left[e^{-\beta\epsilon_n}\frac{1}{V}\int \frac{d^3b_n}{(2\pi)^3}\, e^{-i\boldsymbol{b}_n\cdot(\boldsymbol{k}_n+\boldsymbol{k}_0)+M\ln g(\boldsymbol{b}_n)}\right]. \tag{14}$$

While doing the Laplace transformation, we did not consider the possible dependence of $p_{\max}$ on $E_{\rm tot}$. Therefore, in what follows, the value of $p_{\max}$ will be taken constant, thus it plays the role of a parameter.

3 Partition Function for a Large Number of Collisions

To calculate the integrals over the variables $\boldsymbol{a}_n$ and $\boldsymbol{b}_n$ on the right-hand side of Eq. (14) in the case where the number of collisions for every particle is large, $M \gg 1$, the saddle-point method can be applied. It implies the use of the following approximation:

$$e^{M\ln g(\boldsymbol{a})} = e^{M\sum\limits_{i=1}^{3}\ln\frac{\sin(a_i p_{\max})}{a_i p_{\max}}} \approx e^{-\frac{M}{6}\boldsymbol{a}_n^2 p_{\max}^2}\,. \tag{15}$$

This expression is to be substituted to Eq. (14). The analogous operation is fulfilled in relation to the integration variables $\boldsymbol{b}_n$. Using the Poisson integral, let us rewrite the function $\widetilde{\mathbb{F}}_{2N}$ as follows:

$$\widetilde{\mathbb{F}}_{2N}(\beta,\boldsymbol{P}_{\rm tot};\boldsymbol{k}_1,\ldots,\boldsymbol{k}_{2N}) = V_p\,\delta^3\left(\boldsymbol{P}_{\rm tot} - \sum_{n=1}^{2N}\boldsymbol{k}_n\right)\times$$

$$\times \prod_{n=1}^{N}\left[e^{-\beta\epsilon_n}\frac{1}{V}\left(\frac{6\pi}{Mp_{\max}^2}\right)^{3/2} e^{-\frac{3(\boldsymbol{k}_n-\boldsymbol{k}_0)^2}{2Mp_{\max}^2}}\right]\times$$

$$\times \prod_{n=N+1}^{2N}\left[e^{-\beta\epsilon_n}\frac{1}{V}\left(\frac{6\pi}{Mp_{\max}^2}\right)^{3/2} e^{-\frac{3(\boldsymbol{k}_n+\boldsymbol{k}_0)^2}{2Mp_{\max}^2}}\right]. \tag{16}$$

To calculate the partition function $Z_{2N}(\beta)$ for the canonical ensemble, we use the element of integration volume, as was defined in Eq. (7), and the previous equation:

$$Z_{2N}(\beta,\boldsymbol{P}_{\rm tot}) = V_p\int d^3x e^{-i\boldsymbol{P}_{\rm tot}\cdot\boldsymbol{x}}\times$$

$$\times \prod_{n=1}^{N}\left[(4\pi\alpha)^{3/2}\int\frac{d^3k_n}{(2\pi)^3}e^{-\beta\omega(\boldsymbol{k}_n)+i\boldsymbol{k}_n\cdot\boldsymbol{x}-\alpha(\boldsymbol{k}_n-\boldsymbol{k}_0)^2}\right]\times$$

$$\times \prod_{n=N+1}^{2N}\left[(4\pi\alpha)^{3/2}\int\frac{d^3k_n}{(2\pi)^3}e^{-\beta\omega(\boldsymbol{k}_n)+i\boldsymbol{k}_n\cdot\boldsymbol{x}-\alpha(\boldsymbol{k}_n+\boldsymbol{k}_0)^2}\right], \tag{17}$$

where the δ-function, which reflects the conservation law of total momentum, is presented in the form of a Fourier integral over the variable $\boldsymbol{x}$, and the parameter α is determined as

$$\alpha \equiv \frac{3}{2Mp_{\max}^2}\,. \tag{18}$$

Let us define auxiliary one-particle functions

$$\mathbb{Y}_{a(b)}(\beta,\boldsymbol{x}) \equiv (4\pi\alpha)^{3/2}\int\frac{d^3k}{(2\pi)^3}\times$$

$$\times \exp\left[-\beta\omega(\boldsymbol{k}) + i\boldsymbol{k}\cdot\boldsymbol{x} - \alpha(\boldsymbol{k}\mp\boldsymbol{k}_0)^2\right] \tag{19}$$

for subsystems A and B, respectively. With their help, the partition function $Z_{2N}(\beta, \boldsymbol{P}_{\rm tot})$ from Eq. (17) can be rewritten as follows:

$$Z_{2N}(\beta, \boldsymbol{P}_{\rm tot}) = V_p \int d^3x e^{-i\boldsymbol{P}_{\rm tot}\cdot\boldsymbol{x} + N\ln\mathbb{Y}_a(\beta,\boldsymbol{x}) + N\ln\mathbb{Y}_b(\beta,\boldsymbol{x})} . \tag{20}$$

This expression will be basic for the further research in this work.

3.1. Collision of two identical systems

In the case of the collision between two identical systems, i.e. when all particles in the one system have the initial momentum $\boldsymbol{k}_0$, and all particles in the other system have the initial momentum $-\boldsymbol{k}_0$, the calculation of partition function (20) will be carried out making use of the saddle-point method at $N \gg 1$. We expand the functions $\ln\mathbb{Y}_a(\beta,\boldsymbol{x})$ and $\ln\mathbb{Y}_b(\beta,\boldsymbol{x})$ (see Eq. (19)) in series in the variable $\boldsymbol{x}$ about the point $\boldsymbol{x}=0$ up to the second order inclusive:

$$\ln\mathbb{Y}_a(\beta,\boldsymbol{x}) \approx \ln\mathbb{Y}_a(\beta,0) + \sum_{i=1}^{3}\left[\frac{1}{\mathbb{Y}_a(\beta,\boldsymbol{x})}\frac{\partial\mathbb{Y}_a(\beta,\boldsymbol{x})}{\partial x_i}\right]_{\boldsymbol{x}=0} x_i +$$
$$+\frac{1}{2}\sum_{i,j=1}^{3}\left[\frac{1}{\mathbb{Y}_a(\beta,\boldsymbol{x})}\frac{\partial^2\mathbb{Y}_a(\beta,\boldsymbol{x})}{\partial x_j\partial x_i} -\right.$$
$$\left.-\frac{1}{\mathbb{Y}_a^2(\beta,\boldsymbol{x})}\frac{\partial\mathbb{Y}_a(\beta,\boldsymbol{x})}{\partial x_j}\frac{\partial\mathbb{Y}_a(\beta,\boldsymbol{x})}{\partial x_i}\right]_{\boldsymbol{x}=0} x_j x_i . \tag{21}$$

We consider $\mathbb{Y}_a(\beta,0) = \mathbb{Y}_a(\beta)$ and $\mathbb{Y}_b(\beta,0) = \mathbb{Y}_b(\beta)$ as the one-particle partition functions. In this case, the terms in brackets on the right-hand side of expression (21) are proportional to the statistical averages of k_i, k_ik_j, and so on. We designate such quantities by angle parentheses $\langle\ldots\rangle_a$ and $\langle\ldots\rangle_b$, where the subscripts a and b mean that the averaging is carried out with the use of the one-particle partition function $\mathbb{Y}_a(\beta)$ or $\mathbb{Y}_b(\beta)$, respectively:

$$\langle\ldots\rangle_{a(b)} \equiv \frac{(4\pi\alpha)^{3/2}}{\mathbb{Y}_{a(b)}(\beta)}\int\frac{d^3k}{(2\pi)^3}(\ldots)e^{-\beta\omega(\boldsymbol{k})-\alpha(\boldsymbol{k}\mp\boldsymbol{k}_0)^2} . \tag{22}$$

Thus, we can rewrite expression (21) in terms of the averaged momentum components:

$$\ln\mathbb{Y}_a(\beta,\boldsymbol{x}) \approx \ln\mathbb{Y}_a(\beta) + i\sum_{i=1}^{3}\langle k_i\rangle_a x_i -$$
$$-\frac{1}{2}\sum_{i,j=1}^{3}\Big(\langle k_ik_j\rangle_a - \langle k_i\rangle_a\langle k_j\rangle_a\Big)x_ix_j . \tag{23}$$

It is clear that the analogous expression can be written down for particles from system B as well.

Now, we introduce the correlation function of momenta,

$$C_{ij}^{(r)} \equiv \langle k_ik_j\rangle_r - \langle k_i\rangle_r\langle k_j\rangle_r , \tag{24}$$

where $r = a$ or b, and rewrite expression (23) for system A and the analogous one for system B in the same format:

$$\ln\mathbb{Y}_r(\beta,\boldsymbol{x}) \approx \ln\mathbb{Y}_r(\beta,0) + i\langle\boldsymbol{k}\rangle_r\cdot\boldsymbol{x} - \frac{1}{2}\sum_{i,j=1}^{3}C_{ij}^{(r)}x_ix_j . \tag{25}$$

This allows us to rewrite partition function (20) as follows:

$$Z_{2N}(\beta) \approx \mathbb{Y}_a^N(\beta)\,\mathbb{Y}_b^N(\beta)\,V_p\int d^3x\times$$
$$\times\exp\left[-i\boldsymbol{P}_{\rm tot}\cdot\boldsymbol{x} + i\,2N\langle\boldsymbol{k}\rangle\cdot\boldsymbol{x} - N\sum_{i,j=1}^{3}C_{ij}x_ix_j\right] , \tag{26}$$

where

$$C_{ij} = \frac{1}{2}\left(C_{ij}^{(a)} + C_{ij}^{(b)}\right) , \quad \langle\boldsymbol{k}\rangle = \frac{1}{2}\Big(\langle\boldsymbol{k}\rangle_a + \langle\boldsymbol{k}\rangle_b\Big) . \tag{27}$$

Integrating the Poisson integral on the right-hand side of Eq. (26) brings about a simpler result

$$Z_{2N}(\beta) \approx \mathbb{Y}_a^N(\beta)\,\mathbb{Y}_b^N(\beta)\left(\frac{2\pi}{2N}\right)^{3/2}\frac{V_p}{\left(\det\widehat{C}\right)^{1/2}}\times$$
$$\times\exp\left[-N\left(\boldsymbol{p}_{\rm tot} - \langle\boldsymbol{k}\rangle\right)\cdot\widehat{C}^{-1}\cdot\left(\boldsymbol{p}_{\rm tot} - \langle\boldsymbol{k}\rangle\right)\right] , \tag{28}$$

where $\boldsymbol{p}_{\rm tot} = \boldsymbol{P}_{\rm tot}/2N$.

From expression (28), one can determine the **"common"** one-particle partition function, which simultaneously concerns both systems A and B:

$$z(\beta) = \mathbb{Y}_a(\beta)\,\mathbb{Y}_b(\beta)\left(\frac{\pi}{N}\right)^{\frac{3}{2N}}\left(\frac{V_p}{\left(\det\widehat{C}\right)^{1/2}}\right)^{1/N}\times$$
$$\times\exp\left[-\left(\boldsymbol{p}_{\rm tot} - \langle\boldsymbol{k}\rangle\right)\cdot\widehat{C}^{-1}\cdot\left(\boldsymbol{p}_{\rm tot} - \langle\boldsymbol{k}\rangle\right)\right] , \tag{29}$$

so that we can write down

$$Z_{2N}(\beta) = z^N(\beta)\,. \tag{30}$$

We would like to emphasize the validity of the following limit value:

$$\lim_{N\to\infty}\left(\frac{1}{N}\right)^{1/N} = 1 \quad\Longrightarrow\quad \lim_{N\to\infty}\left(\frac{\pi}{N}\right)^{\frac{3}{2N}} = 1\,.$$

For instance, at $N = 10$, 100, and 1000, the corresponding estimations are $(\pi/N)^{3/2N} = 0.84$, 0.95, and 0.99. As a result, in the case where $\boldsymbol{P}_{\rm tot} = 0$ and provided that the particle number N is rather large, it follows from Eq. (29) that

$$\lim_{N\to\infty} z(\beta) = \mathbb{Y}_a(\beta)\,\mathbb{Y}_b(\beta)\,e^{-\langle \boldsymbol{k}\rangle\cdot\widehat{C}^{-1}\cdot\langle \boldsymbol{k}\rangle}\,.$$

The last result can be simplified for a many-particle system without given initial momentum (e.g., the pion creation when $\boldsymbol{k}_0 = 0$ and the initial momentum distribution is isotropic), namely

$$\lim_{N\to\infty} z(\beta) = \mathbb{Y}_a(\beta) = \mathbb{Y}_b(\beta)\,.$$

3.2. Calculation of correlation matrix C_{ij}

Consider now the calculation of elements of the correlation matrix for the collision of two identical nuclei in the center-of-mass frame, i.e. we adopt the initial momentum $\boldsymbol{k}_0 = (0, 0, k_{0z})$.

We would like to recall the definition of the correlation matrix,

$$C_{ij} \equiv \frac{1}{2}\left[C_{ij}^{(a)} + C_{ij}^{(b)}\right] =$$

$$= \frac{1}{2}\left[\langle k_i k_j\rangle_a + \langle k_i k_j\rangle_b - \langle k_i\rangle_a\langle k_j\rangle_a - \langle k_i\rangle_b\langle k_j\rangle_b\right]\,. \tag{31}$$

First, we note that the average momentum in a perpendicular plane is equal to zero, $\langle k_x\rangle_{a(b)} = \langle k_y\rangle_{a(b)} = 0$. It is evident from definition (22) that the effective average momentum (27) is

$$\langle \boldsymbol{k}\rangle \equiv \frac{1}{2}\Big(\langle \boldsymbol{k}\rangle_a + \langle \boldsymbol{k}\rangle_b\Big) = \left(0, 0, \frac{1}{2}\left(\langle k_z\rangle_a + \langle k_z\rangle_b\right)\right)\,,$$

while

$$\langle k_x k_y\rangle_{a(b)} = \langle k_x k_z\rangle_{a(b)} = \langle k_y k_z\rangle_{a(b)} = 0\,, \tag{32}$$

i.e. all the nondiagonal elements of the correlation matrix are equal to zero. As a result, the matrices $\hat{C}^{(a)}$ and $\hat{C}^{(b)}$ are diagonal; hence, the matrix inverse to $\hat{C}$ is diagonal as well, namely,

$$\widehat{C}^{-1} = \begin{pmatrix} \dfrac{2}{C_{11}^{(a)}+C_{11}^{(b)}} & 0 & 0 \\ 0 & \dfrac{2}{C_{22}^{(a)}+C_{22}^{(b)}} & 0 \\ 0 & 0 & \dfrac{2}{C_{33}^{(a)}+C_{33}^{(b)}} \end{pmatrix}.$$

Consider the one-particle partition function (29) in the center-of-mass system, i.e. in the case where $\boldsymbol{p}_{\rm tot} = \boldsymbol{P}_{\rm tot}/2N = 0$. Bearing the structure of the correlation matrix in mind and taking the value of the average moment $\langle \boldsymbol{k}\rangle$ into account, partition function (29) can be rewritten in the form

$$z(\beta) = \mathbb{Y}_a(\beta)\,\mathbb{Y}_b(\beta)\left(\frac{2\pi}{N}\right)^{\frac{3}{2N}} \times$$

$$\times\left[\frac{V_p}{\prod_{i=1}^3\left(C_{ii}^{(a)}+C_{ii}^{(b)}\right)^{1/2}}\right]^{\frac{1}{N}} e^{-\,C_{zz}^{-1}\langle k_z\rangle^2}\,. \tag{33}$$

On the other hand, the z-component of the average momentum is equal to

$$\langle k_z\rangle = \frac{1}{2}\Big(\langle k_z\rangle_a + \langle k_z\rangle_b\Big) =$$

$$= \frac{(4\pi\alpha)^{3/2}}{2\mathbb{Y}_a(\beta)}\int\frac{d^3k}{(2\pi)^3}k_z\exp\left[-\beta\omega(\boldsymbol{k}) - \alpha(\boldsymbol{k}-\boldsymbol{k}_0)^2\right] +$$

$$+\frac{(4\pi\alpha)^{3/2}}{2\mathbb{Y}_b(\beta)}\int\frac{d^3k}{(2\pi)^3}\,k_z\,\exp\left[-\beta\omega(\boldsymbol{k}) - \alpha(\boldsymbol{k}+\boldsymbol{k}_0)^2\right]. \tag{34}$$

Note that the change of the integration variable $k_z \to -k_z$ in integral (19) at $\boldsymbol{x} = 0$ makes it obviuos that

$$\mathbb{Y}_a(\beta) = \mathbb{Y}_b(\beta)\,. \tag{35}$$

Using this equality, we obtain, from Eq. (34), that

$$\langle k_z\rangle \sim \int\limits_{-\infty}^{\infty}\frac{dk_z}{2\pi}\,k_z\,e^{-\beta\omega(\boldsymbol{k})}\left[e^{-\alpha(k_z-k_{0z})^2}+e^{-\alpha(k_z+k_{0z})^2}\right]. \tag{36}$$

Here, the integrand is no more than a product of even and odd functions. Owing to the symmetry of the integration limits, the result of integration vanishes. At last, we have $\langle k_z\rangle = \frac{1}{2}\left(\langle k_z\rangle_a + \langle k_z\rangle_b\right) = 0$, i.e. $\langle k_z\rangle_a = -\langle k_z\rangle_b$. This result was expectedly obtained in the center-of-mass system, where $\boldsymbol{P}_{\rm tot}$ equals zero.

Taking into account that $\mathbb{Y}_a(\beta) = \mathbb{Y}_b(\beta)$ and the matrices $\hat{C}^{(a)}$ and $\hat{C}^{(b)}$ are diagonal, we draw a conclusion that these matrices coincide, i.e.

$$\widehat{C}^{(a)} = \widehat{C}^{(b)} = \widehat{C}\,. \tag{37}$$

As a consequence, we can simplify Eq. (33) and write down the ultimate expression for the one-particle partition function:

$$\lim_{N\to\infty} z(\beta) = \mathbb{Y}_a(\beta)\,\mathbb{Y}_b(\beta) =$$

$$= (4\pi\alpha)^3 \int \frac{d^3k_a}{(2\pi)^3} \exp\left[-\beta\omega(\boldsymbol{k}_a) - \alpha(\boldsymbol{k}_a - \boldsymbol{k}_0)^2\right]\times$$

$$\times \int \frac{d^3k_b}{(2\pi)^3} \exp\left[-\beta\omega(\boldsymbol{k}_b) - \alpha(\boldsymbol{k}_b + \boldsymbol{k}_0)^2\right], \tag{38}$$

where the approximation

$$\left(\frac{\pi}{N}\right)^{\frac{3}{2N}} \left(\frac{V_p}{\left(\det\widehat{C}\right)^{1/2}}\right)^{\frac{1}{N}} \approx 1$$

fair at large enough N was taken into account.

Hence, on the basis of the approach proposed, we have obtained the following nonequilibrium distribution function:

$$f(\boldsymbol{k}_a, \boldsymbol{k}_b) = \frac{(4\pi\alpha)^3}{z_a(\beta) z_b(\beta)} \exp\left[-\beta\omega(\boldsymbol{k}_a) - \alpha(\boldsymbol{k}_a - \boldsymbol{k}_0)^2\right]\times$$

$$\times \exp\left[-\beta\omega(\boldsymbol{k}_b) - \alpha(\boldsymbol{k}_b + \boldsymbol{k}_0)^2\right], \tag{39}$$

where

$$z_a(\beta) = (4\pi\alpha)^{3/2} \int d^3k\, e^{-\beta\omega(\boldsymbol{k}) - \alpha(\boldsymbol{k} - \boldsymbol{k}_0)^2},$$

$$z_b(\beta) = (4\pi\alpha)^{3/2} \int d^3k\, e^{-\beta\omega(\boldsymbol{k}) - \alpha(\boldsymbol{k} + \boldsymbol{k}_0)^2}. \tag{40}$$

One can see that distribution function (39) demonstrates features associated with the central limit theorem, which manifest themselves in the availability of two Gaussians symmetrically located in the momentum space, if the analysis is carried out in the center-of-mass system of two many-particle systems (nuclei). The approach proposed allows not only the expected general result to be obtained, but also the expression for the dispersion and the distribution centers to be deduced, which can be checked up experimentally in nucleus-nucleus collisions at high energies.

In particular, we note that the increase in the number of collisions M and the accessible volume in the momentum space, which is determined by the parameter $p_{\max}$ (see Eq. (18)), gives rise to the smearing of this effect and the transformation of the distribution into the thermal one.

3.3. One-particle spectrum

Consider the spectrum of particles which are formed when two identical nuclei collide. If every nucleon from this large system, according to the model proposed, takes part in M collisions, the distribution (as well as an arbitrary function $D(\boldsymbol{k}_a, \boldsymbol{k}_b)$ of random variables) can be obtained by averaging the quantity

$$D(\boldsymbol{p}, \boldsymbol{k}_a, \boldsymbol{k}_b) = N\delta^3(\boldsymbol{p} - \boldsymbol{k}_a) + N\delta^3(\boldsymbol{p} - \boldsymbol{k}_b) \tag{41}$$

over the values of $\boldsymbol{k}_a$ and $\boldsymbol{k}_b$, making use of the one-particle partition function (38). Really, in the general case, the value averaged over the ensemble of two subsystems (A and B) is calculated as

$$\langle D\rangle = \frac{(4\pi\alpha)^3}{z_a(\beta) z_b(\beta)} \int \frac{d^3k_a}{(2\pi)^3} \frac{d^3k_b}{(2\pi)^3} D(\boldsymbol{p}, \boldsymbol{k}_a, \boldsymbol{k}_b)\times$$

$$\times e^{-\beta\omega(\mathbf{k}_a) - \alpha(\boldsymbol{k}_a - \boldsymbol{k}_0)^2}\, e^{-\beta\omega(\boldsymbol{k}_b) - \alpha(\boldsymbol{k}_b + \boldsymbol{k}_0)^2}, \tag{42}$$

Then, after averaging the random quantity (41), the spectrum looks like

$$\frac{d^3N}{dp^3} = \langle D\rangle = N\,(4\pi\alpha)^{3/2}\, e^{-\beta\omega(\boldsymbol{p})}\times$$

$$\times \left[\frac{1}{z_a(\beta)}\, e^{-\alpha(\boldsymbol{p} - \boldsymbol{k}_0)^2} + \frac{1}{z_b(\beta)}\, e^{-\alpha(\boldsymbol{p} + \boldsymbol{k}_0)^2}\right]. \tag{43}$$

We distinguish between the one-particle partition functions $z_a(\beta)$ and $z_b(\beta)$, because the region in the momentum space, where the momenta of particles are measured, can be nonsymmetric in a specific experiment. Certainly, in the case of a region symmetric with respect to the zero momentum, the equalities $z_a(\beta) = z_b(\beta) = z_0(\beta)$ take place; in particular, they are valid for an infinite momentum space.

If we consider the initial momenta $\boldsymbol{k}_0 = (0, 0, \pm k_{0z})$, the spectrum can possess two maxima near the two values of the longitudinal momentum: $p_z = k_{0z}$ and $p_z = -k_{0z}$. Let the notation $\boldsymbol{p}_\perp^2$ stand for $p_x^2 + p_y^2$; then, the particle spectrum in the momentum space looks like

$$\frac{d^3N}{dp^3} = \frac{N\,(4\pi\alpha)^{3/2}}{z_0(\beta)}\, e^{-\beta\omega(\boldsymbol{p}) - \alpha\boldsymbol{p}_\perp^2} \times$$

$$\times \left[e^{-\alpha(p_z - k_{0z})^2} + e^{-\alpha(p_z + k_{0z})^2} \right], \tag{44}$$

where

$$z_0(\beta) = (4\pi\alpha)^{3/2} \int d^3k\, e^{-\beta\omega(\boldsymbol{k}) - \alpha\left[\boldsymbol{k}_\perp^2 + (k_z - k_{0z})^2\right]}. \tag{45}$$

3.4. Effective temperature at $\boldsymbol{k}_0 = 0$

At low-energy collisions, when a nonrelativistic behavior can be admitted for particles created in the system after the collision – i.e. if $T = 1/\beta \ll m$ and, as a consequence, $\omega(\boldsymbol{p}) \approx m + \boldsymbol{p}^2/2m$, where m is the particle's mass, – the following estimation for the effective temperature can be obtained:

$$\int \frac{d^3k}{(2\pi)^3} e^{-\frac{\boldsymbol{k}^2}{2mT} - \alpha\boldsymbol{k}^2} =$$

$$= \prod_{i=1}^{3} \int_{-\infty}^{\infty} \frac{dk_i}{2\pi} e^{-\frac{k_i^2}{2mT_{\rm eff}}} = \left(\frac{mT_{\rm eff}}{2\pi} \right)^{3/2}, \tag{46}$$

where

$$\frac{1}{T_{\rm eff}} = \frac{1}{T} + \frac{1}{T_{\rm coll}}, \tag{47}$$

and the collision "temperature" is defined as

$$T_{\rm coll} \equiv \frac{Mp_{\rm max}^2}{3m}. \tag{48}$$

For the thermal length of a wave determined as $\Lambda = \sqrt{2\pi/mT}$ and, respectively, $\Lambda_{\rm eff} = \sqrt{2\pi/mT_{\rm eff}}$, the spectrum can be written down in the form

$$\frac{d^3N}{dp^3} = \frac{N\,\Lambda_{\rm eff}^3}{(2\pi)^3}\, e^{-\frac{\boldsymbol{p}^2}{2mT_{\rm eff}}}. \tag{49}$$

The confinement on the accessible volume in the momentum space and the finiteness of the collision number turn out to effectively reduce the temperature, $T_{\rm eff} \le T$. Really, according to Eq. (47),

$$T_{\rm eff} = \frac{T}{1 + T/T_{\rm coll}} \le T. \tag{50}$$

Thus, the increase of the collision number M is accompanied by the growth of the effective temperature $T_{\rm eff}$ up to its limit value T. It is the reason of why, when the parameter M is large enough, the quantity $T_{\rm eff}$ in Eq. (49) should be replace by T. On the other hand, if $p_{\rm max}$ tends to infinity, spectrum (49) acquires the standard form

$$\lim_{p_{\rm max}\to\infty} \frac{d^3N}{dp^3} = \frac{N\,\Lambda^3}{(2\pi)^3}\, e^{-\frac{\boldsymbol{p}^2}{2mT}}. \tag{51}$$

In the general case ($T \sim m$), the relativistic dispersion relation $\omega(\mathbf{p}) = \sqrt{m^2 + \mathbf{p}^2}$ has to be used. Let us make the inverse Laplace transformation of the many-particle partition function:

$$\Omega_N(E_{\rm tot}) = \int_{c-i\infty}^{c+i\infty} d\beta\, e^{\beta E_{\rm tot}}\, Z_N(\beta). \tag{52}$$

In order to calculate the integral on the right-hand side, we use the saddle-point method. For this purpose, the last expression is rewritten in the form

$$\Omega_N(E_{\rm tot}) = \int_{c-i\infty}^{c+i\infty} d\beta\, e^{\beta E_{\rm tot} + N\log z(\beta)} = \int_{c-i\infty}^{c+i\infty} d\beta\, e^{F(\beta)}, \tag{53}$$

where Eq. (30) was used. Now, the minimum of the function $F(\beta)$ along the imaginary axis and its maximum along the real axis of the variable β are to be determined. The condition for the extremum to take place along the real axis (the variable β is real) looks like

$$\frac{1}{N} E_{\rm tot} = -\frac{1}{z(\beta)} \frac{\partial z(\beta)}{\partial \beta}. \tag{54}$$

The solution of this equation with respect to β gives the value of the system temperature $T = 1/\beta$, which corresponds to the average energy value per particle, namely, $E_{\rm tot}/N$. Here, we may use the explicit form of the one-particle partition function (45) at $\boldsymbol{k}_0 = 0$ to obtain

$$\frac{1}{N} E_{\rm tot} = \frac{(4\pi\alpha)^{3/2}}{z(\beta)} \int \frac{d^3k}{(2\pi)^3}\, \omega(\boldsymbol{k})\, \exp\left[-\beta\omega(\boldsymbol{k}) - \alpha\boldsymbol{k}^2 \right]. \tag{55}$$

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We have obtained a transcendental equation for the parameter β.The temperature value, which is the root of this equation, differs from the value which is determined from the similar equation for the ideal gas

$$\frac{1}{N}\,E_{\rm tot} = \frac{V}{z_{\rm id}(\beta)} \int \frac{d^3k}{(2\pi)^3}\,\omega(\boldsymbol{k})\,e^{-\beta\omega(\boldsymbol{k})},$$

where

$$z_{\rm id}(\beta) = V \int \frac{d^3k}{(2\pi)^3}\,e^{-\beta\omega(\boldsymbol{k})}$$

is the one-particle partition function for the ideal gas.

Hence, Eq. (55) evidences for the dependence of the system temperature on the collision number M per particle and on the $p_{\max}$-value that confines the set of attainable momenta.

Similarly to what was done in the nonrelativistic case, one can introduce the idea of effective temperature $T_{\rm eff}$. By definition, it is the temperature of the ideal gas, at which the average one-particle energy $E_{\rm tot}/N$ is the same:

$$\frac{(4\pi\alpha)^{3/2}}{z(T)} \int \frac{d^3k}{(2\pi)^3}\,\omega(\boldsymbol{k})\,e^{-\frac{\omega(\boldsymbol{k})}{T}-\alpha\boldsymbol{k}^2} = \frac{E_{\rm tot}}{N} =$$

$$= \frac{V}{z_{\rm id}(T_{\rm eff})} \int \frac{d^3k}{(2\pi)^3}\,\omega(\boldsymbol{k})\,e^{-\frac{\omega(\boldsymbol{k})}{T_{\rm eff}}}\,. \tag{56}$$

To illustrate the estimation of the parameter $p_{\max}$, we use the approximate relation

$$E_{\rm tot}/N = \sqrt{m^2 + 3p_{\max}^2}\,. \tag{57}$$

In Fig. 2, the results of calculations of the relation between the system temperature and the corresponding effective one – both are obtained as the solutions of Eq. (56) taking condition (57) into account – are depicted. The horizontal straight line in this figure corresponds to a constant value of one-particle average energy. That is, for this straight line, $\langle\omega(\boldsymbol{k})\rangle$ = const. The intersections of this straight line with a curve determine the temperature which corresponds to this

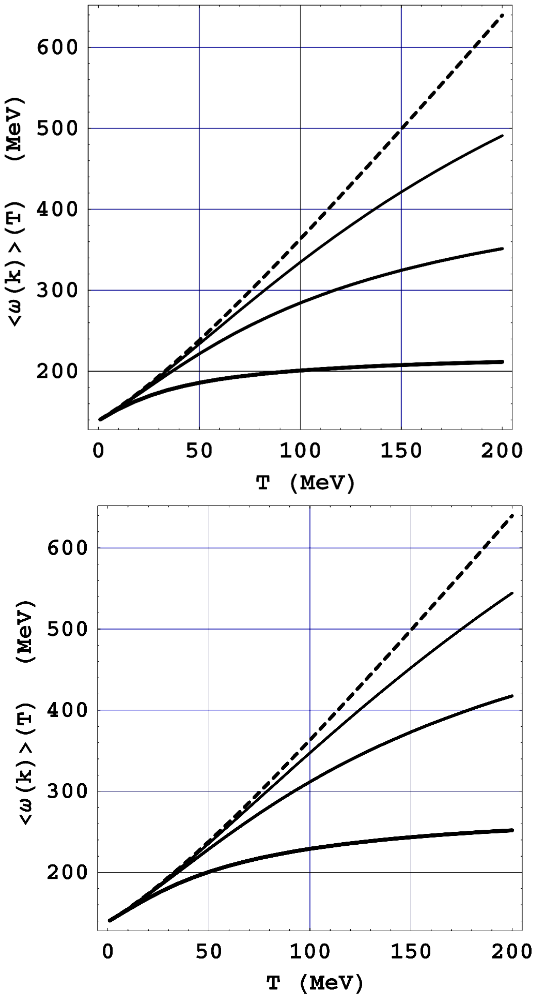

Fig. 2. Average value of one-particle energy after 5 (upper panel) and 10 (lower panel) consecutive scatterings for $E_{\rm tot}$ per particle equal to 200, 400, and 800 MeV (solid curves from bottom to top) and that of the ideal gas of π-mesons (dashed line)

value of the one-particle average energy. Therefore, by fixing the average value of the energy per particle, we come to a conclusion that the effective temperature (the value of the temperature in the ideal gas, the dashed curve) is lower than the actual temperature, which is determined by the intersection with the solid curve, i.e. $T_{\rm eff} \le T$.

Such a situation completely corresponds to the relationship between the effective and actual temperatures in nonrelativistic systems (50). Similarly to the nonrelativistic case, the effective temperature is a limit value for the actual one, which is reached after a sufficiently large number of collisions M occurs and provided that the accessible momentum space is unconfined, i.e. $T_{\rm eff} \to T$, if $M \to \infty$ and/or $p_{\max} \to \infty$.

4 Conclusions

In this work, on the basis of the model proposed for collisions of two relativistic many-particle systems (for example, nuclei), the process of establishment of a thermal equilibrium has been analyzed. This process is parametrized by the number of collisions per particle, which bring about the total randomization of the dynamic degrees of freedom; the main of which are particles' momenta. Note that, in kinetic models in contrast to our approach, the processes of reaching the thermal equilibrium are parametrized by time (i.e. they are considered on the time scale) rather than by the collision number, as was done in our model.

We went further than the authors of works [8, 9], where the systems that are already in the relevant equilibrium state are considered, and the thermal distribution is a result of the energy conservation in a many-particle system, which is mathematically expressed by the availability of the δ-function of the energy (or of the total 4-momentum). In addition to the use of the conservation law of the 4-momentum, we considered successive collisions between particles in the system and obtained the nonequilibrium distribution function, which includes the average number of collisions per particle – before the system decays completely – as a parameter. That is, we consider that the establishment of the thermal equilibrium depends on the average number of collisions per particle in a many-particle system. It is worth noting that the parametrization of a nonequilibrium process by the number of collisions per particle is especially promising in the range of relativistic energies, because the collision number is an invariant under a change of the reference frame.

Another parameter which parametrizes the nonequilibrium distribution function, is the maximal momentum of particles in the system. In other words, it is a certain effective quantity, $p_{\max}$, which characterizes the confinement of the one-particle momentum space. Meanwhile, we can take the dependence of $p_{\max}$ on the azimuth angle into account. Then, we hope for that the distribution function derived in the present paper will help to understand better the effect of the hadronic rescattering in noncentral relativistic collisions of heavy ions on the azimuth anisotropy of the momentum spectra (elliptic flow) [10].

The specific features of the model include our simplification of the differential scattering cross-section of two particles, which takes it approximately as isotropic. It is allowable owing to a rather large number of collisions. Therefore, the procedure of averaging over possible scattering directions is eligible. In other words, it is a manifestation of collective effects in a dense medium which is formed when nuclei collide.

One of the results of the work is the relations between the effective temperature of the system, which is measured making use of the ideal gas model, and the temperature of a nonequilibrium distribution that was obtained in this work.

The main result of the work is the explicit expression (39) for the nonequilibrium distribution function which depends on the collision number and the effective volume of the momentum space.

A comparison of the results obtained theoretically with experimental ones will be carried out elsewhere.

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ТЕРМАЛІЗАЦІЯ У ЗІТКНЕННЯХ ВАЖКИХ ІОНІВ

Д.В. Анчишкін, С.М .Єжов

Р е з ю м е

Запропоновано та досліджено модель ізотропізації та відповідної термалізації в системі, що утворюється внаслідок зіткнення двох N-частинкових систем (ядер). Враховуються двочастинкові зіткнення. Два головних припущення є ознакою моделі: 1) вже після трьох зіткнень відбувається практично повна рандомізація імпульсів окремих частинок, 2) простір можливих значень одночастинкових імпульсів є обмеженим внаслідок скінченності повної енергії багаточастинкової системи. Показано, що ці дві властивості приводять до розподілу значень імпульсів навколо початкових значень і як наслідок до часткової ізотропізації імпульсів частинок. Отримано одночастинкова функцію нерівноважного розподілу в імпульсному просторі.