DESCRIPTION OF NUCLEAR BINDING ENERGY BASED ON THE S-MATRIX FORMALISM AT VARIABLE ELECTRIC AND BARYONIC CHARGES

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The results of the S-matrix formalism of the problem of interaction of two nuclear fragments with J^π = const at variable electric charges and reduced mass are generalized to the case of the fission of a complex system into free nucleons. In this case, an approximate linear dependence of the binding energy of nonmagic nuclei ${}^A_zQ$ on the parameter ZA^{-1} is established with regard for the fact that they are isotopes ($Z = \text{const}$), isotones ($A - Z = \text{const}$), isobars ($A = \text{const}$), or nuclei with a fixed neutron excess number ($A - 2Z = \text{const}$).

It is known that the Schrödinger equation for partial waves includes the energy E and the orbital moment l as fundamental spectral parameters as well as the reduced mass μ and the electric charges ze and Ze of interacting particles. In this case, the resonant (bound) states of a system of fixed charged particles (nuclei) ${}_zX + {}_ZY$ can be described with the help of the trajectory of a pole of the S-matrix in the complex k-plane ($k = \sqrt{E}, 2\mu = \hbar = 1$) at real values of l or in the complex l-plane at real values of E [1].

The analytic structure of the S-matrix in the complex plane of the Coulomb coupling constant $a = Zze^2$ ($2\mu = \hbar = c = 1$) conditioned by complex values of the charges ze and Ze at fixed values of the reduced mass μ and the orbital moment l describes Coulomb effects in the system

$$ {}^{A_1-A}_{\;\;\;z}X + {}^A_ZY \leftrightarrow {}^{\;\;A_1}_{z+Z}Q $$
$$ (A = \text{const}, \quad A_1 = \text{const}, \quad J^\pi = \text{const}), $$

where X, Y, and Q are isobaric nuclear multiplets [2].

In the case $z = \text{const}$ (X is a charged cluster), Y and Q are isobaric nuclei with $J^\pi = \text{const}$ [3]. A successive statement of the method of complex electric charges and its application to nuclear physics are presented in [4, 5].

The analytic properties of the S-matrix in the complex a-plane ($a = \mu$, $2Zze^2 = \hbar = c = 1$) conditioned by complex values of the reduced mass μ describe the systems

$$ {}^{A_1-A}_{\;\;\;z}X + {}^A_ZY \leftrightarrow {}^{\;\;A_1}_{z+Z}Q $$
$$ (Z = \text{const}, \quad z = \text{const}, \quad J^\pi = \text{const}), $$

where X, Y, and Q are isotopic nuclei [6–8]. At $A_1 - A = \text{const}$ (X is a charged cluster), resonance characteristics of a complex isotopic system Q (the width of a resonance Γ_r and the resonance energy E_r) are linked by the relation similar to the Geiger–Nuttall rule. That is, the quantities $\lg \Gamma_r$ and $(E_r)^{-1/2}$ are linearly dependent [6]. If $z = 0$, then the absence of a Coulomb potential barrier results in an increase of the resonance width and an essential change of the relation between the resonance characteristics $\left(\Gamma_r \approx \gamma E_r^{l+1/2}, \gamma > 0\right)$ [7].

In the case where X is a fixed charged cluster, whereas ${}^A_ZY$ are nuclei with $A - Z = \text{const}$ and $J^\pi = \text{const}$, the quasistationary states of a complex isotonic system Q can be described with the help of the trajectory of a pole of the S-matrix in the complex plane $a = \mu Z$ [9]. If complex values of the Coulomb coupling constant $a = \mu Zz$ $\left(2e^2 = \hbar = c = 1\right)$ are caused by complex values of the masses and the charges of the interacting fragments X and Yc $J^\pi = \text{const}$, then the real part of a trajectory of the pole $a(E)$ $(a(E) = \mu(E)Z(E)z(E))$ satisfies the equation

$$ \text{Re}\,a(E) \approx \gamma_1 + \gamma_2 E, \tag{1} $$

where γ_1 and γ_2 are constants [10,11].

Equation (1) is similar to the equations for Re$a(E)$ in the cases μ = const [2]; μ = const, z = const [3]; z = const, Z = const [6, 8], and z = const [9]. In this case, the corresponding constants of equations analogous to (1) depend on the boundary values of the poles $\mu(0)$, $Z(0)$, $z(0)$ and the orbital moment l. By the example of the decay (fission) of the ground states of heavy even-even isotopes, it was also established that a linear dependence between the decay (fission) energy and physical values of the reduced mass of daughter fragments is also conserved in the case where several fixed clusters and the corresponding daughter nuclei-isotopes are formed in the channel [12]. The detailed statement of the S-matrix formalism at complex values of μ, Z, and z and its application to nuclear physics are given in [13].

The aim of the given paper lies in the establishment of the possibility to describe the coupling energy of nonmagic nuclei ${}^A_Z Q$ based on the S-matrix formalism of the A-particle problem at variable values of the reduced mass of nucleons ($A \neq$ const) and the charge number Z. For this purpose, let's introduce the notion of reduced (effective) mass of the A-nucleon system by analogy with such a notion for a two-fragment one, that is,

$$\frac{1}{\mu} = \frac{Z}{m_p} + \frac{A-Z}{m_n}, \tag{2}$$

where m_p and m_n are the proton and neutron masses, respectively.

In the region where charge-independent forces manifest themselves, one can neglect the difference between the proton and neutron masses ($m_p \approx m_n = m$) and, according to (2), present the expression for μ in the form $\mu = mA^{-1}$. Then, the introduction of the notion of reduced mass for the A-particle problem by analogy with a two-particle one allows one to reduce both problems to the single one – the motion of a particle with variable mass μ in the field of external (short-range) forces, whose specific form is inessential for the S-matrix formalism. Indeed, in the case of a system of particles with masses $m_1, m_2, \ldots, m_n$ and momenta $k_1, k_2, \ldots, k_n$, the free Hamiltonian in case of the transition to the center-of-mass system $\left(\sum\limits_{i=1}^{n} k_i = 0\right)$ describes the internal motion of the system and corresponds to the Hamiltonian of a free particle with the mass $\mu = \left(\sum\limits_{i=1}^{n} m_i^{-1}\right)^{-1}$ and the momentum k that represents the mean-square momentum of particles of the system [12].

Let's suppose that, in case of the motion of a charged particle with the mass μ ($\mu = mA^{-1}$) in the field of Coulomb and short-range forces, the Coulomb-nuclear potential is not point-like, but it is described by a finite-depth well with a Coulomb tail. That is, the potential doesn't contain singularities of the r^{-1} type at zero (the case of a nucleus in the form of a uniform charged sphere) [1, p.4, 5]. In this case, based on the results of the S-matrix formalism at variable values of μ and Z [9], the real part of a trajectory of the pole $a(E)$,

$$a(E) = mA^{-1}(E)Z(E),$$

satisfies the equation similar to Eq. (1). Thus, for the energy E equal to

$$E = M(Z;A) - m_p Z - (A-Z)m_n = -E_{\rm b}, \tag{3}$$

where $E_{\rm b}$ and $M(Z;A)$ are, respectively, the binding energy and the mass of the ground state of the system ${}^A_Z Q$, we get

$${\rm Re}a(E) \quad = mZA^{-1}, \tag{4}$$

where Z and A are real.

Then relations (1)–(4) imply that, with regard for the fact that the nuclei ${}^A_Z Q$ belong to isotopes, isobars, isotones, and nuclei with a fixed neutron excess number, their binding energies must be approximate linear functions of the parameter ZA^{-1} (specific charge).

Figures 1 and 2 present the dependence of $E_{\rm b}$ on the parameter ZA^{-1} for nuclei ${}^A_Z Q$ (Z = 60, 61, ... , 74 and Z = 88, 89, ... , 100) [14]. One can see that even-even and even-odd isotopes are located on one line, i.e. their binding energies linearly depend on A^{-1}. In this case, one observes the nearly equidistant location of these lines for both the former and the latter nuclei. A similar situation is also observed for isotopes with odd values of Z, i.e. odd-even and odd-odd ones. The equidistance of the binding energy also conserves in the case of the description of a complex system ${}^A_Z Q$ in the framework of the S-matrix formalism at Z = var, A = const or Z = var, A = var at $A - Z$ = const and $A - 2Z$ = const. It's natural as bound states of the complex system ${}^A_Z Q$ are invariant with respect to the way of their description with the help of the trajectory of an S-matrix pole in the complex μ ($\mu \sim A^{-1}$)-, Z-, or ZA^{-1}-planes.

On the line that represents the dependence of $E_{\rm b}$ on ZA^{-1} for nuclei with $A - 2Z = 16$ (see Fig. 1), the crosses indicate the nuclei ^{138}Pm, ^{140}Sm, ^{142}Eu, ^{162}Ta, and ^{164}W, whose binding energies are derived with the help of the approximate relation

$$2E_{\rm b}(Z;A) \approx E_{\rm b}(Z;A-2) + E_{\rm b}(Z;A+2)$$

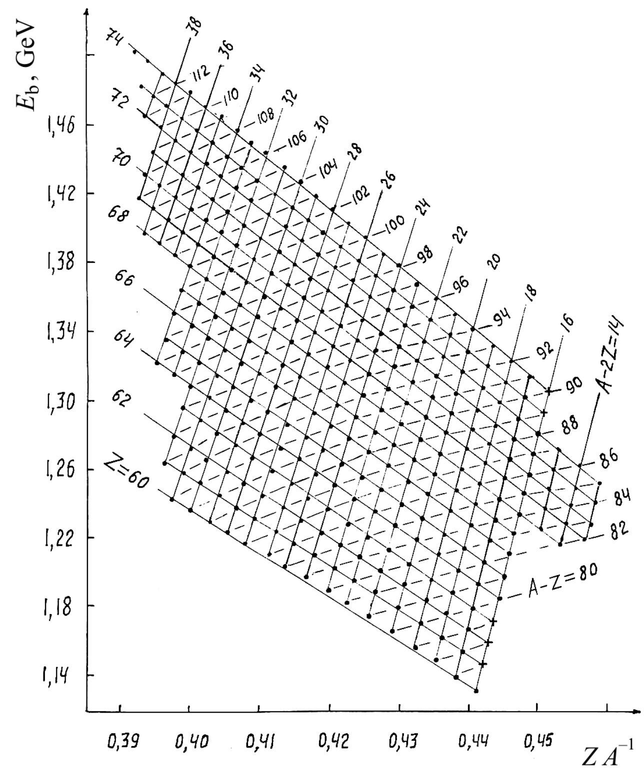

Fig. 1. Binding energy of nuclei ${}^{A}_{z}Q$ versus the specific charge ZA^{-1}. Solid lines pass through isotopes and nuclei with a fixed value of the excess of neutrons, and dashed lines are drawn through isotonic nuclei

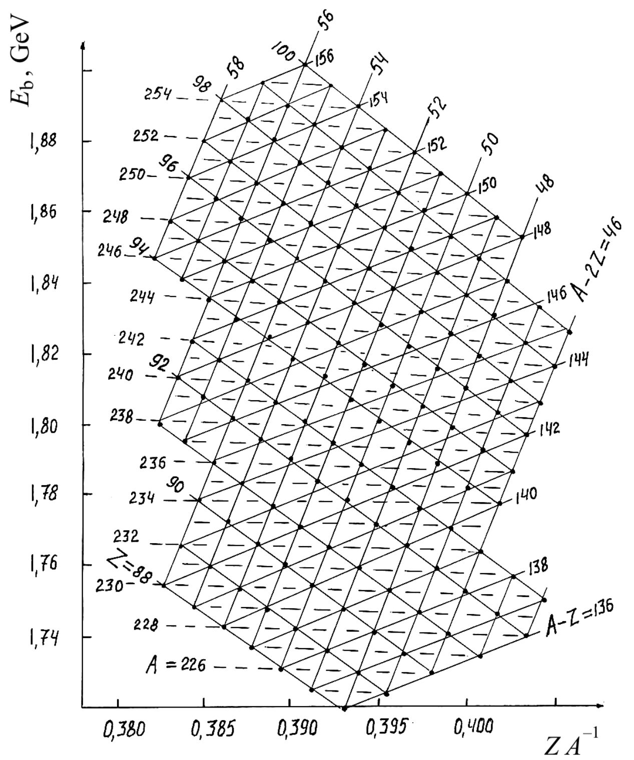

Fig. 2. Binding energy of nuclei ${}^{A}_{z}Q$ versus the specific charge ZA^{-1}. Solid lines pass through isotopes, isotones, and nuclei with a fixed value of the excess of neutrons, and dashed lines are drawn through isobaric nuclei

based on the known values of the binding energies of the nearest nuclei – isotopes of the same evenness. In this case, the values of E_b of the above-mentioned nuclei are equal to 1.142, 1.158, 1.171, 1.290, and 1.303 GeV, respectively. Possible deviations in the calculated binding energies of these nuclei caused by neglecting the difference between the neutron and proton masses in expression (2) amount to $\sim$ 1 MeV.

The average values of the increase of the binding energy $\Delta\overline{E}$ calculated with regard for the known values of the binding energy of even-even nuclei, the isotopes with Z=88, 90, ... , 100, are equal to (see Fig. 2):

1) in the case of the isotope-isotope transition

$$\Delta\overline{E}_Z = E_\mathrm{b}(Z; A+2) - E_\mathrm{b}(Z; A) \approx 12 \ \ \mathrm{MeV};$$

2) in the case of the isotone-isotone transition

$$\Delta\overline{E}_{A-Z} = E_\mathrm{b}(Z+2; A+2) - E_\mathrm{b}(Z; A) \approx 11 \ \ \mathrm{MeV};$$

3) for the nearest nuclei with a fixed neutron excess

$$\Delta\overline{E}_{A-2Z}(Z; A) =$$

$$= E_\mathrm{b}(Z+2; A+4) - E_\mathrm{b}(Z; A) \approx 22 \ \ \mathrm{MeV}.$$

One can see that an increase of the binding energy in the case of the isotope-isotope transition is somewhat higher than that in the case of the isotone–isotone one. This is caused by the fact that an increase of the baryonic charge of the system in the former case is realized by means of the addition of a neutron pair (with antiparallel spins), while a proton pair is added in the latter case, which results in the strengthening of the Coulomb component of the nuclear-Coulomb potential.

Based on the equidistance of the binding energy of even-even nuclei, ($J^\pi = 0^+$) at Z = const, $A - Z$ = const, and $A - 2Z$ = const, the lines in Fig. 2 can be extended to the region of higher values of $Z (Z < 126)$. Such an analytic extension rests on the fact that there are no other magic numbers between the known magic numbers of protons (neutrons) equal to 82 and 126.

The above-stated equidistance of the binding energy of nuclei with $A - 2Z = \mathrm{const}$ yields the approximate relation

$$E_{\rm b}(Z_i;A_i) \approx E_{\rm b}(Z;A) + \Delta\overline{E}_{A-2Z}\frac{Z_i - Z}{2}, \qquad (5)$$

where $Z_i = Z, Z+2, Z+4, \dots$; $A_i = A + 2(Z_i - Z)$.

For example, based on the known binding energy of the basic nucleus ^{256}Fm ($E_{\rm b}(100;256) = 1.9026$ GeV [14]), one can obtain, according to (5), the binding energies of the nuclei $^{A_i}_{Z_i}Q$ (Z_i = 112, 114, 116, 118, 120; A_i=280, 284, 288, 292, 296) equal to 2.035, 2.057, 2.079, 2.101, and 2.123 GeV, respectively. The indicated superheavy nuclei must be located on the extension of the line segment corresponding to the value of the isotopic number $A - 2Z = 56$, at which the ^{256}Fm nucleus is located (see Fig. 2). The binding energies of the isotopes of the indicated nuclei can be obtained by means of the increase of their values by $\Delta\overline{E_{Z_i}} \approx \pm 12$ MeV in the case of the transition to isotopes $^{A_i^*}_{Z_i}Q$ (Z_i = const, $A_i^* = A_i \pm 2, A_i \pm 4, \dots$).

Possible deviations in the calculated binding energies of the indicated nuclei are determined by errors of the increments of the binding energy $\Delta\overline{E}_{A-2Z}$ and $\Delta\overline{E_Z}$ that amount to ~1 MeV.

In conclusion, it is worth noting that the linear behavior of the real part of a trajectory of the S-matrix pole in the complex plane of fundamental parameters of the Schrödinger equation represents a result of the effective radius approximation, i.e. it doesn't depend on the specific form of the interaction potential. This means that, in case of the decay (fission) of a complex system ^A_ZQ, the principal role is played by properties of the parent nuclei, rather than those of daughter fragments. That's why in the region of magic values Z or $A - Z$, a change in the behavior of the short-range potential results in the violation of the above-mentioned linear dependence.

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ОПИС ЕНЕРГІЇ ЗВ'ЯЗКУ ЯДЕР НА ОСНОВІ S-МАТРИЧНОГО ФОРМАЛІЗМУ ЗІ ЗМІННИМИ ЗНАЧЕННЯМИ ЕЛЕКТРИЧНОГО І БАРІОННОГО ЗАРЯДІВ

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Р е з ю м е

Проведено узагальнення результатів S-матричного формалізму задачі про взаємодію двох ядерних фрагментів з J^π = const зі змінними значеннями електричних зарядів і зведеної маси на випадок поділу складеної системи на вільні нуклони. При цьому встановлено приблизну лінійну залежність енергії зв'язку немагічних ядер ^A_zQ від значень параметра ZA^{-1} з урахуванням належності останніх до ізотопів (Z = const), ізотонів ($A - Z$ = const), ізобар (A = const) або ядер з фіксованим значенням надлишку нейтронів ($A - 2Z$ = const).