A SIMULTANEOUS CENTER-OF-MASS CORRECTION OF NUCLEON DENSITY AND MOMENTUM DISTRIBUTIONS IN NUCLEI

A. SHEBEKO, P. GRYGOROV[1]

National Scientific Center "Kharkiv Institute of Physics and Technology"
(1, Akademichna Str., Kharkiv 61108, Ukraine; e-mail: shebeko@kipt.kharkov.ua),

[1]V.N. Karazin Kharkiv National University
(31, Kurchatova Ave., Kharkiv 61108, Ukraine; e-mail: pavel_ grigorov@mail.ru)

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The approach exposed in the recent paper [1] has been applied in studying the center-of-mass (CM) motion effects on the nucleon density and momentum distributions in nuclei. We use and develop a formalism based upon the Cartesian or boson representation, in which the coordinate and momentum operators are expressed through the creation and annihilation operators for oscillator quanta in three different space directions. We are focused upon effects due to the center-of-mass and short-range nucleon correlations embedded in translationally invariant ground-state wavefunctions. The latter are constructed in the so-called fixed center-of-mass approximation, starting with a Slater determinant wave function modified by some correlator e.g., after Jastrow or Villars. It is shown how one can simplify the evaluation of the corresponding expectation values that determine the distributions. The analytic expressions derived here involve the own Tassie–Barker factors for each distribution. As an illustration, numerical calculations have been carried out for a nucleus ^{4}He with the Slater determinant to describe the nucleon $(1s)^4$ configuration composed of single-particle orbitals which differ from harmonic oscillator ones at small distances. Such orbitals simulate a somewhat short-range repulsion between nucleons. Special attention is paid to a simultaneous shrinking of the CM corrected density and momentum distributions as compared with the purely $(1s)^4$ shell nontranslationally invariant ones.

1. Introduction

Treatment of the CM motion has been an attractive subject of exploration in earlier and more recent studies of nuclear theory (see, e.g., [1–9]). Those studies originated from the necessity to remedy a deficiency of the nuclear many-body wave function (WF), namely its lack of translational invariance (TI), wherever shell-model single-particle (s.p.) WF's are used for its construction. This deficiency is important in quite a number of cases.

In the present investigation, we adopt the "fixed-CM approximation" [10, 11] as a recipe to restore TI of a many-body WF which does not possess this property. We apply it when evaluating the elastic form factor (FF) $F(q)$ and the nucleon momentum distribution (MD) $\eta(p)$ for light nuclei, and more specifically for ^{4}He in its ground state (g.s.). Following [1], we prefer to deal with the intrinsic quantities which are determined as expectation values of appropriate (multiplicative) operators that depend on the corresponding Jacobi variables and act on the intrinsic WF's. We have seen in [1] that the intrinsic density distribution (DD) $\rho_{\rm int}(r)$, being defined by the Fourier transform of $F(q)$, does not coincide with the diagonal part of the one-body density matrix (1DM) which is related in a standard manner to the intrinsic MD. In the context, we also note that the term "one-body" used here is somewhat conventional. Let us mention that $F(q)$ and $\eta(p)$ can be related to the different quantities measured via electron–nucleus collisions, respectively, the elastic electron scattering cross sections and the inclusive electron scattering cross sections. First of all, we mean comparatively simple relations in the Born approximation with the plane electron waves. In addition, to the so-called approximation of small interaction times (see [12–14] and refs. therein) the double differential (e, e′) reaction cross section becomes proportional to an integral of $\eta(p)$ over the momentum range that is fixed with a certain combination (the so-called y – scaling variable) of the momentum transfer q and the energy transfer ω (cf. [15]). Of course, in the framework of these approximations, one neglects off–shell effects in the electron scattering on bound nucleons and meson exchange currents (MEC) contributions to an effective electromagnetic (e.m.) interaction with nuclei. The latter should be taken into account (see, e.g., [16, 17]) when describing the electron scattering on nuclei, especially at high momentum transfers (in particular, helping to remove a certain discrepancy between theory and experiment in the vicinity of the first minimum of $|F_{\rm ch}(q)|$ at $q^2 = 10$ fm^{-2} for ^{4}He). Therefore, any comparison with experimental data omitting such physical inputs has a restricted character. Nevertheless, in case of light nuclei, every approximate evaluation of intrinsic quantities, being independent of different constraints originated from reaction mechanisms, can be compared with microscopic ("exact") results. In this respect, our addressing to the alpha particle seems to be perfectly explicable.

The aim of this paper is to show to what extent the approach developed in [1, 12, 18, 19] can be useful in calculations with more realistic WF's than those of the simple harmonic oscillator model (HOM). In this connection, we consider the CM correction of $F(q)$ and $\eta(p)$ treated on an equal physical footing, viz., by using one and the same translationally invariant g.s. WF that incorporates the nucleon-nucleon short-range correlations (SRC). One should note that, despite much interest over the last two decades concerning the MD in nuclei [20–26], its CM correction does not appear to have been properly treated except for certain studies, where harmonic oscillator (HO) wave functions were used (see, e.g., [12]). Note also calculations beyond HOM in [27]. The underlying formalism with basic definitions is exposed in the following section. Section 3 contains the analytic results of our derivations beyond HOM, while the corresponding numerical results are discussed and compared with experimental data in Section 4.

2. The Intrinsic Form Factor, Density and Momentum Distributions with Short-range Correlations Included

By definition, the intrinsic (elastic) FF of a nonrelativistic system with the mass number A and the total angular momentum equal to zero is

$$F(q) = F_{\rm int}(q) \equiv \langle \Phi_{\rm int} \mid \exp[\imath \vec{q}\cdot(\hat{\vec{r}}_1 - \hat{\vec{R}})] \mid \Phi_{\rm int}\rangle, \qquad (1)$$

where $\Phi_{\rm int}$ is the intrinsic WF of the system (nucleus), $\hat{\vec{r}}_1$ the coordinate operator for nucleon number 1, and $\hat{\vec{R}} = A^{-1}\sum_{i=1}^{A}\hat{\vec{r}}_i$ the CM operator.

In the fixed-CM approximation, according to the Ernst, Shakin, and Thaler (EST) prescription [11], the nuclear many-body WF with the total momentum $\vec{P}$ can be written in the form:

$$\mid \Psi_P\rangle = |\vec{P}) \mid \Phi_{\rm int}^{\rm EST}\rangle, \qquad (2)$$

where a round bracket is used to represent a vector in the space of the CM coordinate, so that $|\vec{P})$ means the eigenstate of the total momentum operator $\hat{\vec{P}}$. The intrinsic WF after EST

$$\mid \Phi_{\rm int}^{\rm EST}\rangle = \frac{(\vec{R}=0 \mid \Phi\rangle}{[\langle \Phi \mid \vec{R}=0)(\vec{R}=0 \mid \Phi\rangle]^{1/2}} \qquad (3)$$

is constructed from an arbitrary (in general, translationally non-invariant) WF Φ, by requiring that the CM coordinate $\vec{R}$ be equal to zero. The corresponding FF is the ratio

$$F_{\rm EST}(q) = \frac{A(q)}{A(0)},$$

$$A(q) = \langle \Phi \mid (2\pi)^3 \delta(\hat{\vec{R}}) \exp[i\vec{q}\cdot(\hat{\vec{r}}_1 - \hat{\vec{R}})] \mid \Phi\rangle. \qquad (4)$$

Using the Cartesian representation in which

$$\hat{\vec{r}} = \frac{r_0}{\sqrt{2}}\,(\hat{\vec{a}}^\dagger + \hat{\vec{a}}),\ \hat{\vec{p}} = \imath\frac{p_0}{\sqrt{2}}\,(\hat{\vec{a}}^\dagger - \hat{\vec{a}}) \qquad (5)$$

with the Bose commutation rules,

$$[\hat{a}_i, \hat{a}_j] = 0, \quad [\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij} \quad (i,j = 1,2,3) \qquad (6)$$

and arbitrary real c-numbers r_0 and p_0 that meet the condition

$$r_0 p_0 = 1, \qquad (7)$$

one can show (see [1, 19] and Appendix A) that

$$A(q) = \exp\left(-\frac{\bar{r}_0^2 q^2}{4}\right) U(q), \qquad (8)$$

$$U(q) = \int d\vec{\lambda} \exp\left(-\frac{r_0^2\lambda^2}{4A}\right) F(\vec{v},\vec{s}), \qquad (9)$$

$$F(\vec{v},\vec{s}) = \langle \Phi | \hat{O}_1(\vec{v}+\vec{s}\,)\hat{O}_2(\vec{v})\ldots\hat{O}_A(\vec{v})|\Phi\rangle, \qquad (10)$$

where

$$\hat{O}_\gamma(\vec{x}) = \exp(-\vec{x}^*\hat{\vec{a}}_\gamma^\dagger)\exp(\vec{x}\hat{\vec{a}}_\gamma) \equiv \hat{E}_\gamma^\dagger(-\vec{x})\hat{E}_\gamma(\vec{x}) \qquad (11)$$

$(\gamma = 1, \ldots, A)$

with

$$\vec{s} = \imath\frac{r_0}{\sqrt{2}}\,\vec{q},\ \vec{v} = \imath\frac{r_0}{\sqrt{2}A}(\vec{\lambda} - \vec{q}) \qquad (12)$$

and the renormalized "length" parameter

$$\bar{r}_0 = \sqrt{\frac{A-1}{A}}\,r_0.$$

Further, starting from the definition of intrinsic MD (see [1]),

$$\eta(p) = \langle \Phi_{\rm int} \mid \delta(\hat{\vec{p}}_1 - \hat{\vec{P}}/A - \vec{p}) \mid \Phi_{\rm int} \rangle, \qquad (13)$$

we consider the distribution in the fixed-CM approximation,

$$\eta_{\rm EST}(p) = \frac{\langle \Phi \mid (2\pi)^3 \delta(\hat{\vec{R}}) \delta(\hat{\vec{p}}_1 - \hat{\vec{P}}/A - \vec{p}) \mid \Phi \rangle}{\langle \Phi \mid (2\pi)^3 \delta(\hat{\vec{R}}) \mid \Phi \rangle}, \qquad (14)$$

and the Fourier transform

$$\eta_{\rm EST}(p) = (2\pi)^{-3} \int \exp(-\imath \vec{p}\vec{x}) N(x)/N(0) d\vec{x} \qquad (15)$$

with

$$N(x) = \langle \Phi \mid (2\pi)^3 \delta(\vec{R}) \exp[\imath(\vec{p}_1 - \vec{P}/A)\vec{x}] \mid \Phi \rangle. \qquad (16)$$

We see the certain resemblance between the structure functions $N(x)$ and $A(q)$, viz., both are determined by the expectation values of similar multiplicative operators with one and the same trial WF Φ. Owing to this, using the same algebraic technique, we get

$$N(x) = \exp\left(-\frac{\bar{p}_0^2 x^2}{4}\right) D(x), \qquad (17)$$

$$D(x) = \int d\vec{\lambda} \exp\left(-\frac{r_0^2 \lambda^2}{4A}\right) F(\vec{v}\,', \vec{s}\,'), \qquad (18)$$

where

$$\vec{s}\,' = -\frac{p_0}{\sqrt{2}}\vec{x}, \quad \vec{v}\,' = \frac{\imath r_0}{\sqrt{2}A}(\vec{\lambda} - \imath p_0^2 \vec{x}) \qquad (19)$$

and

$$\bar{p}_0 = \sqrt{\frac{A-1}{A}} p_0.$$

A certain relation of the MD to the corresponding intrinsic density matrix has been shown in [1].

Then let us assume a trial WF,

$$\mid \Phi \rangle = \mid \Phi_{\rm corr} \rangle = \hat{C}(1, 2, \cdots, A) \mid {\rm Det} \rangle \qquad (20)$$

with the Slater determinant

$$\mid {\rm Det} \rangle = \frac{1}{\sqrt{A!}} \sum_{\hat{\mathcal{P}} \in S_A} \epsilon_{\mathcal{P}} \hat{\mathcal{P}} \{ \mid \phi_{p_1}(1) \rangle \cdots \mid \phi_{p_A}(A) \rangle \}. \qquad (21)$$

Here, $\epsilon_{\mathcal{P}}$ is the parity factor for the permutation $\mathcal{P}$, ϕ_a the occupied orbital with the quantum numbers $\{a\}$, and the summation runs over all permutations of the symmetric group S_A.

The A-particle operator $\hat{C} = \hat{C}(\hat{\vec{r}}_\alpha - \hat{\vec{r}}_\beta,\ \hat{\vec{p}}_\alpha - \hat{\vec{p}}_\beta)$ [1] introduces the SRC and meets all necessary requirements of the translational and Galilei invariance, the permutable and rotational symmetry, etc. However, being translationally invariant itself, such model introduction of correlations does not enable one to restore the TI violated with such shell-model WF as the Slater determinant.

What it follows can be used with the Jastrow correlator [28]

$$\hat{C} = \prod_{\alpha<\beta}^{A} f(\hat{\vec{r}}_{\alpha\beta}), \qquad (22)$$

where $f(\hat{\vec{r}}_{\alpha\beta})$ is a two-body correlation factor whose deviation from unity occurs only for small distances $r_{\alpha\beta} = |\vec{r}_\alpha - \vec{r}_\beta|$ less than a correlation radius r_c.

Another popular option goes back to the lectures by Villars [29] (see also [30]) with a unitary operator

$$\hat{C} = \exp(-\imath \hat{G}), \qquad (23)$$

$$\hat{G} = \sum_{\alpha<\beta} \hat{g}(\alpha, \beta), \qquad (24)$$

where the Hermitian operator $\hat{g}(\alpha, \beta)$ acts onto the space of the pair (α, β). In particular, we could follow the simplest Darmstadt ansatz [31]:

$$\hat{g}(\alpha, \beta) = \frac{1}{2}\{\vec{s}\ (\hat{\vec{r}}_{\alpha\beta})\hat{\vec{p}}_{\alpha\beta} + \hat{\vec{p}}_{\alpha\beta}\vec{s}\ (\hat{\vec{r}}_{\alpha\beta})\}, \qquad (25)$$

where $\vec{s}$ is a function of the relative coordinate $\hat{\vec{r}}_{\alpha\beta} = \hat{\vec{r}}_\alpha - \hat{\vec{r}}_\beta$. Its canonically conjugate momentum $\hat{\vec{p}}_{\alpha\beta} = \frac{1}{2}(\hat{\vec{p}}_\alpha - \hat{\vec{p}}_\beta)$.

Keeping in mind similar constructions, we rewrite expectation (10) as

$$F(\vec{v}, \vec{s}) = \langle \Phi(-\vec{v}) \mid \hat{E}_1^\dagger(-\vec{s}) \hat{E}_1(\vec{s}) \mid \Phi(\vec{v}) \rangle, \qquad (26)$$

where

$$\mid \Phi(\vec{x}) \rangle = \hat{E}_1(\vec{x}) \ldots \hat{E}_A(\vec{x}) \mid \Phi \rangle,$$

since $\hat{E}_1(\vec{v} + \vec{s}) = \hat{E}_1(\vec{v})\hat{E}_1(\vec{s})$ and $[\hat{E}_\alpha(\vec{x}), \hat{E}_\beta(\vec{y})] = 0$ $(\alpha, \beta = 1, \ldots, A)$ for any vectors $\vec{x}$ and $\vec{y}$.

[1] Of course, the operator may be spin and isospin dependent

Moreover, we find that

$$\hat{E}(\vec{x})\ \hat{\vec{r}}\ \hat{E}^{-1}(\vec{x}) = \hat{\vec{r}} + \frac{r_0}{\sqrt{2}}\ \vec{x} \tag{27}$$

and

$$\hat{E}(\vec{x})\hat{\vec{p}}\ \hat{E}^{-1}(\vec{x}) = \hat{\vec{p}} - \imath\frac{p_0}{\sqrt{2}}\ \vec{x}. \tag{28}$$

We recall that $E^\dagger \neq E^{-1}$. In other words, $\hat{E}_\alpha(\vec{x})$ is the displacement operator in the space of nucleon states with the label α.

Due to this property, when handling the similarity transformation

$$\hat{C}' = \hat{E}_1(\vec{x})\ldots\hat{E}_A(\vec{x})\hat{C}(\hat{\vec{r}}_\alpha - \hat{\vec{r}}_\beta,\ \hat{\vec{p}}_\alpha - \hat{\vec{p}}_\beta)\times$$

$$\times\hat{E}_1^{-1}(\vec{x})\ldots\hat{E}_A^{-1}(\vec{x}),$$

we get

$$\hat{C}' = \hat{C}(\hat{E}_\alpha(\vec{x})\hat{\vec{r}}_\alpha\hat{E}_\alpha^{-1}(\vec{x}) - \hat{E}_\beta(\vec{x})\hat{\vec{r}}_\beta\hat{E}_\beta^{-1}(\vec{x}),$$

$$\hat{E}_\alpha(\vec{x})\hat{\vec{p}}_\alpha\hat{E}_\alpha^{-1}(\vec{x}) - \hat{E}_\beta(\vec{x})\hat{\vec{p}}_\beta\hat{E}_\beta^{-1}(\vec{x})) =$$

$$= \hat{C}(\vec{r}_\alpha - \vec{r}_\beta, \vec{p}_\alpha - \vec{p}_\beta) = \hat{C}$$

i.e.,

$$\hat{C}' = \hat{C}. \tag{29}$$

We recall that C is a function of <u>all</u> the relative coordinates and their canonically conjugate momenta. It follows from Eqs. (20) and (29) that

$$\mid \Phi_{\rm corr}(\vec{x})\rangle \equiv \hat{E}_1(\vec{x})\ldots\hat{E}_A(\vec{x}) \mid \Phi_{\rm corr}\rangle =$$

$$= \hat{C} \mid {\rm Det}(\vec{x})\rangle. \tag{30}$$

Here, $\mid {\rm Det}(\vec{x})\rangle = \hat{E}_1(\vec{x})\ldots\hat{E}_A(\vec{x}) \mid {\rm Det}\rangle$ is a new Slater determinant composed of the renormalized orbitals,

$$\mid \phi_a(\vec{x};\alpha)\rangle = \hat{E}_\alpha(\vec{x}) \mid \phi_a(\alpha)\rangle \quad (\alpha = 1,\ldots,A), \tag{31}$$

viz.,

$$\mid {\rm Det}(\vec{x})\rangle = \frac{1}{\sqrt{A!}}\sum_{\hat{\mathcal{P}}\in S_A}\epsilon_{\mathcal{P}}\hat{\mathcal{P}}\{\mid \phi_{p_1}(\vec{x};1)\rangle\cdots\mid \phi_{p_A}(\vec{x};A)\rangle\}. \tag{32}$$

In their turn, such orbitals can be evaluated in a concise analytic form, as the initial ones are linear combinations of the HOM orbitals (see Appendix A).

Following (26), we arrive at

$$F_{\rm corr}(\vec{v},\vec{s}) \equiv \langle\Phi_{\rm corr}(-\vec{v}) \mid \hat{E}_1^\dagger(-\vec{s})\hat{E}_1(\vec{s}) \mid \Phi_{\rm corr}(\vec{v})\rangle =$$

$$= \langle{\rm Det}(-\vec{v}) \mid \hat{C}^\dagger\hat{E}_1^\dagger(-\vec{s})\hat{E}_1(\vec{s})\hat{C} \mid {\rm Det}(\vec{v})\rangle. \tag{33}$$

Expressions (8) and (17) with expectations $F(\vec{v},\vec{s})$ and $F(\vec{v}\ ',\vec{s}\ ')$, which are determined by Eq. (33), are a certain base for our calculations.

2.1. Several working formulae: application to ^{4}He

In the special case of the pure HOM $(1s)^4$ configuration occupied by the four nucleons in ^{4}He, we have

$$\mid \Phi_{\rm corr}(\vec{x})\rangle = \mid \Phi_{\rm corr}\rangle = \hat{C} \mid (1s)^4\rangle, \tag{34}$$

taking into account that the HOM g.s. $\mid (1s)^4\rangle$ is the vacuum for the operators $\hat{\vec{a}}_\alpha$ ($\alpha = 1, \ldots, A$). It is the case, where $\mid {\rm Det}(\vec{v})\rangle$ does not depend on $\vec{v}$ coinciding with the initial Slater determinant $\mid (1s)^4\rangle$. Hence,

$$F_{1s}(\vec{v},\vec{s}) = \langle(1s)^4 \mid \hat{C}^\dagger\hat{E}_1^\dagger(-\vec{s})\hat{E}_1(\vec{s})\hat{C} \mid (1s)^4\rangle. \tag{35}$$

In other words, under such a simplification, the function $F(\vec{v},\vec{s})$ in integral (9) becomes independent of $\vec{\lambda}$, and we get

$$U(q) = U_{1s}(q)\int\exp\left(-\frac{r_0^2\lambda^2}{4A}\right)d\vec{\lambda},$$

so that

$$\frac{U_{1s}(q)}{U_{1s}(0)} = \frac{\langle(1s)^4 \mid \hat{C}^\dagger\hat{E}_1^\dagger(-\imath\frac{r_0}{\sqrt{2}}\vec{q})\hat{E}_1(\imath\frac{r_0}{\sqrt{2}}\vec{q})\hat{C} \mid (1s)^4\rangle}{\langle(1s)^4 \mid \hat{C}^\dagger\hat{C} \mid (1s)^4\rangle}. \tag{36}$$

Thus, the FF of interest is

$$F_{\rm EST}(q) = F_{\rm TB}(q)F_{\rm IPM}(q)F_{\rm corr}(q), \tag{37}$$

where, according to Eq.(A.1), we have the Tassie–Barker factor $F_{\rm TB}(q)$ and the HOM FF $F_{\rm HOM}(q)$. The factor

$$F_{\rm corr}(q) = \frac{\langle(1s)^4 \mid \hat{C}^\dagger\hat{E}_1^\dagger(-\imath\frac{r_0}{\sqrt{2}}\vec{q})\hat{E}_1(\imath\frac{r_0}{\sqrt{2}}\vec{q})\hat{C} \mid (1s)^4\rangle}{\langle(1s)^4 \mid \hat{C}^\dagger\hat{C} \mid (1s)^4\rangle} \tag{38}$$

incorporates the SRC in any way.

At this point, one can proceed, at least, along two guidelines. One of them could be based upon the representation

$$\langle (1s)^4 \mid \hat{C}^\dagger \hat{E}_1^\dagger(-\imath\frac{r_0}{\sqrt{2}}\vec{q})\hat{E}_1(\imath\frac{r_0}{\sqrt{2}}\vec{q})\hat{C} \mid (1s)^4\rangle =$$

$$= \langle (1s)^4 \mid \hat{C}_1^\dagger(-\vec{q})\hat{C}_1(\vec{q}) \mid (1s)^4\rangle, \tag{39}$$

where

$$\hat{C}_1(\vec{q}) = \hat{E}_1(\imath\frac{r_0}{\sqrt{2}}\vec{q})\hat{C}(\hat{\vec{r}}_1, \hat{\vec{p}}_1, \cdots)\hat{E}_1^{-1}(\imath\frac{r_0}{\sqrt{2}}\vec{q}) =$$

$$= C\left(\hat{\vec{r}}_1 + \imath\frac{\vec{q}}{2}r_0^2, \hat{\vec{p}}_1 + \frac{\vec{q}}{2}, \dots\right). \tag{40}$$

Other continuation is prompted by the relation

$$\hat{E}_1^\dagger(-\imath\frac{r_0}{\sqrt{2}}\ \vec{q}\ )\hat{E}_1(\imath\frac{r_0}{\sqrt{2}}\ \vec{q}\ ) = \exp\left(\frac{r_0^2 q^2}{4}\right)\exp(\imath\vec{q}\ \hat{\vec{r}}_1),$$

that gives rise to

$$F_{\rm corr}(q) = \exp\left(\frac{r_0^2 q^2}{4}\right)F_C(q), \tag{41}$$

$$F_C(q) = \frac{\langle (1s)^4 \mid \hat{C}^\dagger \exp(\imath\vec{q}\hat{\vec{r}}_1)\hat{C} \mid (1s)^4\rangle}{\langle (1s)^4 \mid \hat{C}^\dagger\hat{C} \mid (1s)^4\rangle}, \tag{42}$$

where $F_C(q)$ is the no CM corrected FF with the correlated g.s. $\hat{C} \mid (1s)^4\rangle$.

Analogously, we find

$$N(x) = N_{\rm TB}(x)\, N_{\rm HOM}(x)\, N_{\rm corr}(x) \tag{43}$$

with the <u>own</u> Tassie–Barker factor

$$N_{\rm TB}(x) = \exp\left(\frac{p_0^2\ x^2}{4A}\right) \tag{44}$$

and

$$N_{\rm HOM}(x) = \exp\left(-\frac{p_0^2\ x^2}{4}\right), \tag{45}$$

$$N_{\rm corr}(x) = \frac{\langle (1s)^4 \mid \hat{C}^\dagger \hat{E}_1^\dagger(\frac{p_0}{\sqrt{2}}\vec{x})\hat{E}_1(-\frac{p_0}{\sqrt{2}}\vec{x})\hat{C} \mid (1s)^4\rangle}{\langle (1s)^4 \mid \hat{C}^\dagger\hat{C} \mid (1s)^4\rangle}. \tag{46}$$

Again, different continuations are possible (cf. the transition from Eq. (38) to Eqs. (39) and (41)). In particular, with the help of

$$\hat{E}_1^\dagger(\frac{p_0}{\sqrt{2}}\vec{x})\hat{E}_1(-\frac{p_0}{\sqrt{2}}\vec{x}) = \exp\left(\frac{p_0^2\ x^2}{4}\right)\exp\left(\imath\hat{\vec{p}}_1\vec{x}\right),$$

we get

$$N_{\rm corr}(x) = \exp\left(\frac{p_0^2\ x^2}{4}\right)N_C(x), \tag{47}$$

$$N_C(x) = \frac{\langle (1s)^4 \mid \hat{C}^\dagger \exp(\imath\hat{\vec{p}}_1\vec{x})\hat{C} \mid (1s)^4\rangle}{\langle (1s)^4 \mid \hat{C}^\dagger\hat{C} \mid (1s)^4\rangle}. \tag{48}$$

The Fourier transform

$$\eta_C(p) = \frac{1}{(2\pi)^3}\int {\rm e}^{-\imath\vec{p}\vec{x}}N_C(x)d\vec{x} \tag{49}$$

gives us the one-body momentum distribution (OBMD) without the CM correction of the model g.s. $\hat{C} \mid (1s)^4\rangle$.

By definition, the intrinsic DD is

$$\rho_{\rm int}(r) = \frac{1}{(2\pi)^3}\int {\rm e}^{-\imath\vec{q}\vec{r}}F_{\rm int}(q)d\vec{q} =$$

$$= \langle \Phi_{\rm int} \mid \delta(\hat{\vec{r}}_1 - \hat{\vec{R}} - \vec{r}) \mid \Phi_{\rm int}\rangle, \tag{50}$$

so that the relations

$$\rho_{\rm EST}(r) = \frac{1}{(2\pi)^3}\int {\rm e}^{-\imath\vec{q}\vec{r}}F_{\rm EST}(q)d\vec{q}$$

and

$$\rho_C(r) = \frac{1}{(2\pi)^3}\int {\rm e}^{-\imath\vec{q}\vec{r}}F_C(q)d\vec{q}$$

are, respectively, the one-body density distribution (OBDD) with the CM correction and the no CM corrected distribution.

Thus, we have shown (with the help of purely algebraic means) that the evaluation of the distributions can be reduced to the well-known treatment. Indeed, the expectation values (42) and (48) occur in all conventional calculations with the many-particle WF (20), i.e., without any CM correction. Diverse methods have been elaborated when evaluating similar quantities (see, e.g., [6, 8, 30–35] and refs. therein). In this work, we confine ourselves to comparatively simple computations for the $(1s)^4$ configuration, where a short-range repulsion between nucleons is introduced in an effective way, viz., modifying the s.p. orbital as in [10]. Respectively, the following WF is used in the next section.

3. Analytic Expressions for the Form Factor, Density and Momentum Distributions with the Single-particle Wave Function beyond HOM

In accordance with [10], we employ the normalized Radhakant, Khadkikar, and Banerjee (RKB) radial orbital for the lowest s.p. state of ^{4}He,

$$\phi^{\rm RKB}(r) = \frac{1}{\sqrt{1+\beta^2}}(\phi_{00}(r) + \beta\phi_{10}(r)), \tag{51}$$

where ϕ_{00} and ϕ_{10} are the normalized HO radial eigenfunctions:

$$\phi_{00}(r) = 2\sqrt{\frac{1}{\sqrt{\pi}b_H}}\frac{r}{b_H}\exp\left(-\frac{r^2}{2b_H^2}\right), \tag{52}$$

$$\phi_{10}(r) = \sqrt{\frac{3!}{\sqrt{\pi}b_H}}\frac{r}{b_H}\left[1-\frac{2}{3}\frac{r^2}{b_H^2}\right]\exp\left(-\frac{r^2}{2b_H^2}\right) \tag{53}$$

for the states with $n=0, l=0$ and $n=1, l=0$, respectively. Here, b_H is the HO parameter, and β is a mixing parameter.

The RKB WF allows one to obtain the following expressions for the density distribution (normalized to unity), for the point proton FF as well as for the MD (also normalized to unity):

$$\rho_{sp}^{\rm RKB}(r) = \frac{1}{(\sqrt{\pi}b_H)^3(1+\beta^2)}\exp\left(-\frac{r^2}{b_H^2}\right)\times$$

$$\times\left[1+\sqrt{\frac{3}{2}}\beta\left(1-\frac{2r^2}{3b_H^2}\right)\right]^2, \tag{54}$$

$$F_{sp}^{\rm RKB}(q) = \frac{1}{1+\beta^2}\exp\left(-\frac{(b_Hq)^2}{4}\right)\times$$

$$\times\left[1+\beta^2+\frac{\beta}{\sqrt{6}}\left(1-\sqrt{\frac{2}{3}}\beta\right)b_H^2q^2+\frac{\beta^2 b_H^4 q^4}{24}\right], \tag{55}$$

$$\eta_{sp}^{\rm RKB}(p) = \frac{b_H^3}{\pi\sqrt{\pi}(1+\beta^2)}\exp(-b_H^2p^2)\times$$

$$\times\left[1-\sqrt{\frac{3}{2}}\beta\left(1-\frac{2}{3}b_H^2p^2\right)\right]^2. \tag{56}$$

3.1. The CM corrected form factor $F(q)$ and its reduction to quadratures

Assuming a Slater determinant as the g.s. $(1s)^4$ of ^{4}He, its FF in the fixed-CM approximation can be written in the form (cf. [10]):

$$F_{\rm EST}(q) = \frac{\int F_{1s}(|\vec{q}+\vec{u}|)F_{1s}^3(u)d\vec{u}}{\int F_{1s}^4(u)d\vec{u}}, \tag{57}$$

where

$$F_{1s}(v) \equiv \int {\rm e}^{i\vec{v}\vec{r}}\phi_{1s}^2(r){\rm d}\vec{r} = \frac{4\pi}{v}\int \phi_{1s}^2(r)\sin vr\; r{\rm d}r$$

is the not CM-corrected FF.

In the case of the RKB-like s.p. WFs whose orbitals are truncated expansions in the radial HO eigenfunctions, the multiple integrals on the r.h.s. of Eq. (63) can be expressed through simple integrals. The respective algebraic technique has been developed in [12] and exposed recently in [37] (see also Appendix A to the present paper). Its application with the RKB orbital enables us to get

$$F^{\rm RKB}(q) = \frac{A^{\rm RKB}(q)}{A^{\rm RKB}(0)}, \tag{58}$$

where

$$A^{\rm RKB}(q) = I_1(q) + I_2(q),$$

$$I_1(q) = \frac{4\pi}{qb_H^4}\exp\left(-\frac{3}{16}q^2b_H^2\right)\times$$

$$\times\int_0^\infty\{B_2\left[\frac{1}{4}(t-\frac{3}{4}\;qb_H)^2\right]M_2^3\left[\frac{1}{4}(t+\frac{1}{4}\;qb_H)^2\right]-$$

$$-B_2\left[\frac{1}{4}(t+\frac{3}{4}qb_H)^2\right]M_2^3\left[\frac{1}{4}(t-\frac{1}{4}qb_H)^2\right]\}\exp(-t^2)tdt,$$

$$I_2(q) = \frac{\pi}{b_H^3}\exp\left(-\frac{3}{16}q^2b_H^2\right)\times$$

$$\times\int_0^\infty\{B_2\left[\frac{1}{4}(t-\frac{3}{4}qb_H)^2\right]M_2^3\left[\frac{1}{4}(t+\frac{1}{4}qb_H)^2\right]+$$

$$+B_2\left[\frac{1}{4}(t+\frac{3}{4}qb_H)^2\right]M_2^3\left[\frac{1}{4}(t-\frac{1}{4}qb_H)^2\right]\}\exp(-t^2)dt,$$

$$A^{\rm RKB}(0)=\frac{4\pi}{b_H^3}\int\limits_0^\infty M_2^4\left[\frac{1}{4}\ t^2\right]\exp(-t^2)t^2dt.$$

The functions $M_2(z)$ and $B_2(z)$ are second degree polynomials of the variable z

$$M_2(z)=m_0+m_1z+m_2z^2$$

$$B_2(z)=h_0+h_1z+h_2z^2,$$

where the constants are related to the mixing parameter β

$$m_0=1+\beta^2,$$

$$m_1=2\sqrt{2/3}\beta(1-\sqrt{2/3}\beta),$$

$$m_2=(2/3)\beta^2,$$

and

$$h_0=m_0+m_1+2m_{2,}$$

$$h_1=m_1+2m_2,$$

$$h_2=m_2.$$

3.2. The CM corrected momentum distribution $\eta(p)$ and its reduction to quadratures

In parallel, starting from Eq.(16), we obtain, for the $(1s)^4$ configuration with the Slater determinant $\mid\Phi\rangle=\mid(1s)^4\rangle$ (see Appendix B to Lect. I in [37]),

$$N_{\rm EST}(x)=\int{\rm d}\vec{k}\ \langle 1s\mid\exp\left(\imath\frac{\vec{k}\hat{\vec{r}}}{A}\right)\exp\left(\imath\frac{A-1}{A}\hat{\vec{p}}\vec{x}\right)\mid 1s\rangle\times$$

$$\times\langle 1s\mid\exp\left(\imath\frac{\vec{k}\hat{\vec{r}}}{A}\right)\exp\left(-\imath\frac{\hat{\vec{p}}\vec{x}}{A}\right)\mid 1s\rangle^3. \tag{59}$$

Again, using representation (5) and splitting the exponents involved in Eq.(67) with the succesive normal ordering of the operators $\hat{\vec{a}}^{\ \dagger}$ and $\hat{\vec{a}}$ (the former are to the left from the latter), we get

$$\frac{N(x)^{\rm RKB}}{N(0)^{\rm RKB}}=\exp\left(-\frac{A-1}{A}\frac{x^2}{4b_H^2}\right)\frac{J(x)}{J(0)}, \tag{60}$$

where the integral $J(x)$ is determined by

$$J(x)=\int\limits_0^\infty\exp\left(-\frac{r_0^2\lambda^2}{4A}\right)g(\lambda^2;x^2)\lambda^2d\lambda,$$

$$g(\lambda^2;x^2)=\left(A_1+\frac{1}{3}A_2\right)B_1^3+\left(A_1+\frac{3}{5}A_2\right)B_1^2B_2+$$

$$+3\left(\frac{1}{5}A_1+\frac{1}{7}A_2\right)B_1B_2^2+\left(\frac{1}{7}A_1+\frac{1}{9}A_2\right)B_2^3,$$

$$A_1=1+\beta^2-\sqrt{\frac{2}{3}}\ \beta\left[1+\sqrt{\frac{2}{3}}\beta\right]\left(\frac{A-1}{A}\right)^2\frac{p_0^2x^2}{2}+$$

$$+\sqrt{\frac{2}{3}}\beta\left[1-\sqrt{\frac{2}{3}}\beta\right]\frac{r_0^2\lambda^2}{2A^2}+$$

$$+\frac{1}{6}\beta^2\left[\left(\frac{A-1}{A}\right)^2\frac{p_0^2x^2}{2}-\frac{r_0^2\lambda^2}{2A^2}\right]^2,$$

$$A_2=\frac{1}{6}\beta^2\left(\frac{A-1}{A}\right)^2\frac{x^2\lambda^2}{A^2},$$

$$B_1=1+\beta^2-\sqrt{\frac{2}{3}}\beta\left[1+\sqrt{\frac{2}{3}}\beta\right]\frac{p_0^2x^2}{2A^2}+$$

$$+\sqrt{\frac{2}{3}}\beta\left[1-\sqrt{\frac{2}{3}}\beta\right]\frac{r_0^2\lambda^2}{2A^2}+\frac{1}{6}\beta^2\left[\frac{p_0^2x^2}{2A^2}-\frac{r_0^2\lambda^2}{2A^2}\right]^2,$$

$$B_2=\frac{1}{6}\beta^2\frac{x^2\lambda^2}{A^4}.$$

Thus, the structure function $N^{\rm RKB}(x)$ can be reduced to one-dimensional integrals similar to those derived for $F^{\rm RKB}(q)$. Here, $A=4$, but we allow A to be changeable, particularly, in order to check that the corresponding distribution

$$\eta_{\rm EST}^{\rm RKB}(p)=\frac{1}{2\pi^2p}\int\limits_0^\infty N^{\rm RKB}(x)/N^{\rm RKB}(0)\sin(px)xdx$$

to the limit $A\to\infty$ yields the not CM corrected distribution (62).

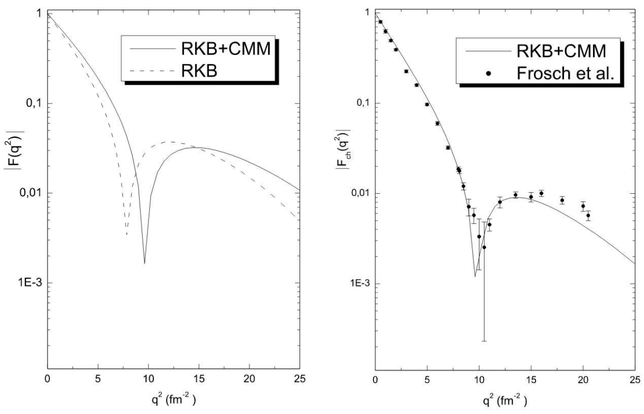

Fig. 1. Point-like FF (left) and the charge FF (right) of ^{4}He. Curves calculated with the RKB WF using the EST prescription (solid) and without the CM-correction (dashed); experimental points from [39]. Other clarifications are given in the text

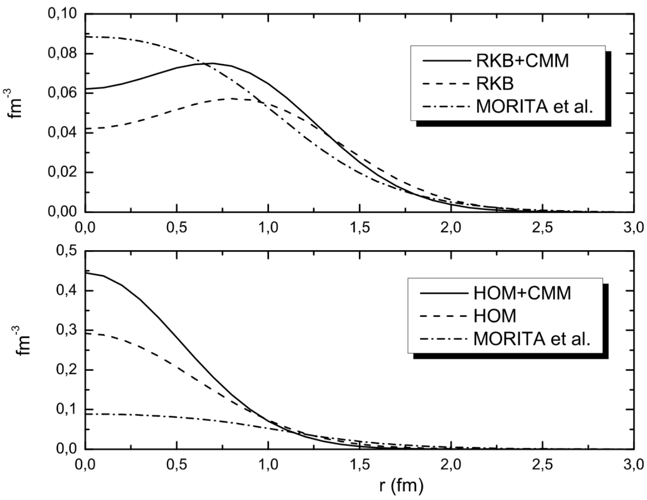

Fig. 2. One-body density distribution (OBDD) for $(1s)^4$ configuration with the RKB orbital in the fixed-CM approximation (solid) and without the CM correction (dashed). For two sets of parameters: $b_H = 0.8532$ fm and $\beta = -0.4738$ (top); $b_H = 0.8532$ fm and $\beta = 0$ (bottom). Dot-dashed line with the parametrization from [41]

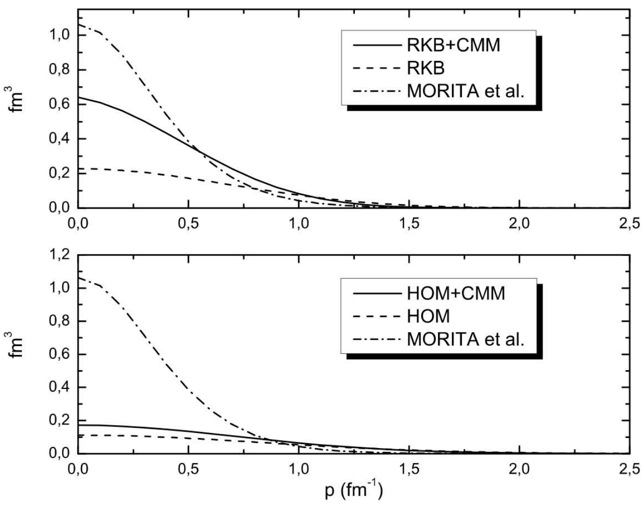

Fig. 3. One-body momentum distribution (OBMD) for the $(1s)^4$ configuration with the RKB orbital. Difference between the curves is the same as in Fig. 2. Dot-dashed line with the parametrization from [42]

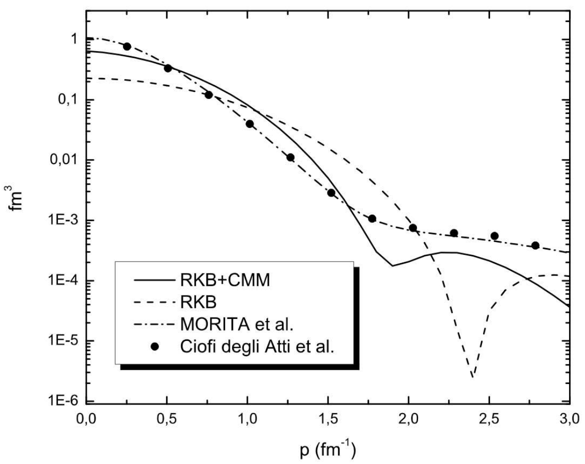

Fig. 4. Variations $\log\eta^{\rm RKB}_{\rm EST}(p)$ (solid), $\log\eta^{\rm RKB}_{sp}(p)$ (dashed), and $\log\eta_{\rm Morita}(p)$ (dot-dashed) with p. Notation $\log\eta_{\rm Morita}(p) = \frac{4}{3}\,^3 W^{\rm SN}(\frac{4}{3}p)$, where the function $W^{\rm SN}(x)$ calculated with the convenient parametrization from [42]. Points are resulted from [24]

4. Results and Discussion

The analytic expressions obtained in Section 2 for the density and momentum distributions and their Fourier transforms are sufficiently general to be applied in different translationally invariant treatments with the SRC included. The corresponding formulae derived in Section 3 in the case of a ^{4}He nucleus have been employed to carry out our the calculations beyond the simple HOM. Their numerical results are displayed in Figs. 1–4.

In Fig. 1, we show the results of calculations of the charge FF, $F_{\rm ch}(q^2) = f_p(q)F(q)$ of the alpha particle, using Eqs. (55) and (58) and considering the Chandra and Sauer prescription [40] for the finite proton size factor $f_p(q)$. The parameters b_H and β have been determined by the least-square fitting to the experimental values [39]: their best-fit values are $b_H = 0.8532$ fm and $\beta = -0.4738$ ($\chi^2 \simeq 13.07$). These values have been utilized in our calculations shown in Figs. 2–4.

As seen in Fig. 1 (its left part), the CM correction leads to a considerable qualitative change of the q-dependence of the FF: its first minimum and second maximum are shifted towards higher q-values. This difference between the solid and dashed curves is due to the different behavior of the respective densities at small distances $r \leq 1$ fm: see Fig. 2, top, where the dashed curve is $\rho_{sp}^{\mathrm{RKB}}(r)$ by Eq. (54), while the solid line is determined by

$$\rho^{\mathrm{RKB}}(r) = \frac{1}{2\pi^2 r}\int\limits_0^\infty F^{\mathrm{RKB}}(q)\sin(qr)qdq. \qquad (61)$$

Moreover, we see that each of the distributions (for the simple HOM orbital on the bottom and for the RKB orbital on the top), after being CM corrected, increases in its central but decreases in its peripheral region. One may say that we encounter a specific effect of shrinking the OBDD owing to the translationally invariant treatment.

In addition, there is a central depression of the density distribution (cf. the upper and lower dashed lines in Fig. 2). Such a change is not unexpected since the RKB WF represents a simple way to allow for some of the effects of short-range repulsion between the nucleons in ^{4}He. These numerical results get an explicit confirmation if we write

$$\rho_{sp}^{\mathrm{RKB}}(0) = \frac{1}{(\sqrt{\pi}b_H)^3}\frac{1}{1+\beta^2}\left(1+\sqrt{\frac{3}{2}}\beta\right)^2. \qquad (62)$$

Evidently, the inequality

$$\rho_{sp}^{\mathrm{RKB}}(0) \leq \rho_{sp}^{\mathrm{RKB}}(0)\mid_{\beta=0} \equiv \rho^{\mathrm{HOM}}(0) = \frac{1}{(\sqrt{\pi}b_H)^3}$$

takes place for negative β values with $\mid \beta \mid < 2\sqrt{6}$.

In parallel, we show in Fig. 3 that the corresponding change of the OBMD has much in common with that for the OBDD, viz., the distribution $\eta_{\mathrm{EST}}^{\mathrm{RKB}}(p)$ turns out to be shrunk in the above sense relative to the distribution $\eta_{sp}^{\mathrm{RKB}}(p)$. Thus, we see a simultaneous shrinking of the density distribution $\rho(r)$ and the momentum distribution $\eta(p)$. As has been shown in [12] (see also [14]), such a simultaneous change of these distributions plays a substantial role in getting a fair treatment of the data on the elastic and inelastic electron scattering of ^{4}He. Let us recall that there the charge FF and the dynamic FF of ^{4}He were calculated using one and the same HOM WF, corrected both with the fixed-CM approximation and the Peierls–Yoccoz prescription [44].

Regarding the properties of these simultaneously corrected distributions in detail, we would like to emphasize a practical consequence of their interpretation. This aspect becomes especially transparent in the case of the simple HOM $(1s)^4$ configuration, where we have

$$\rho_{\mathrm{EST}}^{\mathrm{HOM}}(r) = [\sqrt{\pi}\bar{r}_0]^{-3}\exp(-r^2/\bar{r}_0^2)$$

vs

$$\rho_{sp}^{\mathrm{HOM}}(r) = [\sqrt{\pi}r_0]^{-3}\exp(-r^2/r_0^2)$$

and

$$\eta_{\mathrm{EST}}^{\mathrm{HOM}}(p) = [\sqrt{\pi}\bar{p}_0]^{-3}\exp(-p^2/\bar{p}_0^2)$$

vs

$$\eta_{sp}^{\mathrm{HOM}}(p) = [\sqrt{\pi}p_0]^{-3}\exp(-p^2/p_0^2).$$

Thus, the inclusion of CM corrections gives rise to two independent renormalizations, $r_0 \equiv b_H \to \bar{r}_0 = \sqrt{3/4}r_0$ and $p_0 \equiv {b_H}^{-1} \to \bar{p}_0 = \sqrt{3/4}p_0$, of the oscillatory parameter values, r_0 and p_0 (cf. [12]). Evidently, such changes are not equivalent to a hasty replacement of p_0 by $\sqrt{4/3}p_0$ if one follows the Tassie–Barker recipe with $b_H \to \sqrt{3/4}b_H$ only.

Now, following the conventional way of determining the HOM parameter r_0, as in [12], we use the expansions

$$F_{\mathrm{ch}}(q^2) = 1 - \frac{1}{6}q^2 r_{\mathrm{ch}}^2 + \cdots,$$

$$f_p(q) = 1 - \frac{1}{6}q^2 r_p^2 + \cdots$$

and

$$F(q) = 1 - \frac{1}{6}q^2 r_{rms}^2 + \cdots,$$

where, within the HOM, we have

$$r_{rms}^2 = \frac{3}{2}r_0^2,$$

so that

$$r_{\mathrm{ch}}^2 = \frac{3}{2}r_0^2 + r_p^2,$$

whence

$$r_0^2 \equiv r_{\text{exp}}^2 = \frac{2}{3}\left[r_{\text{ch}}^2 - r_p^2\right]. \tag{63}$$

Doing so for the CM corrected quantities, we find the similar relation

$$\bar{r}_0^2 = \frac{2}{3}\left[r_{\text{ch}}^2 - r_p^2\right] = r_{\text{exp}}^2 \tag{64}$$

with the identical q-dependence $F_{\text{EST}}^{\text{HOM}}(q) = F^{\text{HOM}}(q) = \exp(-q^2 r_{\text{exp}}^2/4)$. At the same time, the difference between the respective OBMDs becomes more considerable than that after the substitution $r_0 \to \sqrt{\frac{A}{A-1}} r_{\text{exp}}$ in $\eta_{sp}^{\text{HOM}}(p) = \frac{r_0^3}{\pi^{3/2}} \exp(-p^2 r_0^2)$, which gives

$$\eta_{sp}^{\text{HOM}}(p) = \left(\frac{A}{A-1}\right)^{3/2} \frac{r_{\text{exp}}^3}{\pi^{3/2}} \exp\left[-\frac{A}{A-1} p^2 r_{\text{exp}}^2\right]$$

vs

$$\eta_{sp}^{\text{HOM}}(p) = \frac{r_{exp}^3}{\pi^{3/2}} \exp(-p^2 r_{\text{exp}}^2).$$

Under the simultaneous CM correction of the OBDD and OBMD, we have

$$\eta_{\text{EST}}^{\text{HOM}}(p) = \left(\frac{A}{A-1}\right)^3 \frac{r_{\text{exp}}^3}{\pi^{3/2}} \exp\left[-\left(\frac{A}{A-1}\right)^2 p^2 r_{\text{exp}}^2\right] \tag{65}$$

vs

$$\eta_{sp}^{\text{HOM}}(p) = \frac{r_{exp}^3}{\pi^{3/2}} \exp(-p^2 r_{\text{exp}}^2), \tag{66}$$

which is equivalent to the substitution $r_0 \to \frac{A}{A-1} r_{\text{exp}}$ in $\eta_{sp}^{\text{HOM}}(p) = \frac{r_0^3}{\pi^{3/2}} \exp(-p^2 r_0^2)$.

Note also that the product $\bar{r}_0 \bar{p}_0 = 1 - A^{-1} \neq 1$, unlike the relation $r_0 p_0 = 1$. In this connection, following [19], let us recall the commutation rules for the intrinsic coordinate $\vec{r}\,' = \vec{r} - \vec{R}$ and conjugate momenta $\vec{p}\,' = \vec{p} - \vec{P}/A$:

$$[\vec{r}\,'_l, \vec{p}\,'_j] = i\delta_{l,j}(1 - 1/A), \quad (l, j = 1, 2, 3). \tag{67}$$

One can show that the corresponding uncertainty principle is related to the deviation from unity. Thus, the uncertainty principle does not contradict the simultaneous shrinking of the density and momentum distributions (see also [37], Lect. I, Suppl. C)

In the case of the RKB function, we get

$$r_{rms}^2 = \frac{3}{2} r_0^2 - \frac{\beta\sqrt{6}}{1+\beta^2}\left(1 - \sqrt{\frac{2}{3}}\beta\right) r_0^2. \tag{68}$$

This means that the short-range repulsion involved in the WF with a negative β leads to some increase in the rms radius, viz., $r_{rms}^{\text{RKB}} > r_{rms}^{\text{HOM}}$. For the values $b_H = 0.8532$ fm and $\beta = -0.4738$, formula (68) yields $r_{rms}^{\text{RKB}} = 1.429$ fm, so that the corresponding charge radius is equal to $r_{\text{ch}}^{\text{RKB}} = 1.667$ fm. Here, we employ the charge proton radius $r_p = 0.86$ fm (see, for example, Appendix 7 in [43]).

The CM correction gives an opposite effect. Indeed, after some calculation, we find that $r_{rms}^{\text{EST}} = 1.309$ fm for the same $b_H = 0.8532$ fm and $\beta = -0.4738$. This yields $r_{\text{ch}}^{\text{EST}} = 1.566$ fm.

The variation of $\log \eta_{\text{EST}}^{\text{RKB}}$ and $\log \eta_{sp}^{\text{RKB}}$ with p is depicted in Fig. 4 for a wider range of momenta. It is seen from Fig. 4 that the allowance of the CM motion improves the description of the available data on the OBMD of an alpha particle. It is further seen from Fig. 4 that, in the translationally invariant quantity according to the fixed-CM prescription, the "seagull" behavior appearing in the variation of the corresponding s.p. one becomes somewhat less pronounced. The dip is diminished, and it moves to smaller values of momentum.

Finally, we would like to point out in connection to the comparison with the s.p. distributions that the CM corrected OBDD and OBMD become closer to the corresponding microscopic ones by using their convenient parametrizations from [41] and [42], as one can see in Figs. 3 and 4. According to communication [42], one has to introduce the factor $(2\pi)^{-3}$ to reproduce the momentum distribution $\eta_{\text{Morita}}(p)$ which is one of the significant results obtained by the Sapporo group. At this point, let us recall that these authors employed the so-called ATMS-method (ATMS is abbreviation of "Amalgamation of Two-body correlations into Multiple Scattering process") to construct the variational WF of the ^{4}He nucleus (see [21] and refs. therein). Along the variational approach, a considerable progress was made when including more the dynamics of the realistic nucleon-nucleon interaction such as the effect of its tensor component (cf. [22]).

5. Concluding Remarks

We have seen how the approach exposed in [1] can be extended to the translationally invariant evaluation of the density and momentum distributions in nuclei. The present analysis shows that the restoration of translational invariance in the Slater determinant WF of ^{4}He by means of the fixed-CM correction (the EST prescription) gives rise, as a whole, to essential changes in the r-, p-, and q-dependences of the OBDD $\rho(r)$, the OBMD $\eta(p)$, and the charge FF $F_{\rm ch}(q)$, respectively. We have seen that the correlation between nucleons induced by the fixation of the center-of-mass of the nucleus results in the simultaneous shrinking of $\rho(r)$ and $\eta(p)$. Meanwhile, this effect has been revealed here beyond the pure HOM extending the available experience.

Also, this study demonstrates the relative importance of the CM and SRC corrections for the same nucleus, viz., the shrinking of the density and momentum distributions owing to the use of translationally invariant g.s. wave functions of ^{4}He versus their broadening after the inclusion of short-range repulsion in these wave functions at small distances $r < 1$ fm. It is true that the latter has been introduced in our calculations in a simple manner. Nevertheless, there are all reasons to believe that the algebraic method employed here might be helpful within more sophisticated approaches, where the short-range correlations are taken into account via the Jastrow factor or other correlation operator (see, e.g., [31]). At present, the corresponding applications are in progress both for ^{4}He and ^{16}O nuclei.

One of us, A.S., is very grateful to M. Grypeos and his colleagues for their hospitality during his visits to Department of Theoretical Physics at Aristotle University of Thessaloniki, where this work was begun. Also, it is a great pleasure for us to thank A. Antonov and M. Gaidarov for sending the numerical values of the momentum distribution displayed by the points in Fig. 4.

APPENDIX A
Details of calculations beyond HOM

Here, we want to illustrate a convenient method for the evaluation of the expectations in question, being aimed at some general (model-independent) results (cf. [1, 19]).

First of all, we have, by recurring the Cartesian representation,

$$\exp\left[\imath\vec{q}\,(\hat{\vec{r}}_1-\hat{\vec{R}})\right]=\exp\ \imath\vec{q}\quad\frac{A-1}{A}\quad\hat{\vec{r}}_1\ \exp\left[-\imath\vec{q}\,\frac{\hat{\vec{r}}_2}{A}\right]\cdots=$$

$$=F_{\rm TB}(q)\ F_{\rm HOM}(q)\exp\ \imath\vec{q}\quad\frac{A-1}{A}\quad\frac{r_0}{\sqrt{2}}\ \hat{\vec{a}}_1^{\dagger}\ \times$$

$$\times\exp\ \imath\vec{q}\quad\frac{A-1}{A}\quad\frac{r_0}{\sqrt{2}}\ \hat{\vec{a}}_1\ \exp\ -\imath\vec{q}\frac{r_0}{\sqrt{2A}}\ \hat{\vec{a}}_2^{\dagger}\ \times$$

$$\times\exp\ -\imath\vec{q}\frac{r_0}{\sqrt{2A}}\ \hat{\vec{a}}_2\ \ \ldots,\qquad\text{(A.1)}$$

$F_{\rm TB}(q)=\exp(\frac{r_0^2q^2}{4A})$, $F_{\rm HOM}(q)=\exp(-\frac{r_0^2q^2}{4})$, where the index α at $\hat{\vec{a}}_\alpha(\hat{\vec{a}}_\alpha^{\dagger})$ is the individual particle number ($\alpha=1,\cdots,A$).

Thereat, the Tassie–Barker factor $F_{\rm TB}(q)$ appears automatically due to a specific structure of the operators involved. In other words, its appearance is independent of any nuclear properties (in general, properties of a finite system). The only mathematical tool that has been used is the Baker–Hausdorff relation

$$\mathrm{e}^{\mathrm{A+B}}=\mathrm{e}^{\mathrm{A}}\ \mathrm{e}^{\mathrm{B}}\ \mathrm{e}^{-\frac{1}{2}[\mathrm{A,B}]}\qquad\text{(A.2)}$$

that is valid with arbitrary operators $\hat{A}$ and $\hat{B}$, for which the commutator $\left[\hat{A},\hat{B}\right]$ commutes with each of them. Further, applying Eq. (A.2) in combination with

$$(2\pi)^3\,\delta\ \ \hat{\vec{R}}\ \ =\int\exp\ \ \imath\vec{\lambda}\hat{\vec{R}}\ \ d\vec{\lambda},\qquad\text{(A.3)}$$

we can show that the expectation value $A(q)$ in Eq. (4) and the expectation value $N(x)$ in Eq. (16) are expressed through one and the same function $F(\vec{x},\vec{y})$ that depends, respectively, on the arguments $\vec{x}=\vec{v}$, $\vec{y}=\vec{s}$ (as in Eq. (9)) and $\vec{x}=\vec{v'}$, $\vec{y}=\vec{s'}$ (as in Eq. (18)). In other words, we have constructed the common generating function for each of them. One should stress that this result has been obtained independently of the model WF Φ.

The algebraic technique shown here turns out to be useful for practical calculations with the Slater determinants like $|\ \Phi\rangle$ (see [18]) or the Slater determinants modified by different correlators (for instance, the Jastrow factor).

In the simplest case of the independent particle model (IPM) $(1s)^4$ configuration for ^{4}He with the Slater determinant $|\ \Phi\rangle=|\ (1s)^4\rangle$, we get, by omitting the nonessential factor $[A!]^{-1}$,

$$A^{\rm IPM}(q)=\exp\ \ -\frac{\bar{r}_0^2q^2}{4}\ \ U^{\rm IPM}(q),\qquad\text{(A.4)}$$

$$U^{\rm IPM}(q)=\int d\vec{\lambda}\exp\ \ -\frac{r_0^2\lambda^2}{4A}\ \ f(\vec{\lambda},\vec{q}\,),\qquad\text{(A.5)}$$

$$f(\vec{\lambda},\vec{q})=\langle 1s\mid\exp\left(-\vec{\alpha}^*\hat{\vec{a}}^{\dagger}\right)\exp\left(\vec{\alpha}\ \hat{\vec{a}}\right)\mid 1s\rangle\times$$

$$\times\langle 1s\mid\exp\left(-\vec{\beta}^*\hat{\vec{a}}^{\dagger}\right)\exp\left(\vec{\beta}\vec{a}\right)\mid 1s\rangle^3,\qquad\text{(A.6)}$$

$$\vec{\alpha}=\imath\frac{r_0}{\sqrt{2A}}[\vec{\lambda}+(A-1)\vec{q}],\ \ \vec{\beta}=\imath\frac{r_0}{\sqrt{2A}}[\vec{\lambda}-\vec{q}],$$

with the renormalized "length" parameter

$$\bar{r}_0=\sqrt{\frac{A-1}{A}}\ r_0.$$

In our case, $r_0=b_H$. We are keeping the notations with a mass number A $(=4)$ to point out a certain trend in the A-dependence.

At this point, let us note that the RKB orbital (or another model orbital) being composed of the basis states of the spherical representation can be written then as a superposition of the basis states $|\ n_1n_2n_3\rangle$ of the Cartesian representation (see, e.g., [38] and refs. therein),

$$|\ n_1n_2n_3\rangle=\frac{1}{\sqrt{n_1!n_2!n_3!}}(\hat{\vec{a}}_1{}^{\dagger})^{n_1}(\hat{\vec{a}}_2{}^{\dagger})^{n_2}(\hat{\vec{a}}_3{}^{\dagger})^{n_3}\ |\ 0\rangle,\qquad\text{(A.7)}$$

where the vector $|\ 0\rangle\equiv|\ 000\rangle$ is the vacuum state with respect to the destruction operators $\hat{a}_i$ $(i=1,2,3)$, e.g.,

$$\hat{\vec{a}}\mid 0\rangle=0.\qquad\text{(A.8)}$$

It is proved that, for the RKB-orbital,

$$|\ 1s\rangle=[1+\beta^2]^{-1/2}[1-(\beta/\sqrt{6})\ \hat{\vec{a}}\ ^{\dagger}\hat{\vec{a}}\ ^{\dagger}\ ]\mid 0\rangle.\qquad\text{(A.9)}$$

Substituting (A.9) into (A.6) (when calculating the ratio $A^{\rm IPM}(q)/A^{\rm IPM}(0)$, the normalization factor $[1+\beta^2]^{-1/2}$ can be omitted), we find

$$\exp(\vec{\chi}\cdot\vec{a}) \mid 1s\rangle = [1-(\beta/\sqrt{6})(\hat{\vec{a}}^{\,\dagger}+\vec{\chi})(\hat{\vec{a}}^{\,\dagger}+\vec{\chi})] \mid 0\rangle \qquad (A.10)$$

for any complex vector $\vec{\chi}$.

Now, after the modest effort, we obtain

$$\langle 1s \mid \exp(-\vec{\chi}^{\,*}\cdot\hat{\vec{a}}^{\,\dagger})\exp(\vec{\chi}\cdot\hat{\vec{a}}) \mid 1s\rangle = 1+\beta^2-\frac{2}{3}\,\beta^2\vec{\chi}^{\,*}\vec{\chi}-$$

$$-\frac{\beta}{\sqrt{6}}\,[\vec{\chi}^{\,*}\vec{\chi}^{\,*}+\vec{\chi}\,\vec{\chi}\,]+\frac{\beta^2}{6}\,(\vec{\chi}^{\,*}\vec{\chi}^{\,*})(\vec{\chi}\,\vec{\chi}). \qquad (A.11)$$

It follows from (A.11), for instance, that

$$\langle 1s \mid \exp(\imath\vec{\alpha}^*\hat{\vec{a}}^{\,\dagger})\exp(\imath\vec{\alpha}\hat{\vec{a}}) \mid 1s\rangle = M_2\left(\frac{\vec{\alpha}^*\vec{\alpha}}{2}\right), \qquad (A.12)$$

where the polynomial $M_2(z)$ is given at the end of Subsection 3.1.

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ОДНОЧАСНА КОРЕКЦІЯ ПРОСТОРОВОГО ТА ІМПУЛЬСНОГО РОЗПОДІЛІВ НУКЛОНІВ В ЯДРАХ НА РУХ ЦЕНТРА МАС

О. Шебеко, П. Григоров

Р е з ю м е

Основну увагу зосереджено на ЦМ-корегуванні одночастинкової матриці густини та її фур'є-перетворень, які містять багату інформацію про властивості основного стану ядра (зокрема про нуклон-нуклонні кореляції). Ці величини є невід'ємною складовою аналізу розсіяння швидких частинок (наприклад електронів) ядерними мішенями. Насамперед, йдеться про розподіл густини нуклонів $\rho(r)$ і відповідний форм-фактор, а також про імпульсний розподіл нуклонів $\eta(p)$ в ядрі. При їх обчислюванні в цій роботі застосовано алгебраїчний метод, що ґрунтується на використанні декартового зображення для операторів координат та імпульсів нуклонів. Цей метод (див. [1]) дозволив не тільки спростити розрахунки очікувань багаточастинкових мультиплікативних операторів, але й, що важливіше, дав змогу виявити нові модельно незалежні зв'язки між розглянутими величинами. Відповідні співвідношення демонструють одночасне звуження обох розподілів $\rho(r)$ та $\eta(p)$ за рахунок виокремлення руху ЦМ. Такий якісний результат підтверджується чисельними розрахунками в наближенні фіксованого ЦМ для ядра ^{4}He. У цьому контексті обговорюються значні відхилення від популярного рецепта Тассі–Баркера як в межах осциляторної моделі, так і поза її межами.