VAN HOVE OVERLAP FUNCTION AND ELASTIC DIFFRACTION SCATTERING OF HADRONS AT HIGH ENERGIES

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Elastic diffraction scattering of hadrons at high transferred momenta has been studied making use of the overlap function. The unitarity condition for the amplitude of elastic diffraction scattering with a modified overlap function has been solved. A comparison between theoretical and experimental differential cross-sections for pp- and $\pi^{\pm}$p-scattering at various energies has been made.

Experimental data measured at the IHEP, FIAN, and ISR accelerators and the collider demonstrate that (i) the total cross-sections grow with the energy; (ii) the differential cross-sections are characterized by a "cusp" structure in the vicinity of $t \approx -0.1$ GeV and a "dip" moving along the energy axis; (iii) the amplitude contains a positive real part; and (iv) polarization effects do not disappear. A large energy range provided by ISR accelerators and the SPS collider makes it possible to verify various phenomenological models which were proposed to describe experimental data for energies below those of ISR. It is known that the calculation of the amplitudes of nuclear processes with low transferred momenta requires going beyond the framework of perturbation theory which is valid only in the range of high transferred momenta. For this reason, various phenomenological models – such as optical, Regge, dual, quasipotential, and control ones, and the U-matrix theory – are widely used for the description of scattering processes with a low transferred momentum [1]. The scattering theory of light nuclei, to the development of which A.G. Sitenko made an essential contribution [2], helps to understand, in many respects, a qualitative picture of problems that exist today in high-energy physics.

One of the existing methods of studying the elastic interactions is a solution of the unitarity condition at a given Van Hove inelastic overlap function [3] which characterizes the overlapping of the wave functions of two inelastic final states obtained from two-particle initial and final states in the elastic process. In other words, the Van Hove overlap function describes a contribution of inelastic processes to the imaginary part of the elastic scattering amplitude making use of the unitarity condition. Provided that the law of the inelastic process is known, then, proceeding from unitarity, one can calculate the corresponding elastic (shadow) scattering; and vice versa, having a certain information concerning the elastic scattering, it is possible to draw a conclusion about the inelastic one. Therefore, the form of the overlap function has a decisive meaning in such an approach. If this function is known for some reason, the fundamental unitarity condition can be considered as an initial dynamical controlling factor while calculating the amplitude of elastic scattering. An approach based on the solution of the unitarity equation was intensively developed in works [1, 4, 5]. In the framework of his model of uncorrelated jets, Van Hove obtained an exponential dependence of this function on the square of the momentum transferred

$$G(t) = \sigma_{\text{in}} e^{\alpha t}. \tag{1}$$

The differential cross-section of elastic scattering at low transferred momenta in the diffraction peak region obtained from Eq. (1) describes – at a qualitatively good level – the available experimental data on the elastic scattering of hadrons at high energies. Moreover, the model under consideration makes a certain prediction with respect to the angular distribution at high transferred momenta, which is also in rather a good qualitative correspondence with experiment. However, this model does not provide a quantitative agreement with experiment. For example, the available experimental data on pp-scattering are hard to be put in a quantitative agreement with the Van Hove model – even in the diffraction cone region – without violating the unitarity condition. In order to determine the shape of the overlap function, the unitarity condition can be used, by substituting the diffraction peak well established experimentally into it; such a procedure was made analytically in works [6–10]. In this case, the overlap function includes the difference of two exponents, rather than a single exponent:

$$G(t) = ae^{\alpha t} - be^{\beta t}, \tag{2}$$

where $\beta = \alpha/2$, and, according to the normalization condition $G(0) = \sigma_{\text{in}}$,

$$a - b = \sigma_{\text{in}}. \tag{3}$$

Note that expression (2) for the overlap function is more realistic than expression (1). In this work, we consider the elastic scattering corresponding to the overlap function (2) in details with the main attention being focused on the scattering with high transferred momenta t which is more sensitive to the $G(t)$-dependence shape than the scattering with low t [9, 10].

1. The amplitude of elastic scattering in the model of uncorrelated jets is expressed by a nonalternating series

$$F(t) \simeq 2i\sqrt{\pi}a \sum_{m=1}^{\infty} \frac{(2m-3)!!(g_0/2)^m}{mm!} \exp\left(\frac{\alpha t}{m}\right), \tag{4}$$

where the quantity $g_0 = 1 - \eta_0^2$ is determined by the partial inelastic cross-section in the s-state, and, according to the unitarity condition, $0 \leq g_0 \leq 1$. At low transferred momenta and near the diffraction peak, this formula can be written down as follows:

$$F(t) \simeq F(0) \exp(a_1 \alpha t + a_2 \alpha^2 t^2 + \dots), \tag{5}$$

where the coefficients a_i ($i = 1, 2, \dots$) are expressed through sums (see works [9,10]). At $|t| > |t_d|$, where $t_d \simeq 1$ (GeV/c)^2 determines the diffraction cone boundary, the summation in Eq. (4) can be replaced – with a high accuracy – by integration. Then, the scattering amplitude reads [9, 12, 13]

$$F(t) \simeq i\sqrt{\pi|\ln g_0|}|t|^{-1}(1 + 0,25(\alpha t \ln g_0)^{-1/2} \times$$

$$\times \exp(-2\sqrt{\alpha t \ln g_0}), \tag{6}$$

where the inequality $g_0 < 1$ is supposed.

In the case of maximal absorption in the s-state, we obtain

$$F(t) \simeq i\alpha(2/3)\Phi\,(3/2, 5/2; \alpha) =$$

$$= i\alpha(2/3)e^{\alpha t}\Phi\,(1, 5/2; -\alpha t)\,, \tag{7}$$

where $\Phi(\alpha, \gamma, z)$ is the degenerate hypergeometric function [11]. The second equality in Eq. (7) is especially convenient for making the qualitative analysis in the diffraction cone region; assuming $\Phi \simeq 1$, we obtain the diffraction pattern $F \sim e^{\alpha t}$. For the momentum transfers beyond the diffraction cone, it is convenient to express the right-hand side of Eq. (7) in terms of the probability integral $\Phi(z)$ [11]. Using the known formulas (9.212.2), (9.215.1), and (9.236.1) from [11] for degenerate hypergeometric functions, we obtain

$$F(t) \simeq i|t|^{-1}\left((\pi/4\alpha t)^{1/2}\Phi\sqrt{\alpha|t|} - e^{\alpha t}\right). \tag{8}$$

Hence, in the Van Hove model, the scattering amplitude beyond the diffraction cone is given by formulas (7) and (8) at $|g_0| < 1$ and $g_0 = 1$, respectively. These results, which are valid in a wider region of momenta transferred in comparison with the case considered by Van Hove, attracted no attention. However, it is the study of the general formulas (6) and (8) that gives a clear general picture [8].

Consider the consequences which follow from using the modified overlap function (2). The parameters of this model are connected with the quantity g_0 by the relation

$$4\pi g_0 \simeq a\alpha^{-1} - b\beta^{-1}. \tag{9}$$

Whence, according to the unitarity condition, we obtain

$$0 \leq a\alpha^{-1} - b\beta^{-1} < 4\pi. \tag{10}$$

The most general consideration discussed in works [7, 10] brought about the lower limit of the overlap function derivative. Using this relation for the total cross-section, we obtain

$$\sigma_t = \frac{2\pi}{k^2}\int_0^{\infty}\left(1 - \sqrt{1 - g_e}\right) d(l + 1/2)^2 =$$

$$= -\frac{2\pi}{k^2}\int_0^{\infty}\left(1 - \sqrt{1 - z}\right)\left[\frac{dg_e}{d(l + 1/2)^2}\right]^{-1} dz. \tag{11}$$

The calculation of the integral gives rise ultimately to the following result:

$$\sigma_t = 8\pi\alpha\left[1-\eta_0+\ln\left(\frac{c\eta_0+\sqrt{1-cg_0}+c-1}{2c}\right)+\right.$$

$$\left.+\frac{1}{\sqrt{c}}\ln\left(\frac{1+\sqrt{c}}{\sqrt{1-cg_0+\eta_0\sqrt{c}}}\right)\right], \eta_0=\sqrt{1-g_0}. \quad (12)$$

In contrast to the nonalternating series (4) in the Van Hove model, the scattering amplitude in model (2) is an alternating series

$$F(t) = 2i\sqrt{\pi}\alpha\sum_{m=1}^{\infty}\frac{(2m-3)!!}{m!}\sum_{n=0}^{m}\binom{m}{n}\left(-\frac{c}{2}\right)^n\times$$

$$\times\frac{x^{m+n}}{m+n}e^{-\frac{\alpha|t|}{m+n}}. \quad (13)$$

Carrying out the Laplace transformation with respect to the quantity $\alpha\,|t|$, one can obtain the following integral representation for the amplitude:

$$\frac{F(t)}{2i\sqrt{\pi}\alpha} = xe^{\alpha t}+2\int_0^{\infty}J_0\left(2z\sqrt{\alpha|t|}\right)\times$$

$$\times\left[1-xe^{-z^2}-\sqrt{1-2xe^{-z^2}+cx^2e^{-2z^2}}\right]zdz. \quad (14)$$

This implies immediately that, at $c=1$,

$$F(t) = 2i\sqrt{\pi}\alpha xe^{\alpha t}. \quad (15)$$

This case was considered in works [8–10]. Note that, if the transferred momenta are low, the scattering amplitude near the diffraction peak can be presented by a formula of type (5). The slope of the diffraction peak in the Van Hove model is equal to $\sigma_{\text{in}}/2\pi g_0 > \sigma_{\text{in}}/2\pi$ which is larger than the experimental value in the case of pp-scattering. In the advanced model (2), this slope is equal to $(\sigma_{\text{in}}-b)/2\pi g_0$, which agrees with experiment.

To calculate the amplitude, we use the results of our works [8–10]. Supposing that $c\neq 1$ (the case $c=0$ corresponds to the model of uncorrelated jets), let us expand the expression in the brackets in Eq. (14) in a series in xe^{-z^2}:

$$\left[1-xe^{-z^2}-\sqrt{1-2xe^{-z^2}+cx^2-e^{-2t^2}}\right]=$$

$$=\sum_{m=2}^{\infty}(m-2)!\left(xe^{-z^2}\right)^m A_m(c). \quad (16)$$

If $z=c=0$, we find

$$A_m(0)=\frac{2m-3}{m!(m-2)!}.$$

Of great interest is the case of scattering beyond the diffraction cone. The relevant amplitudes are

$$\frac{F(t)}{2i\sqrt{2}\alpha\left(1-c^{-1}\right)^{1/4}}=\frac{\sqrt{\pi}}{\gamma e^{-2u\sqrt{\gamma}}}\left[(u+1/2\sqrt{\gamma})\times\right.$$

$$\left.\times\cos\left(2v\sqrt{\gamma}+\frac{\varphi}{2}\right)+v\sin\left(2v\sqrt{\gamma}+\frac{\varphi}{2}\right)\right], \quad (17)$$

where the following notations were introduced:

$$\beta\equiv\left|\ln x\sqrt{c}\right|,\quad \gamma\equiv\alpha\,|t|\,,\quad \varphi_1\equiv\frac{\pi}{2}-\varphi,$$

$$\sqrt{2}u\equiv\sqrt{\varphi_1^2+\beta^2}+\beta,$$

$$\sqrt{2}v\equiv\sqrt{\sqrt{\varphi_1^2+\beta^2}-\beta}.$$

Squaring Eq. (17) and neglecting $\sqrt{\gamma}/2$ in comparison with u, we obtain the expression for the differential cross-section

$$\frac{d\sigma}{dt}\approx 4\pi\alpha^2\left(1-c^{-1}\right)^{\frac{1}{2}}\gamma^{-2}\exp\left(-4u\sqrt{\gamma}\right)\left[\left(\varphi_1^2+\right.\right.$$

$$\left.\left.+\beta^2\right)^{\frac{1}{2}}+\beta\cos\left(4v\sqrt{\gamma+\varphi}\right)+\varphi_1\sin\left(4v\sqrt{\gamma}+\varphi\right)\right] \quad (18)$$

which coincides with the result of work [12], where the issue concerning the oscillations of the differential cross-section with respect to the momentum was mentioned for the first time. Note that these oscillations stem from the choice of the overlap function just in form (2), i.e. when the amplitude is represented by an alternating series, unlike the case of the Van Hove model (1).

2. In a number of works [5–7, 12, 17], the method to solve the unitarity condition for the elastic scattering amplitude of two spinless particles in the high-energy range, provided that the contribution of all inelastic reaction channels (the inelastic overlap function) is given, was proposed, and, on the basis of this solution, the asymptotic behavior of the scattering amplitude was studied in detail. The expansion series of the scattering amplitude in terms of Legendre polynomials were evaluated by the saddle-point method. The results obtained turned out simpler and, to some extent, more general than, e.g., the corresponding expansions in the potential approach or the Regge expansions. Therefore, the approach used in those works is more convenient for studying the angular distribution in a wide range of scattering angles. In the cited works, the spectral density of the overlap function was taken in a general form as

$$\varphi_g(\rho) = a_1(\rho)\, e^{-\beta(\rho)} \tag{19}$$

in the impact parameter representation. Although such an approximation does allow the basic regularities in the behavior of the differential scattering cross-section of two protons to be described, it contradicts the results of numerical calculations of the overlap function for pp-scattering [10], according to which the function $\varphi_g(\rho)$ should most likely be given as a superposition of two exponents (see Section 1):

$$\varphi_g(\rho) = a_1(\rho)\, e^{-\beta(\rho)} - a_2 e^{-2\beta(\rho)}. \tag{20}$$

In this case, the zero point of the overlap function $G(t)$ and the change of its sign determined by numerical calculations [4, 16] are obtained in the natural manner. The solution of unitarity condition in the specific case where the function $\varphi_g(\rho)$ was selected in the form of a superposition of two Gaussian-like profiles ($\beta(\rho) = \rho^2 2b$) was examined in works [7, 9], where the characteristics of elastic scattering were demonstrated to depend substantially on the model parameters. In this section, the unitarity condition [10, 20] for the scattering amplitude is solved, provided that the spectral function $\varphi_g(\rho)$ is given in the general form; in addition, we take here into account the real part of amplitude which can play an essential role in the scattering at large angles [6, 10, 12–25] and allows the fine structure of differential cross-section to be studied. The consideration of a more general problem can enable the study of angular distribution to be carried out in a wider range of momenta transferred.

The solution of the unitarity condition in terms of spectral functions at asymptotically high energies and in the zero-order approximation looks like

$$\varphi(\rho) = 1 - e^{2i\alpha(\rho)} + e^{2i\alpha(\rho)}\left(1 - \sqrt{1 - \varphi_g(\rho)}\right), \tag{21}$$

where an arbitrary real function $\alpha(\rho)$ can be interpreted as a phase shift of purely elastic scattering. We take it in the form [4, 5]

$$2\alpha = -d(\rho) e^{-\psi(\rho)}, \tag{22}$$

and the functions $d(\rho)$ and $\psi(\rho)$ in Eq. (22) and $a_1(\rho)$, $a_2(\rho)$, and $\beta(\rho)$ in Eq. (20) can be regarded as certain even functions of the impact parameter.

The scattering amplitude can be presented in the accepted model [Eqs. (20) and (22)] as a sum of three terms:

$$F_0(t) = i\sqrt{\pi}\int\limits_0^\infty \left(1 - e^{2i\alpha(\rho)} J_0\left(\rho|t|^{\frac{1}{2}}\right)\right)\rho d\rho, \tag{23}$$

$$F_1(t) = i\sqrt{\pi}\int\limits_0^\infty e^{2i\alpha(\rho)-\beta(\rho)} a(\rho) J_0\left(\rho|t|^{\frac{1}{2}}\right)\rho d\rho, \tag{24}$$

$$F_2(t) = i\sqrt{\pi}\int\limits_0^\infty (m-2)! A_m(c)\left[a(\rho)e^{-\beta(\rho)}\right]^m \times$$

$$\times e^{2i\alpha(\rho)} J_0\left(\rho|t|^{\frac{1}{2}}\right)\rho d\rho, \tag{25}$$

where $a(\rho) = a_1(\rho)/2$, $\delta^2 = |1 - c(\rho)|$, $c(\rho) = 4a_2(\rho)/a_1^2(\rho)$, $\beta(\rho) = \rho^2/2b_1$, and $\psi(\rho) = \rho^2/2b_2$. Equation (25) was obtained making use of equality (16). Let $c(\rho) < 1$ for every ρ. Using the values for sums $A_m(c)$ found in works [8–10], Eq. (25) can be rewritten as

$$F_2(t) = \frac{i}{2}\sum_{m=2}^\infty\sum_{n=0}^\infty \frac{i^n A_{mn}}{n! m^{\frac{3}{2}}}, \tag{26}$$

where

$$A_{mn} \equiv \int\limits_0^\infty \frac{2\delta}{(1+\delta)}\left[(1+\delta)ae^{-\beta}\right]^m \left(-de^{-\psi}\right)^n \times$$

$$\times J_0\left(\rho|t|^{\frac{1}{2}}\right)\rho d\rho. \tag{27}$$

One can see that, at $c(\rho) < 1$, the expression $F_2(t)$ in model (20) differs from the corresponding one in model (19) in that the former contains the subintegral factor $2\delta/\sqrt{1+\delta}$ and the replacement $a_1 \longrightarrow a_1(1+\delta)/2$ is made.

3. Compare the results obtained with experimental ones for pp- and $\pi^{\pm}$p-scattering. In models (19), (21), and (24), the differential cross-section in the diffraction cone region is described by five parameters: b_1, a, c, d, and b_2. The parameter a is determined by the optical theorem ($ab_1 \approx \sigma_{\rm in}/4\pi$), knowing the measured b_1-value (it is the slope of the diffraction peak) and the total cross-section. The values of the parameter c can be determined from the exponential factor in the differential cross-section of the scattering at high momenta transferred. The well-measured ratio between the real and imaginary parts of the forward scattering amplitude is determined by the formula [8–13]

$$\delta\left(0\right) \approx -\left(db_2/ab_1\right)\left(1 - ab_1\left(b_1 + b_2\right)\right). \tag{28}$$

The parameter b_2 is determined by fitting the results of calculations to experimental data at low momenta transferred. Expression (21), as well as (18) one, is the zero-order approximation. It is evident from Eq. (27) that the quantity $\delta(0)$ equals zero only if $d = 0$, because, in experiment, $a \approx 1$. But the equality $d = 0$ means that the amplitude in model (18) has a pure imaginary value at every t, which has no sense. Moreover, the substitution $d \to -d$ induces an unsubstantiated change of the sign of $\delta(t)$ at every t. Therefore, it is expedient to take

$$2\alpha\left(\rho\right) = -d_1\left(\rho\right)e^{-\psi_1(\rho)} + d_2\left(\rho\right)e^{-\psi_2(\rho)} \tag{29}$$

instead of expression (21).

As was noted in works [4,9], the formula of type (19) is the first terms of a more exact expansion series for the overlap function obtained in the eikonal approach [10,19]. Really, taking the overlap function in the impact parameter representation in the form

$$\varphi_g\left(\rho\right) = 1 - \exp\left(\,-x_g\left(\rho\right)\right) \tag{30}$$

and considering $x_g = 2ae^{-\beta}$ as the Born term, we see that formulas (18) and (19) correspond to the first and second Born approximations, respectively.

While making a comparison with experiment, we will use the exact expressions (19), (21), (24), (28), and (29):

$$F_0\left(t\right) = -i\sqrt{\pi}b_2\sum_{n=1}^{\infty}\frac{\left(\,-id\right)^n}{n!n}\exp\left(\frac{b_2t}{2n}\right), \tag{31}$$

$$F_1(t) = i\sqrt{\pi}a\sum_{n=0}^{\infty}\frac{(-id)^n}{n!}\left(\frac{n}{b_2}+\frac{1}{b_1}\right)^{-1}\times$$

$$\times\exp\left(\frac{t}{2\left(nb_2^{-1}+b_1^{-1}\right)}\right), \tag{32}$$

$$F_2\left(t\right) = i\sqrt{\pi}2\sum_{m=2}^{\infty}\left(m-2\right)!\sum_{k=1}^{-cm/2}\frac{k\left(1-c\right)^k a^m}{2^{2k(k!)^2(m-2k)!}}\times$$

$$\times\sum_{n=0}^{\infty}\frac{\left(-id\right)^n\exp\left(t/2\left(nb_2^{-1}+mb_1\right)\right)}{n!\left(nb_2^{-1}+mb_1^{-1}\right)}. \tag{33}$$

In models (24), (28), and (29), the solution of the unitarity condition it a sum of three terms [9, 10, 17]:

$$\frac{F_0\left(t\right)}{i\sqrt{\pi}} = -\sum_{n=1}^{\infty}\sum_{l=0}^{\infty}\frac{\left(-id_1\right)^n\left(d_2/d_1\right)^l}{l!\left(n-l\right)!\left(nb_2^{-1}+l\left(b_3^{-1}-b_2^{-1}\right)\right)}\times$$

$$\times\exp\left(\frac{t}{2\left(nb_2^{-1}+l\left(b_3^{-1}-b_2^{-1}\right)\right)}\right), \tag{34}$$

$$\frac{F_1\left(t\right)}{i\sqrt{\pi}} =$$

$$= a\sum_{n=0}^{\infty}\sum_{l=0}^{n}\frac{\left(id_1\right)^n\left(-d_2/d_1\right)^l}{l!(n-l)!\left(nb_2^{-1}+l\left(b_3^{-1}-b_2^{-1}\right)+b_1^{-1}\right)}\times$$

$$\times\exp\left(\frac{t}{2\left(nb_2^{-1}+l\left(b_3^{-1}-b_2^{-1}\right)+b_1^{-1}\right)}\right), \tag{35}$$

$$\frac{F_3\left(t\right)}{i\sqrt{\pi}} = 2\sum_{m=2}^{\infty}\sum_{n=0}^{\infty}\sum_{l=0}^{n}\sum_{k=1}^{E\left(\frac{m}{2}\right)}\frac{\left(id_1\right)^n\left(-d_2/d_1\right)^l}{2^{2k}\left(k!\right)^2 l!\left(n-l\right)!}\times$$

$$\times\frac{(m-2)!a^m k\left(1-c\right)^k}{(m-2k)!\left(nb_2^{-1}+l\left(b_3^{-1}-b_2^{-1}\right)-mb_1^{-1}\right)}\times$$

$$\times\exp\left(\frac{t}{2\left(nb_2^{-1}+l\left(b_3^{-1}-b_2^{-1}\right)+mb_1^{-1}\right)}\right). \tag{36}$$

The scattering amplitude is written down as a sum of three terms which are determined by formulas (31)–(33) and (34)–(36) in the cases of models (28) and (29), respectively. The differential cross-sections calculated by those formulas were compared with experimental data on elastic proton-proton and pion-proton diffraction

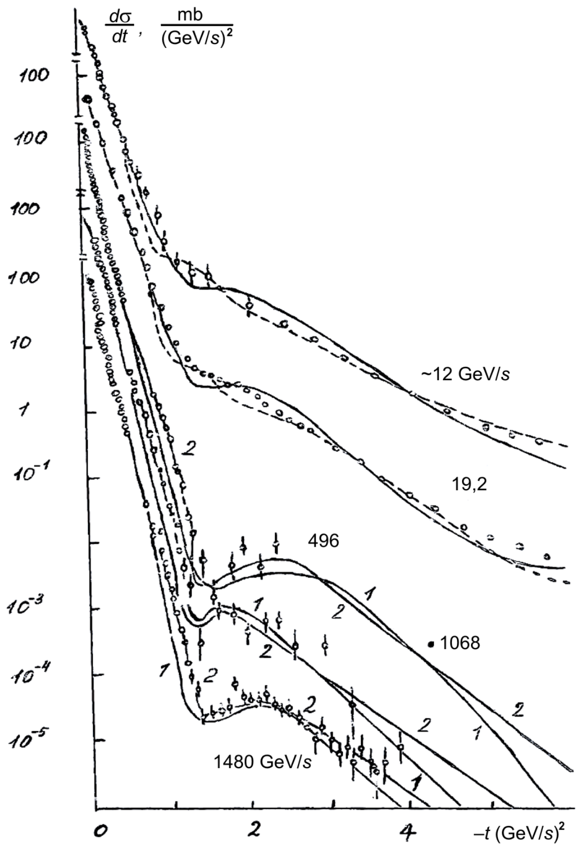

Fig. 1. Differential cross-sections of pp-scattering and the corresponding theoretical curves in models (30)–(32) (solid curves) and (33)–(35) (dashed curves) at various energies

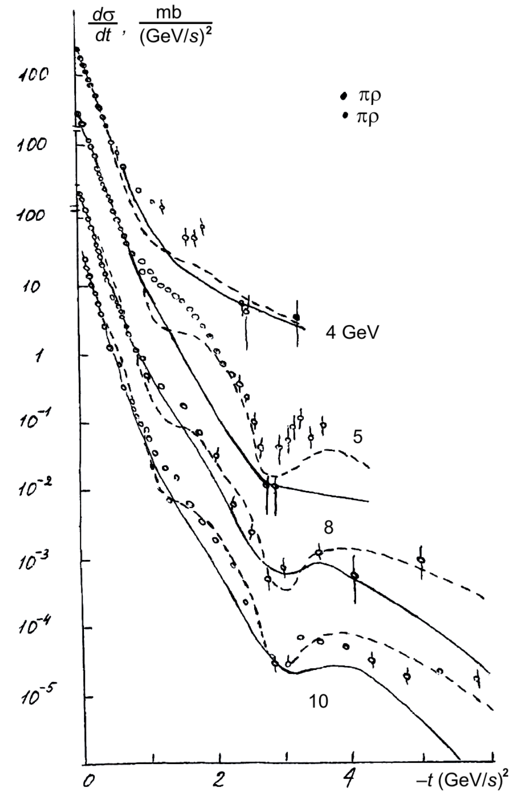

Fig. 2. The same as in Fig. 1, but for $\pi^{\pm}$p-scattering

scattering. The parameters, which enter those formulas, were fitted to obtain the best agreement with the experimental angular distribution.

The parameters for models (31)–(33) and (34)–(36) are given in work [15]. We did not succeed to satisfactorily describe the experimental data measured at ISR energies within the latter model. In Tables 1 and 2, we give the theoretical values for total cross-sections which agree satisfactorily with experimental data (see works [5,16]). In Figs. 1 and 2, the experimental dependences of the differential cross-sections for elastic pp- and $\pi^{\pm}$p-scattering are plotted, as well as the corresponding dependences calculated in models (30)–(32) (solid curves) and (33)–(35) (dashed curves). The calculations for ISR energies were carried out for two

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Interaction type	p_l, GeV/c	$\sigma_t^{\text{experiment}}$	σ_t^{theory}	$\sigma_{\text{el}}^{\text{experiment}}$	$\sigma_{\text{el}}^{\text{theory}}$	$\sigma_{ln}^{\text{experiment}}$	$\sigma_{ln}^{\text{theory}}$
		mb					
pp	19.2	38.9±0.3	38.639	9.69±0.73	9.139	29.21	30.68
	12	39.4±0.6	39.426	9.87	8.63	29.53	30.8
	8	40.0±0.6	39.010	8.79	8.705	31.21	31.51
	5	44.0±1.0	47.805	–	11.17	–	27.515
π^-p	10	26.87±0.08	28.082	–	6.119	–	22.543
	8	27.5±0.3	28.408	–	6.692	–	21.24
	6	28.072±0.025	29.658	5.5	6.736	22.57	22.572
	5	29.12±0.01	30.192	5	7.072	24.12	24.12
	5	26.483±0.01	26.634	5.94	6.084	20.54	20.543
π^+p	4	27.721±0.01	27.394	6.42	6.093	21.304	22.293
	3.5	28.224±0.015	28.83	5.9	6.118	22.324	22.324

T a b l e 2

Interaction type	p_l, GeV/c	$\sigma_t^{\text{experiment}}$	σ_t^{theory}	$\sigma_{\text{el}}^{\text{experiment}}$	$\sigma_{\text{el}}^{\text{theory}}$	$\sigma_{ln}^{\text{experiment}}$	$\sigma_{ln}^{\text{theory}}$
		mb					
pp	1480	43.3±0.6	43.276	7.6±0.3	7.426	35.6	35.85
	1068	42.5±0.5	42.388	7.5±0.3	7.483	35.0	34.905
	496	40.5±0.5	39.881	7.0±0.2	7.454	33.5	32.427
	19.2	38.9±0.3	39.241	9.69±0.73	8.562	29.21	30.679
	12	39.4±0.6	40.295	9.87	9.492	29.53	30.803
	8	40.0±0.6	41.443	8.79	9.79	31.21	32.653
	5	44.0±1.0	46.8	–	10.93	–	35.87
π^-p	10	26.87±0.08	27555	–	5.012	–	22.543
	8	27.5±0.3	26.745	–	4.507	–	22.248
	6	28.07±0.025	26.681	5.5	4.731	22.57	21.95
	5	29.12±0.01	28.516	5	5.657	24.12	22.859
	5	26.483±0.01	25.227	5.94	4.795	20.543	20.432
π^+p	4	27.724±0.010	26.713	6.42	7.192	21.304	21.521
	3.5	28.224±0.015	30.912	5.9	7.815	22.324	23.098

sets of parameters: the data corresponding to curves *1* were calculated to obtain a good agreement of all the quantities $d\sigma/dt$, σ_t, σ_{in}, and $\delta(0)$ with experiment simultaneously, while the calculations of curves *2* included only fitting to angular distributions. Figures 1 and 2 demonstrate that both models describe satisfactorily the structure of the differential elastic scattering cross-section, in particular, the positions and the amplitudes of peaks and second maxima. The analysis of the scattering amplitude in model (30)–(32) testifies that the slope of the differential scattering cross-section $b(t)$ slightly diminishes at low $|t|$ as the value of $|t|$ decreases [9]. Very interesting is the variation of the $\delta(t)$-dependence behavior with the change of the momentum transferred: as the value of $|t|$ grows, the absolute value of $\delta(t)$ first increases and the $\delta(t)$-sign does not change; then, having reached its maximum, the value of $\delta(t)$ drastically decreases, changes its sign, and continues to grow by absolute value. The overlap function in model (19) has a zero point for pp-scattering in the interval $t \approx 0.5 \div 0.7$ (GeV/c)^2, where this t-value diminishes as the energy grows, and a zero point for $\pi^{\pm}$p-scattering in the interval $t \approx 0.8 \div 0.9$ (GeV/c)^2. In model (29), the dependence $G(t)$ becomes zero, with the change of its sign, three times in the considered region of momentum transferred $|t| \leq 6$ (GeV/c)^2: for pp-scattering, at $t_0 \approx 0.5$ and 4.5 (GeV/c)^2; and for $\pi^{\pm}$p-scattering, at $t \approx 1.3$ and 6 (GeV/c)^2. The value of the overlap function $\varphi_g(\rho = 0)$ depends very weakly on the energy for all processes under consideration; it is especially true for model (29), where $\varphi_g(\rho = 0) \approx 0.95$, whereas $\varphi_g(\rho = 0) \approx 0.8 \div 0.9$ in model (19). It means that the hypothesis of geometrical scaling [7–9] of the overlap function is valid even at energies of the order of a few GeV.

The considered overlap functions (19) and (29) differ substantially from a Gaussian-like dependence, especially in the central area. Models (19) and (29) result in a slower – in comparison with the Gaussian-like behavior – decrease of the overlap function in its central region as the value of the impact parameter grows. This means that the corrections for absorption are essential. At larger ρ, dependences (19) and (29) transform into Gaussian-like ones.

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ФУНКЦІЯ ПЕРЕКРИВАННЯ ВАН ХОВА ТА ПРУЖНЕ ДИФРАКЦІЙНЕ РОЗСІЯННЯ АДРОНІВ ВИСОКИХ ЕНЕРГІЙ

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Р е з ю м е

За допомогою функції перекривання досліджено пружне дифракційне розсіяння адронів в умовах передавання великих імпульсів. Розв'язана умова унітарності для амплітуди пружного дифракційного розсіяння з модифікованою функцією перекривання. Отримані значення диференціальних перерізів порівняно з виміряними у експериментах з pp- та $\pi^{\pm}$p-розсіяння для різних енергій.