MASS SPECTRUM AND LEPTONIC DECAY CONSTANTS OF PSEUDOSCALAR MESONS

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A phenomenological model of hadrons has been developed in terms of bilocal meson fields with regard for the renormalization of the wave function of a quark "inside" a meson. A modified Schwinger–Dyson equation for an arbitrary potential has been proposed, the solutions of which are free of ultra-violet divergences but preserve the proper asymptotic properties. A boundary-value problem for the potential of quark–quark interaction in the form of a sum of oscillator and Coulomb-like potentials has been formulated, the solutions of which describe meson spectroscopy and meson–meson interaction. It is shown that, in contrast to other hadron phenomenological models based on the effective action of quantum chromodynamics (QCD), the proposed model describes both the mass spectrum and the leptonic decay constants of pseudoscalar mesons on the same basis and at a good quantitative level.

1. Introduction

In recent years, quark potential models (QPMs) have been used most usefully for the description of hadron spectroscopy (including that of mesons) and the dynamics of their formation and decay. These models are based on the fundamental ideas of QCD with the phenomenological description of quark–quark interaction. This class of models includes, in particular, a bilocal relativistic potential model (BRPM) [1], which is based on two principles: the minimal quantization of gauge fields and a definite choice of the axis of quantization [2].

In this model, the description of the spectroscopy of mesons, which are a bound state of a quark q and an antiquark $\bar{q}$, and the interaction between them is reduced to the solution of the Salpeter equation (SE) for a bound $q\bar{q}$-system and the Schwinger–Dyson equation (SDE) for the phase function of a quark and its one-particle energy "inside" a meson, provided that the potential of quark–quark interaction is given. In this case, the SDE allows one to calculate the "dynamic" mass of the quark for the given potential, which is a measure of spontaneous violation of the chiral symmetry (SVCS). At the same time, the solutions of the SE correspond to the masses and the wave functions of free mesons.

One of the basic issues in QPMs, including the BRPM, is the choice of the shape of the quark–quark interaction potential. This choice is ambiguous, because a consecutive theory that would be valid at all distances is absent. One of the selection criteria is that the quark–quark potential must be universal and independent of quarks' aroma (of course, taking the running coupling constant and quark masses into account).

In the description of meson spectroscopy, quite widely used are linear- and quadratic-in-distance potentials. The reason is that they bring about SVCS and the emergence of the constituent mass of a quark [3, 4].

In work [4], the analysis of the solutions of the SDE and the SE with a linearly growing potential has demonstrated that this approximation cannot give a correct wave function for a pion. This means that the approximation of linearly growing potential does not allow the correct values of the low-energy hadron physics parameters to be reproduced.

Concerning the oscillator potential, there are the rather sound statements that it is a good approximation at distances of the order of the light quark wavelength [3]. It was in this approximation and provided the zero current mass of the quark that the existence of a nontrivial solution for the SDE, which described SVCS and the emergence of the constituent mass of a quark, was proved.

In works [5–7], the oscillator potential was used to show that the BRPM reproduces the mass spectrum of pseudoscalar mesons quantitatively at a satisfactory level. However, the calculated values for the constants of their leptonic decay, which is one of the verification criteria that the wave functions found for mesons are correct, proved to be underestimated by a factor of about four in comparison with experimentally determined ones [6, 7]. The addition of a Coulomb-like term, which describes the one-gluon exchange between quarks, to the oscillator potential has allowed the Coulomb correction to the mass spectrum of pseudoscalar mesons to be estimated. However, the account of those corrections turned out insufficient for the theoretical values of the leptonic decay constants to be increased up to their experimental values.

It should be noted here that the presence of a Coulomb-like term in the potential gives rise to ultraviolet (UV) divergences, the regularization of which is made by applying the standard renormalization routine to the wave function of a quark "inside" a meson.

In this work, in order to obtain a self-consistent description of the spectroscopy of pseudoscalar mesons and their leptonic decay constants and to eliminate UV divergences which emerge if the Coulomb-like term is included into the potential of quark–quark interaction, we propose to modify the BRPM by introducing counterterms into the SDE which are used to renormalize the wave function of a quark "inside" a meson. In so doing, such counterterms are selected to be included, which would not break the asymptotic properties of SDE solutions. Giving the potential in the form of a sum of the oscillator and Coulomb-like potentials makes the boundary-value problem to be formulated, where the SDE contains counterterms. By choosing proper counterterms from physical reasons and solving the obtained modified SDE together with the SE, we demonstrated that the proposed model describes both the mass spectrum and the leptonic decay constants of pseudoscalar mesons at a good quantitative level and on the same basis.

2. Model Equations

The Bethe–Salpeter equation is known to be the most general relativistic equation which describes a state of the bound system formed as a result of quark–antiquark interaction in a meson. In the momentum representation, the equation looks like [1]

$$\Gamma(p,P) = -\imath \int \frac{d^4q}{(2\pi)^4} V(|\mathbf{p}-\mathbf{q}|)\ \not\eta G_{\Sigma_{a,b}} \times$$

$$\times \left(q+\frac{P}{2}\right)\Gamma(q,P)G_{\Sigma_{a,b}}\left(q-\frac{P}{2}\right)\ \not\eta. \quad (1)$$

Here, $\not\eta = \eta_\mu\gamma^\mu$; η_μ is a unit time-like vector directed along the time axis of the gauge field, which, while describing the bound state, is selected proportional to its complete momentum: $\eta_\mu \sim P_\mu$; $G_{\Sigma_{a,b}}(q,P)$ is Green's function of a quark (antiquark) "inside" a meson; and $V(|p-q|)$ is the potential of quark–quark interaction.

In Eq. (1), $\Gamma(q,P)$ is the vertex function, which describes the $\bar{q}q$-system and is connected with its relativistic wave function $\Psi_P(q^\perp,P)$ through the relation

$$\Gamma(q^\perp,P) = -S_a(q^\perp)\Big\{[E_t(q^\perp)-M_P]\Lambda_+^P\Psi_P(q^\perp,P)\Lambda_-^P$$

$$+[E_t(q^\perp)+M_P]\Lambda_-^P\Psi_P(q^\perp,P)\Lambda_+^P\Big\}S_b^{-1}(q^\perp), \quad (2)$$

where

$$\Lambda_\pm^P = \frac{1}{2}\left(1\pm\frac{\not P}{M}\right)$$

are the projection operators;

$$S_{a,b}(q^\perp) = \sin\varphi_{a,b}(q^\perp) + \frac{q^\perp}{|q^\perp|}\cos\varphi_{a,b}(q^\perp)$$

is a transformation matrix of the Foldy–Wouthuysen type [1];

$$E_t(q^\perp) = E_a(q^\perp) + E_b(q^\perp);$$

$$\Psi_P(q^\perp) = \gamma_5\left[L_1(q^\perp) + \frac{\not P}{M_P}L_2(q^\perp)\right];$$

$$q^\perp = q - q^\parallel; \quad q^\parallel = qP/\sqrt{P^2};$$

$$\not P = \gamma_\mu P^\mu; \quad \not q = \gamma_\mu q^\mu;$$

$\varphi_{a,b}$ and $E_{a,b}(q^\perp)$ are the phase functions and one-particle energies, respectively, of a quark (antiquark) "inside" a meson; $L_{1,2}(q^\perp)$ are the components of the wave function; and M_P is the mass of a pseudoscalar meson.

While describing meson spectroscopy and meson–meson interactions, the SE for a coupled $q\bar{q}$-system, which follows from Eq. (1) under definite assumptions, is used as a rule. In our case, this equation can be derived from Eq. (1) by substituting Eq. (2) into the latter and integrating the result with respect to the variable $q^{\|}$. We obtain

$$M_P L_{2(1)}(q^{\perp}) = E_t L_{1(2)}(q^{\perp}) -$$

$$- \int \frac{d^3 q^{\perp}}{(2\pi)^3} V(|p^{\perp} - q^{\perp}|)(c_p^{\mp} c_q^{\mp} + s_p^{\mp} s_q^{\mp} \xi) L_{1(2)}(q^{\perp}), \quad (3)$$

where $c_p^{\pm} = \cos[\varphi_a(q^{\perp}) \pm \varphi_b(q^{\perp})]$, $s_p^{\pm} = \sin[\varphi_a(q^{\perp}) \pm \varphi_b(q^{\perp})]$, $\xi = p^{\perp} q^{\perp} / (|p^{\perp}|\,|q^{\perp}|)$, $p^{\perp} = p - p^{\|}$, and $p^{\|} = pP/\sqrt{P^2}$. In this case, the normalization of the meson wave functions is determined by the following relation:

$$\frac{2N_c}{M_p} \int \frac{d^3 q^{\perp}}{(2\pi)^3} \left[L_1(q^{\perp}) L_2^*(q^{\perp}) + L_2(q^{\perp}) L_1^*(q^{\perp}) \right] = 1, \quad (4)$$

where N_c is the number of color degrees of freedom of a quark (antiquark).

From Eq. (3), it is evident that, in order to solve the SE with a given potential of quark–quark interaction, it is necessary to obtain first the phase function of the quark (antiquark) and its one-particle energy. In the BRPM, they are determined by solving the SDE with the same potential:

$$E_{a,b}(p) \sin \varphi_{a,b}(p) =$$

$$= m_{a,b}^0 + \frac{1}{2} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) \sin \varphi_{a,b}(q),$$

$$E_{a,b}(p) \cos \varphi_{a,b}(p) =$$

$$= p + \frac{1}{2} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) \xi \cos \varphi_{a,b}(q), \quad (5)$$

where $m_{a,b}^0$ is the current mass of the quark.

Quarks "inside" a meson are known to possess a finite energy at low momenta, and to behave as free particles (the asymptotic freedom) at larger momenta. Those conditions for the meson wave function look like

$$\lim_{p \to 0} \varphi_{a,b}(p) = \frac{\pi}{2},$$

$$\lim_{p \to \infty} \varphi_{a,b}(p) = \arcsin \frac{m_{a,b}^0}{\sqrt{{m_{a,b}^0}^2 + p^2}}. \quad (6)$$

Not every potential used in the description of meson spectroscopy and meson–meson interactions allows one to obtain the solutions of Eq. (5), which would satisfy the boundary conditions (6). In this connection, it should be emphasized that, if the potential contains a Coulomb-like term, the SDE solutions for such a potential have UV-divergences.

In order to obtain the SDE solutions for an arbitrary potential, which satisfies Eq. (6) but is free of UV-divergences even if the potential does contain a Coulomb-like term, we have to modify Eq. (5) by introducing the additional functions $f_1(q)$ and $f_2(q)$, which do not change the asymptotic properties of its solutions, into it:

$$E_{a,b}(p) \sin \varphi_{a,b}^m(p) = m_{a,b}^0 [1 - Z_m(p)] +$$

$$+ \frac{1}{2} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) \sin \varphi_{a,b}^m(q),$$

$$E_{a,b}(p) \cos \varphi_{a,b}^m(p) = p[1 - Z(p)) +$$

$$+ \frac{1}{2} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) \xi \cos \varphi_{a,b}^m(q), \quad (7)$$

where

$$Z_m(p) = \frac{1}{2m_{a,b}^0} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) f_1(q),$$

$$Z(p) = \frac{1}{2p} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) \xi f_2(q), \quad (8)$$

and $\varphi_{a,b}^m(p)$ are the solutions of the modified SDE with the given potential of quark–quark interaction. If $f_1(q) = 0$ and $f_2(q) = 0$, Eq. (7) transforms into Eq. (5).

By introducing the notations

$$I_1(p) = \frac{1}{2} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) \sin \varphi_{a,b}^m(q), \quad (9)$$

$$I_2(p) = \frac{1}{2} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) \xi \cos \varphi_{a,b}^m(q), \quad (10)$$

$$I_{11}(p) = \frac{1}{2} \int \frac{\mathbf{dq}}{(2\pi)^3} V(|\mathbf{p} - \mathbf{q}|) f_1(q), \quad (11)$$

$$I_{22}(p)=\frac{1}{2}\int\frac{d\mathbf{q}}{(2\pi)^3}V(|\mathbf{p}-\mathbf{q}|)\xi f_2(q). \tag{12}$$

Eqs. (7) read

$$E_{a,b}(p)\sin\varphi^m_{a,b}(p)=m^0_{a,b}+I_1(p)-I_{11}(p),$$

$$E_{a,b}(p)\cos\varphi^m_{a,b}(p)=p+I_2(p)-I_{22}(p). \tag{13}$$

In Eqs. (11) and (12), the counterterms $I_{11}(p)$ and $I_{22}(p)$ are selected from physical reasons and taking the boundary conditions (6) into account. For definite $I_{11}(p)$ and $I_{22}(p)$, one can obtain a modified SDE with an arbitrary potential, the solutions of which will satisfy conditions (6) and will be free of UV-divergences.

From Eq. (13), we obtain the following expression for the energy of a quark "inside" a meson:

$$E_{a,b}(p)=[m^0_{a,b}+I_1(p)-I_{11}(p)]\sin\varphi^m_{a,b}(p)+$$

$$+[p+I_2(p)-I_{22}(p)]\cos\varphi^m_{a,b}(p). \tag{14}$$

A common solution of Eqs. (3) and (13) obtained for the given counterterms $I_{11}(p)$ and $I_{22}(p)$ allows one to calculate also the leptonic decay constant f_p of a pseudoscalar meson, which plays a fundamental role in the description of low-energy hadron physics. In the framework of the BRPM,

$$f_p=\frac{N_c}{\pi^2\sqrt{\pi}M_p}\int\limits_0^\infty L_2(q^\perp)\sin\left[\frac{\varphi^m_a(q^\perp)+\varphi^m_b(q^\perp)}{2}\right]q^\perp dq^\perp. \tag{15}$$

From Eqs. (3) and (13) and relation (15), it follows that, fixing the counterterms $I_{11}(p)$ and $I_{22}(p)$, which correspond to a definite scheme of SDE modification with the given potential of quark–quark interaction, and fitting the eigenvalues of the SDE and the SE to the most accurate experimental mass values of the known mesons (by varying the values of free parameters, in particular, the masses of current quarks and the parameters of the potential), one can calculate both the mass spectrum and the leptonic decay constants of pseudoscalar mesons.

3. Mass Spectrum and Wave Functions of Pseudoscalar Mesons

To calculate the mass spectrum and the wave functions of mesons on the basis of Eqs. (3) and (13), it is necessary to preliminarily choose the modification scheme for an SDE with a given potential and to fit the values of the free parameters of the model. In works [8, 9], the values of the current mass of quarks ($m^0_{u,d}$, m^0_s, m^0_c, and m^0_b) were fitted by comparing the solutions of the SE with the masses of π, K, D, and B-mesons.

Consider the scheme of SDE modification, according to which the functions $f_1(q)$ and $f_2(q)$ correspond to the asymptotics of its free solution. As a quark–quark potential, we choose the potential that is a sum of the oscillator and Coulomb-like terms:

$$V(|\mathbf{p}-\mathbf{q}|)=\frac{4}{3}\left[(2\pi)^3V_0\Delta_{\mathbf{q}}\delta^3(|\mathbf{p}-\mathbf{q}|)+\frac{4\pi\alpha_s}{|\mathbf{p}-\mathbf{q}|^2}\right], \tag{16}$$

where V_0 and α_s are the parameters of the potential. The potential was selected in form (16), because, first, it is simple enough from the viewpoint of the wave function calculation procedure; second, this potential allowed the quantitative description of the mass spectrum of pseudoscalar mesons to be obtained; and third, its oscillator part confirms a conclusion about the strong splitting between the π- and ρ-meson masses owing to SVCS [3].

It is known that the solutions of the SDE with a potential including a Coulomb-like term contain UV-divergences. To eliminate them and to preserve the asymptotic properties of the solutions, let us take advantage of the modification scheme, according to which the functions $I_{11}(p)$ and $I_{22}(p)$ look like [8, 9]

$$I_{11}(p)=I^O_{11}(p)+I^C_{11}(p),\quad I_{22}(p)=I^O_{22}(p)+I^C_{22}(p), \tag{17}$$

where

$$I^O_{11}(p)=0,\quad I^O_{22}(p)=p\exp(-\lambda p), \tag{18}$$

$$I^C_{11}(q)=\frac{\alpha_s}{2\pi}\int\limits_0^\infty dqV_1(p,q)\frac{m^0}{\sqrt{{m^0}^2+q^2}},$$

$$I^C_{22}(q)=\frac{\alpha_s}{2\pi}\int\limits_0^\infty dqV_2(p,q)\frac{q}{\sqrt{{m^0}^2+q^2}}. \tag{19}$$

Here, λ is an extra free parameter, and

$$V_1(p,q)=\frac{q}{p}\ln\left|\frac{p+q}{p-q}\right|,$$

$$V_2(p,q) = \frac{q}{p}\left[\frac{p^2+q^2}{2pq}\ln\left|\frac{p+q}{p-q}\right| - 1\right].$$

In this case, the SDE with regard for Eqs. (12)–(19) reads

$$\varphi''(p) + \frac{2}{p}\varphi'(p) +$$

$$+\frac{1}{p^2}\sin 2\varphi(p) + 2m^0\cos\varphi(p) - 2p\sin\varphi(p) +$$

$$+[I_1(p) - I_{11}^O(p) - I_{11}^C(p)]\cos\varphi(p) -$$

$$-[I_2(p) - I_{22}^O(p) - I_{22}^C(p)]\sin\varphi(p) = 0, \tag{20}$$

where

$$I_1(q) = \frac{\alpha_s}{2\pi}\int_0^\infty dq V_1(p,q)\sin\varphi(q),$$

$$I_2(q) = \frac{\alpha_s}{2\pi}\int_0^\infty dq V_2(p,q)\cos\varphi(q). \tag{21}$$

In Eq. (20), the prime means a derivative with respect to the variable p.

It becomes evident that, provided the counterterms are given, the solution of Eq. (20) depends on the values of the current mass of the quark m^0 and has to satisfy the boundary conditions (6). By fixing definite values for m^0, the solutions of the modified SDE with the given potential can be obtained. These solutions should be substituted into Eq. (3), the solutions of which, as is known, are the mass spectrum and the wave functions of pseudoscalar mesons. Inserting them into expression (15), one can calculate the leptonic decay constant of pseudoscalar mesons as well.

4. Discussion of Results Obtained and the Conclusion

The boundary-value problem for the SDE and the SE, which describes the mass spectrum of pseudoscalar mesons and meson–meson interactions and which was formulated in this work, comprises a system of ordinary integro-differential equations; the solutions of the latter are found, as a rule, by numerical methods. The method of numerical solution of such equations, which is based on a continuous analog of the Newton method, was expounded in work [11].

To calculate the mass spectrum and the wave functions of pseudoscalar mesons on the basis of Eqs. (3), (4), (6), and (20), the modification scheme for the SDE is to be preliminarily selected and the values of the free parameters of the model to be fitted. In their turn, the values of the free parameters, which depend on the potential shape, are determined by fitting the eigenvalues of the SE to the mass values of the known mesons which have been measured in experiments most reliably.

In Table 1, the values of the free parameters m^0, λ, and V_0, which were obtained by solving numerically the modified SDE and SE with potential (16), are tabulated. The procedure also included the fitting of the obtained solutions for the SE to the masses of π, K, D, and B-mesons at fixed values of the parameter α_s (for all quark aromas, the value of the parameter α_s was the same). The table makes it evident that each α_s-value is associated with definite values of the free parameters.

In Table 2, the mass spectrum of pseudoscalar mesons π, K, D, D_s, B, B_s, and B_c, as well as their leptonic decay constants (f_π, f_K, f_D, f_{D_s}, f_B, f_{B_s}, and f_{B_c}, respectively), calculated in the framework of the modified BRPM with potential (16), are quoted. In the course of calculations, the values of the free parameters m^0, V_0, and λ at fixed values α_s = 0 and 0.4 were taken from Table 1. For the sake of comparison of the results obtained, Table 1 also includes the values of the masses of these mesons and their leptonic decay constants that were calculated in the framework of other phenomenological models (the effective QCD-Hamiltonian, lattice QCD, the relativistic meson model, and others) and their experimental values. From this table, one can see that, in contrast to other phenomenological models where an agreement between the descriptions of the spectroscopy of pseudo-scalar mesons and their interactions was not attained, the modified BRPM with potential (16) self-consistently reproduces the experimental values for both the mass spectrum and the leptonic decay constants of the mesons concerned.

Table 1. Values of free parameters for the modified bilocal relativistic potential model

Parameter	α_s				
	0	0.2	0.4	0.6	0.8
$(4V_0/3)^{1/3}$ (MeV)	289	299	315	340	376
$(\text{MeV})^{-1}$	1483	1161	1030	895	649
$m^0_{u,d}$ (MeV)	2.0	2.3	2.6	2.8	3.0
m^0_s(MeV)	59	68	76	84	92
m^0_c(MeV)	1228	1273	1310	1342	1370
m^0_b(MeV)	4667	4720	4762	4794	4820

T a b l e 2. Mass (in MeV) spectrum of pseudoscalar mesons and their leptonic decay constants in the modified bilocal relativistic potential model

Pseudoscalar meson	Theory $\alpha_s = 0$	Theory $\alpha_s = 0.4$	Experiment [13]	Other models	Leptonic decay constant	Theory $\alpha_s = 0$	Theory $\alpha_s = 0.4$	Experiment	Other models
π	<u>140</u>	<u>138</u>	138	658[12]	f_π	131	131	130.7 [13]	20 [3], 37 [4] 49 [14], 34 [6]
K	<u>494</u>	<u>494</u>	493	825[12]	f_K	168	167	159.8 [13]	47 [6]
D	<u>1866</u>	<u>1869</u>	1869	2079[12], 1880[16]	f_D	337	336	300 [17]	129 [15]
D_s	1882	1860	1968	2131[12]	f_{D_s}	366	367	280 [18]	141 [15]
B	<u>5275</u>	<u>5275</u>	5270	5701[12], 5303[16]	f_B	268	268		163 [15]
B_s	5286	5140		5285[12]	f_{B_s}	287	287		173 [15]
B_c	6209	6080		6495[12]	f_{B_c}	432	432		306 [15]

Thus, the above-presented results imply that the modified BRPM with a potential consisting of a sum of oscillator and Coulomb-like potentials, in contrast to other phenomenological models, describes the spectroscopy and interactions between pseudoscalar mesons at a good quantitative level and from the mutual viewpoint. Whence it follows that the renormalization procedure has probably a general character, so that it should be used in the cases of other potentials of quark–quark interaction as well. Note that the procedure discussed is necessary not only for the elimination of UV-divergences.

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СПЕКТР МАС ПСЕВДОСКАЛЯРНИХ МЕЗОНІВ ТА КОНСТАНТИ ЇХ ЛЕПТОННИХ РОЗПАДІВ

Т.З. Насиров

Р е з ю м е

Розвинуто феноменологічну модель адрона в термінах білокальних мезонних полів з урахуванням перенормування хвильової функції кварка “всередині” мезона. Запропоновано модифіковане рівняння Швінгера–Дайсона з довільним потенціалом, розв’язки якого не мають ультрафіолетових розбіжностей і при цьому зберігають його асимптотичні властивості. Потенціал міжкваркової взаємодії було вибрано у вигляді суми осциляторного та кулоноподібного потенціалів. З цим потенціалом сформульовано межову задачу, розв’язки якої описують спектроскопію мезонів та їхніх взаємодій. Показано, що на відміну від інших феноменологічних моделей адронів, що ґрунтуються на ефективній дії КХД, дана модель з єдиної точки зору на хорошому кількісному рівні описує як спектр мас псевдоскалярних мезонів, так і їхні константи лептонних розпадів.