THERMODYNAMIC REPRESENTATION OF THE PERIODIC SET APPEARING AS A RESULT OF THE HOPF BIFURCATION

A.I. OLEMSKOI, I.A. SHUDA

Sumy State University
(2, Rimskii-Korsakov Str., Sumy 40007, Ukraine; e-mail: alex@ufn.ru)

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On the base of the Hamiltonian formalism, we show that the Hopf bifurcation arrives, in the course of the system evolution, at the creation of a revolving region of the phase plane being bounded by the limit cycle. A revolving phase plane with a set of limit cycles is presented in analogy with a revolving vessel containing superfluid He^4. Within such a representation, the fast varying angle is shown to be reduced to the phase of the complex order parameter, whose modulus squared plays a role of action. Respectively, the vector potential of the conjugate field is reduced to the relative velocity of motion of the limit cycle interior with respect to its exterior. From the physical point of view, this means that the nontrivial self-organized system suppresses entirely the external periodic fields with frequencies ω_0 bounded by a limit ω_{c1}, whereas this field within the domain $\omega_{c1} < \omega_0 < \omega_{c2}$ arrives at the resonance series, whose coordinates and momenta are varied within the periodically distributed domains.

1. Introduction

The conception of phase transition is well known to be one of the fundamental ideas of the contemporary physics. A related picture is based on the Landau scheme, according to which a thermodynamic system driven by slow and monotonic variations of the state parameters of the heat bath, transforms drastically its macroscopic state, if the thermodynamic potential gets one or more additional minima in the space of states [1]. From the mathematical point of view, such a phase transition represents the simplest bifurcation that results in the doubling of thermodynamic steady states.

As is known, a thermodynamic phase transition is a special case of the self-organization process, in the course of which three main parameters, being the order parameter, its conjugate field, and the control parameter, vary in a self-consistent manner [2]. Roughly speaking, a generalization of the thermodynamic picture due to the passage to a synergetic one is stipulated by the extension of the set of state parameters from a single parameter to three ones mentioned above. We may hope that such a generalization will allow one to describe not only the simplest Landau-like bifurcation, but the much more complicated Hopf one, when a limit cycle is created as a continuous manifold instead of a discrete one [3].

Our considerations of this problem have shown [4] that using the whole set of universal deformations within the standard synergetic scheme does not arrive at a stable limit cycle, whereas running out off the standard scheme of self-organization has allowed us to obtain the limit cycle shown in Fig. 1 [5].

In this connection, the question arises: what is the physical reason that the self-organization scheme may not represent the Hopf bifurcation?

This paper is devoted to the answer to the above question. It is appeared that the main reason is that the description of a limit cycle demands to use both the potential and the force of a field conjugated to the order parameter, whereas the standard synergetic scheme uses the force of this field only. Following [6], we show in Section 2 that the fast revolving of the state point in the phase plane induces a gauge field, whose potential is reduced to the relative velocity of motion of the interior domain of the limit cycle with respect to its exterior. Such a picture allows us to study, in Section 3, a revolving phase plane with a set of limit cycles, using the analogy with a revolving vessel containing superfluid He^4 [7, 8]. In Section 4, we conclude that a nontrivial self-organized system suppresses entirely external periodic fields with frequencies ω_0 bounded by a limit ω_{c1}. Within the domain $\omega_{c1} <\omega_0 < \omega_{c2}$, this field arives at the resonance series, whose coordinates and momenta are varied within the periodically distributed domains. Finally, by overcoming the upper boundary ω_{c2}, the system behaves itself as a system without self-organization with coordinates and momenta oscillating with the external frequency ω_0.

2. Hopf Bifurcation within the Canonical Representation

We consider the Hamiltonian dynamics determined by the equations of motion

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i};$$

$$\{q_i\} = q, Q, \quad \{p_i\} = p, P \tag{1}$$

for both fast and slow coordinates q, Q and the conjugate momenta p, P, respectively (hereafter, the dot over a symbol denotes the time derivative). The related Hamiltonian

$$H(q, p; Q, P) = H_s(Q, P) + H_f(q, p; Q) \tag{2}$$

is split into the slow term $H_s(Q, P)$ and the fast one $H_f(p, q; Q)$, and the latter depends also on the slow coordinate.

In accordance with the standard scheme [9], it is natural to pass from the fast variables q, p to the canonical ones, being the rapidly alternating angle φ and the slow varying action η^2. This passage keeps the first term of Hamiltonian (2) to be invariant and transforms the second one according to the relation

$$H'_f(\varphi; \eta, Q) = H_f(q, p; Q) + \dot{Q}\frac{\partial \Psi(q; \eta, Q)}{\partial Q}, \tag{3}$$

whose explicit form is determined by the generating function $\Psi(q; \eta, Q)$ defined by the following constraints:

$$\frac{\partial \Psi(q; \eta, Q)}{\partial q} = p, \quad \frac{\partial \Psi(q; \eta, Q)}{\partial Q} = P,$$

$$\frac{\partial \Psi(q; \eta, Q)}{\partial \eta^2} = \varphi. \tag{4}$$

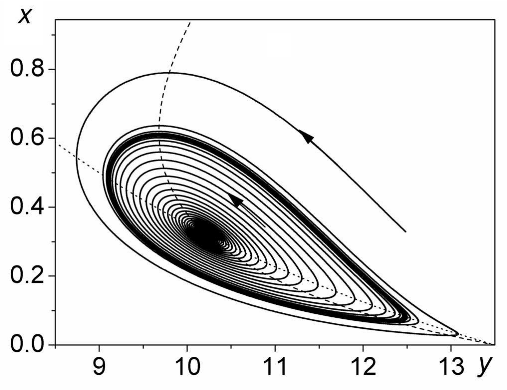

Fig. 1. Limit cycle related to the non-standard self-organization equations $\dot{x} = x\,[y - (1 + rx) - (A - 1)/(1 + \rho x)]$, $\dot{y} = A - y(1+x)$ with $A = 14$, $r = 5$, $\rho = 2$ [5]

Due to fast variations of the angle φ, it is natural to consider the left-hand side in (3),

$$H'(\eta, Q) \equiv \langle H'_f(\varphi; \eta, Q)\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} H'_f(\varphi; \eta, Q)\mathrm{d}\varphi, \tag{5}$$

being average over these variations.

To find the related Hamiltonian, one has to use a one-valued generating function

$$\Phi(\varphi; \eta, Q) \equiv \Psi\Big(q(\varphi; \eta, Q); \eta, Q\Big), \quad 0 \le \varphi \le 2\pi \tag{6}$$

instead of the many-valued one, $\Psi(q; \eta, Q)$. Then, the last factor in Eq. (3) is determined by the relation

$$\frac{\partial \Phi}{\partial Q} = \frac{\partial \Psi}{\partial Q} + p\frac{\partial q}{\partial Q} \tag{7}$$

that expresses the chain rule. Its use gives the averaged term (5) in the following form:

$$H'(\eta, Q) = H(\eta, Q) + \dot{Q}\left\langle \frac{\partial \Phi}{\partial Q} - p\frac{\partial q}{\partial Q}\right\rangle,$$

$$H(\eta, Q) \equiv \langle H_f(q, p; Q)\rangle. \tag{8}$$

As a result, the usage of the canonical angle-action representation leads to a transformation of the averaged Hamiltonian (2):

$$H_{\text{eff}}(\eta; Q, P) \equiv \langle H'(\varphi, \eta; P, Q)\rangle =$$

$$= \mathcal{H}(\eta; Q, P) + \dot{Q}\left(\left\langle \frac{\partial \Phi}{\partial Q} \right\rangle - \left\langle p \frac{\partial q}{\partial Q} \right\rangle\right), \tag{9}$$

where the first term

$$\mathcal{H}(\eta; Q, P) \equiv H(\eta, Q) + H_s(Q, P) \tag{10}$$

depends on the slowly varying values only.

To order to find the equations of motion for the above-mentioned slow variables, we have to use the extremum condition for the effective action [9],

$$S_{\text{eff}}\{Q(t), P(t); \eta(t)\} \equiv \int_{t_{\text{in}}}^{t_{\text{f}}} [P(t)\dot{Q}(t)-$$

$$-H_{\text{eff}}(\eta(t); Q(t), P(t))]dt, \tag{11}$$

whose form is determined by Hamiltonian (9), and t_{in} and t_{f} are the initial and final points of the time interval. The variation of this expression with respect to the momentum gives the equation of motion

$$\dot{Q}_\alpha = \frac{\partial \mathcal{H}}{\partial P_\alpha} \tag{12}$$

keeping the initial form (1) (hereafter, we suppose that the slow variables are vector quantities with components Q_α, P_α). On the other hand, the variation of action (11) with respect to the slow coordinate gives the equation

$$\dot{P}_\alpha = -\frac{\partial \mathcal{H}}{\partial Q_\alpha} + F_{\alpha\beta}\frac{\partial \mathcal{H}}{\partial P_\beta} \tag{13}$$

that is prolonged due to the effective field with a force given by the antisymmetric tensor

$$F_{\alpha\beta} \equiv \frac{\partial A_\beta}{\partial Q_\alpha} - \frac{\partial A_\alpha}{\partial Q_\beta} \tag{14}$$

with the vector potential

$$A_\alpha \equiv \left\langle p_\beta \frac{\partial q_\beta}{\partial Q_\alpha} \right\rangle. \tag{15}$$

As usual, we imply the summation over the repeated index β.

Relations (13)–(15) lead to the conclusion of fundamental importance: a fast variation of the coordinates, whose values depend on the slow variables induces an effective gauge field [6]. For the Hopf bifurcation, this means that such a bifurcation, in the course of the system evolution, leads to the revolving of not only the configuration point, but of the whole domain of the phase plane that is bounded by the limit cycle. In other words, the physical picture of the Hopf bifurcation is that the phase plane behaves as a real object, but not as a mathematical one.

3. Physical Picture of Limit Cycles

We consider a round phase plane that is spanned on the axes of both the coordinate q and momentum p and revolves with the angular velocity $\boldsymbol{\omega}_0$ and the moment of inertia I. From the physical point of view, the value $\boldsymbol{\omega}_0$ determines the external influence frequency, whereas the quantity I is related to the total action on the system under consideration. If the phase circle revolves as a solid plane, the phase point with a coordinate $\mathbf{r}$ has the linear velocity $\mathbf{v}_n = [\boldsymbol{\omega}_0, \mathbf{r}]$.

According to the above-presented consideration, the Hopf bifurcation results in the creation of a limit cycle that induces a gauge field with the vector potential (15) and strength (14) which are reduced to the linear and angular velocities, $\mathbf{w}$ and $\boldsymbol{\omega}$, respectively. These are not equal to the normal values $\mathbf{v}_n$ and $\boldsymbol{\omega}_0$, because a region bounded by the limit cycle revolves with different velocities due to the gauge field effect. Indeed, if one represents the creation of a limit cycle as the ordering with a complex parameter $\phi = \eta e^{i\varphi}$, then the phase gradient $\mathbf{v}_s \equiv s\nabla\varphi$, where $\nabla \equiv \partial/\partial\mathbf{r}$, s being an elementary action, affects in such a manner to compensate a rotation within the domain bounded by the limit cycle: $\mathbf{w} = \mathbf{v}_n - \mathbf{v}_s$. In this way, the relative velocity $\mathbf{w}$ appears as a gradient prolongation $\nabla \Rightarrow \nabla - (i/s)\mathbf{w}$ being caused by the gauge field $\mathbf{w}$. In opposition to the case of the solid plane revolving, the ordering arrives at the non-linear relation $\boldsymbol{\omega} = (1/2)\text{rot}\mathbf{w}$ between the angular $\boldsymbol{\omega}$ and linear $\mathbf{w}$ components of the revolution velocity.

The well-known example of such a behavior is given by the revolving superfluid He^4 [7,8]. Along this line, the effective potential density of the revolving phase plane, including a set of limit cycles, has the following form [10]:

$$E = \Delta E(\eta) + \frac{1}{2}\Big|\left(-is\nabla - \mathbf{w}\right)\eta\Big|^2 + \frac{I}{2}\omega^2. \tag{16}$$

Within the phenomenological scheme, the density of the potential variation due to the creation of a limit cycle is given by the Landau expansion

$$\Delta E(\eta) = A\eta^2 + \frac{B}{2}\eta^4, \tag{17}$$

whose form is fixed by the parameters A, B. The second term of Eq. (16) determines the heterogeneity energy with the gradient, being prolonged by the vector potential $\mathbf{w}$ of the gauge field. The last term is the kinetic energy of the revolving phase plane.

Under an external influence with frequency $\boldsymbol{\omega}_0$, the behavior of the system is defined by the effective potential density

$$\widetilde{E} = E - (I\boldsymbol{\omega}_0 + \mathbf{M})\,\boldsymbol{\omega}_0, \tag{18}$$

whose value is determined with respect to the revolving plane that is characterized by the angular momentum $\mathbf{M} = I\,(\boldsymbol{\omega} - \boldsymbol{\omega}_0)$. The steady-state distributions of the order parameter $\eta(\mathbf{r})$ and the relative velocity $\mathbf{w}(\mathbf{r})$ are given by the condition of extremum of the effective potential

$$\mathcal{E}\{\eta(\mathbf{r}), \mathbf{w}(\mathbf{r})\} = \int \widetilde{E}\Big(\eta(\mathbf{r}), \mathbf{w}(\mathbf{r})\Big)\mathrm{d}\mathbf{r}, \tag{19}$$

where the integration is fulfilled over the whole area of the phase plane. In this way, the boundary conditions are as follows:

– outside of a limit cycle

$$\eta = 0, \quad \nabla\eta = 0, \quad \mathbf{w} = [\boldsymbol{\omega}_0, \mathbf{r}], \quad \boldsymbol{\omega} = \boldsymbol{\omega}_0; \tag{20}$$

– within a limit cycle

$$\eta = \eta_0, \quad \nabla\eta = 0, \quad \mathbf{w} = 0, \quad \boldsymbol{\omega} = 0; \tag{21}$$

– on a limit cycle itself

$$\mathbf{n}\,(-\mathrm{i}s\nabla - \mathbf{w})\,\eta = 0. \tag{22}$$

Here, $\mathbf{n}$ is the unit vector perpendicular to the limit cycle, $\eta_0 = \sqrt{-A/B}$ is the stationary value of the order parameter to be determined by the condition of minimum for expression (17).

According to the above-given expressions, a disordered phase related to the exterior of the limit cycle is characterized by the effective energy density $\widetilde{E}(0) = -(I/2)\boldsymbol{\omega}_0^2$, whereas the ordered phase bounded by this cycle is related to the value $\widetilde{E}(\eta_0) = -(|A|/2)\eta_0^2$. As a result, the condition $\widetilde{E}(\eta_0) = \widetilde{E}(0)$ of the phase equilibrium gives a characteristic value of the revolution velocity

$$\omega_c \equiv \sqrt{\frac{|A|\eta_0^2}{I}} = \sqrt{\frac{A^2}{IB}} \tag{23}$$

determining the energy scale $E_c \equiv I\omega_c^2 = |A|\eta_0^2 = A^2/B$. Moreover, it is useful to introduce two lengths λ and ξ and their ratio $\kappa = \lambda/\xi$ determined by the relations

$$\lambda \equiv \sqrt{\frac{IB}{4|A|}}, \quad \xi \equiv \sqrt{\frac{s^2}{2|A|}};$$

$$\kappa \equiv \sqrt{\frac{I}{I_0}}, \quad I_0 \equiv \frac{2s^2}{B}. \tag{24}$$

Then, measuring the energy density $\widetilde{E}$ in units of E_c, the order parameter η – in η_0, the angle velocity $\boldsymbol{\omega}$ – in ω_c, the linear velocities $\mathbf{v}_n$, $\mathbf{w}$ – in $2\sqrt{2}\lambda\omega_c$, the angular momentum $\mathbf{M}$ – in $2\sqrt{2}I\lambda\omega_c$, and the distance r – in λ, we reduce the energy density (18) to the simple form

$$\widetilde{E} = \Big|\left(-\mathrm{i}\kappa^{-1}\nabla - \mathbf{w}\right)\eta\Big|^2 - \left(\eta^2 - \frac{1}{2}\eta^4\right) -$$

$$- \left(\boldsymbol{\omega}_0 - \frac{1}{2}\boldsymbol{\omega}\right)\boldsymbol{\omega}. \tag{25}$$

Inserting this equality into the total energy (19) and variating the functional obtained, we find the following equations of motion:

$$\kappa^{-2}\nabla^2\eta = -\left(1 - \mathbf{w}^2\right)\eta + \eta^3, \tag{26}$$

$$-\mathrm{rot}\ \mathrm{rot}\ \mathbf{w} = \eta^2\mathbf{w}. \tag{27}$$

As is known [7, 8], the form of solutions of these equations is fixed by the parameter κ given by two last relations in (24). In the usual case, the phase plane is too small for the condition $\kappa \leq 2^{-1/2}$ to be realized, and a single limit cycle (of the type shown in Fig. 1) can be created with the form and the size determined by the external frequency ω_0. A much more diverse situation is realized in the case of the so large phase plane that the inverted condition $\kappa > 2^{-1/2}$ is fulfilled. Then, within the interval $\omega_{c1} < \omega_0 < \omega_{c2}$ bounded by the limit velocities

$$\omega_{c1} \equiv \frac{\ln\kappa}{\sqrt{2}\kappa}\omega_c = \frac{|A|}{4s}\left(\frac{I}{I_0}\right)^{-1}\ln\frac{I}{I_0}, \tag{28}$$

$$\omega_{c2} \equiv \sqrt{2}\kappa\omega_c = |A|/s, \tag{29}$$

the mixed state is realized as a set of round limit cycles periodically distributed over the surface of the revolving phase plane. Each of these cycles has the elementary action $2\pi s$ to reach the maximum value $N_{\max} = 1/\pi\xi^2$ of the cycle density per unit area at $\omega_0 = \omega_{c2}$. With falling down the external velocity near the upper boundary

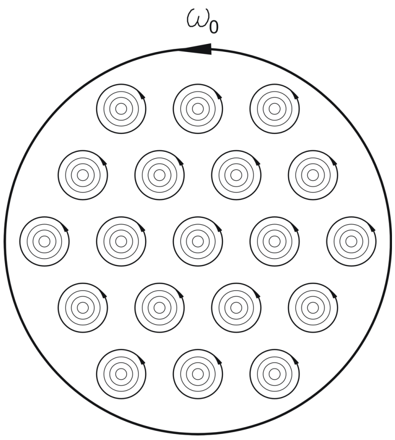

Fig. 2. Limit cycle distribution over the phase plane at $\kappa > 2^{-1/2}$, $\omega_{c1} < \omega_0 < \omega_{c2}$

$(0 < \omega_{c2} - \omega_0 \ll \omega_{c2})$, the limit cycle density decreases according to the equality

$$\frac{N}{N_{\max}} = \frac{\omega_0}{\kappa} - \frac{\overline{\eta^2}}{2\kappa^2} \tag{30}$$

where $\overline{\eta^2}$ averaged over the phase plane is determined by the revolution velocity ω_0:

$$\overline{\eta^2} = \frac{2\kappa}{\beta(2\kappa^2-1)}(\kappa-\omega_0), \quad \beta \equiv \overline{\eta^4}/(\overline{\eta^2})^2 = 0.1596. \tag{31}$$

The average value

$$\overline{\omega} = \omega_0 - \overline{\eta^2}/2\kappa = \omega_0 - (\kappa-\omega_0)/\beta(2\kappa^2-1) \tag{32}$$

is smaller than ω_0 by a value equal to the average of the plane polarization

$$\overline{M} = -\overline{\eta^2}/2\kappa = -(\kappa-\omega_0)/\beta(2\kappa^2-1). \tag{33}$$

The maximum value of the revolution velocity is reached in the cores of limit cycles, and the minimum one, $\omega_{\min} = \omega_0 - \sqrt{2}(\kappa-\omega_0)/(2\kappa^2-1)$, is reached at the centers of the triangles formed by cycles (see Fig. 2).

The average variation of the effective energy (19) caused by the phase plane revolution,

$$\overline{\mathcal{E}} = I\omega_c^2\left(\frac{1}{2} + \overline{\omega^2} - \frac{\overline{\eta^4}}{2}\right) =$$

$$= I\omega_c^2\left[\frac{1}{2} + \overline{\omega}^2 - \frac{(\kappa-\overline{\omega})^2}{1+\beta(2\kappa^2-1)}\right], \tag{34}$$

is the function of the average velocity $\overline{\omega}$, the differentiation with respect to which results in Eq. (32).

Near the lower critical value ω_{c1}, the limit cycle density $N = (\kappa/2\pi)\overline{\omega}$ is not so large, and these cycles can be treated independently. Taking into account that $w(r)$ varies at distances $r \sim 1$ and $\eta(r)$ does at $r \sim \kappa^{-1} \ll 1$, the relative velocity is determined by Eq.(27) with $\eta^2 \approx 1$ and $\kappa \gg 1$:

$$w = -\kappa^{-1}K_1(r), \tag{35}$$

where $K_1(\mathbf{r})$ is the Hankel function of the imaginary argument. Respectively, the order parameter is determined by Eq. (26) with $w = -1/\kappa r$:

$$\eta \simeq cr \quad \text{at} \quad r \ll \kappa^{-1},$$

$$\eta^2 \simeq 1-(\kappa r)^{-2} \quad \text{at} \quad r \gg \kappa^{-1}, \tag{36}$$

where c is a positive constant. According to Eq.(35), we have $w \approx -1/\kappa r$ at $r \ll 1$ and $w \approx -\sqrt{\pi/2\kappa^2}\, r^{-1/2}e^{-r}$ at $r \gg 1$. The dependence $\overline{\omega}(\omega_0)$ is of steadily increasing nature: at $\omega_0 = \omega_{c1}$, it has the vertical tangent and, with increase in ω_0, approaches asymptotically the straight line $\overline{\omega} = \omega_0$. The effective energy per one limit cycle is $(2\pi/\kappa^2)\ln\kappa$, and the value of ω at the cycle center is twice as large as that of ω_{c1}.

4. Conclusions

Within the Hamiltonian formalism, the combined consideration of both the fast and slow sets of dynamical variables shows that the averaging over the angle of the canonical pair angle-action induces an effective gauge field if the fast coordinates depend on the slow ones. For the Hopf bifurcation, this means that such a bifurcation, in the course of the evolution of the system, arrives at the revolution of not only the configuration point, but of the whole region of the phase plane bounded by the limit cycle. In other words, the physical picture of the Hopf bifurcation means that the phase plane behaves itself as a real object, but not only as a mathematical one.

Along this line, a revolving phase plane with a set of limit cycles can be presented in analogy with a revolving vessel containing superfluid He^4. Within the framework of such a representation, the fast varying angle is reduced to the phase φ of the complex order parameter $\phi = \eta e^{i\varphi}$, whose modulus squared η^2 plays a role of the action. In this way, a role of the vector potential of the gauge field is played by the relative velocity $\mathbf{w}$ of the motion of the interior domain of the limit cycle with respect to its exterior, whereas the field force is reduced to the related angular velocity

$\boldsymbol{\omega} = (1/2)\text{rot}\,\mathbf{w}$. By this, the slow variables are reduced to the parameters A and B of the Landau expansion (17).

From the physical point of view, the above-presented picture of the revolving phase plane with one or more limit cycles is interpreted in the following way. The usual case where the canonical pair of a coordinate and a momentum is varied within the whole area of the phase plane is related to the parameter $\kappa \sim 1$, so that the Hopf bifurcation arrives at the creation of a single limit cycle which presents periodic variations in the self-organized system. In the opposite case where the domain of variations in the coordinate and the momentum within a limit cycle is much less than the whole phase plane, the main parameter is $\kappa \gg 1$. If the self-organized system is subjected to an external periodic influence, whose frequencies ω_0 are bounded within the domain $\omega_{c1} - \omega_{c2}$ given by Eqs. (28) and (29), a set of nontrivial resonances is displayed. The mean values of variations in the coordinate and the momentum within these resonances are related to the centers of the limit cycles, whereas the amplitude of variations is fixed by the correlation length ξ given by the second relation in (24). The first of such resonances appears at the external frequency $\omega_0 = \omega_{c1}$ and arrives at the upper frequency $\omega_0 = \omega_{c2}$ with increase in its number to the maximum value $N_{\max} \sim \xi^{-2}$. In this way, the resonance number varies in accordance with Eq. (30), whereas the average resonance frequency is defined by Eq. (32) to arrive at the monotonically increasing dependence of this frequency on the external one.

Shortly speaking, a self-organized system with the parameter $\kappa \gg 1$ suppresses entirely the external periodic fields with frequencies $\omega_0 < \omega_{c1}$. Within the domain $\omega_{c1} < \omega_0 < \omega_{c2}$, this field arrives at the resonance series, whose coordinates and momenta are varied within the periodically distributed domains. With overcoming the upper boundary ω_{c2}, the system under consideration behaves itself as a system without self-organization, whose coordinates and momenta oscillate with the external frequency ω_0.

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ТЕРМОДИНАМІЧНЕ ПРЕДСТАВЛЕННЯ НАБОРУ ГРАНИЧНИХ ЦИКЛІВ, ЩО З'ЯВЛЯЮТЬСЯ ЯК НАСЛІДОК БІФУРКАЦІЇ ХОПФА

О.І. Олемской, І.О. Шуда

Р е з ю м е

У рамках гамільтонового формалізму показано, що в ході еволюції системи біфуркація Хопфа приводить до появи областей фазової площини, що обертаються, обмежених граничними циклами. Фазову площину, яка містить такі цикли, ми описуємо за аналогією з посудиною, що обертається і містить надплинний He^4. Показано, що швидкозмінний кут обертання зводиться до фази комплексного параметра порядку, квадрат модуля якого відіграє роль дії. Відповідно векторний потенціал калібрувального поля зводиться до відносної швидкості руху внутрішньої області, обмеженої граничним циклом, відносно зовнішньої. З фізичної точки зору це означає, що нетривіальна система, яка самоорганізується, подавляє зовнішнє періодичне поле з частотами ω_0, обмеженими верхнею межею ω_{c1}, тоді як в області $\omega_{c1} < \omega_0 < \omega_{c2}$ це поле приводить до серії резонансів, координати та імпульси яких періодично розподілені на фазовій площині.