EXCITON CONDENSATION IN SEMICONDUCTOR QUANTUM WELLS IN NONUNIFORM ELECTRIC FIELD

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The appearance of a structure in the exciton density distribution in a semiconductor double quantum well with the transverse electric field applied is studied in the case where the metal electrode contains a round window. It is suggested that there exists the exciton condensed phase, the free energy of which can be described by the Landau phenomenological model. To determine the exciton density, we used the traditional theory of phase transitions generalized to the case of the finite exciton lifetime and the presence of a pumping and an inhomogeneity of the system. It is shown that the different types of structures appear at a high exciton density: rings of the exciton condensed phase or a periodic distribution of its islands. The behavior of the structures depending on the pumping, the window size, and temperature is analyzed. The obtained results are in agreement with experimental data.

1. Introduction

The interexciton interaction in semiconductor double quantum wells is intensively studied, particularly in connection with the problem of revealing the Bose-Einstein condensation of excitons [1]. Important results were obtained on the investigation of the so-called "indirect excitons" in double quantum wells. In the case of applying the electric field, the recombination of indirect excitons is hampered, and the lifetime becomes long; therefore, great concentrations of excitons can be created. In semiconductor alloys based on GaAs/AlGaAs with double quantum wells at low temperatures, a narrow luminescence line with some specific features was revealed: the line appears at the pumping larger than some threshold value which depends on temperature; the emission intensity of the line depends superlinearly on the pumping, etc. [2, 3]. In the exciton luminescence spectrum in double quantum wells, the luminescence from the laser spot region was revealed to be surrounded by a concentric bright ring of the emission separated from the central spot by a dark region [4, 5]; the distance between the ring and the laser spot enlarges with increase in the pumping, and the ring was observed to be broken down at low temperatures into periodically sited fragments [4].

Different interpretations of the structure formation are developed. A number of explanations of the phenomenon is related to the Bose statistics of excitons. According to [6], the mechanism based on the stimulated Bose statistics of excitons during the relaxation gives rise to the instability in the exciton subsystem. In [7, 8], the authors demonstrated a possibility of the appearance of periodic structures in the solutions of nonlinear quantum equations like a nonlinear Schrodinger equation. But the investigation of the observed pecularities of the structure and its changes depending on the temperature, pumping, and parameters of the exciton system was not performed there.

Another alternative approach to the explanation of the peculiarities in a system with great exciton density was developed in [9–13]. The explanation is not connected with the Bose statistics of excitons. In those works, the traditional phase transition theory generalized to the case of unstable particles and the presence of a pumping was applied. The appearance and properties of the formed structures are related to: 1) the presence of the attraction between excitons, which leads to the condensed phase formation (the possibility to form a condensed phase of indirect excitons was shown in [14]), and 2) the nonequilibrium of the system caused by a finite exciton lifetime.

As was shown in [9], the system of attracting excitons is unstable with respect to the density superlattice formation. The properties of the exciton condensation in 2D systems in the statistical approach generalized to the case of particles with finite lifetime were studied in [11]. The ring fragmentation outside of the laser irradiation region in quantum wells observed in [4] was explained in the statistical approach [10] and in the spinodal decay model generalized for the system of unstable particles [12, 13]. The structure emerging in the case of two interacting laser spots was simulated in [13]. The method

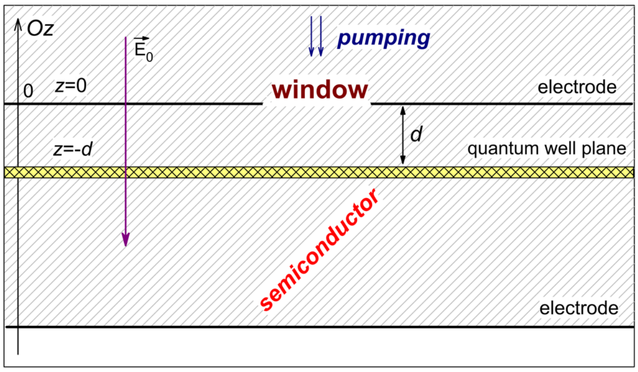

Fig. 1. Scheme of the system

employed in [12, 13] in the study of a nonuniform system allowed one to explain the transition between a fragmented luminescence ring and a continuous one with variation in the temperature or other parameters, the formation of localized spots in the emission caused by defects in the structure, etc. Thus, the appearance of a structure in high-density exciton systems was explained in [9–13] by the processes of self-organization in nonequilibrium systems caused by both the finite value of exciton lifetime and the presence of a pumping. Such explanations do not require the Bose—Einstein condensation of excitons.

Recently, the interesting effects were observed in [16–18]. The excitation and emission of indirect excitons was performed through "a window" in the metallic gate with the diameter of the order of several micrometers. In the case of the excitation intensity growth, a regular ring pattern of equidistant bright spots along the perimeter of the window was observed. At a higher pumping or high temperatures, the structure disappears, which is accompanied by the emission from the ring. With increase in the window size, the structure became complicated. These experiments are explained in the present paper, by continuing the viewpoint of works [9–13].

2. Model of the System

Let us deal with a qualitative picture of the appearance of a structure of the exciton condensed phase in a quantum well in the case of a window in the electrode. As the electric field increases, the position of the indirect exciton level shifts towards low energies. Under the window, the electric field is smaller, than that in the region remote from the window; therefore, a hump of the exciton potential energy arises in the well. Excitons created by light passing through the window roll down in the nonuniform field from the middle of the well towards the region under the window. Due to a finite lifetime, they cannot move far away from the region under the window. As a result, the maximum of the exciton density appears in the well under the perimeter of the window. Thus, as the pumping grows, the condensed phase is formed in the region of the maximum exciton density, i.e., on the ring along the window perimeter, like it was observed in experiments [16–18].

Let us find the exciton potential energy caused by the presence of a window in the metallic electrode above the quantum well plane (Fig. 1). In the external electric field, an exciton gets the additional energy $V = -p_z E_z$, where p_z is the dipole moment of an exciton (the Oz axis is perpendicular to the quantum well plane). In the experiments [16–18], a semiconductor with double quantum well was covered by a metallic mask, in which a round window was located. The uniform electric field of the intensity E_0 was applied perpendicularly to the quantum well plane. Due to the presence of the window in the mask, the electric field is distorted in the immediate proximity to the window. Let us calculate nonuniform addition to the exciton potential energy in the external uniform electric field. Let the upper electrode with the coordinate $z = 0$ have a round window of radius r_0. For the determination of the field, we have to solve the Laplace equation for the potential with the following boundary conditions: the potential of the electrostatic filed is constant at both electrodes; the difference of the potentials of electrodes should be equal to the value, at which the field between electrodes is equal to E_0 far away from the window. For this purpose, we use the solution of the task given in [19] on the field created by the ground plane with a window located in an external electric field of the intensity E_0. As a matter of fact, this solution does not satisfy the condition of the constant potential on the lower electrode. But a change of the potential introduced by the window falls down with moving off from the window as a dipole potential [19], and thus it is small in the region of the lower electrode under conditions that $r_0 \ll L$, where L is the distance between electrodes. Let us consider that $L \gg r_0, z$, i.e., the plane of the well is located significantly nearer to the upper electrode, than to the lower one. In addition, for using the solution of the task [19], in which the environment on both sides of the window is identical, we consider that the upper electrode (the electrode with the window) is inner and is located inside the semiconductor environment. So, the considered system differs slightly from that investigated in the experiments [16–18], but it gives the same qualitative behavior of the potential on the change of both the window radius and the location of the well relative to the electrode. Thus, the additional potential created by the window is

$$\varphi = \frac{E_0 z}{\pi}\left(\text{arctg}\,\frac{r_0}{\sqrt{\xi}} - \frac{r_0}{\sqrt{\xi}}\right), \qquad (1)$$

where $\xi \equiv \frac{1}{2}\left[\rho^2 + z^2 - r_0^2 + \sqrt{\left(\rho^2 + z^2 - r_0^2\right)^2 + 4z^2 r_0^2}\right]$ is the oblate spheroidal coordinate, and ρ is the radial coordinate in the quantum well plane. At great fields, when the electrons and holes are distributed to different wells and the dipole moment does not depend on E_0, the exciton potential energy $V(\rho, z) = p_z(\partial\varphi/\partial z)$. We will characterize the value of the potential energy by a "pulling force" $\lambda = p_z E_0/(kT_c)$. Radial profiles of the exciton energy $V(\rho)$ at different values of z/r_0 are presented in Fig. 2. The coordinate z characterizes the distance from the quantum well to the plane of the electrode with the window. As this distance decreases, the potential hump, which appears due to the presence of the window, enlarges, and the slope becomes steeper. In the region below the window perimeter inside the well, a round potential well for excitons arises (it is shown by the arrow in Fig. 2). The exciton energy in this well is lower, than the energy in regions remote from the window. The depth of the well increases, if the plane of the quantum well approaches the window. The appearance of the well is related with the charge origination nearby edges of the window. In work [18], a shift in the emission spectrum of indirect excitons from the regions below the window perimeter was observed, and this is probably related to the formation of the aforementioned round potential well.

3. Equation for the Exciton Density

We assume the duration of the post-excitation establishment of a quasilocal equilibrium of electrons and holes and their binding into excitons is much smaller than the exciton lifetime and the duration of the establishment of an equilibrium between different regions. In this case, the free energy of a quasilocal state can be regarded as a function of the exciton density. A phenomenological equation for the exciton density n can be written down as

$$\frac{\partial n}{\partial t} = -\text{div}\mathbf{j} + G - \frac{n}{\tau}, \qquad (2)$$

where $G(\mathbf{r})$ is the pumping (the number of excitons created in a unit area for a unit time), τ is the exciton lifetime, $\mathbf{j} = -M\nabla\mu$ is the exciton current density,

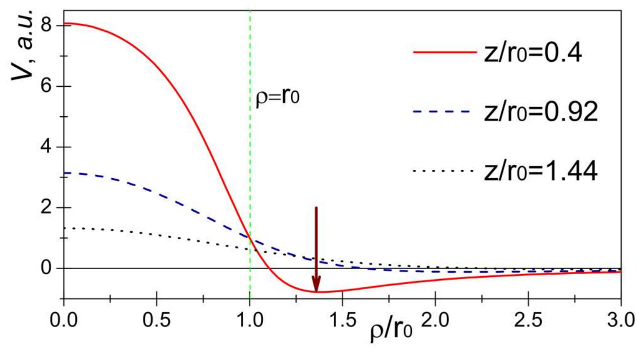

Fig. 2. Radial profile of the exciton potential energy V in a nonuniform electric field as a function of the ratio ρ/r_0 at different values of the parameter z/r_0. The pulling force $\lambda = 30$ for all curves

where μ is the chemical potential, and M is the exciton mobility. For M, we use the Einstein condition $M = nD/(kT)$. The corrections to this condition caused by the Bose statistics of excitons were studied in [20] for quantum wells. But, for the temperatures and the exciton density under investigation, these corrections are not essential. The chemical potential can be expressed in terms of the free energy: $\mu = \delta F/\delta n$. We chose the free energy in the Landau model as

$$F[n] = \int d\mathbf{r}\left[\frac{K}{2}(\nabla n)^2 + f(n) + nV\right]. \qquad (3)$$

The term $\frac{K}{2}(\nabla n)^2$ characterizes the inhomogeneity energy. The exciton additional energy in a nonuniform electric field is taken into account by the term nV. According to the Landau method, we expand the free energy in a power series in $(n - n_c)$ in the vicinity of its minimum:

$$f(n) = f(n_c) + \frac{a}{2}(n - n_c)^2 + \frac{b}{4}(n - n_c)^4. \qquad (4)$$

The parameters a, b, and n_c in (4) are phenomenological and can be obtained from quantum-mechanical calculations of the free energy of the exciton system in the infinite exciton lifetime approximation or by comparing the theory with the experiment. After the substitution of (3) and (4) in Eq. (2), the latter is reduced to

$$\frac{\partial n}{\partial t} = \frac{D}{kT}\nabla\left[n\nabla\left(\frac{df}{dn} - K\Delta n\right) + n\nabla V\right] + G - \frac{n}{\tau}. \qquad (5)$$

We note that, for small densities of excitons, the term $kTn(\ln n - 1)$ has to be added to the free energy (3), which leads to the well-known term $D\Delta n$ in Eq. (5).

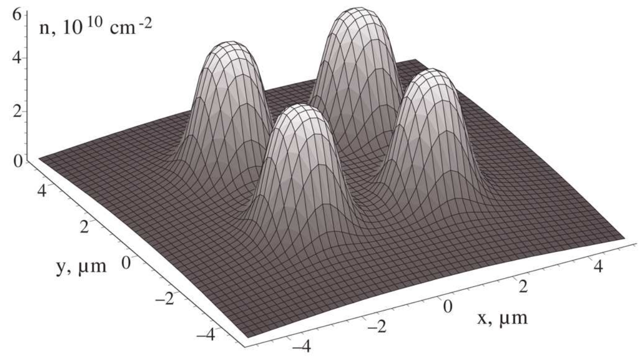

Fig. 3. Exciton density distribution $n(x, y)$ in the quantum well plane at the parameters: $r_0 = 2.5$ μm, $z/r_0 = 1.44$, $T = 1.71$ K, $G\tau = 9.71 \cdot 10^{10}$ cm^{-2}, $\lambda = 30$

But its contribution is inessential at great exciton densities.

Equation (5) is a non-linear 2D phenomenological equation which describes the density distribution of interacting excitons of high concentrations taking into account the pumping and the finite lifetime. We will solve it instead of the Gross—Pitaevsky equation [21], because the wave function loses the coherence at distances of the order or smaller than the distance between excitons. The finite lifetime and the presence of a constant exciton generation lead to new qualitative peculiarities in the phase formation in comparison with the phase formation for stable particles. We assume that Eq. (5) is also applicable, if the condensed phase is the electron-hole liquid. Then, n in the free energy (3) is the density of electron-hole pairs.

For numerical calculations, we choose the units of length, concentration, and time as

$$l_u = \sqrt{\frac{K}{-a}}, \quad n_u = \sqrt{\frac{-a}{b}}, \quad t_u = \frac{kTK}{Da^2 n_u}, \qquad (6)$$

respectively. Then the generation rate and the energy are measured in units of $G_u = n_u/t_u$ and $V_u = -an_u$, respectively. In dimensionless variables, Eq. (5) for the exciton density becomes

$$\frac{\partial n}{\partial t} = \nabla \left[n \nabla \left(-\Delta n + (3n_c^2 - 1)n - \right.\right.$$

$$\left.\left. -3n_c n^2 + n^3 \right) + n \nabla V \right] + G - \frac{n}{\tau}. \qquad (7)$$

4. Calculations and Discussion of the Results

Equation (7) was solved numerically for a 2D system of the shape of a rectangular plate, the sizes of which exceed singificantly the window radius. So, at the further increase of window sizes, the quasiparticle density distribution is almost not changed. The following values of the parameters of the system were chosen: $\tau = 10$ ns, $T_c = 5.7$ K, $n_c = 1.2n_u = 3.33 \times 10^{10}$ cm^{-2}, $Kn_c^2 = 15.7$ meV, $-an_c = 0.826$ meV, and $bn_c^3 = 1.70$ meV. The free path length is $l = \sqrt{D\tau} = 1.41$ μm. The pumping G is constant on the disc with radius r_0 and equals zero outside of the disc. An example of the stationary solution of Eq. (7) for the exciton density is shown in Fig. 3. The high-density points can be attributed to the condensed phase, and the low-density points are assumed to be the gas phase. Thus, the formed fragments are periodically sited islands of the exciton condensed phase, which corresponds to the structure observed in [16–18]. One can see from Fig. 3 that the islands of the condensed phase appear at the boundary of the ring, as it is follows from the qualitative analysis of the influence of the nonuniform potential distribution. A part of an island is under the metal mask and may not be observed in the luminescence emission. The size of the region "hidden" below the mask depends on the distance of the quantum well relative to the upper electrode. While moving off the electrode, the islands shift towards the window center and "stretch" from the electrode. As the well plane approaches the electrode, islands "hide" below the electrode.

Hereinafter, we present the results of studies of the behavior of the structure on changing the external parameters. With broadening the window, the number of islands increases (Fig. 4), because the length of the circle restricting the window increases. At the same time, the shape of the fragments is almost not changed. At some small window radius, only the spot at the center is formed, but the critical value of a pumping, which is necessary for its appearance, is greater than the critical value for the formation of islands in a system with bigger window.

Islands are formed only at the pumping exceeding some threshold, because the condensed phase is formed at large exciton densities. At a small pumping, the structure does not appear. But, at large irradiation intensities, separate islands merge into a continuous ring of the condensed phase (Fig. 5).

Let us examine the temperature dependence in the Landau model. The phase transition occurs, if the coefficient a in the free energy density (4) changes the

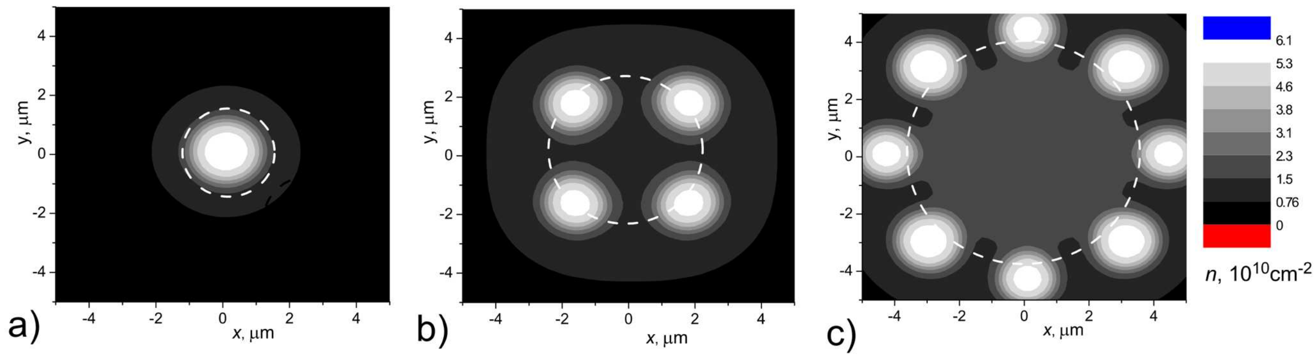

Fig. 4. Exciton density distribution with increase in the window radius. The radius r_0 (in μm) is equal to: *a*) 1.4, *b*) 2.5, *c*) 3.6. Other parameters are the same as in Fig. 3, except for case *a*, where the pumping G is larger (see the text). Case *b* is a density plot of the 3D image in Fig. 3

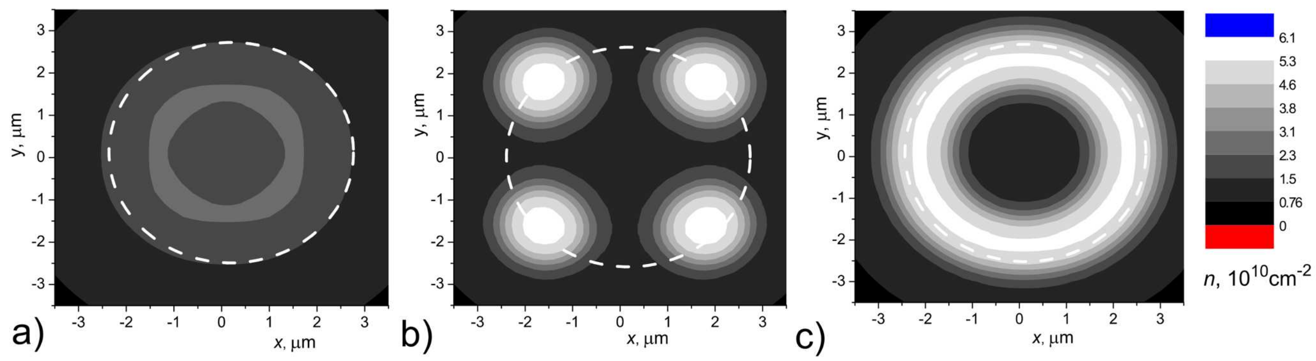

Fig. 5. Exciton density distribution with increase in the pumping. The pumping G equals to: *a*) $0.86G_0$, *b*) G_0, *c*) $1.14G_0$; $G_0\tau = 9.71 \times 10^{10}$ cm^{-2}. Other parameters (r_0, z, T) are the same as in Fig. 3

sign. The temperature dependence of other coefficients can be neglected. In the Landau approximation, the parameter a depends on temperature like

$$a(T) = \alpha \left(1 - \frac{T}{T_c}\right), \tag{8}$$

where T_c is the critical temperature, and $\alpha < 0$. The linear dependence (8) is valid in the framework of the self-consistent field approximation and can not be fulfilled, if fluctuations play the essential role. On the derivation of the equation in dimensionless variables, we used the units (6), but substituting a and T for α and T_c, respectively. If the temperature is measured in T_c, then Eq. (7) for the exciton density in new variables looks as

$$\frac{\partial n}{\partial t} = \frac{1}{T}\nabla\left[n\nabla\left(-\Delta n + (3n_{\rm c}^2 - 1 + T)n - -3n_{\rm c}n^2) + n^3\right) + n\nabla V\right] + G - \frac{n}{\tau}. \tag{9}$$

The numerical calculations show that, with increase in the temperature, separate islands of the condensed phase merge into a continuous ring, and, at higher temperatures, the emission from the window is homogeneous (Fig. 6).

5. Conclusions

In the present paper, we have investigated the exciton condensation in the framework of the traditional phase transition theory for the system in a nonuniform electric field, by taking the effects of a finite exciton lifetime and the presence of a pumping into account. The main results are the following.

(1) At the pumping exceeding some threshold value, the condensed phase of excitons appears in the form of the islands, which are localized nearby the edge of the window, or a continuous ring.

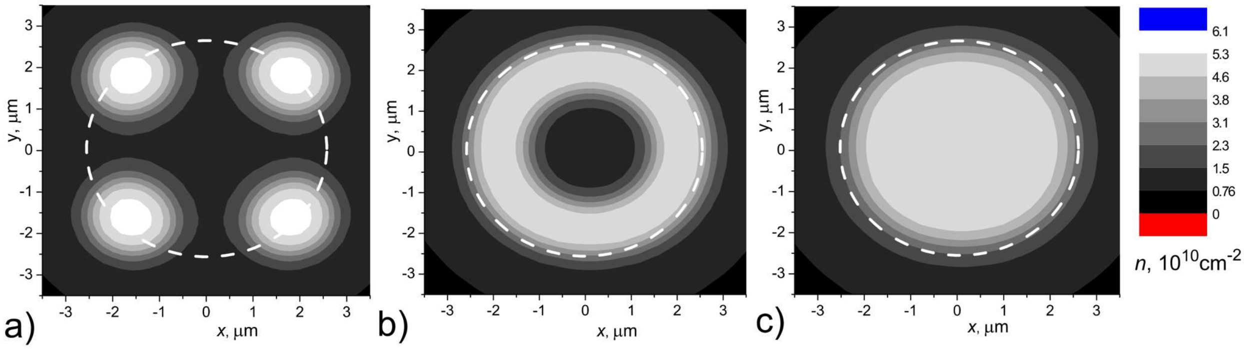

Fig. 6. Exciton density distribution with increase in the temperature. T, K: *a*) 1.71, *b*) 1.94, *c*) 3.14. Other parameters (G, r_0, z) are the same as in Fig. 3

(2) The number of the islands increases with increase in the window radius. A continuous ring can break down into separate periodically sited islands of the condensed phase.
(3) At small sizes of the window, only the spot at the window center appears.
(4) With increase in the pumping and temperature, separate islands merge into a continuous ring.

All the above-mentioned peculiarities were observed in works [16–18]. We have to remark that the Bose—Einstein condensation was not involved to obtain the results, but the quantum statistics of excitons can play some role in the formation of parameters which were used in the phenomenological model. We consider that the formed structure (the formation of islands, their location, the dynamics under changing the parameters) is a consequence of the non-equilibrium state of the system, i.e., the structure is an example of self-organization processes in non-equilibrium systems [22, 23]. In the considered case, the origin of a non-equilibrium state is the finite value of the exciton lifetime. The general theory of the formation of a spatial structure for unstable particles at phase transitions is presented in [24, 25].

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КОНДЕНСАЦІЯ ЕКСИТОНІВ В КВАНТОВИХ ЯМАХ НАПІВПРОВІДНИКІВ У НЕОДНОРІДНОМУ ЕЛЕКТРИЧНОМУ ПОЛІ

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Р е з ю м е

Вивчено появу структури у розподілі густини екситонів в площині квантової ями у напівпровідниках з прикладеним поперечним електричним полем у випадку, коли металічний електрод містить круглий отвір. Вважається, що існує конденсована фаза екситонів, вільна енергія якої може бути описана феноменологічною моделлю Ландау. Для визначення густини екситонів застосовано традиційну теорію фазових переходів, узагальнену на випадок скінченного часу життя екситона, наявності накачки і неоднорідності системи. Показано, що за високої густини екситонів можливі різні типи структури: періодичний розподіл острівців конденсованої фази або кільця конденсованої фази. Проаналізовано поведінку системи зі зміною накачки, розмірів отвору і температури. Отримані результати узгоджуються з експериментальними даними.