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We have developed a theory of the photoconversion in Si solar cells (SCs) with rear and standard contact positions under concentrated illumination, by using quite realistic approximations for the relation between the thickness of a cell and the diffusion length. A comparative analysis of the results is made for the specified geometries. It is shown that the rear-contact Si solar cells can demonstrate a higher efficiency. An agreement between the theory developed and experimental data is obtained.

1. Introduction

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Despite a sufficiently great number of works devoted to the study of the photoconversion efficiency of Si SCs under concentrated illumination (see, e.g., [1–4]), its comparative analysis for the standard and rear geometries of the position of current-collecting contacts has not been performed till now.

In this work with the use of sufficiently general approximations, we analyze theoretically the behavior of the photoconversion efficiency of Si SCs under concentrated illumination for the rear and standard geometries of contacts for SCs, in which the thickness of the base is less than the diffusion length and compare the results with those of works [2, 3]. The difference of the effects of heating for SCs with rear and standard metallizations is taken into account.

The schemes of Si SCs, for which the photoconversion efficiency under concentrated illumination was calculated, are shown in Fig. 1. In Fig. 1, a, b, we present, respectively, SCs with the rear and standard geometries of the position of contacts.

2. Solar Cells with Rear Contacts

2.1. Statement of the problem

The analysis of the photoconversion efficiency of SCs with rear contacts is carried out for the AMO and AM 1.5 conditions with regard for the fact that the diffusion

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length of electron-hole pairs L decreases and can become comparable with the base thickness d with increase in the degree of the solar illumination concentration M. This means that, under such conditions, the excess concentration of electron-hole pairs can noticeably decrease with increase in the distance from the illuminated surface. We consider the following bulk channels of recombination: the Shockley-Reed-Hall recombination, band-to-band radiative recombination, exciton radiative and nonradiative recombinations, Auger interband recombination, and their dependence on the level of excitation (in the scope of the approach developed in works [5, 6]). For the semiconductor-dielectric interface, we calculated, in a self-consistent way, the effective rate of surface recombination (RSR) on the illuminated surface S_0 referred to the boundary of the spatial charge region (SCR) and the quasineutral volume. The calculation was carried out under the assumption that the characteristics of an SCR, which arises near the illuminated surface, are determined by the value and the sign of a built-in charge, the doping level of the base, and the excitation level. We introduced and theoretically determined the effective RSR on the rear surface S_d ; this recombination occurs both on the semiconductor-metal and semiconductor-dielectric interfaces.

We considered the cases where the base has the *n*-type or the *p*-type of conductivity. The calculation is carried out in the one-dimensional approximation in the case where the current-collecting grid is a comb. The one-dimensionality puts certain limitations on the distances between current-collecting electrodes l which must be significantly less than the base thickness d. We also assume that the doping of p^+ - and n^+ - strips, which are positioned on the rear surface in turn (see Fig. 1,*a*), is a stepwise one and denote the thicknesses of strips as, respectively, x_p and x_n .



Fig. 1. Scheme of an SC with rear metallization (a) and standard geometry of contacts (b)

2.2. Open-circuit voltage and short-circuit current

In this case, the open-circuit voltage is determined by the relation

$$V_{\rm OC} = \frac{kT}{q} \ln\left(\frac{(n_0 + \Delta n(d))\Delta n(d)}{n_i^2(T_d)}\right),\tag{1}$$

where n_0 – the concentration of majority charge carriers (for definiteness, of electrons) in the base, $\Delta n(d)$ – the excess concentration of electron-hole pairs in the base on the boundary of the SCR at the rear surface, $n_i(T_d)$ – the concentration of intrinsic charge carriers in the base near the rear surface, and T_d – the temperature of the rear surface.

We determine the value of the excess concentration of electron-hole pairs in the base $\Delta n(x)$ from the the equation of generation-recombination balance

$$S_0 \Delta n(0) + S_d \Delta n(d) + \int_0^d \frac{\Delta n(x)dx}{\tau_{\text{eff}}(x)} = J_{\text{gen}}/q, \qquad (2)$$

where q – the elementary charge, $J_{\text{gen}} = q \int_{0}^{d} g(x) dx$ – the photogeneration current density, g(x) – the generation function of electron-hole paira in an SC, $\Delta n(0)$ – the excess concentration of electron-hole pairs in the base on

the boundary of the SCR near the illuminated surface, and the effective bulk lifetime $\tau_{\text{eff}}(x)$ takes the Shockley– Reed–Hall recombination, square-law recombination, and Auger interband recombination into account.

To determine the quantity $\Delta n(x)$, we solved the equation of diffusion for generated electron-hole pairs with regard for all the above-mentioned mechanisms of recombination in the bulk of the base and on its surfaces. Let us consider the case where d>L, the function of the generation of electron-hole pairs by the solar illumination in the base is approximated by the δ -function, and the normalization of this function is determined by a value of the bulk generation rate integrated over the base thickness. Then the solution of the problem can be reduced to solving a system of transcendental equations. Such an approximation was used, in particular, in work [2] and is substantiated by the fact that the absorption of approximately a half of photons of the solar illumination, which generate electron-hole pairs, occurs in Si at a distance of only 10 μ m from the illuminated surface. While solving the problem, we set the following boundary conditions.

In the mode of short-circuit current:

$$j(x=0) = -S_0 \Delta n(0),$$
 (3)

$$\Delta n(d) \approx 0. \tag{4}$$

In the mode of open circuit:

$$j(x=d) = S_d \Delta n(d). \tag{5}$$

The quantity j(x) is the flux of nonequilibrium electronhole pairs in the plane x = const.

The diffusion equation for excess electron-hole pairs looks as

$$\frac{d}{dx}\left(D_a\frac{d(\Delta n(x))}{dx}\right) - \frac{\Delta n(x)}{\tau_{\text{eff}}(x)} + g(x) = 0,\tag{6}$$

where $D_a = \frac{D_n D_p (n_0 + p_0 + 2\Delta n)}{D_n (n_0 + \Delta n) + D_p (p_0 + \Delta n)}$ – the bipolar diffusion coefficient in the base, p_0 – the concentration of equilibrium holes in the base, D_n and D_p – the diffusion coefficients of electrons and holes, respectively.

Equation (6) was solved by the method of successive approximations in the both cases of a real space dependence of the generation flux and the approximation of the space dependence of the generation flux by the δ function.

On the solution of the equation of continuity (6) in the mode of short-circuit current, we omitted the

recombination term in the zero approximation and determined the flux and the concentration. Then we substituted the formula for the concentration of electronhole pairs in (6) and determined the corrections for the flux and for the concentration of excess electron-hole pairs. In the first approximation, the final expression for the recombination flux at x = d has the form

$$j(x=d) = \frac{J_{\rm gen}/q}{1+S_0 d/D_a} \times$$

$$\times \left[1 - \int_{0}^{d} \frac{(d-x)dx}{D_{a}\tau_{\text{eff}}(x)} + \frac{S_{0}}{D_{a}} \int_{0}^{d} \int_{0}^{x} \frac{(d-x)dx'dx}{D_{a}\tau_{\text{eff}}(x')} \right].$$
(7)

On the solution of the equation of continuity in the mode of open circuit, we assumed in the zero approximation that $\Delta n(x) = \Delta n(0) = \Delta n(d)$ and determined the quantity $\Delta n(0)$ from the equation of generation-recombination balance

$$(S_{0} + S_{d})\Delta n(0) +$$

+ $d\left(\frac{1}{\tau_{v}} + (B_{rad} + B_{nr})(n_{0} + \Delta n(0)) +$
+ $C_{n}(n_{0} + \Delta n(0))^{2} + C_{p}(p_{0} + \Delta n(0))\Delta n(0)\right) \times$

$$\times \Delta n(0) = J_{\rm gen}/q,\tag{8}$$

where τ_v – the lifetime conditioned by the Shockley– Reed–Hall recombination, $B_{\rm rad}$ and $B_{\rm nr}$ – the coefficients of the square-law radiative and nonradiative recombinations, and C_n and C_p – the coefficients of Auger interband recombination for electrons and holes, respectively. In the specific calculations and estimates, we take $B_{\rm rad} = 6.3 \times 10^{-15} {\rm cm}^3/{\rm s}$, $B_{\rm nr} = 1.4 \times 10^{-16}/\tau_v$ ${\rm cm}^3/{\rm s}$, $C_n = (2.8 \times 10^{-31} + 2.5 \times 10^{-22}/(n_0 + \Delta n(0))) {\rm cm}^6/{\rm s}$, $C_p = 10^{-31} {\rm cm}^3/{\rm s}$ [6,7]. In the first approximation, we obtain

$$\Delta n(d) = \Delta n(0) \left(1 - \frac{d^2}{2L^2} \right),\tag{9}$$

where

$$L = \left[D_a \left(\frac{1}{\tau_v} + (B_r + B_{\rm nr})(n_0 + \Delta n(0)) + \right. \right]$$

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$$+C_n(n_0 + \Delta n(0))^2 + C_p(p_0 + \Delta n(0))\Delta n(0) \bigg)^{-1} \bigg]^{1/2}$$
(10)

– the diffusion length of electron-hole pairs at a high excitation level.

We note that formulae (7) and (9) are obtained in a simpler case where the spatial dependence of the generation flux is described by the δ -function.

As seen from (9), the applicability criterion for the given approach is the inequality $d^2/2L^2 < 1$. In particular, at $d^2/2L^2 = 0.5$, the error of calculations of the open-circuit voltage V_{OC} with the use of (9) does not exceed 15 mV, i.e. it is at most 2 %, in the case where the bulk lifetime related to the Shockley–Reed– Hall recombination $\tau_v \geq 1$ ms.

The results of specific calculations showed that it is quite sufficient to apply the first approximation to the calculation of j(d) and $\Delta n(d)$ in the cases where $\tau_v \geq 1$ ms and $d \leq 100 \ \mu$ m, if we set the accuracy of calculations to be about 3 %. At the same time, in the case where $\tau_v \sim 100 \ \mu$ s, it was necessary to use also the higher approximations (up to the convergence of the obtained solutions).

2.3. The effective rate of the surface recombination on the illuminated surface

On the self-consistent determination of the effective RSR on the illuminated surface S_0 , we assume that the front surface possesses a charged dielectric layer with the density of positive surface charge Q_0 , whose value exceeds the charge of electron states on the dielectric-semiconductor interface which are recharged. Then there arises the SCR with depth w_n in the nearsurface layer. This SCR corresponds to the depleting or inversion band bending in the case of a base of the ptype and to the enriching band bending in the case of a base of the *n*-type. Let the surface recombination be realized by the Shockley–Reed–Hall mechanism through discrete recombination-active surface centers with the concentration N_s , the energy position E_s relative to the middle of the energy gap, and the capture coefficients for electrons C_n and holes C_p . Then the effective RSR on the boundary of the near-surface SCR and the quasineutral volume, S_0 , can be determined from the relation

$$S_0 \approx \{C_n C_p N_s (n_0 + p_0 + \Delta n) \exp(\Delta E_{\rm gs}/kT)\} \times$$

$$\times \left\{ C_p \left[(p_0 + \Delta n) \exp(-y_s) + n_i(T) \exp\left(-\frac{E_s}{kT}\right) \right] + \right\}$$

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$$+C_n\left[\left(n_0 + \Delta n\right)\exp(y_s) + n_i(T)\exp\left(\frac{E_s}{kT}\right)\right]\right\}^{-1}, \quad (11)$$

where y_s – the dimensionless nonequilibrium band bending on the surface, $\Delta n = \Delta n(w_n) \approx \Delta n(0)$ – the concentration of excess electron-hole pairs at a distance $x = w_n$ from the geometric surface of an SC, $n_i(T)$ – the concentration of equilibrium charge carriers in intrinsic Si, k – the Boltzmann constant, T– the absolute temperature, $\Delta E_{\rm gs}$ – the narrowing of the energy gap of Si on the surface x = 0 due to the realization of a high concentration of electrons which is induced by the positive charge built in the dielectric. The dependence $y_s(\Delta n)$ can be found from the equation of electroneutrality [8]

$$\frac{Q_0}{\sqrt{2\varepsilon_{\rm Si}\varepsilon_0 kT}} - \left[(n_0 + \Delta n)(e^{y_s} - 1) + (p_0 + \Delta n) \times (e^{-y_s} - 1) + y_s(p_0 - n_0) \right]^{1/2} = 0.$$
(12)

Here, $Q_0 = qN$ – the surface density of the charge built in the dielectric, $\varepsilon_{\rm Si}$ – the dielectric permeability of Si, and ε_0 – the electric constant of vacuum. The joint solution of Eqs. (11) and (12) allows us to determine the character of the interconnection of the quantity S_0 with the surface and bulk parameters of the semiconductor– dielectric system and with the injection level of excess electron-hole pairs in the quasineutral region Δn . The value of $\Delta E_{\rm gs}$ for the given value of n_s was determined with the use of the relations given in [9].

2.4. The effective rate of surface recombination on the rear surface

In order to determine the effective RSR on the rear surface S_d , we will use the approach proposed in [10]. The full value of S_d is equal to the sum of effective RSRs under contacts S_{dm} , on the boundary of the semiconductor-dielectric interface in the p^+ -region S_{dp} , and on the boundary of the semiconductor-dielectric interface in the n^+ -region S_{dn} . Let the levels of doping in the p^+ - and n^+ -regions, p_p and n_n , exceed significantly the levels of doping and excitation in the base $(n_0 \text{ and } \Delta n)$, i.e. the criteria $p_p \gg n_0 + \Delta n$ and $n_n \gg n_0 + \Delta n$ are fulfilled. Then

$$S_d = S_d^0 \left(1 + \frac{\Delta n(d)}{n_0} \right), \tag{13}$$

where $S_d^0 = S_{dm}^0 + S_{dp}^0 + S_{dn}^0$, and the indicated quantities are determined in the general case by taking

the degeneration and narrowing of bands into account in such a way. The sum of the effective RSRs under and outside of contacts in the n^+ -regions, $S_p = S_{dm}^{0n} + S_{dp}^0$, is equal to

$$S_{p} = v_{p} \frac{n_{0}}{n_{n}} F_{1/2}(Z_{n}) e^{\Delta E_{g}^{(n^{+})} - Z_{n}} \left[\left(S_{pm} \operatorname{ch} \left(\frac{x_{n}}{L_{p}} \right) + v_{p} \operatorname{sh} \left(\frac{x_{n}}{L_{p}} \right) \right) \right] + v_{p} \operatorname{sh} \left(\frac{x_{n}}{L_{p}} \right) + v_{p} \operatorname{ch} \left(\frac{x_{n}}{L_{p}} \right) + v_{p} \operatorname{sh} \left(\frac{x_{n}}{L_{p}} \right) \right) + \left(S_{pn} \operatorname{ch} \left(\frac{x_{n}}{L_{p}} \right) + v_{p} \operatorname{sh} \left(\frac{x_{n}}{L_{p}} \right) \right) (0.5 - m_{n}) / \left(S_{pn} \operatorname{sh} \left(\frac{x_{n}}{L_{p}} \right) + v_{p} \operatorname{ch} \left(\frac{x_{n}}{L_{p}} \right) \right) \right].$$
(14)

Here, m_n – the relative share of the metallization by contacts in the n^+ -regions, $v_p = D_p/L_p$, D_p , and L_p – the diffusion velocity, diffusion coefficient, and diffusion length of holes in the n^+ -region, respectively, N_c – the effective density of states in the conduction band of Si, $\Delta E_g^{(n^+)}$ – the change of the energy gap width of Si in the n^+ -region as a result of its doping (in units of kT), $S_{pm} = V_{pT}/4$, S_{pn} – the effective RSR of holes on the semiconductor–dielectric interface, V_{pT} – the thermal velocity of holes, and the quantity Z_n is determined from the equation

$$n_n = N_c F_{1/2}(Z_n), (15)$$

where $F_{1/2}(Z_n)$ – the Fermi–Dirac integral of the order of 1/2.

Analogously, the sum of the effective RSRs under and outside of contacts in the p^+ -regions, $S_n = S_{dm}^{0p} + S_{dn}^0$, is determined as

$$S_{n} = v_{n} \frac{n_{0}}{p_{p}} F_{1/2}(Z_{p}) e^{\Delta E_{g}^{(p^{+})} - Z_{p}} \left[\left(S_{nm} \operatorname{ch} \left(\frac{x_{p}}{L_{n}} \right) + v_{n} \operatorname{sh} \left(\frac{x_{p}}{L_{n}} \right) \right) m_{p} / \left(S_{nm} \operatorname{sh} \left(\frac{x_{p}}{L_{n}} \right) + v_{n} \operatorname{ch} \left(\frac{x_{p}}{L_{n}} \right) \right) + \left(S_{np} \operatorname{ch} \left(\frac{x_{p}}{L_{n}} \right) + v_{n} \operatorname{sh} \left(\frac{x_{p}}{L_{n}} \right) \right) (0.5 - m_{p}) / \left(S_{np} \operatorname{sh} \left(\frac{x_{p}}{L_{n}} \right) + v_{n} \operatorname{ch} \left(\frac{x_{p}}{L_{n}} \right) \right) \right].$$

$$(16)$$

Here, m_p – the relative share of the metallization by contacts in the p^+ -regions, $S_{nm} = V_{nT}/4$, V_{nT} – the thermal velocity of electrons, S_{np} – the effective RSR of electrons on the semiconductor-dielectric interface, $v_n = D_n^+/L_n^+$, D_n^+ , and L_n^+ – the diffusion velocity, diffusion coefficient, and diffusion length of electrons in the p+-region, respectively, N_v – the effective density of states in the valence band of Si, $\Delta E_g^{(p^+)}$ – the change of the energy gap width in Si in the p^+ -region in units of kT, and the quantity Z_p is determined from the equation

$$p_p = N_v F_{1/2}(Z_p). (17)$$

Relations (14) and (16) are written in the case where the area of the p^+ - and n^+ -regions are identical, and $m_n = m_p$, but they are easily generalized to the case where they are different.

2.5. Photoconversion efficiency

The photoconversion efficiency of an SC with unit area with rear contacts is defined as

$$\eta = \frac{J_{\rm SC} V_{\rm OC}}{P} K,\tag{18}$$

where $J_{\rm SC} = qj(d)$ – the short-circuit current density which is numerically equal, in the given case, to the short-circuit current, P – the incident illumination power, and $K = K_1K_2$ – the resulting fill factor for the voltage-current characteristic which is equal to the product of two factors. The first factor, K_1 , is determined for the optimum load resistance under assumption that the series resistance is zero, and the second one, K_2 , is determined by the series resistance R_s .

First, we find K_1 . For $d \ll L$, the excess concentration of electron-hole pairs is practically constant in the whole base and does not depend on the coordinate x. Let us assume that the illumination and the direct bias supply act equivalently. Then, in the case where the resulting current is nonzero, the current density

$$J(V) = J_{\text{gen}} - q \frac{d\Delta n^*}{\tau_{\text{eff}}(\Delta n^*)} - q \left[S_0(\Delta n^*) + S_d(\Delta n^*)\right] \Delta n^*$$
(19)

where

$$\Delta n^* = -\frac{n_0}{2} + \sqrt{\left(\frac{n_0}{2}\right)^2 + n_i^2(T) \exp\left(\frac{qV}{kT}\right)}.$$
 (20)

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In this case, the maximum of the electric power of the SC is determined from the condition

$$\frac{d(J(V)V)}{dV} = 0. \tag{21}$$

This allows us to determine the photovoltage corresponding to the mode with maximum power and then to determine the photocurrent density J_m from (19). Then the quantity K_1 can be defined as

$$K_1 = \frac{J_m V_m}{J_{\rm SC} V_{\rm OC}}.$$
(22)

We note that relation (22) can be used also in the case where $L \approx d$. This is related to the fact that the quantity K_1 is determined, in the first turn, by the value of $V_{\rm OC}$.

We now calculate K_2 . In the given geometry, the series resistance is defined, generally saying, by three components: the layer resistance of heavily doped regions, resistance of a contact grid, and transient resistance of contacts which will be neglected in what follows. First, we will determine the value related to the layer resistance, by using the approach developed in [11]. The calculation is carried out in the case where the n^+ - and p^+ -regions alternate on the rear surface, and current-collecting contacts have the form of a comb (Fig. 1,a). Then, for the optimized situation when the difference of the photovoltages under contacts and between contacts is significantly less than kT/q, we can write

$$K_2 = \frac{2L_c}{l} \tanh\left(\frac{l}{2L_c}\right),\tag{23}$$

where $L_c = \left[q \left(\frac{l_1}{\mu_n N_n l} + \frac{l_2}{\mu_p N_p l} \right)^{-1} V_{\text{OC}} / J_{\text{SC}} \right]^{1/2}$ - the

effective collection length, $l = l_1 + l_2$ – the distance between contacts, l_1 and l_2 – the distances between the boundaries of the n^+ - and p^+ -regions and the corresponding contacts, μ_n and μ_p – the mobilities of electrons and holes in the n^+ - and p^+ -regions, $N_n =$ $n_n x_n$ and $N_p = p_p x_p$ – the surface excesses of electrons and holes in the n^+ - and p^+ -regions. In the case where the width of a metal strip is l_n and the relative metallization of the rear surface is m, we have the relation $l \approx l_n (1-m)/m$.

Taking into account that $l/2L_c \ll 1$ in the optimized case, relation (23) can be written in the standard form as

$$K_2 = 1 - \frac{R_{s1}I_{\rm SC}}{V_{\rm OC}},$$
 (24)

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where $R_{s1} = \frac{1}{12q} \left(\frac{l_1}{\mu_n N_n} + \frac{l_2}{\mu_p P_p} \right)$ – the series resistance related to the finite layer resistance of the n^+ - and p^+ regions, and $I_{\rm SC}$ – the short-circuit current. As shown in [10], the limits of the validity of (24) are narrower than those of (23). This is related to the fact that the resistance R_{s1} is distributed, by its nature, and can be considered as concentrated only under the condition $l/2L_c \leq 0.1$.

Then we take the resistance of a contact grid into account. In the given case of a countercomb grid, we can make a simple estimate of its resistance with regard for a variation of the collected current over the contact length, by assuming that the resistance of a contact grid $R_{s2} \approx (2R_1 + R_2/N_c)/2$, where R_1 – the resistance of a bus, R_2 – the resistance of each of the "fingers", and N_c – their number. The numerical estimates in the case where the area of an SC is 1 cm^2 show that the resistance R_{s1} , which is determined by the layer resistance, can be made to be less than R_{s2} . If we decrease the area of an SC of the square form and conserve the width and thickness of the "fingers" and the buses, the resistance R_{s2} will be decreased proportionally to the ratio of edges, and the resistance R_{s1} will be increased at the same ratio. Therefore, we can really neglect the resistance of a contact grid R_{s2} relative to R_{s1} only for small areas of SCs.

3. Case of the Standard Geometry of Contacts

In order to simulate SCs with the standard geometry of contacts, we use the approach developed in [4]. There the limiting case where the criterion $L \gg d$ is satisfied even for arbitrary values of M was considered. In this case, the excess concentration of electron-hole pairs is constant over the SC base thickness. Really, due to the switching-on of higher-order mechanisms of recombination at high levels of the injection, the effective diffusion length decreases with increase in M. Therefore, the excess concentration of electron-hole pairs decreases by depending on the distance between the front and rear surfaces of an SC, and this effect should be taken into account.

In this case, the open-circuit voltage is determined by the relation

$$V_{\rm OC} = \frac{kT}{q} \ln\left(\frac{(n_0 + \Delta n(d))\Delta n(0)}{n_i^2(T_0)}\right),\tag{25}$$

where $n_i(T_0)$ – the concentration of intrinsic charge carriers in the base near the illuminated surface, and T_0 – the temperature of the illuminated surface. Let the inequality L > d be satisfied. Like the case of SCs with rear contacts, the distribution of excess charge carriers in the base can be determined by the method of successive approximations while solving the equation of ambipolar diffusion. In the first approximation, the ratio of the excess concentrations $\Delta n(0)$ and $\Delta n(d)$ is determined by relations (9) and (10). The value of $\Delta n(0)$ is determined from the solution of the equation of generation-recombination balance (8).

The quantity J_{SC} under the AM0 condition (on the approximation of the solar spectrum by the emission of an absolutely black body with a temperature of 5800 K) for a Si SC can be written as [4]

$$J_{\rm SC} = 0.4505(1-m)M \int_{0}^{1} \frac{f(\theta)d\theta}{\theta^4 \left[\exp\left(\frac{2.21}{\theta}\right) - 1\right]},$$
 (26)

where m – the coefficient of shadowing by contacts, $\theta = \lambda/\lambda_x$, λ – the illumination wavelength, λ_x – the wavelength corresponding to the red edge of a photoeffect in Si, and $f(\theta)$ – the collection coefficient of electrons and holes defined in [4].

It is worth noting that the formula for the collection coefficient obtained in [4] considers the losses on the recombination in the heavily doped front region and the full collection of the generation-active illumination by a semiconductor.

Because the separation of electron-hole pairs occurs near the illuminated surface x = 0, the short-circuit current under the standard geometry of contacts is practically independent of a change in the diffusion length with increase in the injection level. In this case, only the open-circuit voltage $V_{\rm OC}$ will be changed. Therefore, the formula for the photoconversion efficiency η can be written, in this case, as

$$\eta = \eta_0 \frac{V_{\rm OC}}{V_{\rm OC0}},\tag{27}$$

where η_0 – the expression for the photoconversion efficiency obtained in [4], and $V_{\rm OC0}$ – the value of the open-circuit voltage in the case where $\Delta n(d) = \Delta n(0)$.

4. Discussion of the Obtained Results

In Fig. 2, we give the effective RSR on the illuminated surface S_0 versus the excess concentration of electronhole pairs $\Delta n(0)$. The parameter of the theoretical curves is $N = Q_0/q$ (in units of cm⁻²). Curves 1-4 differ from curves 1'-4' by that the base has the *p*-type of conductivity in the first case and the *n*-type in the



Fig. 2. Effective rate of surface recombination on the illuminated surface S_0 versus the excess concentration of electron-hole pairs $\Delta n(0)$. The used parameters: $p_0(n_0)=10^{14}$ cm⁻³; $N_s = 10^{11}$ cm⁻²; $E_s = 0$, $C_p = C_n = 10^{-9}$ cm³/s; N, cm⁻²: 1, 1' - 10^{10} ; 2, 2' - 3×10^{10} ; 3, 3' - 10^{11} ; 4, 4' - 3×10^{11}

second case. As seen from Fig. 2, S_0 increases with the excitation level. At sufficiently small values of a builtin charge and at moderate excitation levels, the value of S_0 is greater in the case of a high-resistance base of the *p*-type than that for a high-resistance base of the *n*-type, which is related to the realization of the initial depletion with respect to holes in the former case and the enrichment with respect to electrons in the latter case. However, at sufficiently great $N \ge 10^{11}$ cm⁻², the dependences of S_0 on the excitation level for bases of the *p*- and *n*-types at the same *N* coincide. Moreover, the linear dependence $S_0(\Delta n)$ is observed in a sufficiently wide region of excitation levels.

It should be noted that the temperature T of an SC grows with increase in the illumination concentration degree M. If the dependence $n_i(T)$ is known, we can calculate the photoconversion efficiency in the given case as well. For the self-consistent determination of T, it is necessary to take the construction of an SC and the heat removal conditions into account. It is a sufficiently complicated problem we did not solve. In specific calculations, we considered, as a rule, that the forced cooling of SCs happens, and their temperature equals 300 K irrespective of m.

In Fig. 3, we present the dependence of the photoconversion efficiency η on M for the AM 1.5 (curves 1–4) and AM0 (curves 1'–4'') conditions для SCs with the rear position of contacts at the minimized



Fig. 3. Photoconversion efficiency versus the illumination concentration degree for the AM 1.5 and AM0 conditions for SCs with rear metallization. The used parameters: $n_0=10^{14}$ cm⁻³, $m = 0.2, d = 10^{-2}$ cm, T = 300 K, $N = 3 \times 10^{11}$ cm⁻², $D_a = 15$ cm²/s, $x_p = x_n = 2 \times 10^{-4}$ cm, $\mu_p = \mu_n = 100$ cm²/(V×s), $p_p = n_n = 3 \times 10^{18}$ cm⁻³, $l_n = 30 \ \mu$ m, τ_v , s: 1, 1' - 3×10⁻³; 2, 2' - 10⁻³; 3, 3' - 3×10⁻⁴; 4, 4' - 10⁻⁴

value of the effective RSR on the illuminated surface S_0 . On the construction of the figure, we took that, under the AM0 conditions, $J_{\text{gen}}(M=1) = 54 \text{ mA/cm}^2$, $P = 0.135 \text{ W/cm}^2$ and, under the AM 1.5 conditions, $J_{\text{gen}}(M=1) = 43.6 \text{ mA/cm}^2$, $P = 0.1 \text{ W/cm}^2$. The parameter of the curves is the bulk lifetime τ_v .

As seen from Fig. 3, each of the curves has a maximum. Its physical nature is related, on the one hand, to the increase in the voltage drop on the series resistance with increase in the short-circuit current and, on the other hand, to the decrease in $\Delta p(d)$ as compared with $\Delta p(0)$ due to the enhancement of the bulk recombination on the switching-on of nonlinear mechanisms of recombination with increase in M. As seen, at $\tau_v = 10^{-3}$ s, the photoconversion efficiency maximum is positioned at $M \sim 4 \times 10^2$, and the values of η at the maximum are, respectively, 30 and 28 % for the AM 1.5 and AM0 conditions. As τ_v decreases, the position of the maximum shifts to the side of lower values of M, and the values of η at the maximum decrease. For example, at $\tau_v = 10^{-4}$ s, the photoconversion efficiency maximum is positioned at $M \sim 2 \times 10^2$, and the values of η at the maximum for the AM 1.5 and AM0 conditions are, respectively, 28 and 26 %. The calculated curves given in Fig. 3 agree with the experimental value, $\eta \approx 28\%$, and with the theoretical dependence of η on M obtained in work [2].



Fig. 4. Experimental [3] and theoretical dependences $\eta(M)$, K(M), and $V_{\rm OC}(M)$ for SCs with rear metallization. The used parameters: $n_0=4.6\times10^{14}$ cm⁻³, $d=1.5\times10^{-2}$ cm, $\tau_v=7.8\times10^{-4}$ s, T=300K, $J_{\rm gen}(M=1)=35.8$ mA/cm², $R_s=0.03$ Ohm, m=0.4

It is worth noting that the short-circuit current and the photoconversion efficiency for SCs with the rear position of contacts depend rather strongly on S_0 . For example, S_0 equals approximately 3 cm/s for the parameters used in the construction of Fig. 2 and M = 1. If $N=10^{10}$ cm⁻² and $N_s=3\times10^{11}$ cm⁻², then S_0 equals 160 cm/s for M = 1, and η at the maximum decreases from 30 to 26% under the AM 1.5 condition. In the case where $N=10^{10}$ cm⁻² and $N_s=10^{12}$ cm⁻², S_0 equals 540 cm/s for M = 1, and η at the maximum decreases to 20%.

In Fig. 4, we give the experimental dependences of the photoconversion efficiency η , the fill factor K of the voltage-current characteristic, and the opencircuit voltage V_{OC} on the concentration degree of the solar illumination M, which are taken from [3], and the corresponding calculated dependences (curves 1-3) constructed with the use of the theory of SCs with rear metallization developed in the present work. In the calculations, such parameters as the concentration of equilibrium electrons in the base, bulk lifetime, thickness and area of an SC were taken from work [3]. We varied the value of S_d , current density $J_{\text{gen}}(M=1)$, and the series resistance R_s . The calculated curves 1? and 2' correspond to $R_s = 0$. As seen from Fig. 4, the experimental and theoretical dependences in the range of M from 1 to 100 are in good agreement. The realization of a maximum on the curves $\eta(M)$ and K(M) and their further decrease with increase in M are related to the influence of the series resistance. This is well seen, in particular, from the comparison of the theoretical curves constructed with and without regard for R_s .



Fig. 5. Photoconversion efficiency versus the illumination concentration degree for the AM0 conditions for SCs with standard metallization. The used parameters: $n_0=10^{14}$ cm⁻³, $d=10^{-2}$ cm, T=300 K, $D_a=15$ cm²/s, $x_p=x_n=2\times10^{-4}$ cm, $\mu_p=\mu_n=100$ cm²/(V×s), $p_p=n_n=3\times10^{18}$ cm⁻³, $l_n=30$ μ m, $\tau_v=10^{-3}$ s, m: 1-0.05; 2-0.1; 3-0.15; 4-0.2

As seen from the dependences given in Figs. 3 and 4 and from the results of experimental studies of the photoconversion efficiency of SCs with rear metallization (works [2,3]), the maximum values of the photoconversion efficiency can vary in the wide limits (from 20 to 30% under the AM 1.5 condition). In order to obtain the maximum values, it is necessary to satisfy the following conditions: 1) to ensure the small thickness of the base as compared with the effective diffusion length of electron-hole pairs; 2) to minimize the value of the effective RSR on the illuminated surface, S_0 ; 3) to ensure the most complete capture of the photoactive illumination; 4) to minimize the series resistance.

In Fig. 5, we show the dependence of η on M under the AM0 condition for the standard geometry of contacts, whose parameter is m. As seen from the figure, these dependences have also a maximum. The position of the maximum and the value of the efficiency at the maximum can be governed by two reasons. The first reason consists in the growth of the series resistance with M at the given parameters of a contact grid. In order to decrease the series resistance, it is necessary to use a grid with smaller meshwidth, by increasing a degree of the shadowing by contacts. This allows us to decrease R_s , but leads to a decrease in the short-circuit current and, hence, to a decrease in the photoconversion efficiency. The other reason consists in the decrease of both the quantity of $\Delta n(d)$ as compared with $\Delta n(0)$ and, hence, the open circuit voltage as a result of the reduction of the effective diffusion length at sufficiently high degrees of the solar illumination concentration. The calculations show that if the bulk lifetime conditioned by the Shockley–Reed–Hall recombination τ_v exceeds 1 ms, then the main reason for both the appearance of the maximum and the reduction of the photoconversion efficiency is the first one. But if the values of τ_v are less than 1 ms, both the first and second reasons are responsible for the appearance of the maximum. We note that we neglect the resistance of a contact grid as compared with the layer resistance of heavily doped regions on the construction of Figs. 3 and 5.

It is seen from the comparison of Figs. 3 and 5 that the higher values of the photoconversion efficiency can be realized, in principle, in SCs with rear metallization. This is related to the fact that a contact grid with smaller meshwidth, which allows one to decrease the influence of the series resistance at high degrees of the concentration M in an SC with the standard position of contacts, leads to a decrease in both the short-circuit current and, hence, the photoconversion efficiency, whereas the expenditures on the shadowing are absent in SCs with rear metallization. At the same time, the fulfillment of the inequality L < d causes the stronger decrease in the photoconversion efficiency of SCs with rear metallization, which is related to the significant decrease in the short-circuit current. For SCs with standard metallization, the decrease in the efficiency happens mainly due to a slower decrease in the open-circuit voltage.

Finally, we discuss the influence of the heating of SCs under concentrated illumination on the photoconversion efficiency for the rear and standard positions of currentcollecting contacts. The decrease in the photoconversion efficiency on heating occurs, mainly, due to a decrease in the open-circuit voltage. We will perform the estimation of this diminution for M = 300, by assuming that the temperature on the illuminated surface is 100 °C, and the temperature on the rear surface is 50 °C. In this case, the photoconversion efficiency at the parameters used in this work decreases for SCs with rear metallization by 7% as compared with the corresponding value at 300 K, whereas the diminution is 20% for SCs with standard metallization. Even if we say nothing about the technological advantages of the position of a contact grid on the rear surface, which excludes its damage under concentrated illumination, it is obvious that SCs with rear metallization in the presence of heating have also the advantages of the purely physical content as compared with SCs with standard metallization.

5. Conclusions

The experimental data obtained in [2], where the record photoconversion efficiency factor of about 28% was attained under concentrated illumination, and the results of the theoretical analysis carried out in the present work allow us to conclude that Si SCs with rear metallization are sufficiently perspective for practical applications. At the same time, their advantages as compared with SCs with the standard geometry of contacts can be realized only in SCs, whose thickness is less than the diffusion length, on the minimization of the effective rate of surface recombination on the illuminated surface.

It is shown that the developed theory describes well the experimental results obtained in [2, 3] on the photoconversion efficiency in Si SCs with rear metallization, which testifies to the reality of the used assumptions.

It is established that heating affects Si SCs with rear and front metallizations in different ways. In particular, the open circuit voltages for SCs with rear and standard metallizations are determined, respectively, by the temperature of the rear surface and by that of the illuminated surface.

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ОСОБЛИВОСТІ ФОТОПЕРЕТВОРЕННЯ ПРИ КОНЦЕНТРОВАНОМУ ОСВІТЛЕННІ В КРЕМНІЄВИХ СОНЯЧНИХ ЕЛЕМЕНТАХ ДЛЯ СТАНДАРТНОЇ І ТИЛЬНОЇ ГЕОМЕТРІЙ РОЗТАШУВАННЯ КОНТАКТІВ

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Резюме

У досить реальних наближеннях щодо співвідношення між товщиною кремнієвих сонячних елементів (СЕ) та довжи-

ною дифузії в них розвинуто теорію фото-перетворення при концентрованому освітленні для тильної та стандартної геометрій струмозбірних контактів. Проведено порівняльний аналіз отриманих результатів для двох вказаних геометрій. Показано, що кремнієві СЕ з тильною металізацією, в принципі, можуть мати більшу ефективність фотоперетворення. Отримано задовільне узгодження розвинутої теорії з експериментом.