FORCED FLOW OF A CONDUCTING VISCOUS FLUID THROUGH A POROUS MEDIUM INDUCED BY A ROTATING DISK WITH APPLIED MAGNETIC FIELD

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We study the forced flow of an electrically conducting viscous incompressible fluid bounded by the porous medium and an infinite impervious rotating disk. A uniform magnetic field is applied in the direction normal to the flow. It is assumed that the flow between the disk and the porous medium is governed by the Navier–Stokes equations and that in the porous medium by the Brinkman equations. The flows in the two regions are matched at the interface by assuming that the velocity and stress components are continuous at it. At the interface (the boundary between the porous medium and the clear fluid), a modified set of boundary conditions suggested by Ochao–Tapia and Whittaker is used. The analytic expressions for the velocity and the shearing stress are obtained, and the effects of various parameters on them are examined.

1. Introduction

The requirements of modern technology have stimulated the interest in the studies of fluid flows which involve the interaction of several phenomena. In the present work, we consider the case where a viscous fluid flows over a porous surface because of its importance in many engineering problems (e.g., the flow of a liquid in a porous bearing [1] and the water flow in river beds), in petroleum technology (the movement of natural gas, oil, and water through the oil reservoirs), in chemical engineering (the processes of filtration and purification), etc. Cunningham and Williams [2] also reported several geophysical applications of flows in porous media, viz. porous rollers and its natural occurrence in the flow of rivers through porous banks and beds, and the flow of oil through underground porous rocks.

The mathematical theory of the flow of a fluid through a porous medium was initiated by Darcy [3]. For a steady flow, he assumed that viscous forces were in equilibrium with external forces due to the pressure difference and the body forces. Later on, Brinkman [4] proposed a modification of the Darcy's law for porous media. In most of the examples, the fluid flows through the porous medium have two regions. In region I, the fluid is free to flow and, in region II, the fluid flows through the porous medium. To link the flows in two regions, the matching conditions are required at their interface. Coupled flows of this type with different geometries and several kinds of matching conditions have been examined by several authors, viz. William [5] and Ochoa-Tapia et al. [6–7]. Srivastava et al. [8] discussed the flow and the heat transfer of a viscous fluid confined between a rotating plate and a porous medium, by assuming that the flow in the porous medium was governed by the Brinkman equation [4] and the free flow is described by the Navier–Stokes equations. The problem (in which the liquid occupies the semiinfinite region on one side of the disk and the motion is axially symmetric) concerning the steady forced flow of an incompressible viscous fluid against a rotating disk was studied in [9]. A complete review of this paper and some related works has been given in [10]. Recently, Chaudhary et al. [11] discussed the flow of a viscous incompressible fluid confined between a rotating disk and a porous medium. The subject of hydromagnetics has attracted the attention of many authors due not only to its own interest, but also due to many applications to the problems of geophysics and astrophysics. It is desirable to extend many of the available viscous hydrodynamic solutions to include the effects of a magnetic field in those cases where the viscous fluid is electrically conducting. In view of the wide applications in industrial and other technological fields, the problem of a flow near a rotating disk has been extended to hydrodynamics initially by Sparrow et al. [12] and Katukani [13]. Kumar et al. [14] and Watanabe et al. [15] studied a MHD flow near a rotating disk. The computational analysis of a MHD flow near a rotating disk was carried out by Ariel [16].

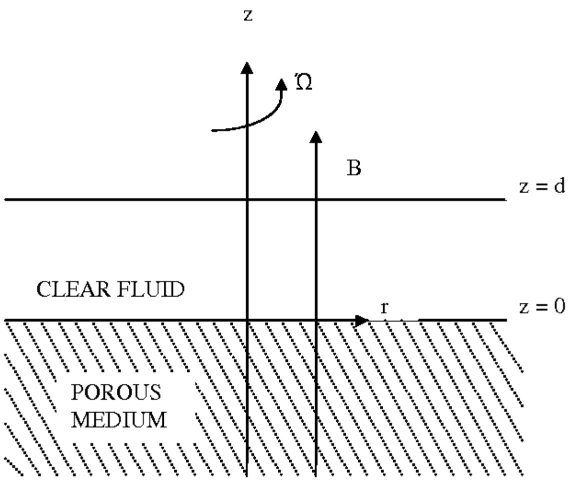

Fig. 1. Schematic of the problem

v disk near it. The gap between them is filled with an incompressible electrically conducting fluid. In what follows, we will describe the variations in the velocity and the shear stress in the flow induced in the fluid by the rotating disk in the presence of a magnetic field.

2. Formulation of the Problem

We consider the steady flow of an incompressible viscous fluid confined between a rotating disk and a porous medium fully saturated with the fluid. Let (r^*, θ^*, z^*) be the set of cylindrical polar coordinates, and let the disk rotate with angular velocity Ω about an axis $r^* = 0$ and be positioned at $z^* = d$. A uniform magnetic field B is applied in the direction normal to the flow. The problem we consider here is represented geometrically by Fig. 1. We assume that the magnetic Reynold's number is small so that the induced magnetic field can be neglected. The conductivity of the fluid is not very large. Since, no external electric field is applied, and the effect of polarization of the ionized fluid is negligible. Thus, it can be assumed that the electric field is zero. The region $z^* \leq 0$ is filled with the porous material and is fully saturated with the liquid. The region $0 \leq z^* \leq d$ is called region I, the region $z^* \leq 0$ is called region II, and $z^* = 0$ is the interface between the two regions.

The Navier–Stokes equations and the continuity equation in region I look as

$$\rho[u^* u^*_r + w^* u^*_z - v^{*2}/r^*] =$$

$$= -p^*_r + \mu[\nabla^2 u^* - u^*/r^{*2}] - \sigma B^2 u^*, \quad (1)$$

$$\rho[u^* v^*_r + w^* v^*_z - u^* v^*/r^*] =$$

$$= -p^*_\theta/r^* + \mu[\nabla^2 v^* - v^*/r^{*2}] - \sigma B^2 v^*, \quad (2)$$

$$(r^* u^*)_r/r^* + w^*_z = 0 \quad (3)$$

where ρ is the density, p is the pressure, σ is the electrical conductivity and B is the intensity of the magnetic field. The velocity components are u^*, v^*, and w^* in the r^*, θ^*, and z^* directions, respectively. The porous region $z^* < 0$ is called region II, and the flow in this region is governed by the Brinkman equations [4]. These equations together with the continuity equation are given by

$$-P^*_r + \mu_e(\nabla^2 U^* - U^*/r^{*2}) - \mu U^*/k - \sigma B^2 U^* = 0, \quad (4)$$

$$-P^*_\theta/r^* + \mu_e(\nabla^2 V^* - V^*/r^{*2}) - \mu V^*/k - \sigma B^2 V^* = 0, \quad (5)$$

$$(r^* U^*)/r^* + W^*_z = 0 \quad (6)$$

where k is the porous medium permeability, μ_e is the effective viscosity for the Brinkman flow model, which is different from μ, the viscosity of the fluid, P^* is the pressure in the porous medium, and U^*, V^*, and W^* are the velocity components in the porous medium in the r^*, θ^*, and z^* directions, respectively. Givler and Altobelli [17] have determined experimentally μ_e for a steady flow through the wall-bounded porous medium, and their result shows that $\mu_e = (7.5^{+3.4}_{-2.4})\mu$. The boundary conditions of the problem are

$$u^* = ar^*, \quad v^* = r^*\Omega, \quad w^* = 0, \quad \text{at} \quad z^* = d, \quad (7)$$

$$U^* \to 0, \quad V^* \to 0, \quad \text{as} \quad z^* \to -\infty. \quad (8)$$

We use the matching condition at the interface as suggested by Ochao−Tapia and Whittaker [6–7]. These conditions, which are investigated theoretically and experimentally, state that the equation requires a discontinuity in the shear stresses, while retaining the continuity of the velocity. The steady fully developed laminar flow in the parallel plate and the cylindrical channel partially filled with the porous medium and partially with the clear fluid was investigated by Kuznetsov [18] using the matching conditions from [6–7]. Using these conditions, Srivastava [19] has also discussed the flow of a viscous fluid confined between a torsionally oscillating disk and the porous medium fully saturated with the liquid. At the interface of the porous medium and the clear fluid $z^* = 0$, we assume the velocity components and the pressure are continuous and the jumps in the shear stresses $\tau_{z\theta}$ and τ_{zr} as given by Ochao–Tapia and Whittaker [6–7]. In our notation, these assumptions can be written as

$$\left.\begin{array}{l} u^* = U^*, v^* = V^*, w^* = W^*, p^* = P^*, \text{at } z^* = 0, \\ \mu_e U_z^* - \mu u_z^* = \beta\mu U^*/\sqrt{k}, \\ \mu_e V_z^* - \mu v_z^* = \beta\mu V^*/\sqrt{k}, \text{at } z^* = 0. \end{array}\right\} \quad (9)$$

3. Equation of Motion

We assume that the velocity components in region I have the form

$$\left.\begin{array}{l} u^* = r^*\Omega f'(y),\ v^* = r^*\Omega g(y),\ w^* = -2d\Omega f(y), \\ \rho^* = -\mu\Omega p_1(y),\ y = z^*/d, \end{array}\right\} \quad (10)$$

where the prime denotes the differentiation with respect to y. This form of the velocity components satisfied the equation of continuity (3). Substituting Eq. (10) in Eqs. (1) and (2), we get the following equations of motion in the directions of r and θ, respectively:

$$R[(f'^2) - 2ff'' - g^2] = f''' - M^2 f', \quad (11)$$

$$2R[f'g - fg'] = g'' - M^2 g, \quad (12)$$

where R (Reynolds number) $= \rho\Omega d^2/\mu$, and M (Hartmann number) $= \sqrt{\frac{\sigma B^2 d^2}{\mu}}$. The equation in the direction of z serves merely to determine the axial pressure gradient and hence is not given. We assume the following form of velocity components for region II:

$$U^* = r^*\Omega F'(y), \quad V^* = r^*\Omega G(y),$$

$$W^* = -2d\Omega F(y), \quad P^* = -\mu\Omega P_1(y). \quad (13)$$

The forms of velocities in Eq. (13) are chosen so that the equation of continuity (6) is satisfied. Substituting Eq. (13) in Eqs. (4) and (5), we get the equations in the directions of r and θ, respectively:

$$\gamma^2 F''' - (\sigma^2 + M^2)F' = 0, \quad (14)$$

$$\gamma^2 G'' - (\sigma^2 + M^2)G = 0. \quad (15)$$

Here, σ (Darcy number) $= d/\sqrt{k}$, and $\gamma^2 = \mu_e/\mu$. The boundary conditions (7), (8), and (9) at interface can be written as

$$f = 0, \quad f' = S, \quad g = 1, \quad \text{at} \quad y = 1, \quad (16)$$

$$F' \to 0, \quad G \to 0, \quad \text{as} \quad y \to -\infty, \quad (17)$$

$$\left.\begin{array}{l} f = F, \quad f' = F'\gamma^2 F'' - f'' = \beta\sigma F', \quad \text{at} \quad y = 0 \\ g = G, \quad \gamma^2 G' - g' = \beta\sigma G, \quad \text{at} \quad y = 0 \end{array}\right\}, \quad (18)$$

where $S = a/\Omega$ is the dimensionless forced parameter assumed to be small ($S \leq 1$).

4. Solution of the Problem

The solutions of Eqs. (14) and (15) satisfying the boundary conditions (17) are given as

$$F'(y) = Ae^{\alpha y}, \quad F(y) = (A/\alpha)e^{\alpha y} + C,$$

$$G(y) = Be^{\alpha y}, \quad (19)$$

where $\alpha = \sqrt{\frac{\sigma^2 + M^2}{\gamma^2}}$. The integration constants A, B, and C can be determined from the matching conditions (18). In our present effort, we use the approximation of the small Reynold's number for the equations involving the viscosity. We consider the distance d between the rotating disc and the porous interface as small, hence Reynold's number may be also taken small. For small values of R, a regular perturbation scheme can be developed for Eqs. (11) and (12) by expanding f and g in powers of R as

$$f = \sum_{n=0}^{\infty} R^n f_n, \quad g = \sum_{n=0}^{\infty} R^n g_n. \quad (20)$$

As f and g have to be matched with equation (19) at the interface, the constants A, B and C must also be expanded in power of R as:

$$A = \sum_{n=0}^{\infty} R^n A_n, \quad B = \sum_{n=0}^{\infty} R^n B_n, \quad C = \sum_{n=0}^{\infty} R^n C_n. \quad (21)$$

Using this perturbation scheme, we get the solutions of Eqs. (11) and (12) for region I as

$$f'(y) = a_3 e^{My} + a_4 e^{-My} + R[(k_1 - d_{15}y)e^{My} +$$

$$+(k_2+d_{16}y)e^{-My}-d_{13}e^{2My}+d_{14}e^{-2My}-d_{17}], \quad (22)$$

$$f(y)=\frac{a_3}{M}e^{My}-\frac{a_4}{M}e^{-My}+a_5+$$

$$+R\left[\left(\frac{k_1}{M}+\frac{d_{15}}{M^2}-\frac{d_{15}}{M}y\right)e^{My}-\right.$$

$$-\left(\frac{k_2}{M}+\frac{d_{16}}{M^2}+\frac{d_{16}y}{M}\right)e^{-My}-$$

$$\left.-\frac{d_{13}}{2M}e^{2My}-\frac{d_{14}}{2M}e^{-2My}-d_{17}y+k_3\right], \quad (23)$$

$$g(y)=a_1e^{My}+a_2e^{-My}+R[(h_1+d_6y)e^{My}+$$

$$+(h_2-d_5y)e^{-My}-d_4], \quad (24)$$

and the solutions of Eq. (19) in the porous medium are given by

$$F'(y)=e^{\alpha y}\left[(a_3-a_4)\frac{M}{a}\right]+Re^{\alpha y}\left[\frac{1}{a}(k_1M-k_2M-\right.$$

$$\left.-2Md_{13}-2Md_{14}-d_{15}+d_{16})\right], \quad (25)$$

$$F(y)=e^{\alpha y}\left[(a_3-a_4)\frac{M}{a\alpha}\right]-$$

$$-a_3\left(\frac{M}{a\alpha}-\frac{1}{M}\right)+a_4\left(\frac{M}{a\alpha}+\frac{1}{M}\right)+a_5+$$

$$+R\left[\frac{e^{\alpha y}}{a\alpha}d_{20}+\frac{1}{2M^2}(2k_1M-2k_2M-\right.$$

$$\left.-d_{13}M-d_{14}M+2d_{15}-2d_{16}+2M^2k_3)-\frac{1}{\alpha a}d_{20}\right], \quad (26)$$

$$G(y)=2Me^{\alpha y}/[(a+M)e^M+(M-a)e^{-M}]+$$

$$+Re^{\alpha y}\left[\frac{1}{a}(h_1M-h_2M-d_5+d_6)\right], \quad (27)$$

where

$$\alpha=\sqrt{(\sigma^2+M^2)/\gamma^2}, \quad a=\gamma^2\alpha-\beta\sigma,$$

$$a_1=(a+M)/[e^M(a+M)+e^{-M}(m-a)],$$

$$a_2=(M-a)/[e^M(a+M)+e^{-M}(m-a)],$$

$$a_3=S(M+a)/[e^M(M+a)+e^{-M}(M-a)],$$

$$a_4=S(M-a)/e^M(M+a)+e^{-M}(M-a),$$

$$a_5=(a_4e^{-M}-a_3e^M)/M,$$

$$d_1=4a_2a_3+4a_1a_4, \quad d_2=2a_2a_5M,$$

$$d_3=-2a_1a_5M, \quad d_4=d_1/M^2,$$

$$d_5=d_2/2M, \quad d_6=d_3/2M, \quad d_7=d_5-d_6-ad_4,$$

$$h_1=[d_4(M+a)+d_5(M+a)e^{-M}-d_6(M+a)e^M-$$

$$-h_3(M+a)e^M]/[e^M(M+a)+e^{-M}(M-a)]+h_3,$$

$$h_2=[d_4(M-a)+d_5(M-a)e^{-M}-d_6(M-a)e^M-$$

$$-d_7e^M]/[e^M(M+a)+e^{-M}(M-a)],$$

$$h_3=d_7/(M-a), \quad d_8=a_1^2+a_3^2, \quad d_9=a_2^2+3a_4^2,$$

$$d_{10}=2a_3a_5M, \quad d_{11}=2a_4a_5M,$$

$$d_{12}=2a_1a_2+2a_3a_4, \quad d_{13}=d_8/3M^2,$$

$$d_{14}=d_9/3M^2, \quad d_{15}=d_{10}/2M,$$

$$d_{16}=d_{11}/2M, \quad d_{17}=d_{12}/M^2,$$

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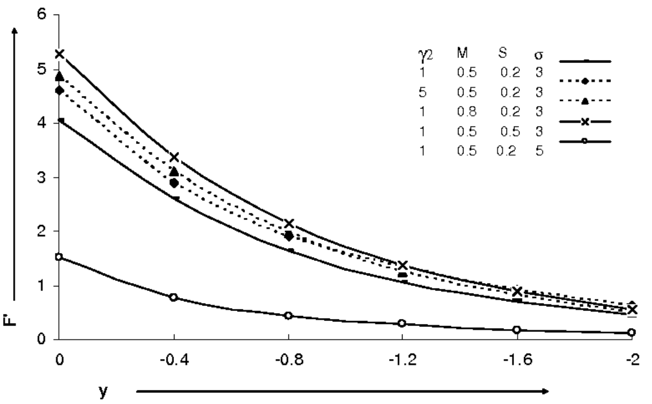

Fig. 2. Radial (F') velocity components in the porous medium for $\beta = 0.5$ and $R = 0.2$

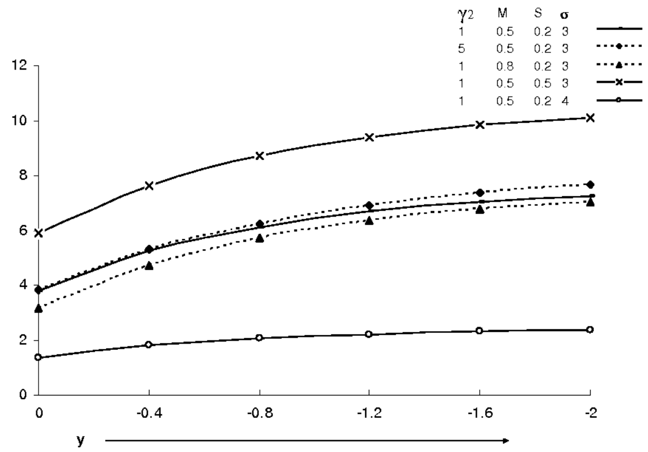

Fig. 3. Axial ($-F$) velocity components in the porous medium for $\beta = 0.5$ and $R = 0.2$

$$d_{18} = d_{13}(2M - a) + d_{14}(2M + a) + d_{15} - d_{16} - ad_{17},$$

$$d_{19} = d_{13}e^{2M} - d_{14}e^{-2M} + d_{15}e^{M} - d_{16}e^{-M} + d_{17},$$

$$k_1 = (M^2 - a^2)d_{19}/[e^M(M + a) + e^{-M}(M - a)+$$

$$+d_{18}e^M] + d_{18}/(M - a),$$

$$k_2 = (M - a)d_{19}/[e^M(M + a)+$$

$$+e^{-M}(M - a) + d_{18}e^M],$$

$$k_3 = [e^M(2d_{15}M - 2d_{15} - 2k_1M) + e^{-M}(2k_2M+$$

$$+2d_{16}M + 2d_{16}) + d_{13}Me^{2M} + d_{14}Me^{-2M}+$$

$$+2M^2d_{17}]/2M^2,$$

$$d_{20} = k_1M - k_2M - 2Md_{13} - 2Md_{14} - d_{15} + d_{16}.$$

Having determined the velocity fields, we can calculate the shear stresses at the rotating disc as

$$[\tau_{rz}]_{z=1} = \frac{\mu\Omega r}{d} f''(y) = \frac{\mu\Omega r}{d} f_1''(1), \qquad (28)$$

where

$$f''(1) = a_3Me^M - a_4Me^{-M} + R[M(k_1 - d_{15})e^M-$$

$$-d_{15}e^M - M(k_2 + d_{16})e^{-M} + d_{16}e^{-M}-$$

$$-2Md_{13}e^{2M} - 2Md_{14}e^{-2M}],$$

and

$$[\tau_{z\theta}]_{z=1} = \frac{\mu\Omega r}{d} g'(1), \qquad (29)$$

where

$$g'(1) = a_1Me^M - a_2Me^{-M} + R[M(h_1 + d_6)e^M+$$

$$d_6e^M - M(h_2 - d_5)e^{-M} - d_5e^{-M}].$$

5. Discussion

We have determined the velocities and the shear stress of the MHD flow of a viscous, incompressible, electrically conducting fluid through a porous medium induced by an impervious rotating disk. This enables us to carry out the numerical computations for the velocities and the shear stress at the rotating disk for various values of the Hartmann number (M), ratio of viscosities (γ^2), forced parameter (S), and Darcy number (σ). The velocity components in the porous medium against the distance from the interface ($-y$) have been plotted in Figs. 2, 3, and 4 for various values of the parameters which are consistent with the results in [17,19]. Figure 2 reveals that the radial velocity components (F') are maximum at the interface and decay exponentially, as we enter the porous medium, by vanishing at a large distance from the interface. It is observed that the radial velocity

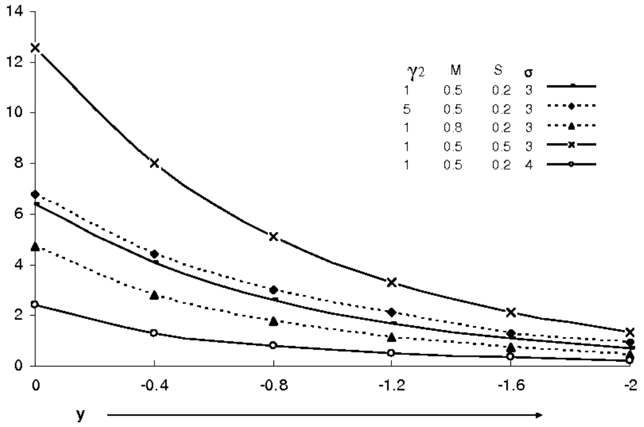

Fig. 4. Transverse ($-G$) velocity components in the porous medium for $\beta = 0.5$ and $R = 0.2$

component increases with the magnetic parameter (M) and the forced parameter (S), but it decreases with increase in σ. The radial velocity falls sharply with increase in the Darcy number. We conclude that the radial velocity in the porous medium increases if the magnetic field is strong. The depth of penetration is greater for $\gamma^2 = 1$ as compared to that for $\gamma^2 = 5$. The axial velocity component in the porous medium has been shown in Fig. 3. We have drawn ($-F$) against the distance from the interface for $R = 0.2$ and $\beta = 0.5$, by taking the different values of γ^2, M, S, and σ. As seen, the axial velocity component at a large distance from the interface does not vanish. A boundary layer is formed at the interface, whose thickness is reduced with increase in σ and attains a constant value. It decreases with increase in M and σ, but it increases with γ^2 and S. As the large parameter grows, its magnitude increases sharply. It is concluded that the rotation of a disk near a porous medium fully saturated with a fluid extracts the fluid from the porous medium. This fact may be used by geologists to extract fluids from the porous ground or rocks. We have plotted the graph of the transverse velocity ($-G$) in the porous medium (Fig. 4) against the distance from the interface for $R = 0.2$ and $\beta = 0.5$, taking the different values of γ^2, M, S, and σ. It is

The shear stress components $[f_1''(1)]$ and $[g'(1)]$ for $R = 0.2$ and $\beta = 0.5$

γ^2	S	M/σ	$f_1''(1)$			$g'(1)$		
			3	5	7	3	5	7
2	0.5	0.5	0.2101	0.2029	0.1989	0.4979	0.4810	0.4708
2	0.7	0.5	0.3852	0.3776	0.3733	0.7741	0.7563	0.7137
5	0.5	0.5	1.4160	1.4048	1.3975	4.9239	4.8575	4.8298
2	0.5	0.8	0.3291	0.3223	0.3220	1.4641	1.4069	1.3216

observed that the transverse velocity decreases exponentially, as we enter the porous medium. It decreases with increase in both M and σ, whereas the inverse effect is observed for γ^2 and S. Further, we observed that if the magnetic field is strong, the transverse velocity in the porous medium decreases. It is concluded that the flow in the porous medium in the transverse direction reaches a maximum value at the interface and decays exponentially, as we enter into the porous medium, by vanishing as $y \to \infty$.

In the Table, we have presented the shear stress components at the rotating disk. The Table shows that they increase with γ^2, S, and M but decrease with increase in σ. Further, it is observed that if we take $\gamma^2 = 1$, $\beta = 0$, $M = 0$, and $S = 0$ in our analysis and $\phi = \lambda = 1$ and $\alpha = 1$ in work [20], the results of both the studies are comparable. Further, if we take $M = 0$ and $S = 0$ in our analysis, the results reduced to those in [11].

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1. D.D. Joseph and L.N. Tao, J. Appl. Mech., Trans. ASME, Series E **88**, 753 (1966).

2. R.E. Cunningham and R.J. Williams, *Diffusion in Gases and Porous Media* (Plenum Press, New York, 1980).

3. H. Darcy, *The Flow of Fluids through Porous Media* (McGraw-Hill, New York, 1937).

4. H.C. Brinkman, Appl. Sci. Res. **A1**, 27 (1947).

5. W.O. William, Arch. Rat. Mech. Anal. **51**, 239 (1973).

6. Ochao–Tapia and S. Whittaker, Int. J. Heat Mass Transfer **38**, 2635 (1995).

7. Ochao–Tapia and S. Whittaker, Int. J. Heat Mass Transfer **38**, 2647 (1995).

8. A.C. Srivastava and B.R. Sharma, J. Math. Phys. Sci. **26**, 539 (1992).

9. H. Schlichting and Z. Truckenbrodt, ZAMM **32**, 97 (1952).

10. F.K. Moore, *Advances in Applied Mechanics* (Academic Press, New York, 1956).

11. R.C. Chaudhary and P.K. Sharma, Int. J. of App. Mech. and Eng. **9(3)**, 607 (2004).

12. E.M. Sparrow and R.D. Cess, J. Appl. Mech. Trans. ASME **29**, 181 (1962).

13. T. Katukani, J. Phys. Soc. Jpn. **17**, 1496 (1962).

14. S.K. Kumar, W.I. Thacker, and L.T. Watson, Computers Fluids **16**, 183 (1988).

15. T. Watanabe and T. Oyama, ZAMM **71**, 522 (1991).

16. P.D. Ariel, ZAMM **82**, 235 (2002).

17. R.C. Givler and S.A. Altobelli, J. Fluid Mech. **258**, 355 (1994).

18. A.V. Kuznetsov, Appl. Sci. Res. **56**, 53 (1996).

19. A.C. Srivastava, ZAMP **50**, 529 (1999).

20. A.C. Srivastava and B. Barman, Proc. Nat. Acad. Sci. India **67(A)**, 165 (1997).

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РУХ ПРОВІДНОЇ В'ЯЗКОЇ РІДИНИ КРІЗЬ ПОРУВАТЕ СЕРЕДОВИЩЕ У МАГНІТНОМУ ПОЛІ, СПРИЧИНЕНИЙ ОБЕРТАННЯМ ДИСКА

Б.К. Шарма, А.К. Джха, Р.С. Чаудхарі

Р е з ю м е

Досліджено рух електропровідної в'язкої нестисливої рідини, обмеженої поруватим середовищем та протяжним непроникним диском, що обертається. Вся система перебуває в однорідному магнітному полі, перпендикулярному до напрямку потоку. Вважається, що рух рідини описується рівняннями Нав'є–Стокса, а порувате середовище – рівняннями Брінкмана. Течії в двох областях пов'язані умовами на межі, які вимагають неперервності швидкості та зсувних напружень. На межі поділу "порувате середовище – чиста рідина" використано також модифіковані межові умови Очао–Тапіа та Уіттекера. Одержано аналітичні вирази для швидкостей і зсувних напружень та досліджено вплив на ці величини різних параметрів.