SUGAWARA—SOMMERFIELD CONSTRUCTION FOR THE $O(3)$-INVARIANT GOLDSTONE MODEL

O.I. BATSULA

M.M. Bogolyubov Institute for Theoretical Physics, Nat. Acad. Sci. of Ukraine
(14b, Metrolohichna Srt., Kyiv 03143, Ukraine)

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It is shown that the Sugawara—Sommerfield relation between currents and the energy-momentum tensor, which is valid for some integrable two-dimensional field theories and for a model of self-interacting charged scalar mesons, can be generalized to the non-Abelian $O(3)$-invariant Goldstone model.

An important ingredient of many two-dimensional conformal field theories is the expression of the energy-momentum tensor in terms of currents, via the formula proposed in [1]:

$$\Theta_{\mu\nu} \propto tr(j_\mu j_\nu - \frac{1}{2}\eta_{\mu\nu}j^2). \tag{1}$$

A nice example of the Spontaneous Breakdown of Global Symmetry is provided by the Goldstone model [2]. Let us consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{m^2}{2}\phi_i\phi_i - \frac{\lambda}{4}(\phi_i\phi_i)^2, \tag{2}$$

where the sum over repeated indices is implied. Here, ϕ_i are real scalar fields, $i = 1, 2, 3$.

Now we focus our attention on the case $m^2 < 0$. The potential

$$V = \frac{m^2}{2}\phi_i\phi_i + \frac{\lambda}{4}(\phi_i\phi_i)^2 \tag{3}$$

is minimum for

$$|\phi_0| = (\phi_1^2 + \phi_2^2 + \phi_3^2)^{1/2} = (-\frac{m^2}{\lambda})^{1/2}. \tag{4}$$

Thus, the system has an infinite number of ground states which "lie" on the surface of a three-dimensional sphere. In fact, the Lagrangian is invariant under rotations of this sphere, i.e. under global rotations of 3 scalar fields

$$\delta\phi_a = \varepsilon_{ab}\phi_b, \qquad \varepsilon_{ab} = -\varepsilon_{ba}. \tag{5}$$

It is also important that the currents

$$j^\mu_{ab} = \phi_a\partial^\mu\phi_b - \phi_b\partial^\mu\phi_a,$$

$$j^\mu_a = \varepsilon_{abc}j^\mu_{bc},$$

$$j^\mu_{12} = j^\mu_3 = (\phi_1\partial^\mu\phi_2 - \phi_2\partial^\mu\phi_1),$$

$$j^\mu_{23} = j^\mu_1 = (\phi_2\partial^\mu\phi_3 - \phi_3\partial^\mu\phi_2),$$

$$j^\mu_{31} = j^\mu_2 = (\phi_3\partial^\mu\phi_1 - \phi_1\partial^\mu\phi_3), \tag{6}$$

defined by the field equations

$$\Box\phi_i + m^2\phi_i + \lambda(\phi_k\phi_k)\phi_i = 0 \tag{7}$$

are just the usual Noether currents [3] for the global $O(3)$ symmetry, whose conservation,

$$\partial_\mu j^\mu_a = 0, \tag{8}$$

follows from the invariance of Lagrangian (2) with respect to the global symmetry group [3,4].

We now introduce the currents

$$j_i^{-\mu} = \frac{1}{\sqrt{2}}(j^\mu_i - \frac{n_i}{2}\partial^\mu S),$$

$$j_i^{+\mu} = \frac{1}{\sqrt{2}}(j^\mu_i + \frac{n_i}{2}\partial^\mu S), \tag{9}$$

where

$$S = \phi_i\phi_i, \quad n_i = \frac{\phi_i}{\sqrt{s}}, \quad n_in_i = 1. \tag{10}$$

Then we find

$$j_i^{+\mu}j^+_{i\mu} + j_i^{-\mu}j^-_{i\mu} = S(\partial^\mu\phi_i\partial_\mu\phi_i), \tag{11}$$

and Lagrangian (2) of the Goldstone model can be rewritten as

$$\mathcal{L} = \frac{j_i^{+\mu}j^+_{i\mu} + j_i^{-\mu}j^-_{i\mu}}{2S} - \frac{m^2S}{2} - \frac{\lambda S^2}{4}. \tag{12}$$

Furthermore,

$$j_i^{+\mu}j_i^{+\nu} + j_i^{-\mu}j_i^{-\nu} = j^\mu_i j^\nu_i + \frac{\partial^\mu S\partial^\nu S}{4} =$$

$$= \partial^\mu \phi_i \partial^\nu \phi_i (\phi_k \phi_k) \tag{13}$$

and

$$\partial^\mu \phi_i \partial^\nu \phi_i = \frac{j_i^{+\mu} j_i^{+\nu} + j_i^{-\mu} j_i^{-\nu}}{S} = \frac{j_i^\mu j_i^\nu}{S} + \frac{\partial^\mu S \partial^\nu S}{4S}. \tag{14}$$

It is easy now to express the energy-momentum tensor for the Goldstone model,

$$T^{\mu\nu} = \partial^\mu \phi_i \partial^\mu \phi_i - \frac{1}{2}\eta^{\mu\nu} \times$$

$$\times \left[\partial^\alpha \phi_i \partial_\alpha \phi_i - m^2 \phi_i \phi_i - \frac{\lambda}{2} (\phi_i \phi_i)^2 \right], \tag{15}$$

in terms of these currents as

$$T^{\mu\nu} = \frac{j_i^{+\mu} j_i^{+\mu} + j_i^{-\mu} j_i^{-\nu}}{S} - \frac{1}{2}\eta^{\mu\nu} \times$$

$$\times \left[\frac{j_i^{+\alpha} j_{i\alpha}^{+} + j_i^{-\alpha} j_{i\alpha}^{-}}{2S} - m^2 S - \frac{\lambda}{2} S^2 \right]. \tag{16}$$

Thus, we generalized the Sugawara—Sommerfield construction for a relativistic model of self-interacting charged scalar mesons to the non-Abelian $O(3)$-invariant Goldstone model [5].

It is easy to show that Lagrangian (12) and the energy-momentum tensor (16) are invariant under the duality transformations

$$\widetilde{j}_i^{+\mu} = j_i^{+\mu} \cos\alpha + j_i^{-\mu} \sin\alpha,$$

$$\widetilde{j}_i^{-\mu} = j_i^{-\mu} \cos\alpha - j_i^{+\mu} \sin\alpha \tag{17}$$

of the currents $j_i^{+\mu}$ and $j_i^{-\mu}$.

Finally, the dynamical laws can be expressed through the equations for the currents

$$\Box S + 2m^2 S + 2\lambda S^2 - \frac{\partial^\mu S \partial_\mu S}{2S} = 2\frac{j_i^\mu j_{i\mu}}{S}, \tag{18}$$

$$\partial_\mu j_i^\mu = 0$$

or

$$\Box S + 2m^2 S + 2\lambda S^2 = 2\frac{j_i^{+\mu} j_{i\mu}^{+} + j_i^{-\mu} j_{i\mu}^{-}}{S}, \tag{19}$$

$$\partial_\mu (j_i^{-\mu} + j_i^{+\mu}) = 0$$

using the equation of motion (7).

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ПОБУДОВА СУГАВАРИ – ЗОММЕРФЕЛЬДА ДЛЯ $O(3)$ ІНВАРІАНТНОЇ МОДЕЛІ ГОЛДСТОУНА

О.І. Бацула

Р е з ю м е

Показано, що співвідношення Сугавари – Зоммерфельда між струмами і тензором енергії-імпульсу, яке виконується для деяких інтегровних 2-вимірних моделей та для моделі самодіючих заряджених скалярних мезонів, може бути узагальнено для неабелевої $O(3)$ інваріантної моделі Голдстоуна.