TRIPLE-PRODUCT CORRELATIONS IN THE PROCESSES OF $B^{\pm}$-MESON DECAY INTO TWO, D^* AND $K^{*\pm}$, VECTOR MESONS

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The T-violating triple-product correlations in the decay processes $B^{\pm} \to D^* K^{*\pm}$, $D^* \to D\pi^0$, $D\gamma$, $D \to f$, where the neutral $D(D^*)$ meson is a superposition of $D^0(D^{*0})$ and $\bar{D}^0(\bar{D}^{*0})$ ones, have been studied. In the framework of the standard model, it has been shown that the large T-violating asymmetries ($\sim$30% for the weak phase $\gamma = 62°$) are possible for such final hadronic states f that the decay $D^0 \to f$ is a doubly Cabibbo-suppressed mode, while $\bar{D}^0 \to f$ is a Cabibbo-allowed one.

1. Introduction

In the framework of the standard model (SM), the violation of CP symmetry in weak interactions arises owing to the presence of a phase in the Cabibbo—Kobayashi—Maskawa (CKM) quark-mixing matrix [1]. The available experimental data testify that it is very probable that the CKM phase is a prevailing source of CP violation in the flavor-changing processes. However, our knowledge about the origin of CP violation is still incomplete, because it is known that the CKM phase cannot explain the observable magnitude of asymmetry between the matter and antimatter in the Universe [2]. Therefore, the search for new sources of CP violation is one of the challenging problems for B-factories.

The nonzero asymmetry between the probabilities for the pair of CP-conjugate decay processes is known to evidence for CP violation most directly. For channels with two (pseudo-)scalar mesons or one (pseudo-)scalar and one vector meson in the final state, only CP asymmetries of this kind can be observed. However, for states with a more complicated spin structure or with a larger number of elementary particles, the asymmetries in the distributions of kinematic variables can also be used to study the effects of CP violation. Although, while discussing the CP violation effects, the main attention is traditionally paid to studying the asymmetry between the partial probabilities of decay processes, there also exists a signal of another type, which will evidence for CP violation and can potentially enable one to find a physics beyond the SM. For example, using the processes of B-meson decay into a pair of vector mesons $B \to V_1 V_2$, it is possible to study triple-product correlations $\vec{q} \cdot (\vec{\epsilon}_1^* \times \vec{\epsilon}_2^*)$, where $\vec{q}$ is the momentum of one of the vector mesons in the rest frame of the B meson, and $\vec{\epsilon}_{1,2}$ are the polarization vectors of V_1 and V_2. Such correlations are odd with respect to the time inversion operation (T); therefore, according to the CPT theorem, a nonzero magnitude of those correlations will also be a signal that CP symmetry is violated. An important feature of T-odd correlations is a possibility for the interaction to generate nonzero values for those quantities in the final state, even if CP symmetry is not violated. Hence, in order to observe a signal of direct T violation, it is necessary to measure the asymmetry between T-odd correlations of a pair of CP-conjugate decay processes. Experimental studies of such correlations in the decay processes $B \to V_1 V_2$ have already been started at B-factories [3].

In the framework of the SM, some of the triple-product correlations in the processes of B-meson (neutral and charged) decay into a pair of vector mesons, $B \to V_1 V_2$, were considered in work [4–7]. The effect of direct T-violation in the processes of decay $B \to V_1 V_2$ into the ground states of vector mesons was found in those works to be small almost without exception. The triple-product correlations for radially excited vector mesons were analyzed in detail in work [8]; the analysis was made in the framework of the approach which is based on the hypothesis of generalized factorization. Moreover, the authors of works [4, 8, 9] assert that T-violating triple-product correlations are maximal, if the differences between the interaction phases from different mechanisms of decay processes are zero in the final state. However, this statement is not always correct, as will be shown in this article.

In this paper, we consider the T-violation effects in the decay processes of B^- meson into a pair of vector $\tilde{D}^{*0}$ and K^{*-} mesons, namely,

$$B^- \to \widetilde{D}^{*0}(\to \widetilde{D}^0(\to f)\,\pi^0)K^{*-}(\to K\,\pi), \qquad (1)$$

$$B^- \to \widetilde{D}^{*0}(\to \widetilde{D}^0(\to f)\,\gamma)K^{*-}(\to K\,\pi), \qquad (2)$$

$$B^- \to \widetilde{D}^{*0}(\to \widetilde{D}^0(\to \bar{f})\,\pi^0)K^{*-}(\to K\,\pi), \qquad (3)$$

and

$$B^- \to \widetilde{D}^{*0}(\to \widetilde{D}^0(\to \bar{f})\,\gamma)K^{*-}(\to K\,\pi). \qquad (4)$$

Hereafter, the notation $\tilde{D}^0(\tilde{D}^{*0})$ designates a superposition of D^0 and $\bar{D}^0$ mesons (D^{*0} and $\bar{D}^{*0}$ ones). The Cabibbo-allowed modes of the $\bar{D}^0$-meson decay will play the role of final states f. Accordingly, they will be doubly Cabibbo-suppressed modes of the $\bar{D}^0$-meson decay; e.g., $f = K^+\pi^-$, $K^{*+}\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, and so on. It should be noted that a neutral vector D^{*0} meson has two basic decay modes, namely, into the $D^0\pi^0$ (with a relative width of 62%) and $D^0\gamma$ (38%) pairs [10]. Therefore, as will be shown below, the combined analysis of those two modes of the D^*-meson decay increases the number of useful events and, owing to the phase difference between those two modes which was pointed at in work [11], opens new opportunities in studying the T-violation effects in the decay $B^\pm \to D^*K^{*\pm}$.

2. Asymmetries of Triple-Product Correlations

The decay of a B meson into a pair of vector mesons, $B \to V_1\,V_2$, is characterized by three amplitudes. In the transverse basis [6], these decay amplitudes correspond to linearly polarized states of vector mesons, which are polarized either longitudinally (0) or transversely to the direction of their motion, being polarized in parallel ($\|$) or normally ($\perp$) to each another. The states 0 and $\|$ are P-even, while the state $\perp$ is P-odd.

A B^- meson can decay into the final state $D^{*0}K^{*-}$ owing to the transition $b \to c\bar{u}s$, or into the state $\bar{D}^{*0}K^{*-}$ owing to the transition $b \to u\bar{c}s$. In the SM, the decay amplitudes for each of three possible helicity states are

$$A_\lambda(B^- \to D^{*0}\,K^{*-}) \equiv A_{c\lambda} = |V_{cb}V_{us}^*|\,a_{c\lambda}\,e^{i\delta_{c\lambda}},$$

$$A_\lambda(B^- \to \overline{D}^{\,*0}\,K^{*-}) \equiv A_{u\lambda} = |V_{ub}V_{cs}^*|\,a_{u\lambda}\,e^{i(\delta_{u\lambda}-\gamma)},$$

where the helicity index λ takes the values $\{0, \|, \perp\}$, $a_{c\lambda}$ and $a_{u\lambda}$ are positive parameters, $\delta_{c\lambda}$ and $\delta_{u\lambda}$ are strong-interaction phases, V_{ij} are the elements of the CKM matrix, and $\gamma = \arg(V_{ub}^*)$ [12]. Provided that D^{*0} and $\bar{D}^{*0}$ decay into a common final state, the amplitudes of the $B^- \to D^{*0}\ K^{*-}$ and $B^- \to \bar{D}^{*0}\ K^{*-}$ transitions would interfere with each other, owing to which the T-odd CP violation can appear in these decay processes. Note that this type of interference has been proposed to separate the weak phase γ [13, 14].

Let us designate the helicity amplitudes of the cascade decay processes (1)–(4) as $A_\lambda^{f\pi}$, $A_\lambda^{f\gamma}$, $A_\lambda^{\bar{f}\pi}$, and $A_\lambda^{\bar{f}\gamma}$, respectively, and the helicity amplitudes of the corresponding CP-conjugate decay processes as $\bar{A}_\lambda^{\bar{f}\pi}$, $\bar{A}_\lambda^{\bar{f}\gamma}$, $\bar{A}_\lambda^{f\pi}$, and $\bar{A}_\lambda^{f\gamma}$, respectively. Then, neglecting small mixing effects in the system $D^0 - \bar{D}^0$ [15] and taking the effective phase difference of π between two modes of D^{*0} decay into $D^0\pi^0$ and $D^0\gamma$ [11] into account, the helicity amplitudes of those processes can be written down as

$$A_\lambda^{f\pi(\gamma)} = (r_{Df} \pm \mathsf{z}_\lambda^-)\,A_{c\lambda}, \qquad (5)$$

$$A_\lambda^{\bar{f}\pi(\gamma)} = (1 \pm r_{Df}\,\mathsf{z}_\lambda^-\,e^{-2i\delta_{Df}})\,A_{c\lambda}, \qquad (6)$$

$$\overline{A}_\lambda^{\bar{f}\pi(\gamma)} = \pm\sigma_\lambda(r_{Df} \pm \mathsf{z}_\lambda^+)\,A_{c\lambda}, \qquad (7)$$

and

$$\overline{A}_\lambda^{f\pi(\gamma)} = \pm\sigma_\lambda(1 \pm r_{Df}\,\mathsf{z}_\lambda^+\,e^{-2i\delta_{Df}})\,A_{c\lambda}, \qquad (8)$$

where $\sigma_0 = \sigma_\| = 1$, $\sigma_\perp = -1$,

$$\delta_{Df} \equiv \arg(\frac{A(\overline{D}^0 \to f)}{A(D^0 \to f)}), : r_{Df} \equiv \sqrt{\frac{\mathrm{Br}(D^0 \to f)}{\mathrm{Br}(\overline{D}^0 \to f)}},$$

$\mathsf{z}_\lambda^\pm \equiv r_{B\lambda}\,e^{i(\delta_\lambda \pm \gamma)}$, $\delta_\lambda \equiv \delta_{B\lambda} + \delta_{Df}$, $\delta_{B\lambda} \equiv \delta_{u\lambda} - \delta_{c\lambda}$, and $r_{B\lambda} \equiv |A_{u\lambda}/A_{c\lambda}|$. The upper signs in expressions (5)–(8) correspond to processes (1), (3), and relevant CP-conjugate ones; the lower signs correspond to processes (2), (4), and relevant CP-conjugate ones.

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The differential probabilities for the cascade processes of B-meson decay into a pair of vector mesons with the following transition of either the both into two pseudoscalar mesons or one vector meson into two pseudoscalar mesons and the other vector meson into a pseudoscalar meson and a photon were obtained in works [5,6,16]. But we are more interested in the consideration of the ratio between the differential probability of decay (1) and the probability of decay (3) (between the differential probability of decay (2) and the probability of decay (4)), as well as of the corresponding ratios for CP-conjugate decay processes, namely,

$$\frac{d^3\mathcal{R}_{f,\,D\pi(\gamma)}}{d\cos\theta_1 d\cos\theta_2 d\Phi} \equiv \frac{1}{\Gamma_{\bar{f},\,D\pi(\gamma)}} \frac{d^3\Gamma_{f,\,D\pi(\gamma)}}{d\cos\theta_1 d\cos\theta_2 d\Phi}, \tag{9}$$

and

$$\frac{d^3\overline{\mathcal{R}}_{\bar{f},\,D\pi(\gamma)}}{d\cos\theta_1 d\cos\theta_2 d\Phi} \equiv \frac{1}{\overline{\Gamma}_{f,\,D\pi(\gamma)}} \frac{d^3\overline{\Gamma}_{\bar{f},\,D\pi(\gamma)}}{d\cos\theta_1 d\cos\theta_2 d\Phi}, \tag{10}$$

because they are free of many theoretical and experimental uncertainties. In Eqs. (9) and (10), we suppose that there is no CP violation in modes (3) and (4). Really, it follows from Eqs. (6) and (8) that the expectable CP violation in those modes is very insignificant, because $r_{Df} r_{B\lambda} \approx 0.01$. The probability of the decay processes (1) and (2), as well as the effects of direct CP violation in decay processes (1)–(4), essentially depends on the magnitudes of the parameters r_{Df} and $r_{B\lambda}$. A detailed discussion concerning the values of those parameters is given in work [17]. Now, we only emphasize that, in this paper, the numerical calculations will be carried out for the final state $f = \mathrm{K}^+\pi^-$, for which $r_{D(\mathrm{K}^+\pi^-)} = 0.060 \pm 0.002$ [18]; at the same time, for $r_{B\lambda}$, along with the expected value $r_{B\lambda} = 0.2$ for each polarization state of vector mesons [19], other values are also examined. In the framework of the following consideration of processes (1)–(4), the small effects of CP violation in decay processes (3) and (4) will also be neglected.

The ratio between the differential probability of decay (1) and the probability of decay (3), expressed in a helicity coordinate system, looks like

$$\frac{d^3\mathcal{R}_{f,\,D\pi}}{d\cos\theta_1 d\cos\theta_2 d\Phi} = \frac{9}{16\pi}\Big(2R_0^\pi \cos^2\theta_1 \cos^2\theta_2 + \Big(R_\parallel^\pi \cos^2\Phi + R_\perp^\pi \sin^2\Phi - \xi_\parallel^\pi \sin 2\Phi\Big)\sin^2\theta_1 \sin^2\theta_2 + \left(\zeta^\pi\cos\Phi - \xi_0^\pi \sin\Phi\right)\sin 2\theta_1 \sin 2\theta_2/\sqrt{2}\Big), \tag{11}$$

and the ratio between the differential probability of decay (2) and the probability of decay (4) takes the form

$$\frac{d^3\mathcal{R}_{f,\,D\gamma}}{d\cos\theta_1 d\cos\theta_2 d\Phi} = \frac{9}{32\pi}\Big(2R_0^\gamma \sin^2\theta_1\cos^2\theta_2 - \Big(R_\parallel^\gamma \times \cos^2\Phi + R_\perp^\gamma \sin^2\Phi - \xi_\parallel^\gamma \sin 2\Phi\Big)\sin^2\theta_1 \sin^2\theta_2 + \Big(R_\parallel^\gamma + R_\perp^\gamma\Big)\sin^2\theta_2 - \frac{\zeta^\gamma\cos\Phi - \xi_0^\gamma \sin\Phi}{\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2\Big), \tag{12}$$

where θ_1 is an angle between the direction of motion of the D meson after the decay $\mathrm{D}^* \to \mathrm{D}\pi$ or $\mathrm{D}^* \to \mathrm{D}\gamma$ and the direction, which is opposite to the direction of motion of the B meson in the rest system of the D^* meson; θ_2 is an angle between the direction of motion of the K meson after the decay $\mathrm{K}^* \to \mathrm{K}\pi$ and the direction, which is opposite to the direction of motion of the B meson in the rest system of the K^* meson; and Φ is an angle between the planes of decays $\mathrm{D}^* \to \mathrm{D}\pi$ (or $\mathrm{D}^* \to \mathrm{D}\gamma$) and $\mathrm{K}^* \to \mathrm{K}\pi$ in the rest system of the B meson. The quantities $R_\lambda^{\pi(\gamma)}$, $\xi_{0,\parallel}^{\pi(\gamma)}$, and $\zeta^{\pi(\gamma)}$ look like

$$R_\lambda^{\pi(\gamma)} = R_{c\lambda}\left(\left(r_{Df} \pm x_\lambda^-\right)^2 + \left(y_\lambda^-\right)^2\right), \tag{13}$$

$$\xi_i^{\pi(\gamma)} \equiv \frac{\Im\left(A_\perp^{f\pi(\gamma)}\left(A_i^{f\pi(\gamma)}\right)^*\right)}{\sum_{\lambda=0,\parallel,\perp}\left|A_\lambda^{\bar{f}\pi(\gamma)}\right|^2},$$

and

$$\zeta^{\pi(\gamma)} \equiv \frac{\Re\left(A_\parallel^{f\pi(\gamma)}\left(A_0^{f\pi(\gamma)}\right)^*\right)}{\sum_{\lambda=0,\parallel,\perp}\left|A_\lambda^{\bar{f}\pi(\gamma)}\right|^2}, \tag{14}$$

where $i = \{0, \parallel\}$; $x_\lambda^\pm$ and $y_\lambda^\pm$ are, respectively, the real and imaginary parts of the complex quantities $\mathsf{z}_\lambda^\pm$; and $R_{c\,0}$, $R_{c\,\parallel}$, and $R_{c\,\perp}$ ($R_{c\lambda} \equiv a_{c\lambda}^2/\sum_{\lambda=0,\parallel,\perp} a_{c\lambda}^2$) are the polarization fractions (longitudinal, parallel, and perpendicular to the transverse one) of the vector meson in decays (3) and (4). Note that the longitudinal polarization fraction of vector mesons has already been measured and amounts to $R_{c0} = 0.86 \pm 0.06 \pm 0.03$ [20].

Ratio (10) looks like expressions (11) or (12), where the substitutions $\Phi \to -\Phi$, $R_\lambda^{\pi(\gamma)} \to \bar{R}_\lambda^{\pi(\gamma)}$, $\xi_i^{\pi(\gamma)} \to \bar{\xi}_i^{\pi(\gamma)}$, and $\zeta^{\pi(\gamma)} \to \bar{\zeta}^{\pi(\gamma)}$ are made. The quantity $\bar{R}_\lambda^{\pi(\gamma)}$ is equal to

$$\overline{R}_\lambda^{\pi(\gamma)} = R_{c\lambda}\left(\left(r_{Df} \pm x_\lambda^+\right)^2 + \left(y_\lambda^+\right)^2\right), \tag{15}$$

and the expressions for $\bar{\xi}_i^{\pi(\gamma)}$ and $\bar{\zeta}^{\pi(\gamma)}$ are identical to expression (14) with the substitutions $A_\lambda^{f\pi(\gamma)} \to \bar{A}_\lambda^{\bar{f}\pi(\gamma)}$ and $A_\lambda^{\bar{f}\pi(\gamma)} \to \bar{A}_\lambda^{f\pi(\gamma)}$, respectively.

The analysis of the total angular distributions of decays (1) and (2) and the corresponding CP-conjugate decays makes it possible to determine the values for the quantities $R_\lambda^{\pi(\gamma)}$, $\bar{R}_\lambda^{\pi(\gamma)}$, $\xi_i^{\pi(\gamma)}$, $\bar{\xi}_i^{\pi(\gamma)}$, $\zeta^{\pi(\gamma)}$, and $\bar{\zeta}^{\pi(\gamma)}$; the same procedure for decays (3) and (4) allows one to find the values of the parameters $R_{c\lambda}$ and $\delta_{c\lambda}$. In such a way, we can study the observables $R_\lambda^{\pi(\gamma)} - \bar{R}_\lambda^{\pi(\gamma)}$ and $\zeta^{\pi(\gamma)} - \bar{\zeta}^{\pi(\gamma)}$, the nonzero values of which would evidence for CP violation, as well as the observable $\xi_i^{\pi(\gamma)} + \bar{\xi}_i^{\pi(\gamma)}$, the nonzero values of which would evidence for both CP and T violation. A capability to observe the CP violation effects in the course of studying the quantities $R_\lambda^{\pi(\gamma)} - \bar{R}_\lambda^{\pi(\gamma)}$ was examined in work [17].

The observable $\xi_i^{\pi(\gamma)}$ is a coefficient of the T-odd triple-product correlation $\vec{q}\cdot(\vec{\epsilon}_1^{\,*} \times \vec{\epsilon}_2^{\,*})$ of the decay $B^- \to D^*K^{*-}$, where $\vec{q}$ is the momentum of one of the vector mesons in the rest frame of the B^- meson, and $\vec{\epsilon}_{1,2}$ are the polarization vectors of D^* and K^* mesons. However, since we consider the ratio of the differential probability of decay (1) to the probability of decay (3) and the ratio of the differential probability of decay (2) to the probability of decay (4), it is the quantities $A_{i\,T}^{\pi(\gamma)}$ and $\bar{A}_{i\,T}^{\pi(\gamma)}$, connected with $\xi_i^{\pi(\gamma)}$ and $\bar{\xi}_i^{\pi(\gamma)}$ by the relations $\xi_i^{\pi(\gamma)} \equiv A_{i\,T}^{\pi(\gamma)}\mathcal{R}_{f,\,D\pi(\gamma)}$ and $\bar{\xi}_i^{\pi(\gamma)} \equiv \bar{A}_{i\,T}^{\pi(\gamma)}\bar{\mathcal{R}}_{\bar{f},\,D\pi(\gamma)}$, that virtually characterize the effect of T violation. From Eqs. (5), (6), and (14), it follows that the observable $\xi_i^{\pi(\gamma)}$ looks like

$$\xi_i^{\pi(\gamma)} = -\sqrt{R_{c\perp}R_{c\,i}}\Bigg(r_{Df}^2\sin(\delta_{c\,i} - \delta_{c\perp}) \pm$$

$$\pm r_{Df}\,r_{B\perp}\sin(\chi_i + \gamma) \pm r_{Df}\,r_{B\,i}\sin(\varphi_i - \gamma) +$$

$$+ r_{B\perp}\,r_{B\,i}\sin(\delta_{u\,i} - \delta_{u\perp})\Bigg), \tag{16}$$

where $\varphi_i \equiv \delta_{ui} - \delta_{c\perp} + \delta_{Df}$, $\chi_i \equiv \delta_{ci} - \delta_{u\perp} - \delta_{Df}$, the upper signs correspond to decay (1), and the lower to decay (2). In order to obtain $\bar{\xi}_i^{\pi(\gamma)}$, we must multiply Eq. (16) by (-1) and substitute γ by $-\gamma$. It is important to emphasize that if the parameters $r_{B\lambda}$ have identical values for all polarization states of vector mesons, i.e. if $r_{B0} = r_{B\parallel} = r_{B\perp}$, the observable $\xi_i^{\pi(\gamma)}$ equals zero at the zero difference between the strong final-state-interaction phases. Therefore, the statement that the effects of direct T violation are maximal in the absence of final-state interaction (see, e.g., work [8]) is not true for this case.

Let us designate the observables that characterize the value of T-odd effects in processes (1) and (2), which arise owing to the final-state interaction of these processes, as $\mathcal{R}_{iT}^\pi$ and $\mathcal{R}_{iT}^\gamma$, respectively. Then

$$\mathcal{R}_{i\,T}^{\pi(\gamma)} \equiv \mp(\xi_i^{\pi(\gamma)} - \overline{\xi}_i^{\pi(\gamma)}) =$$

$$= \pm 2\sqrt{R_{c\perp}R_{c\,i}}\Bigg(r_{Df}^2\,\sin(\delta_{c\,i} - \delta_{c\perp}) + r_{B\perp}\,r_{B\,i}\,\times$$

$$\times\sin(\delta_{u\,i} - \delta_{u\perp}) \pm r_{Df}\,(r_{B\,i}\sin\varphi_i + r_{B\perp}\sin\chi_i)\cos\gamma\Bigg). \tag{17}$$

Really, it follows from Eq. (17) that $\mathcal{R}_{iT}^\pi$ and $\mathcal{R}_{iT}^\gamma$ can differ from zero even if the weak phase γ disappears. Additionally, let us use the notations $\mathcal{A}_{iT}^\pi$ and $\mathcal{A}_{iT}^\gamma$ for the observables that characterize the magnitudes of direct T-violation effects in those decay processes (1) and (2) which originate from the availability of the phase in the CKM quark-mixing matrix, i.e. from CP violation. So, then

$$\mathcal{A}_{i\,T}^{\pi(\gamma)} \equiv \mp(\xi_i^{\pi(\gamma)} + \overline{\xi}_i^{\pi(\gamma)}) =$$

$$= 2\,r_{Df}\sqrt{R_{c\perp}R_{c\,i}}(r_{B\perp}\cos\chi_i - r_{B\,i}\cos\varphi_i)\sin\gamma. \tag{18}$$

It follows from Eq. (18) that the signal of direct T violation would differ from zero only if $\gamma \neq 0$, i.e. owing to CP violation. The upper signs in formulas (17)–(18) corresponds to decay (1), and the lower ones to decay (2). It is remarkable that, as follows from Eq. (18), two asymmetries $\mathcal{A}_{i\,T}^\pi$ and $\mathcal{A}_{i\,T}^\gamma$ are identical by value. Therefore, studying decays (1) and (2) simultaneously provides an opportunity to observe the combined asymmetry $\mathcal{A}_{iT}^\pi + \mathcal{A}_{iT}^\gamma = 4r_{Df}\sqrt{R_{c\perp}R_{ci}}(r_{B\perp}\cos\chi_i - r_{Bi}\cos\varphi_i)\sin\gamma$.

Note that the potential dependence of the parameters $r_{B\lambda}$ on the polarization state of vector mesons substantially affects the amplitude of direct T violation effects (it can be seen from Eq. (18)). If the parameters $r_{B\lambda}$ are identical for all polarization states of vector mesons, i.e. if $r_{B0} = r_{B\|} = r_{B\perp}$, the quantities $\mathcal{A}_{iT}^{\pi(\gamma)}$ are

$$\mathcal{A}_{i\,T}^{\pi(\gamma)} = 4\, r_{Df}\, r_{B\perp} \sqrt{R_{c\perp} R_{c\,i}}\, \sin((\varphi_i + \chi_i)/2)\, \times$$

$$\times \sin((\delta_i + \delta_\perp)/2)\, \sin\gamma, \tag{19}$$

and, due to this circumstance, they can disappear both in the approximation of zero strong-interaction phases and at some relations between the latter, e.g., $\delta_{ci} + \delta_{ui} = \delta_{c\perp} + \delta_{u\perp}$. This feature is of importance, because there exists a widespread statement in the literature that the effects of direct T violation become maximal, provided that there is no difference between strong final-state-interaction phases [4, 8, 9], in contrast to direct CP violation effects. The given example illustrates that this statement is not always correct. On the other hand, if the $r_{B\lambda}$ values differ substantially for different polarization states of vector mesons, then the quantities $\mathcal{A}_{iT}^{\pi(\gamma)}$ look like

$$\mathcal{A}_{i\,T}^{\pi(\gamma)} = 2\, r_{Df}\, (r_{B\perp} - r_{B\,i}) \sqrt{R_{c\perp} R_{c\,i}}\, \sin\gamma \tag{20}$$

in the zero strong-interaction phase approximation.

Thus, the magnitude of the direct T violation effect in decays (1) and (2) substantially depends on the weak phase γ and the strong final-state-interaction phases of those processes, as well as on whether the quantities $r_{B\lambda}$ become identical for all polarization states of vector mesons or their values differ considerably for different polarizations.

The expected amplitude of the asymmetries of triple-product correlations in decays (1) and (2) depends, in general, on the values of $r_{B\lambda}$ and γ, the final-state-interaction phases in the B- and D-meson decay processes, and the polarization fraction of vector mesons in decays (3) and (4). The phases $\delta_{c\lambda}$, $\delta_{u\lambda}$, and δ_{Df}, which arise owing to the hadron final-state interaction, cannot be calculated reliably with the help of the known methods, so that they should be determined experimentally. Therefore, carrying out our simplified calculations, we shall proceed from an arbitrary choice of those phases. From Eq. (19) and experimental data [20], it follows that

$$|\mathcal{A}_{0\,T}^{\pi(\gamma)}| \leq 1.39\, r_{D\,f}\, r_{B\,\perp}\, |\sin\gamma|.$$

This means that $\left|\mathcal{A}_{0T}^{\pi(\gamma)}\right| \leq 0.013$ for the mode $f = \mathrm{K}^+\pi^-$ and if $r_{B\perp} = 0.18$ and $\gamma = 62°$ [21]. However, this small value does not indicate that the effects of T violation in decays (1) and (2) are also small, because, as was mentioned above, the parameters, e.g., $\kappa_{iT}^{\pi(\gamma)}$ and κ_{iT} which are connected with the parameters $\mathcal{A}_{iT}^{\pi(\gamma)}$ by the relations

$$\mathcal{A}_{i\,T}^{\pi(\gamma)} \equiv \kappa_{i\,T}^{\pi(\gamma)}\, \mathcal{R}^{\pi(\gamma)}$$

and

$$\mathcal{A}_{i\,T}^{\pi(\gamma)} \equiv \kappa_{i\,T}\, (\mathcal{R}^{\pi} + \mathcal{R}^{\gamma})/2\,,$$

where

$$\mathcal{R}^{\pi(\gamma)} \equiv \frac{\mathcal{R}_{f,\,D\pi(\gamma)} + \overline{\mathcal{R}}_{\bar f,\,D\pi(\gamma)}}{2} =$$

$$= r_{Df}^2 + \sum_{\lambda=0,\|,\perp} R_{c\lambda} r_{B\lambda} (r_{B\lambda} \pm 2\, r_{D\,f} \cos\delta_\lambda \cos\gamma),$$

would better testify to the amplitude of T-violation effects. Then, from Eq. (19) and experimental data [20], it follows that

$$|\kappa_{0\,T}| \leq 1.39\, r_{D\,f}\, r_{B\,\perp}\, |\sin\gamma| / (r_{D\,f}^2 + r_{B\,\perp}^2).$$

In its turn, for the mode $f = \mathrm{K}^+\pi^-$ and provided that $r_{B\perp} = 0.18$ and $\gamma = 62°$ [21], this inequality means that $|\kappa_{0T}| \leq 0.37$ for each channel of the D^{*0}-meson decay. Thus, while considering two channels of the D^{*0}-meson decay into the $\mathrm{D}^0\pi^0$ and $\mathrm{D}^0\gamma$ pairs, we may expect for a considerable effect of T violation in the decay processes $\mathrm{B}^{\pm} \to \mathrm{D}^*\mathrm{K}^{*\pm}$.

The problem of extracting T-violation effects in the processes of B-meson decay into the pair of vector mesons is known to be difficult [8], because, along with the effects of direct T violation which arise owing to the presence of the weak phase γ, there also exist T-odd effects which are caused by the final-state interaction in these processes. We emphasize that the combined study of the decay modes $\mathrm{D}^{*0} \to \mathrm{D}^0\pi^0$ and $\mathrm{D}^{*0} \to \mathrm{D}^0\gamma$ allows those T-odd effects to be extracted from decays (1) and (2). Really, it follows from Eq. (17) that

$$\Delta\mathcal{R}_{i\,T} \equiv \mathcal{R}_{i\,T}^{\pi} - \mathcal{R}_{i\,T}^{\gamma} = 4\sqrt{R_{c\perp} R_{c\,i}} \Big(r_{Df}^2\, \times$$

$$\times \sin(\delta_{c\,i} - \delta_{c\perp}) + r_{B\perp} r_{B\,i} \sin(\delta_{u\,i} - \delta_{u\perp}) \Big) \tag{21}$$

and

$$\mathcal{R}^{\pi}_{i\,T}+\mathcal{R}^{\gamma}_{i\,T}=4\,r_{Df}\sqrt{R_{c\perp}R_{c\,i}}\Big(r_{B\,i}\sin\varphi_i+$$

$$+r_{B\perp}\sin\chi_i\Big)\cos\gamma. \qquad (22)$$

The quantity $\Delta\mathcal{R}_{iT}$ describes the contribution made by the mechanisms of transitions b $\rightarrow$ c and $b\rightarrow$ u into T-odd effects, while sum (22) describes the contribution from the interference of those mechanisms to the same effects. According to Eq. (21), the measurement of the quantity $\Delta\mathcal{R}_{iT}$ enables one to determine the phase difference $\delta_{ui}-\delta_{u\perp}$; therefore, by measuring the value of $R^{\pi(\gamma)}_{\lambda}-\bar{R}^{\pi(\gamma)}_{\lambda}$ [17], we can also obtain the values for the phases φ_i and χ_i. Moreover, the measurement of the quantity $\mathcal{R}^{\pi}_{iT}+\mathcal{R}^{\gamma}_{iT}$ allows one to determine the phase γ (with the help of Eq. (22)).

To study the effects of direct T violation in decays (1) and (2), we may also use – along with the quantities $\mathcal{A}^{\pi}_{iT}$ and $\mathcal{A}^{\gamma}_{iT}$ – other parameters, e.g., $Q^{\pi(\gamma)}_{iT}$ which is a ratio between the quantity describing the effects of direct T violation and the quantity describing T-odd effects caused by the strong final-state interaction in those processes:

$$Q^{\pi(\gamma)}_{i\,T}\equiv\frac{W^{\pi(\gamma)}_{i\,(-)}}{W^{\pi(\gamma)}_{i\,(+)}}, \qquad (23)$$

$$W^{\pi(\gamma)}_{0\,(\pm)}\equiv(\int\limits_0^{\pi}d\Phi-\int\limits_{\pi}^{2\pi}d\Phi)\int\limits_D d\cos\theta_1\times$$

$$\times\int\limits_D d\cos\theta_2\,\mathcal{W}^{\pi(\gamma)}_{\pm}(\theta_1,\theta_2,\Phi),$$

$$W^{\pi(\gamma)}_{\|\,(\pm)}\equiv(\int\limits_0^{\frac{\pi}{2}}-\int\limits_{\pi/2}^{\pi}+\int\limits_{\pi}^{3\pi/2}-\int\limits_{3\pi/2}^{2\pi})d\Phi\int\limits_S d\cos\theta_1\times$$

$$\times\int\limits_S d\cos\theta_2\,\mathcal{W}^{\pi(\gamma)}_{\pm}(\theta_1,\theta_2,\Phi),$$

$$\int\limits_{D(S)}f(\theta)\,d\cos\theta\equiv\int\limits_{-1}^{0}f(\theta)\,d\cos\theta\mp\int\limits_{0}^{1}f(\theta)\,d\cos\theta,$$

$$\mathcal{W}^{\pi(\gamma)}_{\pm}(\theta_1,\theta_2,\Phi)\equiv\frac{d^3\mathcal{R}_{f,\,D\pi(\gamma)}}{d\cos\theta_1 d\cos\theta_2 d\Phi}\pm$$

$$\pm\frac{d^3\overline{\mathcal{R}}_{\bar f,\,D\pi(\gamma)}}{d\cos\theta_1 d\cos\theta_2 d\Phi}.$$

Integrating over the angular variables and applying Eq. (16), we obtain an equivalent expression for the quantity $Q^{\pi(\gamma)}_{iT}$, namely,

$$Q^{\pi(\gamma)}_{i\,T}=\frac{\mathcal{A}^{\pi(\gamma)}_{i\,T}}{\mathcal{R}^{\pi(\gamma)}_{i\,T}}=\pm(r_{f\perp}\cos\chi_i-r_{f\,i}\cos\varphi_i)\sin\gamma\times$$

$$\times\Big(\sin(\delta_{c\,i}-\delta_{c\perp})+r_{f\perp}r_{f\,i}\sin(\delta_{u\,i}-\delta_{u\perp})\pm$$

$$\pm(r_{f\,i}\sin\varphi_i+r_{f\perp}\sin\chi_i)\cos\gamma\Big)^{-1}, \qquad (24)$$

where $r_{f\lambda}\equiv r_{B\lambda}/r_{D\,f}$, the upper signs correspond to decay (1), and the lower one to decay (2).

It is clear that the exact values for expression (24), as well as for (17) and (18), can be obtained only after the strong-interaction phases $\delta_{c\lambda}$, $\delta_{u\lambda}$, and δ_{Df}, the weak phase γ, and the parameters $r_{f\lambda}$ have been determined. We estimate the quantities $Q^{\pi(\gamma)}_{i\,T}$ for two possible relations between the strong-interaction phases and the parameters $r_{f\lambda}$. They can be fulfilled for both the longitudinal and parallel polarizations of vector mesons. For example, if $\delta_{\|}\simeq\delta_{\perp}$ and $r_{f\|}\simeq r_{f\perp}$, then the quantities $Q^{\pi(\gamma)}_{i\,T}$ are determined by the formula

$$Q^{\pi(\gamma)}_{\|\,T}=\frac{\pm\rho_{f\|}\sin\delta_{\|}\sin\gamma}{1\pm\rho_{f\|}\cos\delta_{\|}\cos\gamma}, \qquad (25)$$

where $\rho_{f\lambda}\equiv 2r_{f\lambda}/(1+r^2_{f\lambda})$. The estimations of the quantity $\Delta Q_{\|T}\equiv Q^{\pi}_{\|T}-Q^{\gamma}_{\|T}$, which were obtained from Eq. (25) for $f=K^+\pi^-$, $\gamma=62°$, and various values of the parameters $\delta_{\|}$ and $r_{B\|}$, are quoted in Table 1. The results presented in this table testify that the difference $Q^{\pi}_{\|T}-Q^{\gamma}_{\|T}$ is large in the whole ranges of variation that were considered for the parameters $\delta_{\|}$ and $r_{B\|}$; hence, it can be observed experimentally. Note that if the parameter $r_{B\|}$ grows, the value of $\Delta Q_{\|T}$ falls down for every fixed value of the strong-interaction phase $\delta_{\|}$.

T a b l e 1. Values of the quantity $\Delta Q_{\|T}$ at $\gamma=62°$

$\delta_{\|}$, degree	$r_{B\,\|}=0.06$	$r_{B\,\|}=0.12$	$r_{B\,\|}=0.18$	$r_{B\,\|}=0.24$
10	0.39	0.28	0.20	0.15
30	1.06	0.79	0.56	0.43
60	1.62	1.27	0.94	0.73
90	1.77	1.41	1.06	0.83

For the other variant of the relation between the strong-interaction phases, namely, $\delta_{c0} = \delta_{u0} = \delta_{Df} = 0$ and $\delta_{c\perp} \simeq \delta_{u\perp}$, the quantities $Q_{0T}^{\pi(\gamma)}$ are determined by the formula

$$Q_{0\,T}^{\pi(\gamma)} = \frac{\pm(r_{f0} - r_{f\perp})\cot\delta_{c\perp}\sin\gamma}{1 + r_{f0}r_{f\perp} \pm (r_{f0} + r_{f\perp})\cos\gamma}. \qquad (26)$$

In this case, the values of $Q_{0T}^{\pi(\gamma)}$ depend critically on the strong-interaction phase $\delta_{c\perp}$ and the difference between the parameters r_{B0} and $r_{B\perp}$. The estimations of the difference $\Delta Q_{0T} \equiv Q_{0T}^{\pi} - Q_{0T}^{\gamma}$, which were obtained from Eq. (26) for $f = K^+\pi^-$, $\gamma = 62°$, various values of the parameters $\delta_{c\perp}$ and r_{B0}, and $r_{B\perp} = 0.09$ or 0.21, are quoted in Table 2. The results presented in this table testify that the difference $Q_{0T}^{\pi} - Q_{0T}^{\gamma}$ can be both positive (if $r_{B0} > r_{B\perp}$) and negative (if $r_{B0} < r_{B\perp}$); its values lie within a wide range (from −7.14 to 4.14 in the considered ranges of variation of the parameters $\delta_{c\perp}$, r_{B0}, and $r_{B\perp}$). Moreover, since T-odd effects are proportional to sin $\delta_{c\perp}$ and the effects of direct T violation to cos $\delta_{c\perp}$, the absolute value of the quantity ΔQ_{0T} becomes smaller when $\delta_{c\perp}$ changes from 10° to 80°.

Thus, in the case of the latter relation between the strong-interaction phases, the possibility of experimental observation of the quantities Q_{0T}^{π} and Q_{0T}^{γ} will be determined, first of all, by the amplitudes of the parameters r_{B0} and $r_{B\perp}$, as well as by the value of the strong-interaction phase $\delta_{c\perp}$.

3. Conclusions

The effects of T violation in the processes of $B^{\pm}$-meson decay into the pair of linearly polarized vector mesons D^* and $K^{*\pm}$ have been considered. For studying the triple-product correlations in those processes, we propose to use two modes of the vector D^{*0}-meson decay, namely, $D^{*0} \to D^0\pi^0$ and $D^{*0} \to D^0\gamma$, followed by the transition of the D^0 meson into doubly Cabibbo-suppressed states. The combined analysis of those two modes of the D^{*0}-meson decay increases the number of useful events and, owing to a tiny phase difference between those two modes, allows the contribution of the transition $b \to c$ to T-odd triple-product correlations to be separated from that of the transition $b \to u$. We found that the amplitude of the direct T violation effects depends substantially not only on the weak phase γ and the strong final-state-interaction phases of these processes, but also on whether the ratios between the decay amplitudes of $B^-\to\bar{D}^{*0}K^{*-}$ and $B^-\to D^{*0}K^{*-}$ are identical for all polarization states of the vector mesons or their values differ substantially for different polarizations. We have demonstrated that, in the framework of the standard model, a large effect of T-violation (of about 30% at $\gamma = 62°$) is possible.

T a b l e 2. The values of the quantity ΔQ_{0T} at $\gamma = 62°$ and $r_{B\perp} = 0.09(0.21)$

$\delta_{c\perp}$, deg.	$r_{B\,0} = 0{,}06$	$r_{B\,0} = 0.12$	$r_{B\,0} = 0.18$	$r_{B\,0} = 0.24$
10	−2.57 (−7.14)	1.51 (−2.10)	3.20 (−0.47)	4.14 (0.35)
30	−0.78 (−2.18)	0.46 (−0.64)	0.98 (−0.14)	1.26 (0.11)
60	−0.26 (−0.73)	0.15 (−0.21)	0.33 (−0.05)	0.42 (0.04)
80	−0.08 (−0,22)	0.05 (−0.07)	0.10 (−0.01)	0.13 (0.01)

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ПОТРІЙНІ КОРЕЛЯЦІЇ У ПРОЦЕСАХ РОЗПАДУ $B^{\pm}$-МЕЗОНІВ НА ПАРУ ВЕКТОРНИХ D^*- ТА $K^{*\pm}$-МЕЗОНІВ

В.А. Ковальчук

Р е з ю м е

Ми дослідили потрійні кореляції, які порушують T-симетрію, у процесах $B^{\pm} \to D^* K^{*\pm}$, $D^* \to D\,\pi^0$, $D\,\gamma$, $D \to f$, де нейтральний $D(D^*)$-мезон є суперпозицією $D^0(D^{*0})$ та $\overline{D}^{\,0}(\overline{D}^{\,*0})$. Показано, що у рамках стандартної моделі можливі великі асиметрії, що порушують T-симетрію ($\sim$30% для слабкої фази $\gamma = 62°$) для кінцевих адронних станів f, таких, що $D^0 \to f$ є подвійно пригніченою модою Кабіббо, тоді як $\overline{D}^{\,0} \to f$ дозволена мода Кабіббо.