INFRARED BEHAVIOR OF GLUON AND GHOST PROPAGATORS IN QCD IN THE LANDAU GAUGE

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A non-perturbative formalism of the generalized effective action is used for deriving the Schwinger–Dyson equations. In order to clear the domain of integration in the functional integral from gauge copies, a restriction to the Gribov horizon due to Zwanziger is implemented. In this approach, the asymptotic behavior of a gluon propagator and the propagator of Faddeev–Popov ghosts at small momenta is studied. Such a behavior is obtained as a result of solving the coupled Schwinger–Dyson equations in the zeroth and first-order approximations. The qualitative agreement of these results with the ones obtained before is demonstrated, and the quantitative difference in some coefficients is found.

1. Introduction

It is widely believed that Quantum Chromodynamics (QCD) is the theory which describes the strong interactions. Physical phenomena at large momenta transferred are very well described by perturbation theory as the coupling becomes small. Asymptotic freedom allows to picture high energy quarks and gluons as weakly interacting particles. This picture, however, starts to break down at intermediate momenta and is surely inadequate at energies below a few hundred MeV. At such scales, the interaction is strong enough to invalidate perturbation theory, and one has to employ completely different methods to study non-perturbative phenomena like confinement or chiral symmetry breaking. One such method is the study of Schwinger–Dyson equations (SDEs).

The Schwinger–Dyson equations are the equations of motion for Green's functions of field theory. In QCD, the study of the non-perturbative behavior of these functions is of interest for several reasons. One of such reasons is that the confinement mechanism like the Gribov–Zwanziger scenario [1, 2, 14] is related to the infrared behavior of the ghost and gluon propagators.

Many attempts have been made to understand the gluon propagator behavior through SDEs. In the late 1970s, Mandelstam initiated the study of a gluon SDE in the Landau gauge [3]. Neglecting the ghost fields contribution and imposing the cancelations of certain terms in the gluon polarization tensor, he found a highly singular gluon propagator in the infrared region. This enhanced gluon propagator was appraised for many years in the literature, firstly because it provided a simple picture of the quark confinement, since it is possible to derive an inter-quark potential from it that rises linearly with the separation, and, second, because the gluon propagator, which is singular as $1/q^4$, has enough strength to support the dynamical chiral symmetry breaking. However, these results are discarded by simulations of QCD on the lattice [5], where it is shown that the gluon propagator is probably infrared finite.

Later, infrared finite solutions were also found in the Schwinger–Dyson approach [4]. Considering coupled gluon and ghost SDEs, it was shown that the gluon propagator is suppressed and the ghost propagator is enhanced in the infrared region. The brief overview of the results on infrared properties of the gluon and ghost propagators from SDEs existing in today's literature is made in Section 2.

In the main part of this work (Section 3), the formalism of the generalized effective action proposed by Cornwall, Jackiw, and Tomboulis (CJT) [6] is investigated. The effective action which depends on the gluon and ghost propagators and the Gribov mass parameter is obtained. In Section 4, we will construct SDEs for the gluon and ghost propagators, as well as for the Gribov parameter in the one-loop approximation. Notice that the SDEs obtained in our work are not exactly the same ones discussed in the literature [7–10]. Let us clarify this point.

The starting point in most papers is a full system of SDEs. It is a coupled nonlinear system of equations which contains dressed propagators, as well as dressed vertex functions, on the right-hand side. In order to obtain a closed system of equations, it is necessary to specify suitable approximations for these equations. There are many truncation schemes developed for this purpose, for example those replacing dressed vertices by their tree-level expressions. Furthermore, in this formulation, the Gribov's prescription of cutting off the functional integral at the Gribov horizon does not change the Schwinger–Dyson equations, but rather resolves an ambiguity in the solution of these equations.

On the other hand, we restrict ourselves in our presentation to the two-loop approximation of the effective action and obtain a closed system of SDEs without any other truncation schemes. These three equations are derived as a result of nullifying the functional derivatives of the generalized effective action with respect to the gluon and ghost propagators and the Gribov parameter. The restriction to the (first) Gribov region appears explicitly via the modification of the free gluon propagator, which enters the gluon SDE explicitly. Therefore, two other SDEs are affected via the dressed gluon propagator as well.

In Section 5, the zeroth-order analysis of the derived SDEs is carried out. It is noteworthy that, in this order, we get the results for a gluon propagator and the horizon condition similar to those previously obtained by Gribov. It is shown that the ghost propagator obtained in this order and Gribov's one indicate the same qualitative behavior $1/q^4$ in the infrared region. However, they differ in some coefficient.

The full analysis of the derived one-loop SDEs is developed in Section 6. The infrared critical exponents are obtained, as well as the numerical coefficients for the propagators. In the last Section (7), the discussion of the obtained results is presented.

2. Nowadays State of Affairs

The infrared behavior of the Landau gauge gluon and ghost propagators is an interesting and hot subject. Up to now, there are two competitive viewpoints on this problem.

Let us represent the gluon and ghost propagators, respectively, in the following form:

$$G^{ab}_{\mu\nu}(q) = C_G \frac{\widetilde{G}(q^2)}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\delta^{ab}, \tag{1}$$

$$\mathcal{D}^{ab}(q) = C_{\mathcal{D}} \frac{\widetilde{\mathcal{D}}(q^2)}{q^2}\,\delta^{ab}. \tag{2}$$

Here, C_G and $C_{\mathcal{D}}$ are dimensionless constants. We will seek for the gluon and ghost dressing functions in the form of simple power laws

$$\widetilde{G}(q^2) \propto \left(q^2\right)^{-\alpha_G} \tag{3}$$

and

$$\widetilde{\mathcal{D}}(q^2) \propto \left(q^2\right)^{-\alpha_{\mathcal{D}}}, \tag{4}$$

respectively. The coefficients α_G and $\alpha_{\mathcal{D}}$ are called the infrared critical exponents or anomalous dimensions.

The first viewpoint is as follows. Using the gluon propagator SDE, as well as the ghost propagator SDE, it is claimed that $\alpha_G + 2\alpha_{\mathcal{D}} = -(4-d)/2$, [7,8], i.e., for the space-time dimension $d = 4$, one gets $\alpha_G + 2\alpha_{\mathcal{D}} = 0$. The interesting point here is that self-consistency forces an interrelation of the exponents such that they depend on one parameter $k = \alpha_{\mathcal{D}} = -\alpha_G/2$ only. The value of the exponent k is in the range $0.5 < k \le 1$, [7–10], depending on details of the truncation of the set of SDEs.

Some of the authors report on the ambiguous result [7]:

$$\alpha_G = -2,\ \alpha_{\mathcal{D}} = 1 \text{ and } \alpha_G \approx -1.1906,\ \alpha_{\mathcal{D}} \approx 0.5953. \tag{5}$$

It is worth noting that the Kugo—Ojima confinement criterion [11] in terms of infrared exponents requires $k > 0$, which means that the ghost propagator should be more singular and the gluon less singular than a simple pole. On the other hand, Gribov–Zwanziger confinement scenario [1, 12–14] requires the same condition for the ghost propagator, but it needs $k > 0.5$ for the gluon dressing function. Thus, the available results for the exponents are in good agreement with these two widespread confinement scenarios.

However, there is another point of view on this problem. Some of the authors [15] report that $\alpha_G + 2\alpha_{\mathcal{D}} \neq 0$, and the correct value of $\alpha_{\mathcal{D}}$ is close to zero, while α_G is close to 1. Thus, they claim that the ghost dressing function is finite and differs from zero in the infrared limit.

The recent numerical results, i.e. lattice simulations, also are not unambiguous [16].

The aim of the present paper is independently determine the infrared critical exponents.

3. Cornwall–Jackiw–Tomboulis Formalism

We start to consider this problem with making use of the formalism of the generalized effective action proposed by J. Cornwall, R. Jackiw, and E. Tomboulis [6]. This generalization of the effective action, $\Gamma(\phi, G)$, depends not only on ϕ – a possible expectation value of the quantum field $\Phi(x)$ – but also on $G(x,y)$ – a possible expectation value of a bilocal operator $T\Phi(x)\Phi(y)$ – time ordered product of two fields. The physical solutions require

$$\frac{\delta\Gamma(\phi,G)}{\delta\phi(x)} = 0, \qquad \frac{\delta\Gamma(\phi,G)}{\delta G(x,y)} = 0. \tag{6}$$

In our derivation, we identify $\Phi(x)$ with the gluon field, so $G(x,y)$ is the gluon propagator. We introduce two more quantities – the ghost propagator $\mathcal{D}(x,y)$ and the so-called Gribov parameter γ. Parameter γ, known also as the Gribov mass, characterizes the restriction of the domain of integration in the functional integral to the so-called Gribov horizon. This restriction is necessary due to the existence of the Gribov copies which imply that the Landau condition, $\partial_\mu A_\mu = 0$, does not uniquely fix the gauge, and the equivalent gauge copies still exist in the domain of integration of the functional integral [1, 18].

We start from the generating functional for Green's functions of nonlocal composite fields:

$$Z(J,K) = e^{\frac{i}{\hbar}W(J,K)} = \int D\Phi e^{\frac{i}{\hbar}S_{\rm eff}}, \tag{7}$$

$$S_{\rm eff} = -\frac{1}{4}\int d^4x \left(F^a_{\mu\nu}(x)\right)^2 + \int d^4x\, d^4y \overline{C}^a(x) J^{ab}(x,y) C^b(y) +$$

$$+\frac{1}{2}\int d^4x\, d^4y A^a_\mu(x) K^{ab}_{\mu\nu}(x,y) A^b_\nu(y) + S_z(\Phi(x)), \tag{8}$$

where

$$S_z(\Phi(x)) = \int d^4x \{g\gamma^2 f^{abc}(A^a_\mu \varphi^{bc}_\mu + A^a_\mu \overline{\varphi}^{bc}_\mu) - \lambda^a(\partial_\mu A^a_\mu) -$$

$$-\overline{C}^a \partial_\mu (D_\mu C)^a - \overline{\varphi}^{ac}_\mu \partial_\nu (D_\nu \varphi_\mu)^{ac} + \overline{\omega}^{ac}_\mu \partial_\nu (D_\nu \omega_\mu)^{ac} +$$

$$+g(\partial_\nu \overline{\omega}^{ac}_\mu) f^{abm} (D_\nu C)^b \varphi^{mc}_\mu + 4\gamma^4(N^2-1)\} \tag{9}$$

is the expression which characterizes the Zwanziger's formulation of the Gribov horizon [2] (with g being the strong coupling constant). It is BRST-invariant together with the first term of (8). The functional differential $D\Phi$ labels the product of differentials of all fields entering the integrand (effective action (8)):

$$D\Phi \equiv dA dC d\overline{C} d\lambda d\varphi d\overline{\varphi} d\omega d\overline{\omega}. \tag{10}$$

Let us clarify the notations above. Here, A is the gluon field with components A^a_μ. Fields C and $\overline{C}$ are the Faddeev–Popov ghosts which have components C^a and $\overline{C}^a$. Multiplier λ^a enforces the constraint $\partial_\mu A_\mu = 0$ characteristic of the Landau gauge. The fields $\varphi \equiv -\frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$ and $\overline{\varphi} \equiv \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)$ are the pair of boson complex fields with components $\varphi^a_i \equiv \varphi^{ac}_\mu \equiv f^{abc}\varphi^b_\mu$, and f^{abc} are the structure constants of the SU(N) gauge group. Here, we use the single index $i \equiv (\mu, c)$ for the pair of mute indices, and i takes on $f = d(N^2-1)$ values ($d = 4$ stands for the dimension of space). The fields ω and $\overline{\omega}$ are the Grassmann fields which have the same components as φ and $\overline{\varphi}$. The vector indices μ, ν take on d values, while the color indices a, b refer to the adjoint representation of the SU(N) group and take on $N^2 - 1$ values.

We introduced two bilocal sources $K^{ab}_{\mu\nu}(x,y)$ and $J^{ab}(x,y)$ for the gluon field A^a_μ and for the ghosts $\overline{C}^a$ and C^a, respectively.

Let $G^{ab}_{\mu\nu}(x,y)$ be a possible expectation value of $TA^a_\mu(x)A^b_\nu(y)$, and let $\mathcal{D}^{ab}(x,y)$ stand for a possible expectation value of $TC^a(x)\overline{C}^b(y)$:

$$G^{ab}_{\mu\nu}(x,y) \equiv \left\langle TA^a_\mu(x)A^b_\nu(y)\right\rangle, \tag{11}$$

$$\mathcal{D}^{ab}(x,y) \equiv \left\langle TC^a(x)\overline{C}^b(y)\right\rangle. \tag{12}$$

It follows from Eqs. (7) and (8) that

$$\frac{\delta W(J,K)}{\delta K^{ab}_{\mu\nu}(x,y)} = \frac{1}{2}\hbar\, G^{ab}_{\mu\nu}(x,y), \tag{13}$$

$$\frac{\delta W(J,K)}{\delta J^{ab}(x,y)} = -\hbar\, \mathcal{D}^{ba}(y,x). \tag{14}$$

The effective action $\Gamma(G,\mathcal{D})$ is defined as a Legendre transform of W(J,K):

$$\Gamma(G,\mathcal{D}) = W(J,K) - \frac{\hbar}{2}\int d^4x\, d^4y G^{ab}_{\mu\nu}(x,y) K^{ab}_{\mu\nu}(x,y) +$$

$$+\hbar\int d^4x\, d^4y \mathcal{D}^{ba}(y,x) J^{ab}(x,y). \tag{15}$$

We eliminate K and J in favor of G and $\mathcal{D}$ using the next obvious relations:

$$K^{ab}_{\mu\nu}(x,y) = -\frac{2}{\hbar}\frac{\delta\Gamma(G,\mathcal{D})}{\delta G^{ab}_{\mu\nu}(x,y)} \tag{16}$$

and

$$J^{ab}(x,y) = \frac{1}{\hbar}\frac{\delta\Gamma(G,\mathcal{D})}{\delta\mathcal{D}^{ba}(y,x)}. \tag{17}$$

Then, according to (15), we have

$$\exp\frac{i}{\hbar}\Gamma(G,\mathcal{D}) = \exp\frac{i}{\hbar}W(J,K) \times$$

$$\times\exp\{-\frac{i}{2}\int d^4x\, d^4y G^{ab}_{\mu\nu}(x,y) K^{ab}_{\mu\nu}(x,y) +$$

$$+i\int d^4x\, d^4y \mathcal{D}^{ba}(y,x) J^{ab}(x,y)\}. \tag{18}$$

In order to shorten our notations, we will omit the dependence on coordinates and the corresponding integration, where it is obvious, by keeping it in mind. Taking into account (7), (8) together with (16) and (17), we have

$$\exp\frac{i}{\hbar}\Gamma(G,\mathcal{D})=\int D\Phi\exp\frac{i}{\hbar}\{-\frac{1}{4}\left(F^a_{\mu\nu}\right)^2-\frac{1}{\hbar}A^a_\mu\frac{\delta\Gamma}{\delta G^{ab}_{\mu\nu}}A^b_\nu+$$

$$+\frac{1}{\hbar}\overline{C}^a\frac{\delta\Gamma}{\delta\mathcal{D}^{ba}}C^b+G^{ab}_{\mu\nu}\frac{\delta\Gamma}{\delta G^{ab}_{\mu\nu}}+\mathcal{D}^{ba}\frac{\delta\Gamma}{\delta\mathcal{D}^{ba}}+S_z(\Phi)\}. \quad (19)$$

After straightforward computations, we can present the first term in the exponent in the last expression as

$$-\frac{1}{4}\left(F^a_{\mu\nu}\right)^2=\frac{1}{2}A^a_\nu(iD_F^{-1})^{ab}_{\mu\nu}A^b_\mu-gf^{abc}\left((\partial_\mu A^a_\nu)A^b_\mu A^c_\nu\right)-$$

$$-\frac{1}{4}g^2f^{abc}f^{apr}A^b_\mu A^c_\nu A^p_\mu A^r_\nu. \quad (20)$$

Here, D_F is a free gluon propagator:

$$(iD_F^{-1})^{ab}_{\mu\nu}\equiv\left(\partial^2g_{\mu\nu}-\partial_\mu\partial_\nu\right)\delta^{ab}. \quad (21)$$

The series expansion for $\Gamma(G,\mathcal{D})$ is

$$\Gamma(G,\mathcal{D})=\frac{1}{2}i\hbar Tr\ln(G^{-1})^{ab}_{\mu\nu}+\frac{1}{2}i\hbar Tr\left[(\widetilde{D}_F^{-1})^{ab}_{\mu\nu}G^{ab}_{\mu\nu}\right]+$$

$$+\Gamma_2(G)-i\hbar Tr\ln(\mathcal{D}^{-1})^{ab}-i\hbar Tr\left[(S_F^{-1})^{ab}\mathcal{D}^{ba}\right]+$$

$$+\Gamma_2(\mathcal{D})+\text{const}, \quad (22)$$

where S_F is a free ghost propagator, and $\widetilde{D}_F$ is a free gluon propagator modified with respect to the Zwanziger term:

$$\widetilde{D}_F^{-1}=D_F^{-1}+2g^2\gamma^4N(i\partial^2)^{-1}. \quad (23)$$

The quantity Γ_2 is given by all two-particle irreducible vacuum graphs and is of order $\hbar^2$, because the number of loops corresponds to powers of $\hbar$. The quantities $\Gamma_2(G)$ and $\Gamma_2(\mathcal{D})$ are determined by the expansion in powers of $\hbar$:

$$\Gamma_2(G)=\hbar^2\Gamma_2^{(1)}(G)+O(\hbar^3), \quad (24)$$

$$\Gamma_2(\mathcal{D})=\hbar^2\Gamma_2^{(1)}(\mathcal{D})+O(\hbar^3). \quad (25)$$

Let us introduce the notation

$$\Gamma'(G,\mathcal{D})\equiv-i\hbar Tr\ln(\mathcal{D}^{-1})^{ab}+\hbar^2\Gamma_2^{(1)}(\mathcal{D})-\hbar^2\mathcal{D}^{ba}\frac{\delta\Gamma_2^{(1)}(\mathcal{D})}{\delta\mathcal{D}^{ba}}+$$

$$+\frac{1}{2}i\hbar Tr\ln(G^{-1})^{ab}+\hbar^2\Gamma_2^{(1)}(G)-\hbar^2G^{ab}_{\mu\nu}\frac{\delta\Gamma_2^{(1)}(G)}{\delta G^{ab}_{\mu\nu}}, \quad (26)$$

Apart from the constant $\frac{i}{2}\hbar Tr1$, expressions (19) and (20) and the expansions (22), (24), and (25) yield

$$\Gamma'(G,\mathcal{D})=-i\hbar\ln\int D\Phi\exp\frac{i}{\hbar}\{4\gamma^4(N^2-1)V+$$

$$+\frac{i}{2}A^a_\nu(G^{-1}-2g^2\gamma^4N(i\partial^2)^{-1})^{ab}_{\mu\nu}A^b_\mu+i\overline{C}^a(\mathcal{D}^{-1}-$$

$$-S_F^{-1})^{ab}C^b+g\gamma^2f^{abc}(A^a_\mu\varphi^{bc}_\mu+A^a_\mu\overline{\varphi}^{bc}_\mu)-\lambda^a(\partial_\mu A^a_\mu)+$$

$$+\overline{\Theta}_i\partial^2\Theta_i-gf^{abc}(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu-gf^{acb}\overline{C}^a\partial_\mu A^c_\mu C^b+$$

$$+gf^{acb}\overline{\omega}^a_i\partial_\mu A^c_\mu\omega^b_i-gf^{acb}\overline{\varphi}^a_i\partial_\mu A^c_\mu\varphi^b_i+g(\partial_\nu\overline{\omega}^{ac}_\mu)f^{abm}\times$$

$$\times(\partial_\nu C)^b\varphi^{mc}_\mu-\frac{1}{4}g^2f^{abc}f^{apr}A^b_\mu A^c_\nu A^p_\mu A^r_\nu-\hbar A^a_\mu\frac{\delta\Gamma_2^{(1)}(G)}{\delta G^{ab}_{\mu\nu}}A^b_\nu+$$

$$+\hbar\overline{C}^a\frac{\delta\Gamma_2^{(1)}(\mathcal{D})}{\delta\mathcal{D}^{ba}}C^b+g^2f^{abm}(\partial_\nu\overline{\omega}^{ac}_\mu)A^\lambda_\nu C^{\lambda b}\varphi^{mc}_\mu. \quad (27)$$

Here, V is the space-time volume of the system, and we denote the sum of three terms of a similar form by means of one term:

$$\overline{\Theta}_i\partial^2\Theta_i\equiv\left(-\overline{C}^a\partial^2C^a-\overline{\varphi}^{ac}_\mu\partial^2\varphi^{ac}_\mu+\overline{\omega}^{ac}_\mu\partial^2\omega^{ac}_\mu\right). \quad (28)$$

Thus, we have the expression (27) for the effective action, and we have to evaluate the functional integral. But this expression includes not only terms quadratic on the fields we are able to integrate, but also cubic ones and terms of the fourth order in fields. In order to compute it, one has to expand the cubic and higher-order terms in powers of some small parameter in usual way. Following [6], we will use $\hbar$ (to be precise, we use not $\hbar$, but $\sqrt{\hbar}$) as such a small parameter, and this procedure is known as the loop-expansion. We will retain only terms up to $\hbar=\left(\sqrt{\hbar}\right)^2$, this corresponds to the two-loop approximation.

In order to select terms by powers $\hbar$, one has to change the scale of all 8 fields:

$$\Phi\left(x\right)\rightarrow\sqrt{\hbar}\,\Phi\left(x\right). \quad (29)$$

After rescaling the fields, one can represent the series expansion in powers of the small parameter $\hbar$ for (27) in such a way:

$$\Gamma'\left(G,\mathcal{D}\right)=-i\hbar\ln\int D\Phi\hbar^4e^{\left\{\frac{i}{\hbar}4\gamma^4(N^2-1)V+S^q\right\}}\times$$

$$\times\left(1+\sqrt{\hbar}S^{(\sqrt{\hbar})}+\hbar S^{(\hbar)}+\frac{1}{2}\left(\sqrt{\hbar}S^{(\sqrt{\hbar})}+\hbar S^{(\hbar)}\right)^2+o(\hbar)\right), \quad (30)$$

where S^q denotes quadratic terms in the fields:

$$S^q \equiv -\frac{1}{2}A^a_\nu\left(G^{-1}-2g^2\gamma^4N(i\partial^2)^{-1}\right)^{ab}_{\mu\nu}A^b_\mu-\overline{C}^a\left(\mathcal{D}^{-1}-S_F^{-1}\right)^{ab}C^b+$$

$$+ig\gamma^2f^{abc}(A^a_\mu\varphi^{bc}_\mu+A^a_\mu\overline{\varphi}^{bc}_\mu)-i\lambda^a(\partial_\mu A^a_\mu)+\overline{\Theta}_i i\partial^2\Theta_i. \quad (31)$$

The term $S^{(\sqrt{\hbar})}$ denotes the cubic ones:

$$S^{(\sqrt{\hbar})} \equiv i(-gf^{abc}(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu - gf^{acb}\overline{C}^a\partial_\mu A^c_\mu C^b+$$

$$+gf^{acb}\overline{\omega}^a_i\partial_\mu A^c_\mu\omega^b_i - gf^{acb}\overline{\varphi}^a_i\partial_\mu A^c_\mu\varphi^b_i+$$

$$+g(\partial_\nu\overline{\omega}^{ac}_\mu)f^{abm}(\partial_\nu C)^b\varphi^{mc}_\mu), \quad (32)$$

and

$$S^{(\hbar)} \equiv i(-\frac{1}{4}g^2f^{abc}f^{apr}A^b_\mu A^c_\nu A^p_\mu A^r_\nu - A^a_\mu\frac{\delta\Gamma_2(G)}{\delta G^{ab}_{\mu\nu}}A^b_\nu+$$

$$+\overline{C}^a\frac{\delta\Gamma_2(\mathcal{D})}{\delta\mathcal{D}^{ba}}C^b + g^2f^{abm}(\partial_\nu\overline{\omega}^{ac}_\mu)A^\lambda_\nu C^{\lambda b}\varphi^{mc}_\mu) \quad (33)$$

denotes the terms of the fourth order in the fields.

Carrying the term with γ in front of the sign of the integral in Eq. (30), then collecting the terms in powers of $\hbar$ in the last multiplier of this equation, and retaining only the terms of the order of $\hbar$, we obtain the two-loop contribution to the effective action:

$$\Gamma^{'}(G,\mathcal{D}) = 4\gamma^4(N^2-1)V-$$

$$-i\hbar\ln\int D\Phi\hbar^4e^{S^q}\left[1+\hbar\left(S^{(\hbar)}+\frac{1}{2}(S^{(\sqrt{\hbar})})^2\right)\right]. \quad (34)$$

Expanding in $\hbar$, we obtain

$$\Gamma^{'}(G,\mathcal{D}) - 4\gamma^4(N^2-1)V =$$

$$= -i\hbar\ln\int D\Phi\hbar^4e^{S^q} - i\hbar^2\left\langle S^{(\hbar)}+\frac{1}{2}(S^{(\sqrt{\hbar})})^2\right\rangle, \quad (35)$$

The first term is the result of the integration of e^{S^q} over all 8 fields (see notations (10)), (31), and (28). This gives, omitting the unimportant constants $-i\hbar\ln\hbar^4$ and $\frac{i}{2}\hbar Tr\ln[\partial_\mu\partial_\nu]$:

$$-i\hbar\ln\int D\Phi\hbar^4e^{S^q} = -i\hbar\ln\left[\mathrm{Det}(\mathcal{D}^{-1})^{ab}\right]. \quad (36)$$

Note that the integration over the fields $\varphi,\overline{\varphi}$, and $\omega,\overline{\omega}$ gives unity, as well as over A and λ.

The second term on the right-hand side of (35) is the sum of all vacuum expectation values over all the field configurations included inside brackets $\langle\rangle$. One has to write down all these averages explicitly, making possible use of the Wick theorem for the computing every vacuum expectation value. Thus, all configurations with non-zero expectation values, which contribute to the second term on the right-hand side of Eq. (35), give

$$\left\langle S^{(\hbar)}+\frac{1}{2}(S^{(\sqrt{\hbar})})^2\right\rangle = -\frac{i}{4}g^2f^{abc}f^{apr}\left\langle A^b_\mu A^c_\nu A^p_\mu A^r_\nu\right\rangle -$$

$$-i\frac{\delta\Gamma_2(G)}{\delta G^{ab}_{\mu\nu}}\left\langle A^a_\mu A^b_\nu\right\rangle + i\frac{\delta\Gamma_2(\mathcal{D})}{\delta\mathcal{D}^{ba}}\left\langle\overline{C}^aC^b\right\rangle -$$

$$-\frac{1}{2}g^2f^{abc}f^{prs}\left\langle(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu(\partial_\rho A^p_\sigma)A^r_\rho A^s_\sigma\right\rangle -$$

$$-\frac{1}{2}g^2f^{acb}f^{prs}\left\langle\overline{\Theta}^a(\partial_\mu A^c_\mu\Theta^b)\overline{\Theta}^p(\partial_\nu A^r_\nu\Theta^s)\right\rangle. \quad (37)$$

Let us consider these expectation values, making use of the Wick theorem. Here, we write down only the result of computations.

$$-\frac{1}{2}g^2f^{abc}f^{prs}\left\langle\partial^{(x)}_\mu A^a_\nu(x)A^b_\mu(x)A^c_\nu(x)\partial^{(y)}_\rho A^p_\sigma(y)A^r_\rho(y)A^s_\sigma(y)\right\rangle=$$

$$=\frac{1}{2}g^2(f^{abc})^2\{-G_{\nu\rho}(x,y)\left(\partial^{(x)}_\mu G_{\nu\sigma}(x,y)\right)\left(\partial^{(y)}_\rho G_{\mu\sigma}(x,y)\right)+$$

$$+G_{\mu\rho}(x,y)\left(\partial^{(x)}_\mu G_{\nu\sigma}(x,y)\right)\left(\partial^{(y)}_\rho G_{\nu\sigma}(x,y)\right)\}. \quad (38)$$

For four gluon fields at the same space-time point $A^a_\mu(x)$, we get

$$\frac{i}{4}g^2f^{abc}f^{apr}\left\langle A^b_\mu(x)A^c_\nu(x)A^p_\mu(x)A^r_\nu(x)\right\rangle = \frac{i}{4}g^2(f^{abc})^2\times$$

$$\times\{G_{\mu\nu}(x,x)G_{\mu\nu}(x,x) - G_{\mu\mu}(x,x)G_{\nu\nu}(x,x)\}. \quad (39)$$

For anticommuting ghosts (the first three fields are located at the same space-time point, say x, and let y be a space-time point for the location of the last three fields), we get

$$-\frac{1}{2}g^2f^{acb}f^{prs}\left\langle(\partial^{(x)}_\mu\overline{C}^a)A^c_\mu C^b|_{(x)}(\partial^{(y)}_\nu\overline{C}^p)A^r_\nu C^s|_{(y)}\right\rangle=$$

$$= -\frac{1}{2}g^2(f^{acb})^2\{\partial^{(x)}_\mu\mathcal{D}(y,x)\partial^{(y)}_\nu\mathcal{D}(x,y)G_{\mu\nu}(x,y)\}. \quad (40)$$

Now with a close analogy to the ghosts, for the Grassmann fields $\omega,\overline{\omega}$, we get

$$\frac{1}{2}g^2f^{acb}f^{prs}\left\langle(\partial^{(x)}_\mu\overline{\omega}^a_i)A^c_\mu\omega^b_i|_{(x)}(\partial^{(y)}_\nu\overline{\omega}^p_j)A^r_\nu\omega^s_j|_{(y)}\right\rangle=$$

$$= \frac{1}{2}g^2(f^{acb})^2\{\partial^{(x)}_\mu G^\omega_{ji}(y,x)\partial^{(y)}_\nu G^\omega_{ij}(x,y)G_{\mu\nu}(x,y)\}. \quad (41)$$

Here, $G^{\omega}_{ij}(x,y)$ is a tree level propagator for the fields $\omega,\overline{\omega}$, and the indices i,j take on $d(N^2-1)$ values. For the pair of complex conjugate bosonic fields $\varphi,\overline{\varphi}$, we have

$$\frac{1}{2}g^2 f^{acb}f^{prs}\left\langle(\partial^{(x)}_{\mu}\overline{\varphi}^a_i)A^c_{\mu}\varphi^b_i|_{(x)}(\partial^{(y)}_{\nu}\overline{\varphi}^p_j)A^r_{\nu}\varphi^s_j|_{(y)}\right\rangle=$$

$$=-\frac{1}{2}g^2(f^{acb})^2\{\partial^{(x)}_{\mu}G^{\varphi}_{ij}(x,y)\partial^{(y)}_{\nu}G^{\varphi}_{ij}(x,y)G_{\mu\nu}(x,y)\},\quad(42)$$

where $G^{\varphi}_{ij}(x,y)$ is a tree level propagator for the fields $\varphi,\overline{\varphi}$, and the indices i,j are the same ones as in the previous case.

Finally, for the quantity $\Gamma'(G,\mathcal{D})$ from expressions (35)–(42), one has

$$\Gamma'(G,\mathcal{D})=\frac{i}{2}\hbar Tr1+4\gamma^4(N^2-1)V-i\hbar Tr\ln(\mathcal{D}^{-1})^{ab}-$$

$$-\hbar^2\{G^{ab}_{\mu\nu}\frac{\delta\Gamma^{(1)}_2(G)}{\delta G^{ab}_{\mu\nu}}+\mathcal{D}^{ba}\frac{\delta\Gamma^{(1)}_2(\mathcal{D})}{\delta\mathcal{D}^{ba}}+\frac{i}{4}g^2(f^{abc})^2\times$$

$$\times(G_{\mu\nu}G_{\mu\nu}-G_{\mu\mu}G_{\nu\nu})+\frac{i}{2}g^2(f^{acb})^2[-\partial^{(x)}_{\mu}\mathcal{D}\partial^{(y)}_{\nu}\mathcal{D}G_{\mu\nu}-$$

$$-\partial^{(x)}_{\mu}G^{\omega}_{ji}\partial^{(y)}_{\nu}G^{\omega}_{ij}G_{\mu\nu}+\partial^{(x)}_{\mu}G^{\varphi}_{ij}\partial^{(y)}_{\nu}G^{\varphi}_{ij}G_{\mu\nu}]+\frac{i}{2}g^2(f^{abc})^2\times$$

$$\times[-G_{\nu\rho}(\partial_{\mu}G_{\nu\sigma})(\partial_{\rho}G_{\mu\sigma})+G_{\mu\rho}(\partial_{\mu}G_{\nu\sigma})(\partial_{\rho}G_{\nu\sigma})]\}.\quad(43)$$

As follows from the explicit definition of the propagators G^{ω}_{ij} and G^{φ}_{ij} (it can be found during the explicit integration of expression (36) for S^q with notations (31) and (28)), $G^{\omega}_{ij}\equiv G^{\varphi}_{ij}$. Inasmuch as these propagators enter Eq. (43) via terms of the equivalent form, their whole contributions cancel

$$-\partial^{(x)}_{\mu}G^{\omega}_{ij}\partial^{(y)}_{\nu}G^{\omega}_{ij}G_{\mu\nu}+\partial^{(x)}_{\mu}G^{\varphi}_{ij}\partial^{(y)}_{\nu}G^{\varphi}_{ij}G_{\mu\nu}=0.\quad(44)$$

Thus, the effective action has no dependence on the auxiliary unphysical fields $\varphi,\overline{\varphi}$ and $\omega,\overline{\omega}$.

Finally, taking into account that, for the gauge group SU(N),

$$f^{ade}f^{bde}=N\delta^{ab},\quad(45)$$

the explicit expression (22) for the effective action reads as

$$\Gamma(G,\mathcal{D})=4\gamma^4(N^2-1)V+\frac{1}{2}i\hbar Tr[\ln(G^{-1})^{ab}_{\mu\nu}+$$

$$+(\widetilde{D}^{-1}_F)^{ab}_{\mu\nu}G^{ab}_{\mu\nu}-1]-i\hbar Tr\left[\ln(\mathcal{D}^{-1})^{ab}+(S^{-1}_F)^{ab}\mathcal{D}^{ba}-1\right]-$$

$$-\frac{1}{2}i\hbar^2g^2N\delta^{cc}\{\frac{1}{2}(G_{\mu\nu}G_{\mu\nu}-G_{\mu\mu}G_{\nu\nu})-\partial^{(x)}_{\mu}\mathcal{D}\partial^{(y)}_{\nu}\mathcal{D}G_{\mu\nu}-$$

$$-G_{\nu\rho}(\partial_{\mu}G_{\nu\sigma})(\partial_{\rho}G_{\mu\sigma})+G_{\mu\rho}(\partial_{\mu}G_{\nu\sigma})(\partial_{\rho}G_{\nu\sigma})\}.\quad(46)$$

It is convenient to rewrite expression (46) in the momentum space, using the Fourier-transformed propagators defined as follows:

$$G(p)=\int d^4xe^{\frac{i}{\hbar}p(x-y)}G(x-y)\quad(47)$$

(Inasmuch as we seek for the ground state, we consider the case of translation-invariant solutions, i.e., we take $G(x,y)$ to be a function only of $x-y$, and similarly for the other propagators.)

In the momentum space, Eq. (46) divided by the volume V takes the form

$$\frac{1}{V}\widetilde{\Gamma}(G,\mathcal{D})=4\gamma^4(N^2-1)+$$

$$+\frac{1}{2}i\hbar\int\frac{d^4p}{(2\pi\hbar)^4}\left(Tr\ln G^{-1}(p)+Tr\widetilde{D}^{-1}_F(p)G(p)-1\right)-$$

$$-i\hbar\int\frac{d^4p}{(2\pi\hbar)^4}\left(Tr\ln\mathcal{D}^{-1}(p)+TrS^{-1}_F(p)\mathcal{D}(p)-1\right)-$$

$$-\frac{i}{4}\hbar^2g^2N\delta^{cc}\int\frac{d^4p\,d^4r}{(2\pi\hbar)^8}\{G_{\mu\nu}(p)G_{\mu\nu}(r)-G_{\mu\mu}(p)G_{\nu\nu}(r)\}-$$

$$-\frac{i}{2}g^2N\delta^{cc}\int\frac{d^4p\,d^4r}{(2\pi\hbar)^8}\{\mathcal{D}(p)\mathcal{D}(r)G_{\mu\nu}(p-r)p_{\mu}r_{\nu}+$$

$$+G_{\nu\rho}(p)G_{\nu\sigma}(p+r)G_{\mu\sigma}(r)(p+r)_{\mu}r_{\rho}-$$

$$-G_{\mu\rho}(p)G_{\nu\sigma}(p+r)G_{\nu\sigma}(r)(p+r)_{\mu}r_{\rho}\}.\quad(48)$$

Thus, we have the two-loop generalized effective action written in the momentum representation.

4. Schwinger–Dyson Equations

In order to obtain a closed system of equations for the gluon propagator $G^{ab}_{\mu\nu}(p)$, the ghost propagator $\mathcal{D}^{ab}(p)$, and the Gribov parameter γ, one has to take variational derivatives of the effective action (48) with respect to these parameters and equate each of them to zero (see Eq. (6)).

Before doing this, we recall that, in the momentum representation, expression (23) for the free boson propagator modified with respect to the Zwanziger term is given by

$$\widetilde{D}^{-1}_F(p)=D^{-1}_F(p)+2g^2\gamma^4N\frac{i\hbar^2}{p^2}.\quad(49)$$

Hereafter, we put $\hbar=1$.

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Varying expression (48) with respect to γ, we get

$$\frac{1}{V}\frac{\delta\widetilde{\Gamma}(G,\mathcal{D})}{\delta\gamma}=0=$$

$$=4\gamma^3\left[4(N^2-1)-g^2N\int\frac{d^4p}{(2\pi)^4}Tr\frac{G^{ab}_{\mu\nu}(p)}{p^2}\right]. \qquad (50)$$

After rejecting the trivial solution $\gamma=0$, it looks as

$$1=\frac{Ng^2}{4(N^2-1)}\int\frac{d^4p}{(2\pi)^4}Tr\frac{G^{ab}_{\mu\nu}(p)}{p^2}. \qquad (51)$$

The variation with respect to $\mathcal{D}^{ab}(q)$ gives

$$\frac{1}{V}\frac{\delta\widetilde{\Gamma}(G,\mathcal{D})}{\delta\mathcal{D}^{ab}(q)}=0=i\left(\mathcal{D}^{-1}(q)-S_F^{-1}(q)\right)^{ab}+$$

$$-\frac{i}{2}g^2N\delta^{ab}[\int\frac{d^4r}{(2\pi)^4}\mathcal{D}(r)G_{\mu\nu}(q-r)q_\mu r_\nu+$$

$$+\int\frac{d^4p}{(2\pi)^4}\mathcal{D}(p)G_{\mu\nu}(p-q)p_\mu q_\nu]. \qquad (52)$$

Due to the property $G(r)=G(-r)$, this expression, after the changing of mute indices $p\to r$ and $\mu\to\nu$ in the last term, can be written in the form

$$\left(\mathcal{D}^{-1}(q)\right)^{ab}=\left(S_F^{-1}(q)\right)^{ab}+$$

$$+Ng^2\delta^{ab}\int\frac{d^4r}{(2\pi)^4}\mathcal{D}(r)G_{\mu\nu}(q-r)q_\mu r_\nu. \qquad (53)$$

The variation with respect to $G^{ab}_{\mu\nu}(q)$ gives

$$\frac{1}{V}\frac{\delta\widetilde{\Gamma}(G,\mathcal{D})}{\delta G^{ab}_{\mu\nu}(q)}=0=\frac{i}{2}\left(-G^{-1}(q)+\widetilde{D}_F^{-1}(q)\right)^{ab}_{\mu\nu}-\frac{i}{2}g^2N\delta^{ab}\times$$

$$\times\int\frac{d^4p}{(2\pi)^4}\mathcal{D}(p)\mathcal{D}(p-q)p_\mu(p-q)_\nu+\text{(gluon loops)}. \qquad (54)$$

Some simplifications can be done in the expressions above. In Eq. (51), we take the trace on color and Lorentz indices. In the Landau gauge, the gluon propagator is transverse in vector indices:

$$G^{ab}_{\mu\nu}(p)=G(p^2)\mathcal{P}_{\mu\nu}(p)\delta^{ab}, \qquad (55)$$

where

$$\mathcal{P}_{\mu\nu}(p)\equiv\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right] \qquad (56)$$

is the transverse projector. In Eq. (53), we shift the integration variable according to $r\to r+q$ and use Eq. (55). The similar shift of the integration variable will be applied to Eq. (54).

Now we are ready to write down a closed system of equations of motion for the Gribov parameter and the ghost and gluon propagators:

$$1=\frac{3}{4}Ng^2\int\frac{d^4p}{(2\pi)^4}\frac{G(p)}{p^2}; \qquad (57)$$

$$\left(\mathcal{D}^{-1}(q)\right)^{ab}=\left(S_F^{-1}(q)\right)^{ab}+$$

$$+Ng^2\delta^{ab}\int\frac{d^4r}{(2\pi)^4}\mathcal{D}(q+r)G(r^2)\left[\frac{r^2q^2-(qr)^2}{r^2}\right]; \qquad (58)$$

$$\left(G^{-1}(q)\right)^{ab}_{\mu\nu}=\left(\widetilde{D}_F^{-1}(q)\right)^{ab}_{\mu\nu}-g^2N\delta^{ab}\times$$

$$\times\int\frac{d^4p}{(2\pi)^4}\{\mathcal{D}(p+q)\mathcal{D}(p)(p+q)_\mu p_\nu\}+\text{(gluon loops)}. \qquad (59)$$

It is worth noting that the Gribov parameter γ does not enter its equation of motion (57) explicitly, but through the modified free gluon propagator $\widetilde{D}_F(q)$ (according to (49)) which is present in Eq. (59) for the gluon propagator.

We wish to determine the asymptotic form of the propagators at low momenta. For this purpose, let the external momentum in the SDEs written above be asymptotically small. In this case, the loop integration will be dominated by asymptotically small loop momenta, so the propagators inside the integrals can also be replaced by their asymptotic values. As one can see from Section 2, where it is argued that the ghost propagator should be more singular and the gluon one should be less singular than a simple pole, the gluon loops integration at low momenta in Eq. (59) can be neglected.

Let us consider Eq. (58). One of the forms of the horizon conditions that guaranties the absence of the Gribov copies looks as [17]

$$\lim_{q\to 0}\left[q^2\mathcal{D}(q^2)\right]^{-1}=0.$$

To impose this condition, we divide Eq. (58) by q^2 and obtain (after the factorization of color indices)

$$0=i+Ng^2\int\frac{d^4r}{(2\pi)^4}\mathcal{D}(r^2)G(r^2)\left[1-\cos(\hat{qr})\right]. \qquad (60)$$

Subtracting this equation from the previous one (58), we get

$$\mathcal{D}^{-1}(q^2)=-Ng^2\int\frac{d^4r}{(2\pi)^4}G(r^2)[\mathcal{D}(r^2)-$$

$$-\mathcal{D}((q+r)^2)]\left[\frac{r^2q^2-(qr)^2}{r^2}\right]. \quad (61)$$

We now turn to the SDE for the gluon propagator (59) with comments below it. We apply the transverse projector, take the trace on color and Lorentz indices, and obtain, taking into account Eq. (49),

$$G^{-1}(q^2) = iq^2 + 2Ng^2\gamma^4\frac{i}{q^2} -$$

$$-\frac{Ng^2}{3}\int\frac{d^4p}{(2\pi)^4}\mathcal{D}((p+q)^2)\mathcal{D}(p^2)\left[\frac{p^2q^2-(pq)^2}{q^2}\right]. \quad (62)$$

We note that, in the limit $q \to 0$, the first term vanishes.

In order to simplify the further analysis and compare our results with those of other authors, we rewrite SDEs for the gluon and ghost propagators in the Euclidean space as

$$\mathcal{D}^{-1}(q^2) = Ng^2\int\frac{d^4p}{(2\pi)^4}G(p^2)\left[\mathcal{D}(p^2)-\mathcal{D}((q+p)^2)\right]\times$$

$$\times\left[\frac{p^2q^2-(pq)^2}{p^2}\right]. \quad (63)$$

$$G^{-1}(q^2) = 2g^2\gamma^4N\frac{1}{q^2} + \frac{Ng^2}{3}\int\frac{d^4p}{(2\pi)^4}\mathcal{D}((p+q)^2)\mathcal{D}(p^2)\times$$

$$\times\left[\frac{p^2q^2-(pq)^2}{q^2}\right]. \quad (64)$$

This system of SDEs for the gluon and ghost propagators in the Landau gauge, written in the Euclidean space, will be analyzed in next sections.

5. Zeroth-order Analysis of SDEs

We now consider the first approximation which corresponds to retaining only the first term on the right-hand side of (59) without any changes in other equations. It is in fact the zeroth-order approximation. In Eq. (59), the contribution in this order gives

$$\left(G^{(1)}(q)\right)^{ab}_{\mu\nu} = \frac{q^2}{q^4+2Ng^2\gamma^4}\delta^{ab}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right). \quad (65)$$

To see this, one has to inverse the left-hand side and the first term on the right-hand side of Eq. (59) and to rewrite the result in the Euclidean space.

After the substitution of this approximation to the Euclidean form of Eq. (57), we get

$$1 = \frac{3}{4}Ng^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^4+2g^2N\gamma^4}. \quad (66)$$

We note that the gluon propagator (65) and the gap equation (66) were first derived by Gribov [1]. Furthermore, the so-called horizon condition (66) was obtained by making use of the zeroth-order approximation (65), so we see that SDE for the Gribov parameter (57) defines the full horizon condition.

We now turn to the ghost SDE and apply the zeroth-order approximation (65) to it. For this purpose, it is more convenient to use the Euclidean form of Eq. (53). Substituting the explicit expression for $G_{\mu\nu}(q-r)$ (according to (65)) in the horizon condition (66), one gets

$$\mathcal{D}^{-1}(q) = Ng^2q_\mu q_\nu\int\frac{d^4r}{(2\pi)^4}\frac{1}{r^4+\lambda^4}\left(g_{\mu\nu}-\frac{r_\mu r_\nu}{r^2}\right)\times$$

$$\times\left[1-r^2\mathcal{D}(q-r)\right] \quad (67)$$

with $\lambda^4 \equiv 2Ng^2\gamma^4$.

It is a nonlinear integral equation for the $\mathcal{D}$ -function, and we will solve it now. One of the possible ways to do this is the iteration scheme with a free ghost propagator $S_F(q) = 1/q^2$ as the zeroth-order approximation, $\mathcal{D}_{(0)}(q)$, of $\mathcal{D}(q)$. In other words, in order to obtain the first-order approximation, $\mathcal{D}^{-1}_{(1)}(q)$, of $\mathcal{D}^{-1}(q)$, we have to replace $\mathcal{D}(q-r) \to \mathcal{D}_{(0)}(q-r) \equiv 1/(q-r)^2$ on the right-hand side of Eq. (67). Doing this, we get

$$\mathcal{D}^{-1}_{(1)}(q) = Ng^2q_\mu q_\nu\int\frac{d^4r}{(2\pi)^4}\frac{1}{r^4+\lambda^4}\left(g_{\mu\nu}-\frac{r_\mu r_\nu}{r^2}\right)\frac{q^2-2qr}{(q-r)^2}. \quad (68)$$

We note that that exactly the same expression was used by Gribov in order to obtain the infrared behavior of the ghost propagator [1].

We emphasize that while Gribov in fact solved the equation for the ghost propagator approximately, we derive some integral equation and look for the exact solutions to it.

After the integration of the right-hand side of (68) for the first-order approximation of $\mathcal{D}(q)$, one gets [18]

$$\mathcal{D}_{(1)}(q) = \frac{128\pi\lambda^2}{3g^2N}\frac{1}{q^4}. \quad (69)$$

We note that the further use of such iterational scheme for Eq. (67) results in an incompatible system of equations. Thus, the natural question appears: Is the iterational procedure used for solving Eq. (67) admissible? The obvious answer is "no", because this iteration procedure is not convergent. Consequently, the perturbation theory does not work, and the result obtained above is incorrect in the general case, though it gives a qualitatively proper behavior. In order to solve Eq. (67) correctly, some another method is needed.

Let us try to solve the equation for the ghost propagator (67) written in the form similar to (63) (with the explicit gluon propagator (65)) in the infrared region:

$$\left[g\mathcal{D}(q^2)\right]^{-1} = Ng\int \frac{d^4r}{(2\pi)^4}\left[\mathcal{D}(r^2) - \mathcal{D}\left((q+r)^2\right)\right] \times$$

$$\times\frac{\left[r^2q^2-(qr)^2\right]}{r^4+\lambda^4}. \qquad (70)$$

In order to solve this equation, we seek for a solution in the form

$$g\mathcal{D}(q^2) = \frac{A}{\left(q^2\right)^{1+\alpha}}. \qquad (71)$$

Here, A is some coefficient, and α is called the infrared critical exponent. After the evaluation of the integrals, one gets

$$A^2 = \frac{32\pi^2\lambda^4}{N}. \qquad (72)$$

Thus, the exact solution of Eq. (67) which describes the infrared behavior of the ghost propagator is

$$\mathcal{D}(q^2) = 4\sqrt{2}\frac{\pi\lambda^2}{g\sqrt{N}}\frac{1}{q^4}. \qquad (73)$$

We note that the previously obtained solution (69) qualitatively coincides with ours, but has another coefficient. Thus, our zeroth-order analysis has qualitatively confirmed that, in the infrared region, the gluon propagator is suppressed (65), while the ghost propagator is enhanced (73).

6. Full Analysis of the Derived One-loop SDEs

We now turn to SDEs (63) and (64). One can solve these equations using the "weak-angle-dependence" approximation for ghosts:

$$\mathcal{D}\left((q+p)^2\right) \approx \mathcal{D}(q^2)\Theta(q^2-p^2) + \mathcal{D}(p^2)\Theta(p^2-q^2). \qquad (74)$$

Here, $\Theta(x)$ is the ordinary step function. Substituting this approximate expression to the ghost SDE (63) and gluon SDE (64) and then carrying out the angles integration, one gets

$$\mathcal{D}^{-1}(q^2) = Aq^2\int\limits_0^{q^2} dp^2\, p^2 G(p^2)\left[\mathcal{D}(p^2) - \mathcal{D}(q^2)\right], \qquad (75)$$

$$G^{-1}(q^2) = \frac{\lambda^4}{q^2} + B\left[\int\limits_0^{q^2} dp^2 p^4 \mathcal{D}(p^2)\mathcal{D}(q^2) + \int\limits_{q^2}^{\infty} dp^2 p^4 \mathcal{D}^2(p^2)\right], \qquad (76)$$

with

$$A \equiv \frac{3Ng^2}{16(2\pi)^2}, \quad B \equiv \frac{Ng^2}{16(2\pi)^2}, \quad \lambda^4 \equiv 2Ng^2\gamma^4. \qquad (77)$$

Solutions can be found in the power form as

$$G(q^2) = C_G(q^2)^{-\alpha}, \qquad (78)$$

$$\mathcal{D}(q^2) = C_{\mathcal{D}}(q^2)^{-\beta}, \qquad (79)$$

where C_G and $C_{\mathcal{D}}$ are dimensionless constants, and α and β are some exponents. It turned out that the straightforward substitution of anzatz (78), (79) in Eqs. (75), (76) suffers from certain mathematical difficulties, and it is more convenient to do first some transformations in the equations above. We divide Eq. (75) by q^2, then carry out the differentiation with respect to q^2 in the derived equation and in Eq. (76), and obtain

$$\frac{1}{q^4}\mathcal{D}^{-1}(q^2) + \frac{1}{q^2}\mathcal{D}^{-2}(q^2)\mathcal{D}'(q^2) = A\mathcal{D}'(q^2)\int\limits_0^{q^2} dp^2 p^2 G(p^2). \qquad (80)$$

$$G^{-2}(q^2)G'(q^2) = \frac{\lambda^4}{q^4} - B\mathcal{D}'(q^2)\int\limits_0^{q^2} dp^2\, p^4\mathcal{D}(p^2). \qquad (81)$$

Now anzatz (78), (79) can be applied. We substitute these power functions into our equations and carry out all the integrations (under the assumption that $\beta \neq 3$ and $\alpha \neq 2$). Then we get

$$(-\beta+1)C_{\mathcal{D}}^{-1}(q^2)^{\beta-2} + \frac{\beta A C_{\mathcal{D}} C_G}{-\alpha+2}(q^2)^{-\beta-\alpha+1} = 0; \qquad (82)$$

$$-\alpha C_G^{-1}(q^2)^{\alpha-1} - \lambda^4(q^2)^{-2} - \frac{\beta B C_{\mathcal{D}}^2}{-\beta+3}(q^2)^{-2\beta+2} = 0. \qquad (83)$$

We equate the exponents of q^2 on both sides of these equations and obtain the following equations for the parameters α and β:

$$\beta - 2 = -\beta - \alpha + 1 \qquad (84)$$

$$\alpha - 1 = -2 = -2\beta + 2. \qquad (85)$$

They give us a unique solution

$$\alpha = -1 \qquad \beta = 2. \qquad (86)$$

In terms of the infrared critical exponents

$$\alpha_G = \alpha - 1, \quad \alpha_{\mathcal{D}} = \beta - 1 \tag{87}$$

(see (3), (4)), one has

$$\alpha_G = -2, \quad \alpha_{\mathcal{D}} = 1. \tag{88}$$

We now turn to Eqs. (82) and (83) in order to obtain the coefficients C_G and $C_{\mathcal{D}}$. Thus, we get

$$-C_{\mathcal{D}}^{-1} + \frac{2}{3} A C_{\mathcal{D}} C_G = 0, \tag{89}$$

$$C_G^{-1} - \lambda^4 - 2BC_{\mathcal{D}}^2 = 0. \tag{90}$$

Along with notations (77), this gives

$$C_G = \frac{1}{\lambda^4}\left[1 + 3BA^{-1}\right] = \frac{2}{\lambda^4}, \tag{91}$$

$$C_{\mathcal{D}} = \sqrt{\frac{3}{2} A^{-1} C_G^{-1}} = 4\frac{\pi\lambda^2}{g\sqrt{N}} \tag{92}$$

with $\lambda^4 \equiv 2Ng^2\gamma^4$, where γ is the Gribov parameter.

Consequently, the solution of SDEs (63) and (64) presents the following infrared behavior of the gluon and ghost propagators, respectively:

$$G(q^2) = \frac{2}{\lambda^4} q^2, \tag{93}$$

$$\mathcal{D}(q^2) = 4\frac{\pi\lambda^2}{g\sqrt{N}} \frac{1}{q^4}. \tag{94}$$

Thus, one can see that the one-loop results are in a good qualitative agreement with the tree-level ones. Coefficients (91) and (92) obtained in this order differ from the zeroth-order ones (65), (73) by a numerical multiplier and, hopefully, are more precise.

As can be checked by the numerical computation, approximation (74) has no influence on the qualitative behavior of the propagators, but rather gives some additional constant terms unimportant for the asymptotics.

7. Discussion and Conclusions

The formalism of generalized effective action with the implemented restriction to the Gribov horizon was used to investigate the infrared behavior of the gluon and ghost propagators. The system of the SDEs for the gluon propagator (59), the ghost propagator (58), and the Gribov parameter (57) has been derived in this approach. These equations are obtained using the two-loop approximation for the generalized effective action.

The SDE for the Gribov parameter is, in fact, the full horizon condition which is reduced to the Gribov's one in the zero order. Furthermore, the same expression for the gluon propagator, as Gribov first pointed out, follows in this order. The expression for the ghost propagator, following from the zeroth-order approximation, confirms the qualitative behavior, but differs from the Gribov's one by some coefficient.

The obtained SDEs are solved in the first-order approximation as well. The infrared critical exponents and the coefficients for the gluon and ghost propagators are obtained. Thus, we have derived the expressions which describe the infrared behavior of the gluon propagator (93) and the ghost one (94). The solution for the infrared exponents was obtained before [7], whereas the coefficients for the propagators in this order are first derived in the present work.

We note that the first-order results agree with the zeroth-order ones with different numerical coefficients at small momenta. For the gluon and ghost propagators, this difference is reduced to $1/2$ and $\sqrt{2}$, respectively.

Thus, cutting off the measure of the functional integral at the (first) Gribov region leads to the infrared enhancement of the ghost propagator and the suppression of the gluon one. Such a behavior of the propagators supports the picture of color confinement [1, 2, 14].

It would be interesting to study the possibility of generating the gluon mass [21] in the effective action formalism of CJT, when the Zwanziger horizon condition is implemented [19].

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1. V.N. Gribov, Nucl. Phys. B **139**, 1 (1978).
2. D. Zwanziger, Nucl. Phys. B **323**, 513 (1989).
3. S. Mandelstam, Phys. Rev. D **20**, 3223 (1979).
4. D. Atkinson and J.C.R. Bloch, Phys. Lett. A **13**, 1055 (1998); Phys. Rev. D **58**, 094036 (1998).
5. P. Marenzoni, G. Martinelli, N. Stella, e M. Testa, Phys. Lett. B **318**, 511 (1993); C. Alexandrou, Ph. de Forcrand, and E. Follana, Phys. Rev. D **65**, 114508 (2002); D **65**, 117502 (2002); F. D. R. Bonnet et al., Phys. Rev. D **64**, 034501 (2001); D **62**, 051501 (2000); D. B. Leinweber et al. (UKQCD Collaboration), Phys. Rev. D **58**, 031501 (1998); C. Bernard, C. Parrinello, and A. Soni, Phys. Rev. D **49**, 1585 (1994); P. O. Bowman et al., [arXiv:hep-lat/0402032] and the references therein.
6. J. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

7. D. Zwanziger, Phys. Rev. D **65**, 094039 (2002).
8. C. Lerche and L. von Smekal, Phys. Rev. D **65**, 125006 (2002).
9. C. S. Fischer and R. Alkofer, Phys. Rev. D **67**, 094020 (2003).
10. C. S. Fischer and R. Alkofer, Phys. Lett. B **536**, 177 (2002); C. S. Fischer, R. Alkofer, and H. Reinhardt, Phys. Rev. D **65**, 094008 (2002); R. Alkofer, C. S. Fischer, and L. von Smekal, Acta Phys. Slov. **52**, 191 (2002).
11. T. Kugo and I. Ojima Prog. Theor. Phys. Suppl. **66**, 1 (1979).
12. A. Cucchieri and D. Zwanziger, Phys. Rev. D **65**, 014002 (2002).
13. D. Zwanziger, Phys. Rev. Lett. **90**, 102001 (2003).
14. D. Zwanziger Nucl. Phys. B **518**, 237 (1998).
15. Ph. Boucaud, Th. Bruntjen, J.P. Leroy, A. Le Yaouanc, A.Y. Lokhov, J. Micheli, O. Pene, and J. Rodriguez-Quintero, [arXiv:hep-ph/0604056 v1]; A. C. Aguilar and A. A. Natale, Int. J. Mod. Phys. A **20**, 7613 (2005).
16. P. J. Silva and O. Oliveira, Phys.Rev. D **74**, 034513 (2006); O. Oliveira and P. J. Silva, [hep-lat/0609036]; E.-M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, A. Schiller, and I. L. Bogolubsky, [hep-lat/0609043]; A. Cucchieri, A. Maas, and T. Mendes, [hep-lat/0701011]; A. Cucchieri and D. Zwanziger, [hep-lat/0012024]; Ph. Boucaud, Th. Bruntjen, J.P. Leroy, A. Le Yaouanc, A.Y. Lokhov, J. Micheli, O. Pene, J. Rodriguez-Quintero, and C. Roiesnel, [hep-lat/0602006].
17. D. Zwanziger, Nucl. Phys. B **399**, 477 (1993).
18. R.F. Sobreiro and S.P. Sorella, [arXiv:hep-th/0504095 v1].
19. D. Dudal, R.F. Sobreiro, S.P. Sorella, and H. Verschelde, Phys. Rev. D **72**, 014016 (2005).
20. J.A. Gracey, Phys. Lett. B **632**, 282 (2006).
21. V.P. Gusynin and V.A. Miransky, Phys. Lett. B **76**, 585 (1978).

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ІНФРАЧЕРВОНА ПОВЕДІНКА ГЛЮОННОГО ТА ДУХОВОГО ПРОПАГАТОРІВ У КХД В КАЛІБРОВЦІ ЛАНДАУ

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Р е з ю м е

Непертурбаційний формалізм узагальненої ефективної дії застосовано для побудови системи рівнянь Швінгера–Дайсона. Для звільнення області інтегрування функціонального інтегралу від калібрувальних копій введено обмеження до горизонту Грібова методом, запропонованим Цванцігером. В такому підході розглянуто асимптотичну поведінку глюонного пропагатора та пропагатора духів Фаддеєва–Попова при малих імпульсах. Таку поведінку отримано в результаті розв'язання системи рівнянь Швінгера–Дайсона в нульовому та першому наближеннях. Показано якісне узгодження цих результатів з тими, які було отримано раніше, і знайдено їх кількісну відмінність на деякий числовий коефіцієнт.