ON SPECTRAL DISTRIBUTIONS OF RADIATION BY HIGH-ENERGY ELECTRONS IN THE FIELD OF A CRYSTAL ATOMIC STRING

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Formulae for the spectral and spectral-angular densities of radiation by relativistic electrons in an external nonuniform field are explored in the quasiclassical approach by taking the recoil and nondipole radiation factors into account. To compute with the formulae, a numerical method is set out. The extreme cases relevant to the radiation from an angle-type trajectory and to the constant-field approximation are considered. The results of exact and approximate calculations are presented for the radiation by high-energy electrons in the field of a single atomic string into the crystal.

1. Introduction

The description of the radiation by a high-energy electron moving in a matter and an external field in a large spatial region along the direction of a particle motion was given in [1, 2]. If the particle collides within this region with many atoms of the medium, the interaction will happen with them differently than that with the isolated centers carried far apart. In this case, both an increase and a decrease of the efficiency under the particle-medium interaction are possible.

The essential feature of the electron radiation in a matter at high energies is that, with growth of the electron energy and with decrease of the radiated photon frequency, the effective coupling constant of the interaction between a particle and medium atoms, which are in the range of the coherent length for the radiation process, is quickly increased [2]. The validity of the Born approximation, which is widely used for the description of the electron radiation in the field of atoms, is fast broken in this case, and it is necessary to go beyond the scope of the Born perturbation theory.

There is a number of methods to describe the electromagnetic processes in a matter at high energies in external fields beyond the scope of the Born perturbation theory. This can be made, in particular, on the basis of the eikonal and quasiclassical approximations in quantum electrodynamics and methods based on the classical theory of radiation [2]. The adaptation of these methods to fields of a complicated configuration such as the field of a crystal lattice is the intricate problem. Therefore, the construction of simplified models and the development of numerical methods are substantial to account the particle interaction with such fields and a matter. The problem is considerably simplified in the case where the radiation by relativistic electrons in a matter is dipole or the constant field approximation is used, which gives a possibility to apply formulae of the synchrotron radiation theory, and also when the particle trajectory is approximated by the angle-type trajectory. The adaptability of these approximations for the description of the radiation by particles in a matter is essential. The paper is devoted to the examination of this problem.

The present investigation is based on a variant of the quasiclassical representation for a spectral-angular distribution of the radiation intensity of a high-energy electron in a nonuniform external field, for which this distribution with regard for the recoil effect at radiation is determined by a set of electron classical trajectories in this field [2,3]. Such quantum-electrodynamics expression for the radiation density through classical trajectories is rather convenient in some cases, because it allows reducing the quantum-electrodynamics problem on radiation to the problem of solving the classical equations of motion of an electron in an external field, by executing the calculations of some integrals along these classical trajectories. In this case, the possibility to solve this problem with simple numerical methods applied to the equations of motion is appeared. This is rather important at studying the radiation in a field with complicated configuration.

The problem becomes especially acute for studying the radiation by high-energy electrons in aligned crystals, when the character of particle motion is rather complicated and the effects of recoil and a nondipole behavior at radiation are of importance. Therefore, we

pay the considerable attention to the adaptability of the offered numerical method and calculations with simplified equations for the electron radiation in aligned crystals.

2. Radiation Cross-Section in External Field

Within the quasiclassical approximation of quantum electrodynamics, the differential cross-section of radiation by a relativistic electron in the nonuniform external field can be written down as [2, 3]

$$\frac{d\sigma}{d\omega do} = \int d^2\rho \frac{1}{\omega}\frac{dE}{d\omega do} \ , \tag{1}$$

where ω is the photon energy, do is the element of a solid angle in the direction of radiation, $dE/d\omega do$ is the spectral-angular radiation density by the electron that moves along the trajectory $\mathbf{r}(t,\boldsymbol{\rho})$ in an external field, and $\boldsymbol{\rho}$ is the impact parameter, at which the electron incomes into the field. (We use the system of units, for which the Planck constant and the light velocity are accepted equal to unity.) The quantity $dE/d\omega do$ is defined by the relation

$$\frac{dE}{d\omega do} = \frac{e^2}{8\pi^2}\frac{\varepsilon^2+\varepsilon'^2}{\varepsilon'^2}\omega^2\left[|\mathbf{n}\times\mathbf{I}|^2 + \frac{m^2}{\varepsilon^2}\frac{\omega^2\,|I_0|^2}{\varepsilon^2+\varepsilon'^2}\right] \ , \tag{2}$$

where e is the electron charge, $\varepsilon' = \varepsilon - \omega$ is the final electron energy, $\mathbf{n}$ is the unit vector in the direction of radiation,

$$\mathbf{I} = \int\limits_{-\infty}^{\infty} dt\mathbf{v}(t)e^{i\omega'[t-\mathbf{nr}(t)]} \ , \ I_0 = \int\limits_{-\infty}^{\infty} dt e^{i\omega'[t-\mathbf{nr}(t)]} \ , \tag{3}$$

$\omega' = \varepsilon\omega/\varepsilon'$, and $\mathbf{v}(t) = d\mathbf{r}/dt$ is the velocity vector of the electron.

Equation (2) takes into account the effects due to the recoil at radiation. The effects due to the quantum character of electron motion in an external field are neglected in Eq. (2). Its essential feature is the fact that, in the quasiclassical approximation, the spectral-angular radiation density is defined by classical trajectories of an electron in the external field. Particles with different starting conditions have different trajectories in the nonuniform external field, so formula (1) includes the integration over these starting conditions.

At high electron energies, the typical values of scattering and radiating angles in the external field are small in comparison with unity. Then the expansion can be carried out in Eq. (2) in these angles in the general form without using any specific law of a particle motion. For this purpose, we take advantage of the relation

$$\mathbf{v}(t) \approx \mathbf{v}_0\left[1 - \frac{v_\perp^2(t)}{2v_0^2}\right] + \mathbf{v}_\perp(t), \tag{4}$$

which defines the deflection of a particle trajectory in the external field relative to the initial direction of the motion $\mathbf{v}_0$. The value $\mathbf{v}_\perp(t)$ in Eq. (4) is a small deflection of a particle trajectory orthogonal to $\mathbf{v}_0$ ($v_\perp \ll v_0$, $\mathbf{v}_\perp \cdot \mathbf{v}_0 = 0$). Thus, it is assumed that $\mathbf{v}_\perp = 0$ at $t \to -\infty$. Equation (4) considers the modification of both transverse and longitudinal velocities. Terms of the order $v_\perp^4/v_0^4$ are neglected here.

Below we consider that the vector $\mathbf{v}_\perp(t)$, as well $\boldsymbol{\theta}$ which defines the radiation angle, is referred to the initial direction of electron motion $\mathbf{v}_0$. In other words, we will carry out all further evaluations in the frame of reference, in which the z-axis is directed along $\mathbf{v}_0$. Then the spectral-angular radiation density in Eq. (2) will be described as

$$\frac{dE}{d\omega do} \approx \frac{e^2}{8\pi^2}\frac{\varepsilon^2+\varepsilon'^2}{\varepsilon'^2}\omega^2 K \ , \tag{5}$$

where

$$K = |\mathbf{J}|^2 + \gamma^{-2}\frac{\omega^2}{\varepsilon^2+\varepsilon'^2}\,|J|^2 \ . \tag{6}$$

The quantities $\mathbf{J}$ and J from the above equation for K to within the terms of the order of γ^{-2}, $v_\perp^2$, and θ^2 can be expressed in the following form:

$$\mathbf{J} = \int\limits_{-\infty}^{\infty} dt\,[\mathbf{v}_\perp - \boldsymbol{\theta}]\exp\left\{\frac{i\omega'}{2}\left[\frac{t}{\gamma^2} + \int\limits_{-\infty}^{t} d\tilde{t}\left(\mathbf{v}_\perp(\tilde{t}) - \boldsymbol{\theta}\right)^2\right]\right\},$$

$$J = \int\limits_{-\infty}^{\infty} dt \exp\left\{\frac{i\omega'}{2}\left[\frac{t}{\gamma^2} + \int\limits_{-\infty}^{t} d\tilde{t}\left(\mathbf{v}_\perp(\tilde{t}) - \boldsymbol{\theta}\right)^2\right]\right\} \ . \tag{7}$$

Note that the representation of formula (5) for the spectral-angular radiation density has the advantage in comparison with formula (2) due to the explicit aspect for the smallness order of included terms.

3. Algorithm of Numerical Simulation

On the basis of the general formula (5), one can also develop the numerical methods for the calculation of the spectral-angular radiation density. This is especially important for the interaction of a particle with fields of complicated configurations, like with the field of a crystal lattice. For this purpose, one needs to develop the

numerical methods to solve the equations which determine both the trajectory and the velocity time dependences for a particle in an external field, and to calculate the time integrals $\mathbf{J}$ and J in Eq. (6).

As usual, in the numerical calculations of the integrals $\mathbf{J}_T$ and J_T on the time interval, during which a particle interacts with the external field, one can utilize the procedure of their transformation into the corresponding sums through the relation (see, e.g., [4–6])

$$J_T = \int\limits_0^T dt f(t) = \Delta \sum_{n=1}^{N} f(t_n) \ , \tag{8}$$

where $T = N\Delta$, and $f(t_n)$ is the value of the integrand on the trajectory interval n. (The partition of a time interval T into N small fragments can be nonuniform in general.) The complexity of this method for the considered problem lies in the numerical integration of fast oscillating functions. The calculation accuracy can be increased by adding the number of partition intervals N. At the same time, however, the database needed for integrand functions extends sharply, which makes the fulfillment of accurate evaluations problematic even with modern computers.

The numerical method of evaluation of the integrals $\mathbf{J}$ and J described below shortens this procedure by means of utilizing the analytical technique at some stages. The method consists in the following. In $\mathbf{J}$ and J, we replace the integration variable t by ξ which is defined by the relation

$$\xi = t\gamma^{-2} + \int\limits_{-\infty}^{t} d\tilde{t} \left(\mathbf{v}_\perp(\tilde{t}) - \boldsymbol{\theta}\right)^2 . \tag{9}$$

Such a replacement in the integrand functions of $\mathbf{J}$ and J allows us to transform the nonlinear function of time to a linear function of the integration variable ξ in the exponent phase. Substituting relation (9) into expression (7), we can rewrite the integrals $\mathbf{J}$ and J as

$$\mathbf{J} = \int d\left[\mathbf{r}_\perp(t) - \boldsymbol{\theta} t\right] e^{\frac{i}{2}\omega'\xi}, \quad J = \int dt e^{\frac{i}{2}\omega'\xi}, \tag{10}$$

where $\mathbf{r}_\perp = \int_{-\infty}^{t} d\tilde{t}\mathbf{v}_\perp(\tilde{t})$, and the function $t = t(\xi)$ is determined by Eq. (9). After the integration by parts, we find that

$$\begin{aligned} \mathbf{J} &= -\frac{i\omega'}{2} \int\limits_{-\infty}^{\infty} d\xi \left[\mathbf{r}_\perp(t) - \boldsymbol{\theta} t\right] e^{\frac{i}{2}\omega'\xi} \ , \\ J &= -\frac{i\omega'}{2} \int\limits_{-\infty}^{\infty} d\xi t(\xi) e^{\frac{i}{2}\omega'\xi}. \end{aligned} \tag{11}$$

Further, we will describe the calculation procedure for $\mathbf{J}$ only, as it is the same for J. Let us divide the integration region over ξ into the three ones: $\xi < 0$, $0 < \xi < \xi_T$, and $\xi > \xi_T$, where ξ_T is the value of ξ related to the time T. The t-dependence of ξ is linear at $\xi < 0$ and at $\xi > \xi_T$. So the integration of $\mathbf{J}$ over ξ can be carried out analytically upon these intervals (it is supposed that a particle interacts with the external field at the time interval $0 \le t \le T$ only). Thus, $\mathbf{J}$ looks like

$$\mathbf{J} = \mathbf{J}_0 + \mathbf{J}_T + \mathbf{J}', \tag{12}$$

where

$$\mathbf{J}_0 = \frac{i}{\omega'} \frac{\boldsymbol{\theta}}{\gamma^{-2} + \theta^2} \ , \quad \mathbf{J}' = \frac{i}{\omega'} \frac{\mathbf{v}'_\perp - \boldsymbol{\theta}}{\gamma^{-2} + \left(\mathbf{v}'_\perp - \boldsymbol{\theta}\right)^2} e^{\frac{i\omega'}{2}\xi_T},$$

$$\mathbf{J}_T = \int\limits_0^{\xi_T} d\xi \left[\mathbf{r}_\perp(t) - \boldsymbol{\theta} t\right] e^{\frac{i\omega'}{2}\xi}. \tag{13}$$

The subsequent computation is reduced to the estimation of the integral $\mathbf{J}_T$. For this purpose, we divide the time interval T into N segments $0 < t_1 < t_2 < \ldots < t_i < \ldots < t_N = T$. According to (9), these subintervals correspond to ones of ξ: $0 < \xi_1 < \xi_2 < \ldots < \xi_i < \ldots < \xi_T$ (note that we need to solve the equations of motion to find the values ξ_i, while the set of values of ξ_i will be different at each $\boldsymbol{\theta}_i$). Then the integral $\mathbf{J}_T$ can be written as

$$\mathbf{J}_T = \sum_{n=0}^{N-1} \int\limits_{\xi_n}^{\xi_{n+1}} d\xi \left[\mathbf{r}_\perp\left(t(\xi)\right) - \boldsymbol{\theta} t(\xi)\right] e^{\frac{i\omega'}{2}\xi} \ . \tag{14}$$

The electron trajectory in the external field is a smoothly varying function of time (assuming that the external field has a similar behavior with respect to the coordinates). So, for small intervals of ξ corresponding to ones of time t_i, the trajectory can be approximated by a polynomial in powers of $(\xi - \xi_i)$. If we use the third-power polynomial in such an approximation in the interval n of ξ

$$\mathbf{r}_\perp(\xi) = \mathbf{r}_n^{(0)} + \mathbf{r}_n^{(1)}(\xi - \xi_n) + \mathbf{r}_n^{(2)}(\xi - \xi_n)^2 + \mathbf{r}_n^{(3)}(\xi - \xi_n)^3, \tag{15}$$

then the expansion coefficients $\mathbf{r}_n^{(m)}$ can be defined by the particle's coordinates and velocities at the beginning $\mathbf{r}_\perp(\xi_n)$, $\mathbf{v}_\perp(\xi_n)$ and at the end $\mathbf{r}_\perp(\xi_{n+1})$, $\mathbf{v}_\perp(\xi_{n+1})$ of the interval $\xi_n \le \xi \le \xi_{n+1}$ (values of $\mathbf{r}_\perp(\xi_n)$ and $\mathbf{v}_\perp(\xi_n)$ are determined by the solution of the equations of motion). Using a polynomial of this kind for the

trajectory approximation has the following advantages. First, the trajectory $\mathbf{r}_\perp(\xi_n)$ and velocity $\mathbf{v}_\perp(\xi_n)$ are smoothly varying functions of ξ, in this case, throughout the interval $0 \le \xi \le \xi_T$. In other words, the transition from one interval of ξ to another one has no discontinuities of the trajectory and the velocity. Second, the integration of each term in Eq. (14) can be performed analytically. A similar approximation is also fulfilled for the quantity $t(\xi)$ in Eq. (14). As a result, the calculation of $\mathbf{J}_T$ is reduced to the solution of algebraic equations to find the constants $\mathbf{r}_n^{(m)}$ in Eq. (15) and to calculate the sum in Eq. (14). All these techniques allow a considerable speeding up of the computation procedure due to a decrease in the number of trajectory dissections N without losing the accuracy.

4. Results and Discussion

Based on the method stated above, it is possible to develop a procedure for numerical calculations of the spectral and spectral-angular radiation densities by relativistic electrons and positrons in fields of complex configurations such as a crystal lattice. We will exhibit the main computing stages and the obtained results by a simple example that is relevant to the evaluation of the spectral radiation distributions for an electron impact with a separate atomic string in the crystal under a small angle ψ to its z-axis. Such a problem arises in the case of the penetration of a particle through a crystal near to one of the crystalline axes [2]. In this case, the electron sequentially collides with various atomic strings in the crystal that are oriented in parallel with the crystalline axis. If we can neglect the interference effect of waves which are radiated at the electron collisions with different atomic strings, then the radiation can be assumed to be independent on different strings, and the problem is transformed to that of the radiation in the field only of a single string. It is essential that, for the analysis of the electron radiation in a crystal, one needs to take into account the recoil and nondipole effects for many of typical experimental situations (see, e.g., [7]). Both these effects are accounted by formula (5). This is the case, for example, when an electron beam with an energy of 100 GeV enters into the crystal at the angle ψ with respect to one of the crystalline axes which is about several times more than the critical axial-channeling angle ψ_c.

The high-energy electron motion in the field of a crystal atomic string near to its axis is mainly determined by the continuous potential which is the potential of a string averaged along this axis. The particle's momentum component that is in parallel to the string axes is conserved in such a field. Thus, the motion in a cross plane is defined by the two-dimensional equation [2, 8]

$$\ddot{\boldsymbol{\rho}} = -\frac{e}{\varepsilon}\frac{\partial}{\partial \boldsymbol{\rho}} U(\boldsymbol{\rho}) \ , \tag{16}$$

where $\boldsymbol{\rho} = (x, y)$ are the coordinates in the plane that is orthogonal to the string axis, ε is the electron energy, and $U(\boldsymbol{\rho})$ is the continuous potential of a string. In the calculations, we used the approximation for the continuous potential of an atomic string by the function

$$U(\rho) = \begin{cases} U_0 \dfrac{1 - \rho^2/a^2}{1 + c\rho^2/R_e^2}, & \rho < a, \\ 0, & \rho > a, \end{cases} \tag{17}$$

where a is a half of the average distance between atomic strings in the crystal, and R_e is the screening radius for the potential of a separate atom in the crystal. The function of such a kind allows one to achieve a good approximation of real the continuous potential for the atomic string by fitting the coefficients U_0 and c in Eq. (17) (see, e.g., [2]). In particular, at values of the parameters $U_0 = 935$ eV and $c = 0.805$, function (17) gives a good ($\sim 10\%$) approximation for the continuous potential of atomic string $\langle 111 \rangle$ in tungsten crystals at the temperature $T = 300$ K. Far from a string axis ($\rho > a$), we have supposed its potential equals zero in view of that $2a$ represents the average distance between atomic strings in a crystal.

Equation (16) specifies the time dependence for transverse components of the particle velocity and a trajectory in the atomic string field. In the problem considered by us, the parallel beam of particles falls on the atomic string at a small angle ψ to its axis. Thus, the particle trajectories will be determined only by the string impact parameter b [2]. In view of that and according to Eq. (1), the spectral distribution of photons radiated by a unit electron flow in an atomic string field can be written as

$$\frac{d\sigma}{d\omega} = L\psi \int\limits_{-\infty}^{\infty} db \frac{1}{\omega} \int do \frac{dE}{d\omega do} \ , \tag{18}$$

where L is the string length, and $dE/d\omega do$ is the spectral-angular energy distribution (5) of photons radiated by an electron relevant to the trajectory $\mathbf{r}(t, b)$. In the numerical calculations, the time dependence of the transverse electron velocity in the field of an atomic string is determined by a numerical solution of the motion equations (16). To compute the values $\mathbf{J}_T$ and

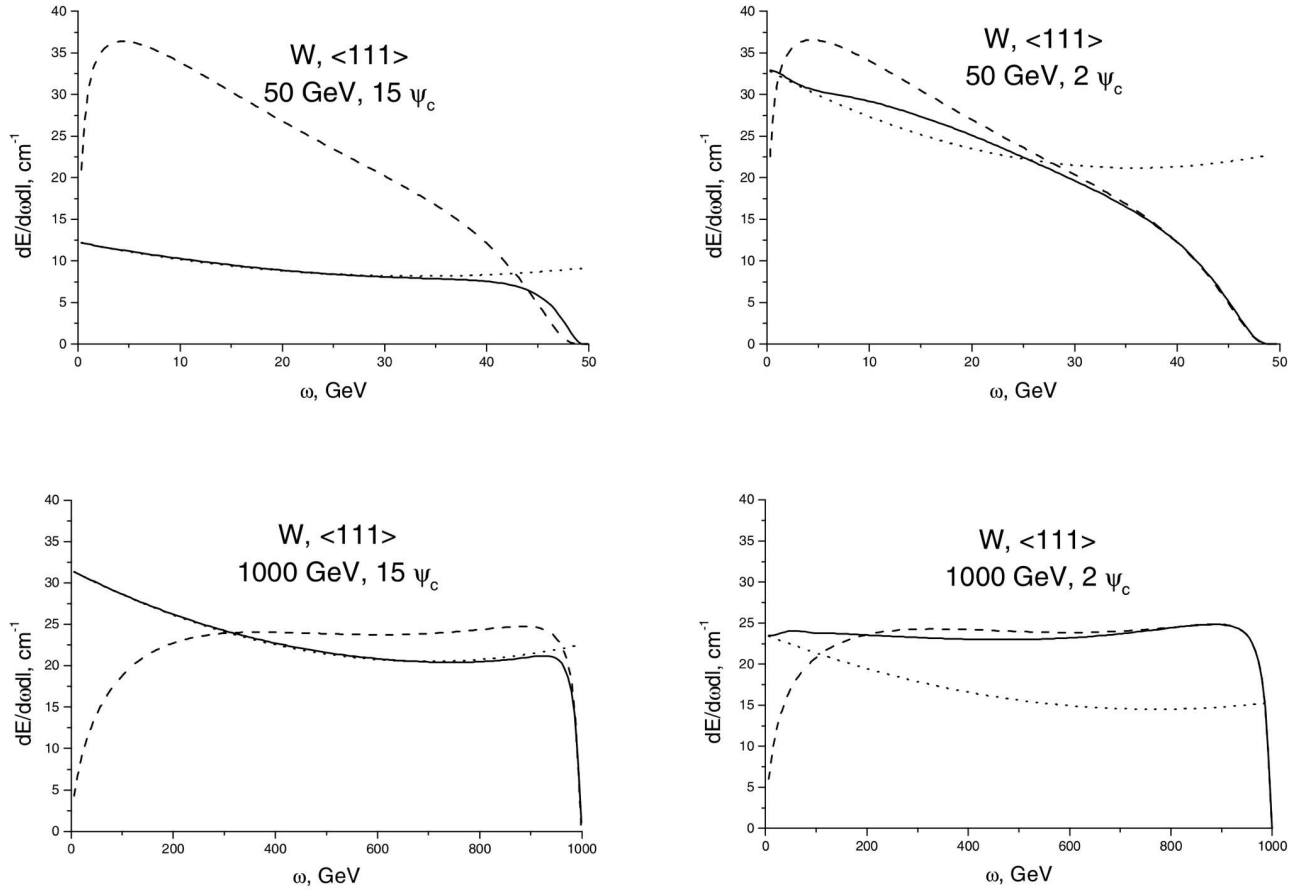

Fig. 1. Spectral densities of the radiation by electrons with energies of 50 GeV and 1000 Gev incident on an atomic string at the angles $\psi = 15\psi_c$ and $\psi = 2\psi_c$ to axis $\langle 111 \rangle$ of a tungsten crystal. Solid lines are the results of numerical calculations using Eq. (18), dashed lines are the results of calculations in the constant field approximation, and dotted lines correspond to the angle-form trajectory approximation

J_T, an electron trajectory in the field of an atomic string (i.e., within $\rho \leq a$) was divided into $N \sim 20$ segments. The beginning in each segment matched to the time t_n. For each t_n, ξ_n was calculated to build up the base of data on $\mathbf{r}_\perp(t_n)$ and $\mathbf{v}_\perp(t_n)$. The coefficients $\mathbf{r}_n^{(m)}$ included in (15) and the value of sum (14) were calculated by using this database. These calculations were carried out for various impact parameters b and angles of radiation θ_x and θ_y. Then, the numerical integration of (18) was fulfilled.

In Fig. 1, the solid lines are the results of computing the spectra of the radiation by electrons with the energies $\varepsilon = 50$ GeV and $\varepsilon = 1$ TeV when the initial beam falls on an atomic string into a tungsten crystal at the angles $\psi = 15\psi_c$ and $\psi = 2\psi_c$ to axis $\langle 111 \rangle$. Along the ordinate and abscissa axes, we show $\omega d\sigma/d\omega dl$ in cm^{-1} and ω in GeV, respectively.

The description of the electron radiation in the field of an atomic string can be carried out in some cases by using simplified models based on such approximations as the angle-form trajectory and the constant-field approximation. Utilizing these approximations allows one to essentially simplify the radiation definition at the interaction of particles with an atomic string. This is of importance for studying the particle radiation under actual particle dynamics in a crystal. The analysis of the possibilities to use these simplified formulae for the description of the fast particle radiation in crystals is very important. To this end, we compare the results computed within a numerical method by formula (5) with those obtained in the approximations of an angle-form trajectory and a constant field.

The angle-form trajectory approximation corresponds to the case where the electron radiation in the

field of an atomic string has the formation length $l_c = \frac{2\varepsilon\varepsilon'}{m^2\omega}(1+\gamma^2\theta_e^2+\gamma^2\theta^2)^{-1}$ that is large in comparison with the length of the order a/ψ, on which the electron interacts with an atomic string, where θ_e is the scattering angle of the electron. In this case, as shown in [2], the spectral density of the radiation is defined by the formula

$$\frac{dE}{d\omega} = \frac{2e^2}{\pi}\frac{\varepsilon'}{\varepsilon} \times \\ \times \left[\frac{2\xi^2(1+\omega^2/2\varepsilon\varepsilon')+1}{\xi\sqrt{\xi^2+1}} \ln\left(\xi+\sqrt{\xi^2+1}\right) - 1\right], \quad (19)$$

where $\xi = \gamma\theta_e/2$ and $\theta_e = \theta_e(b)$. Note that, for high-energy electrons and the small energies of radiated photons, the condition of applicability of this formula, $l_c \gg a/\psi$, holds always. The radiation spectrum (19) does not depend, in this case, on the trajectory details in the atomic string field and is determined only by the particle scattering angle $\theta_e(b)$.

The constant field approximation corresponds to the case where the conditions $l_c \ll a/\psi$ and $\gamma\theta_e \gg 1$ are satisfied. In this case, the radiation cross-section according to the relation

$$\frac{d\sigma}{d\omega} = \frac{2\pi L}{\omega}\int\limits_0^\infty \rho d\rho \frac{dE(u)}{dtd\omega} \quad (20)$$

can be interlinked with the spectral density of synchrotron radiation normalized to a unit path of a particle moving in a constant field

$$\frac{dE}{dtd\omega} = -\frac{e^2}{2\sqrt{\pi}}\frac{\varepsilon^2+\varepsilon'^2}{\varepsilon\varepsilon'}\frac{\omega}{\gamma^2} \times \\ \times \left[\left(1-\frac{\omega^2}{\varepsilon^2+\varepsilon'^2}\right)\int\limits_u^\infty du'\Phi(u') + \frac{2}{u}\frac{\partial}{\partial u}\Phi(u)\right], \quad (21)$$

where $u = \gamma^{-2}(\omega'/|\dot{\mathbf{v}}|)^{2/3}$, $|\dot{\mathbf{v}}| = \left|\frac{e}{\varepsilon}\frac{\partial}{\partial\rho}U(\rho)\right|$, and $\Phi(u)$ is Airy function.

For $\omega \ll \varepsilon$, this formula transforms into the classical result referred to the radiation by relativistic electrons in the field of an atomic string of the crystal [9]. Note that the quantity $dE(u)/dtd\omega$ in Eq. (20) depends only on the integration variable ρ. The radiation cross-section (20) does not depend on the angle of the particle incidence on a string. The value of ψ defines, however, the applicability requirement for formula (21).

Let's note that formulae (19) and (21) are considerably simpler than the exact formulae founded with the use of relation (5). Therefore, the adaptation of the former for the analysis of the electron radiation in the atomic string field allows the essential speeding up of calculations in comparison with formula (5).

The dashed and dotted lines in Fig. 1 correspond, respectively, to the calculations by formulae (20) and (19). The graphs demonstrate that, at large angles of electron incidence on a string, the exact calculation results for the radiation cross-sections are in a good accordance with those obtained in the angle-form trajectory approximation almost through the whole frequency range of radiated photons. This is explained by the fact that, at large angles of the particle incidence on a string, the requirement of the applicability of formula (19) is satisfied practically for the whole frequency range of radiated photons.

The situation at small angles of the particle incidence on a string is rather different from that of the electron radiation in a string field at large values of the angle ψ. Namely, in the range of low frequencies, the exact calculations approach the results obtained in the angle-form trajectory approximation and considerably diverge with those obtained in the constant-field approximation. The reason is that, the strong modification of an atom-string field along the electron trajectory in the region of coherent length takes place in a low frequency range. Hence, the requirements of applicability for the formulas used by the synchrotron radiation theory are not fulfilled. The other situation occurs for the high-frequency range of radiated photons. In this case, the atomic string field varies smoothly on the coherent length of radiation. So, the results of exact calculations approach those obtained in the constant-field approximation. Note that, under the constant-field approximation, the electron radiation cross-section in the atomic string field of a crystal does not depend on the angle ψ. The results of exact calculations, as show the graphs in Fig. 1, have a weak dependence of the radiation cross-section on ψ at $\psi \sim \psi_c$. This is caused by that the value ψ defines the region of applicability for the formulae obtained in the constant-field approximation.

The applicability region of the formulae describing the radiation from an angle-form trajectory is extended with increase in the particle energy. Note that, at the same time, the coherent length of radiation is rapidly increased as well. Thus, if the electron collides with several atomic strings in the crystal in the scope of the coherent length of radiation, it is necessary also to take into account the influence of the multiple scattering of a particle on atomic strings on the radiation alongside with the above-considered effects (the recoil at radiation, the nondipole behavior of radiation, etc.). Such an investi-

gation, however, is beyond the frame of the present work. We only remark that, in principle, it can be considered on the basis of the above-considered numerical calculations by applying them to the case of the motion of a particle in the periodic field of crystal atomic strings. At this, however, the calculation volume will be considerably increased.

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ПРО СПЕКТРАЛЬНІ РОЗПОДІЛИ ВИПРОМІНЮВАННЯ ЕЛЕКТРОНІВ ВЕЛИКОЇ ЕНЕРГІЇ В ПОЛІ ЛАНЦЮЖКА АТОМІВ КРИСТАЛА

В.П. Лапко, М.Ф. Шульга, О.С. Єсаулов

Р е з ю м е

Досліджуються квазікласичні формули для спектрального і спектрально-кутового розподілів випромінювання релятивістських електронів у зовнішньому неоднорідному полі з урахуванням ефекту віддачі при випромінюванні та недипольності випромінювання. Викладено метод чисельних розрахунків за цими формулами. Розглянуто граничні випадки цих формул, що відповідають випромінюванню із траєкторії у вигляді кута і наближенню постійного поля. Проведено зіставлення результатів точних і наближених розрахунків за цими формулами на прикладі випромінювання електронів великої енергії в полі окремого ланцюжка атомів кристала.