ENERGY WEIGHTED SUMS FOR COLLECTIVE EXCITATIONS IN NUCLEAR FERMI-LIQUID

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Model independent, m_1, adiabatic, m_{-1}, and high-energy, m_3, energy weighted sums (EWS) for the isoscalar and isovector nuclear excitations are investigated within the framework of the kinetic theory adapted to the description of a two-component nuclear Fermi-liquid. For both the adiabatic and scaling approximations, the connection of the EWS m_{-1} and m_3 to the nuclear stiffness coefficients and the first- and zero-sound velocities is established. We study the enhancement factor κ_I in the energy weighted sum m_1' for the isovector excitations and provide the reasonable explanation of the experimental exceeding of the 100% exhaustion of sum m_1' for the isovector giant dipole resonances. We show the dependence of the enhancement factor κ_I on the nuclear mass number A and analyze its dependence on the Landau's isovector amplitude F_1'.

1. Introduction

The strength function is the basic characteristic which determines the behavior of a quantum system in an external periodic field $U_{\rm ext} = \lambda(t)\hat{q}$ $(\lambda(t) = \lambda_0 e^{-i\omega t} + c.c.$, here, $\hat{q}$ is the transition operator)

$$S(E) = \sum_{n\neq 0} |\langle\Psi_n|\ \hat{q}\ |\Psi_0\rangle|^2 \delta(E - E_n), \quad E = \hbar\omega, \qquad (1)$$

where Ψ_n and E_n are the eigenfunctions and the eigenenergies of the total Hamiltonian $\hat{H}$, respectively. Using the strength function $S(E)$, one can calculate the moments m_k (EWS)

$$m_k = \int dE\, S(E)\, E^k = \sum_{n\neq 0} |\langle\Psi_n|\ \hat{q}\ |\Psi_0\rangle|^2 (E_n - E_0)^k. \qquad (2)$$

Here, for convenience, we have included the ground state energy $E_0 = 0$ into the energy factor. Special role of the EWS m_k is caused by its connection to the transport characteristic of the system. For example, the sums m_{-1} and m_{-3} determine the stiffness and mass coefficients for the collective excitations in the system [1]. Determined via the properties of the ground state of the system, the sum m_1 plays a specific role. In many cases, it does not depend on the model used for the description of the collective motion. This allows one to test the results of theoretical calculations as well as the correctness and the completeness of the experimental data.

During a few years, the significant attention was paid to the analysis of EWS for the giant multipole resonances (GMR) [2–10]. Nuclear giant resonances exhaust a significant part of EWS (sometimes near 100%) and establish the relatively simple connection between the values of m_k and the basic characteristics of the GMR. However, some problems occur while researching the EWS for the isovector giant dipole resonances which are best investigated experimentally. The connected problem is that the sum m_1 is not model independent because of the dependence of the effective nuclear forces on the nucleon velocity. Thus, for the theory to agree with the experimental data, one has to include a phenomenological enhancement factor to the sum m_1 [6]. As a consequence, this leads to the modification of other sums m_k and can affect the definition of the nuclear transport characteristics.

In this work, we study the EWS m_k for the isovector collective excitations in heavy nuclei and nuclear matter. Our approach is based on the kinetic Landau—Vlasov's theory adapted to a two-component nuclear Fermi-liquid. In Section 2, we consider the connection between the EWS m_k and the linear response function. The connection of the Landau's theory of the Fermi-liquid to the hydrodynamical model and to the scaling approximation is shown in Section 3 [11, 12]. In Section 4, we apply our approach to finite nuclei. The main conclusions of the work are formulated in Section 5.

2. Linear Response Function and EWS

Let us consider the response of a nucleus on an external field $U_{\rm ext}(t)$ periodic in time which is switched on adiabatically at $t = -\infty$:

$$U_{\rm ext}(t) = \lambda_0 e^{-i(\omega+i0)t}\hat{q} + \lambda_0^* e^{i(\omega-i0)t}\hat{q}^*, \qquad (3)$$

where $\hat{q}$ is the Hermitian operator,

$$\hat{q} = \sum_{i=1}^{A} \hat{q}(\vec{r}_i, \tau_i), \tag{4}$$

A is the mass number, and τ_i is the isotopic variable. If $\lambda_0 \ll 1$, then quantum mechanical expectation of the operator $\hat{q}$ takes the following form (see [13])

$$\langle \hat{q} \rangle = \chi(\omega)\lambda_0 e^{-i\omega t} + \chi^*(\omega)\lambda_0^* e^{i\omega t}, \tag{5}$$

where $\chi(\omega)$ is the linear response function

$$\chi(\omega) = \sum_n |\langle \Psi_n | \hat{q} | \Psi_0 \rangle|^2 \times$$

$$\times \left[\frac{1}{E_n - E_0 - \hbar\omega - i0} + \frac{1}{E_n - E_0 + \hbar\omega + i0} \right]. \tag{6}$$

Let us introduce the polarization response function

$$\chi^{(\pi)}(\omega) = \mathrm{Re}\chi(\omega) =$$

$$= -2 \sum_n |\langle \Psi_n | \hat{q} | \Psi_0 \rangle|^2 \frac{E_n - E_0}{(\hbar\omega)^2 - (E_n - E_0)^2}. \tag{7}$$

It is easy to establish the connection between the EWS m_k and the linear response function $\chi(\omega)$. Let us take the Taylor expansion of the function $\chi^{(\pi)}(\omega)$ in a series in $\hbar\omega$ as $\omega \to 0$ (adiabatic expansion) and in a series in $(\hbar\omega)^{-1}$ as $\omega \to \infty$ (high-frequency expansion). Using (2) and (7), we have

$$\chi^{(\pi)}(\omega)\Big|_{\omega\to 0} = 2 \left[m_{-1} + (\hbar\omega)^2 m_{-3} + ... \right], \tag{8}$$

$$\chi^{(\pi)}(\omega)\Big|_{\omega\to \infty} = -\frac{2}{(\hbar\omega)^2} \left[m_1 + (\hbar\omega)^{-2} m_3 + ... \right]. \tag{9}$$

Below, we will pay a special attention to the investigation of the sums m_{-1}, m_1, and m_3. Using these sums, one can define two averaged energies of collective motion

$$\tilde{E}_1 = \sqrt{\frac{m_1}{m_{-1}}} \quad \text{and} \quad \tilde{E}_3 = \sqrt{\frac{m_3}{m_1}}. \tag{10}$$

It is easy to see that the closeness of the energies $\tilde{E}_1$ and $\tilde{E}_3$ to each other determines the exhaustion of the EWS m_k by one state Ψ_n (see (2)). If the effective nuclear forces do not depend on the nucleon velocity, then the sum m_1 can be easily calculated and takes the form which does not depend on the model of collective motion. Namely,

$$m_1 = \frac{1}{2} \left\langle \Psi_0 \left| \left[\hat{q}, \left[\hat{q}, \hat{H} \right] \right] \right| \Psi_0 \right\rangle =$$

$$= \sum_{n\neq 0} |\langle \Psi_n | \hat{q} | \Psi_0 \rangle|^2 (E_n - E_0) =$$

$$= \frac{\hbar^2}{2m} \int d\vec{r}\, \rho_{\mathrm{eq}}(\vec{r}) |\vec{\nabla}\hat{q}(\vec{r})|^2, \tag{11}$$

where $\rho_{\mathrm{eq}}(\vec{r})$ is the nucleon density for the ground state of the nucleus,

$$\rho_{\mathrm{eq}}(\vec{r}) = \left\langle \Psi_0 \left| \sum_{i=1}^{A} \delta(\vec{r} - \vec{r}_i) \right| \Psi_0 \right\rangle.$$

(Here, and in the following, the symbol "eq" means that the proper value is related to the equilibrium (basic) state of the nucleus.) Expression (11) is the so-called model independent EWS rule. If only one (collective) state $\Psi_{n=G}$ exhausts the sum rule (11), i.e.,

$$m_1 \approx |\langle \Psi_G | \hat{q} | \Psi_0 \rangle|^2 (E_G - E_0), \tag{12}$$

then we have $\tilde{E}_1 \approx \tilde{E}_3$ from (2), (10).

The low-frequency (adiabatic) sum m_{-1} is connected to the nuclear stiffness under the adiabatic slow deformation of the nucleus, in another words under the deformation that does not lead to the quantum transitions between nuclear levels. To reveal this connection, we will evaluate the energy variation ΔE of the nuclear ground state in an external *static* field $U_{\mathrm{ext}} = \lambda_0 \hat{q}$ for $\lambda_0 \to 0$. Using the quantum perturbation theory for the calculation of the wave function Ψ of Hamiltonian $\hat{H}' = \hat{H} + \lambda_0 \hat{q}$ in the second order in the small parameter λ_0, we obtain

$$\Delta E_{\mathrm{ad}} = \langle \Psi | \hat{H} | \Psi \rangle - \langle \Psi_0 | \hat{H} | \Psi_0 \rangle = \lambda_0^2 m_{-1}. \tag{13}$$

Let us calculate the variation of the nuclear form parameter $Q = \langle \Psi | \hat{q} | \Psi \rangle$ in the external field $\lambda_0 \hat{q}$,

$$\Delta Q = Q = \langle \Psi | \hat{q} | \Psi \rangle - \langle \Psi_0 | \hat{q} | \Psi_0 \rangle = 2\lambda_0 m_{-1}, \tag{14}$$

where we have assumed $\langle \Psi_0 | \hat{q} | \Psi_0 \rangle = 0$. From (13) and (14), we find the nuclear stiffness parameter $C_{Q,\mathrm{ad}}$ with respect to the adiabatic change of the nuclear form as

$$C_{Q,\mathrm{ad}} = \frac{\partial^2 \Delta E_{\mathrm{ad}}}{\partial Q^2} = \frac{1}{2\, m_{-1}}. \tag{15}$$

Let us now consider the high-frequency sum m_3. We introduce the wave function $\Psi_{\rm sc}$ which is obtained from the wave function of the nuclear ground state Ψ_0 by means of the scale transformation (scaling-approximation),

$$\Psi_{\rm sc} = e^{\nu[\hat{H},\hat{q}]}\Psi_0, \tag{16}$$

where ν is the small parameter of the scale transformation.

In the case of a many-particle wave function Ψ_0 given by the determinant built on the one-particle wave functions $\phi_\alpha(\vec{r})$, the exponential operator of the scale transformation in (16) acts on each function $\phi_\alpha(\vec{r})$ independently. For example, at the quadrupole deformation

$$\hat{q} = \sum_{i=1}^{A}(r_i^2 - 3z_i^2),$$

one can see from (16) that $\Psi_{\rm sc}$ is also a determinant which is built on the one-particle functions $\phi_{\alpha,{\rm sc}}(\vec{r})$ obtained by the scale transformation of coordinates. Namely,

$$\phi_{\alpha,{\rm sc}}(\vec{r}) \equiv \phi_{\alpha,{\rm sc}}(x,y,z) = \phi_\alpha(e^{\tilde{\nu}}x,\ e^{\tilde{\nu}}y,\ e^{-2\tilde{\nu}}z), \tag{17}$$

where $\tilde{\nu} = -2\hbar^2\nu/m$. As can be seen from (17), the scale transformation does not violate the orthonormalization of the wave functions. By means of (16), the energy change ΔE can be found within the scaling approximation as

$$\Delta E = \langle\Psi_{\rm sc}|\ \hat{H}\ |\Psi_{\rm sc}\rangle - \langle\Psi_0|\ \hat{H}\ |\Psi_0\rangle = \nu^2\, m_3. \tag{18}$$

Using (16), we obtain the connection between the parameter of scale transformation ν and the deformation parameter Q:

$$Q = \langle\Psi_{\rm sc}|\ \hat{q}\ |\Psi_{\rm sc}\rangle - \langle\Psi_0|\ \hat{q}\ |\Psi_0\rangle = 2\,\nu\, m_1. \tag{19}$$

Finally, using (18) and (19), we obtain the nuclear stiffness coefficient $C_{Q,{\rm sc}}$ in the scaling approximation as

$$C_{Q,{\rm sc}} = \frac{\partial^2\Delta E}{\partial q^2} = \frac{m_3}{2\,m_1^2}, \tag{20}$$

which differs significantly from the adiabatic one $C_{Q,{\rm ad}}$ (15). The reasons of such a difference will be made clear in the next section.

3. Response Function and EWS for Nuclear Fermi-liquid

It is necessary to make some additional assumptions for the practical calculation of the linear response function $\chi(\omega)$ and the corresponding EWS m_k. We will restrict ourselves to the Landau's approximation for a nuclear Fermi-liquid and use the linearized Landau—Vlasov equation [14]. In the two-component nuclear Fermi-liquid, it is necessary to consider two possibilities: isoscalar excitations (when protons and neutrons move in phase) and isovector excitations (when protons and neutrons move in antiphase).

3.1. Isoscalar excitations

For the nuclear matter in a volume V in the case of isoscalar excitations, the linearized kinetic Landau—Vlasov equation has the same form as that for a one-component Fermi-liquid [12]

$$\frac{\partial}{\partial t}\delta f + \vec{v}\cdot\vec{\nabla}_r\delta f - \vec{\nabla}_p f_{\rm eq}\cdot\vec{\nabla}_r(\delta U_{\rm self} + U_{\rm ext}) = 0, \tag{21}$$

where $\delta f = \delta f_n + \delta f_p \equiv \delta f(\vec{r},\vec{p};t)$ is the variation of the nucleon distribution (δf_n for neutrons and δf_p for protons) in a phase space, $\vec{v}$ is the nucleon velocity, $f_{\rm eq} = f_{{\rm eq},n} + f_{{\rm eq},p} \equiv f_{\rm eq}(\vec{r},\vec{p})$ is the equilibrium distribution function, $\delta U_{\rm self} \equiv \delta U_{\rm self}(\vec{r},\vec{p};t)$ is a variation of the self-consistent mean field. The subscripts at $\vec{\nabla}$ in (21) indicate the variables of differentiation. The variation of the self-consistent field $\delta U_{\rm self}$ depends on the effective nucleon-nucleon interaction $v_{\rm int}$. In the case of the homogeneous nuclear matter, it is given by

$$\delta U_{\rm self} = \int \frac{2V d\vec{p}'}{(2\pi\hbar)^3}\ v_{\rm int}(\vec{p},\vec{p}')\ \delta f(\vec{r},\vec{p}';t), \tag{22}$$

where the additional factor 2 at the numerator is due to the spin degeneration.

The effective interaction $v_{\rm int}(\vec{p},\vec{p}')$ is connected to the Landau's interaction amplitudes F_l [12]

$$v_{\rm int}(\vec{p},\vec{p}') = \frac{1}{N_{\rm F}}\sum_{l=0}^{\infty}F_l P_l(\cos\theta_{pp'}). \tag{23}$$

Here, $P_l(x)$ are the Legendre polynomials, $\theta_{pp'}$ is the angle between the vectors $\vec{p}$ and $\vec{p}'$ and $N_{\rm F}$ is the density of states near the Fermi surface,

$$N_{\rm F} = -4\pi\int\frac{2Vp^2}{(2\pi\hbar)^3}\ \frac{\partial f_{\rm eq}}{\partial\varepsilon_p}\ dp = \frac{Vm^*p_{\rm F}}{\pi^2\hbar^3}, \tag{24}$$

where $\varepsilon_p = p^2/2m^*$, m^* is the effective mass of a nucleon (the definition of m^* is given below), and $p_{\rm F}$ is the Fermi momentum. In (24), we have used the equilibrium Fermi distribution function $f_{\rm eq} = \theta(\varepsilon_{\rm F} - \varepsilon_p)$, where $\theta(x)$ is the Heaviside step function and $\varepsilon_{\rm F} = p_{\rm F}^2/2m^*$ is the Fermi energy. The presence of components with $\ell \neq 0$ in sum (23) is caused by the dependence of the nuclear forces on the nucleon velocities. Below, we will restrict ourselves to the most important case where

$$F_0 \neq 0, \quad F_1 \neq 0, \quad F_{l\geq 2} = 0. \tag{25}$$

Note, that the interaction amplitude F_1 determines the effective mass of a nucleon [14]

$$m^* = (1 + F_1/3)m. \tag{26}$$

We will introduce a variation of the nucleon density $\delta\rho \equiv \delta\rho(\vec{r}, t)$ and the isoscalar velocity field $\vec{u} = \vec{u}(\vec{r}, t)$, which are connected to a variation of the distribution function $\delta f = f - f_{\rm eq} \equiv \delta f(\vec{r}, \vec{p}; t)$ by the relations

$$\delta\rho = \int \frac{2\,d\vec{p}}{(2\pi\hbar)^3}\delta f,$$

$$\vec{u} = \frac{1}{\rho}\int \frac{2\,d\vec{p}}{(2\pi\hbar)^3}\frac{\vec{p}}{m}\delta f \approx \frac{1}{\rho_{\rm eq}}\int \frac{2\,d\vec{p}}{(2\pi\hbar)^3}\frac{\vec{p}}{m}\delta f, \tag{27}$$

where

$$\rho \equiv \rho(\vec{r}, t) = \int \frac{2\,d\vec{p}}{(2\pi\hbar)^3} f(\vec{r}, \vec{p}; t),$$

$$\rho_{\rm eq} \equiv \rho_{\rm eq}(\vec{r}) = \int \frac{2\,d\vec{p}}{(2\pi\hbar)^3} f_{\rm eq}(\vec{r}, \vec{p}) \tag{28}$$

is the nucleon density. The velocity field $\vec{u}$ and the variation of the nucleon density $\delta\rho$ satisfy the continuity relation

$$\frac{\partial}{\partial t}\delta\rho + \vec{\nabla}\rho\,\vec{u} = 0. \tag{29}$$

To check this relation, we calculate the zero-moment of the kinetic equation (21). Multiplying Eq. (21) by $2d\vec{p}/(2\pi\hbar)^3$ and integrating over $\vec{p}$, we obtain

$$\frac{\partial}{\partial t}\delta\rho + \vec{\nabla}_r\frac{m}{m^*}\rho\,\vec{u} + \int \frac{2d\vec{p}}{(2\pi\hbar)^3} f_{\rm eq}\vec{\nabla}_r\cdot\vec{\nabla}_p\delta U_{\rm self} = 0. \tag{30}$$

Using Eqs. (23)–(25) and (28), we have

$$\delta U_{\rm self} = \frac{V}{N_{\rm F}}\left(F_0\,\delta\rho + \frac{F_1}{p_{\rm F}^2} m\rho\,\vec{p}\cdot\vec{u}\right). \tag{31}$$

Substituting Eq. (31) into Eq. (30) and taking the definition of m^* (26) into account, we derive the continuity equation (29).

To solve the kinetic equation (21), we assume that the external field is given by a plane wave $\lambda_0 e^{i(\vec{q}\cdot\vec{r}-\omega t)}$. Then the solution of Eq. (21) can be presented as [14]

$$\delta f \equiv \delta f_{\vec{q}}(\vec{r}, \vec{p}; t) = -\frac{\partial f_{\rm eq}}{\partial\varepsilon_p}\nu_{\vec{q}}(\vec{p})\,e^{i(\vec{q}\cdot\vec{r}-\omega t)}, \tag{32}$$

where $\nu_{\vec{q}}(\vec{p})$ is the unknown function. Substituting Eq. (32) into Eq. (21), we obtain the following equation for $\nu_{\vec{q}}(\vec{p})$:

$$(\omega - \vec{q}\cdot\vec{v})\,\nu_{\vec{q}}(\vec{p}) + \vec{q}\cdot\vec{v}\int \frac{2V d\vec{p}'}{(2\pi\hbar)^3}\,v_{\rm int}(\vec{p}, \vec{p}')\,\frac{\partial f_{\rm eq}}{\partial\varepsilon_p'}\times$$

$$\times\nu_{\vec{q}}(\vec{p}) + \lambda_0\,\vec{q}\cdot\vec{v} = 0. \tag{33}$$

Let us expand the function $\nu_{\vec{q}}(\vec{p}, t)$ in a power series in the multipolarity l of a Fermi surface distortion:

$$\nu_{\vec{q}}(\vec{p}) = \sum_{l=0}^{\infty} P_l(\cos\theta_{pq})\,\nu_l, \tag{34}$$

where θ_{pq} is the angle between the vectors $\vec{p}$ and $\vec{q}$. Using Eqs. (23), (24), (34), and (33), we obtain the infinite set of equations for the amplitudes ν_l [15]:

$$\nu_l + (2l+1)\sum_{l'}\frac{Q_{ll'}(s)}{2l'+1}F_{l'}\nu_{l'} - \lambda_0(2l+1)Q_{l0}(s) = 0. \tag{35}$$

Here, $s = \omega/qv_{\rm F}$ and

$$Q_{ll'}(s) = \frac{1}{2}\int_{-1}^{1} dx\,P_l(x)\frac{x}{x-s}P_{l'}(x). \tag{36}$$

With regard for condition (25), (35) yields

$$\nu_0(s) = \frac{Q_{00}(s)(1 + F_1/3)}{1 + F_1/3 + Q_{00}(s)(F_0 + F_0F_1/3 + F_1s^2)}\lambda_0, \tag{37}$$

where we have used the relations [15]

$$Q_{10}(s) = s\,Q_{00}(s), \qquad Q_{11}(s) = s\,Q_{10}(s) + \frac{1}{3}. \tag{38}$$

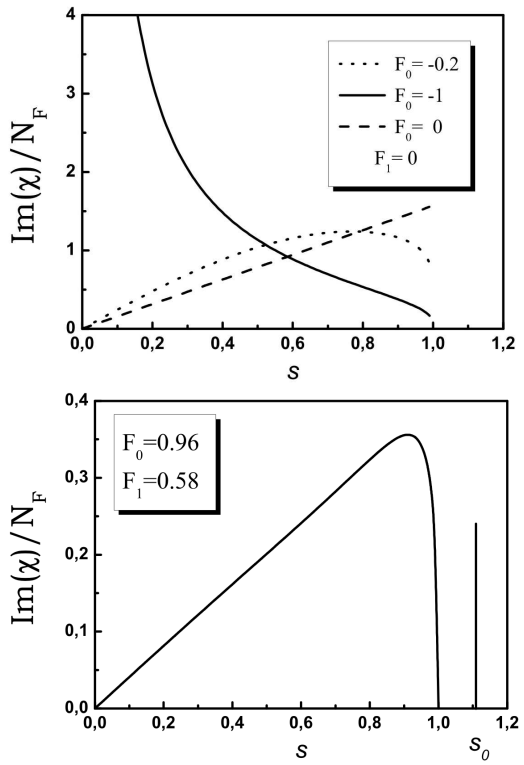

Fig. 1. Dependences of the strength function $\mathrm{Im}\chi(s)$ on the dimensionless parameter s for isoscalar excitations: the upper panel is for the Landau damping regime, $-1 < F_0 \leq 0$; the lower panel is for the zero-sound regime, $F_0 > 0$

The Legendre function of the second kind $Q_{00}(s)$ can be calculated by the use of Eq. (36). Taking the additional condition of the analytical extension of $Q_{00}(s)$ into the complex plane s into account, we can represent the function $Q_{00}(s)$ as

$$Q_{00}(s) = 1 + \frac{s}{2}\,\ln\left|\frac{s-1}{s+1}\right| + i\frac{\pi}{2}s\,\theta(1-|s|). \tag{39}$$

Let us evaluate the density-density response function assuming $\hat{q} = e^{-i\vec{q}\cdot\vec{r}}$ in Eqs. (3)–(5). Using the definition of the linear response function $\chi(\omega)$ from (5) and relations (27), (32), and (34), we obtain

$$\chi(\omega) = \frac{\left\langle e^{-i\vec{q}\cdot\vec{r}}\right\rangle}{\lambda_0 e^{-i\omega t}} =$$

$$= \frac{1}{\lambda_0 e^{-i\omega t}} \int d\vec{r} \int \frac{2d\vec{p}}{(2\pi\hbar)^3} e^{-i\vec{q}\cdot\vec{r}}\delta f(\vec{r},\vec{p};t) =$$

$$= \frac{1}{\lambda_0} N_{\mathrm{F}}\nu_0(s). \tag{40}$$

Finally, taking (37) into account, we obtain the density-density response function as

$$\chi(\omega) = \frac{\overline{Q}_{00}(s)}{1-\kappa(s)\overline{Q}_{00}(s)}, \tag{41}$$

where

$$\kappa(s) = -\frac{1}{N_{\mathrm{F}}}\left(F_0 + \frac{F_1}{1+F_1/3}s^2\right),\ \overline{Q}_{00}(s) = N_{\mathrm{F}}Q_{00}(s).$$

Function (41) has the same form as the collective linear response function in the general theory of collective motion (see, e.g., [18]). The quantity $\overline{Q}_{00}(s)$ is the intrinsic response function, and $\kappa(s)$ plays the role of the effective interaction parameter.

In Fig. 1, we present the dissipative response function

$$\chi^{(d)}(\omega) = \mathrm{Im}\chi(\omega) \tag{42}$$

which is obtained from Eq. (41) for two regimes: the Landau damping regime $-1 < F_0 < 0$, the upper panel, and the zero-sound regime $F_0 > 0$, the lower panel. Note that the zero-sound mode is damped at $s < 1$ (the Landau damping [16]). Here, the zero-sound wave propagates in phase with some particles, and the energy transfer averaged over time from the wave to particles can be positive. The non-damped sound wave exists in a Fermi liquid under the assumption $s > 1$ only. The dimensionless velocity of a sound wave s is determined by the Landau's dispersion equation [14]

$$1 - \kappa(s)\overline{Q}_{00}(s) = 0. \tag{43}$$

If the dispersion equation (43) is satisfied, both the response function (41) and the sound wave amplitude grow to infinity, and the sound wave propagation cannot be described within the framework of a linear response theory. The analysis showed [14, 16] that the solution to Eq. (43) exists (for real values of s) at $F_0 > 0$ only. This is illustrated in Fig. 1. As can be seen from the lower panel of Fig. 1, the isolated root of Eq. (43) exists at $s > 1$ (zero-sound) for $F_0 > 0$ only.

It is easy to see from the dispersion equation (43) that the velocity of the zero-sound increases monotonically with the interaction parameter F_0.

It is useful to consider the solution of the dispersion equation (43) at the asymptotic regime $s \to \infty$ (or $F_0 \to \infty$). Let us use the asymptotic expansion of the Legendre function of the second kind

$$Q_{00}(s)|_{s\to\infty} = -\frac{1}{3s^2} - \frac{1}{5s^4} - \frac{1}{7s^6} - .... \tag{44}$$

From relations (43) and (44), we find the velocity of the zero-sound wave $u_0 = s\,v_{\rm F}$ at $F_0 \to \infty$:

$$u_0|_{F_0\to\infty} = \ \ s|_{F_0\to\infty}\, v_{\rm F} = \sqrt{\frac{F_0}{3mm^*}p_{\rm F}^2}. \qquad (45)$$

Formula (45) can be compared with that for the velocity u_1 of the normal sound (first-sound) in a classical liquid

$$u_1 = \sqrt{\frac{K}{9m}},$$

where K is the incompressibility coefficient. For the Fermi-liquid, the incompressibility coefficient is given by [12]

$$K = 6\,\frac{p_{\rm F}^2}{2m^*}\,(1+F_0) \approx 220\,{\rm MeV}, \qquad (46)$$

and we obtain

$$u_1 = \sqrt{\frac{K}{9m}} = \sqrt{\frac{1+F_0}{3mm^*}p_{\rm F}^2}. \qquad (47)$$

Taking Eqs. (45)–(47) into account, we derive

$$u_0|_{F_0\to\infty} = u_1|_{F_0\to\infty}\,. \qquad (48)$$

This result means that the velocities of the zero- and first sounds in Fermi-liquid coincide at a significant, $F_0 \gg 1$, repulsion between the particles.

Using the expansions of the polarization response function $\chi^{(\pi)}(\omega) = {\rm Re}\chi(\omega)$ (8) and (9) and expression (41), one can find the EWS m_{-1}, m_1 and m_3 for the Fermi-liquid (see also [7]) as

$$m_{-1} = \frac{A}{2}\frac{9}{K},\ m_1 = \hbar^2\frac{A}{2m}\,q^2,\ m_3 = \hbar^4\frac{A}{2}\frac{K'}{9m^2}\,q^4. \qquad (49)$$

Here, we have introduced the renormalized (due to the Fermi surface distortion effect) incompressibility coefficient $K' = K + 24\varepsilon_{\rm F}/5$, see [11]. Using relations (10) and (49), we can derive the average excitation energy (the energy centroids of giant isoscalar resonances) in the adiabatic, $\tilde{E}_1$, and scaling $\tilde{E}_3$, approximations:

$$\tilde{E}_1 = \hbar\sqrt{\frac{K}{9m}}\,q, \qquad \tilde{E}_3 = \hbar\sqrt{\frac{K'}{9m}}\,q. \qquad (50)$$

Using the dispersion relation $\tilde{E} = \hbar\tilde{u}\,q$ between the excitation energy of a sound wave, $\tilde{E}$, and the sound velocity, $\tilde{u}$, and applying Eqs. (50) and (26), we obtain the sound velocity in the adiabatic, $\tilde{u}_1$, and scaling, $\tilde{u}_3$, approximations:

$$\tilde{u}_1 = \sqrt{\frac{(1+F_0)p_{\rm F}^2}{3mm^*}}\,, \qquad \tilde{u}_3 = \sqrt{\frac{(9/5+F_0)p_{\rm F}^2}{3mm^*}}\,. \qquad (51)$$

By comparing Eq. (47) and Eq. (51), it can be seen that the sound velocity in the adiabatic approximation, $\tilde{u}_1$, coincides with the first sound one, u_1, and that the sound velocity in the scaling approximation, $\tilde{u}_3$, exceeds u_1 significantly. The origin of this effect is the same as in the case of the nuclear stiffness coefficients $C_{Q,{\rm ad}}$ and $C_{Q,{\rm sc}}$, see Eqs. (15) and (20).

To clarify the nature of this effect, we will return to the kinetic equation (33) and consider the recurrence method of its solution. For a simplification, we neglect the external field in (33) assuming $\lambda_0 = 0$ and use, instead of (34), the following expansion of the amplitude $\nu_{\vec{q}}(\vec{p},t)$ into a series in the multipolarity l of a dynamic Fermi surface distortion:

$$\nu_{\vec{q}}(\vec{p}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\nu_{lm}(q)\,Y_{lm}(\hat{p}). \qquad (52)$$

Substituting amplitude (52) into Eq. (33), using expressions (23) and (24), multiplying then Eq. (33) by the spherical function $Y^*_{lm}(\hat{\vec{p}})$, and integrating over the angles of the unit vector $\hat{\vec{p}} = \vec{p}/p$, we obtain the following equation for amplitudes ν_{lm}:

$$\omega\nu_{lm} - v_{\rm F}q\sum_{l'm'}G'_l\left\langle lm\left|\hat{\vec{q}}\cdot\hat{\vec{p}}\right|l'm'\right\rangle\nu_{l'm'} = 0. \qquad (53)$$

Here, $\hat{\vec{q}} = \vec{q}/q$, $G_l = 1 + F_l/(2l+1)$,

$$\left\langle lm\left|\hat{\vec{q}}\cdot\hat{\vec{p}}\right|l'm'\right\rangle \equiv$$

$$\equiv C(lm,l'm') = \int d\Omega_{\vec{p}}Y^*_{lm}(\hat{\vec{p}})\cos\theta_{qp}Y_{l'm'}(\hat{\vec{p}}) =$$

$$= (-1)^m\frac{\sqrt{(2l+1)(2l'+1)}}{3}\,\langle ll'00|10\rangle\times$$

$$\times\,\langle ll'm,-m'|1,m-m'\rangle\,, \qquad (54)$$

where $\langle l_1l_2m_1m_2|lm\rangle$ are the Clebsch—Gordan coefficients. We will restrict ourselves to the longitudinal sound waves with $\nu_{l,m\neq0} = 0$ [16]. Taking condition (25) for ν_{l0} into account, we obtain the following chain of recurrence equations from (53):

$$s\nu_{00} - \frac{1}{\sqrt{3}}G_1\nu_{10} = 0,$$

$$s\nu_{10} - \frac{1}{\sqrt{3}}G_0\nu_{00} - \frac{2}{\sqrt{15}}G_2\nu_{20} = 0, \tag{55}$$

$$s\nu_{20} - \frac{2}{15}G_1\nu_{10} - \frac{3}{\sqrt{35}}G_3\nu_{30} = 0,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$s\nu_{l0} - \frac{1}{3}\sqrt{4l^2-1}\,|\langle ll-100|10\rangle|^2\,\nu_{l-1,0} -$$

$$-\frac{1}{3}\sqrt{(2l+1)(2l+3)}\;|\langle ll+100|10\rangle|^2\,\nu_{l+1,0} = 0.$$

Under some additional assumptions, the infinite chain of Eqs. (55) can be cut-off to obtain the analytical solution. We consider two important cases. (i) Neglecting the Fermi surface distortions with multipolarity $l \geq 2$ in Eqs. (55), we obtain the solution

$$\omega = \frac{1}{\sqrt{3}}v_{\rm F}q\sqrt{G_0G_1}. \tag{56}$$

Consequently, the sound speed is given by

$$u = \omega/q = \frac{1}{\sqrt{3}}v_{\rm F}\sqrt{G_0G_1} = \sqrt{\frac{(1+F_0)p_{\rm F}^2}{3mm^*}}. \tag{57}$$

This result coincides with that for the first sound velocity u_1 of Eq. (47). Thus, the first sound regime corresponds to the excitations which preserve the spherical symmetry of the Fermi surface and leads to a displacement of the Fermi sphere as a whole. (ii) If we consider three first equations in (55) and neglect the Fermi surface distortions with multipolarity $l \geq 3$, then the solution to Eqs. (55) (now closed) gives the eigenfrequency

$$\omega = \frac{1}{\sqrt{3}}v_{\rm F}q\sqrt{(G_0+4/5)G_1} \tag{58}$$

and the sound velocity

$$u = \omega/q = \frac{v_{\rm F}}{\sqrt{3}}\sqrt{(G_0+4/5)G_1} = \sqrt{\frac{(9/5+F_0)p_{\rm F}^2}{3mm^*}}. \tag{59}$$

The sound velocity given by Eq. (59) coincides with $\tilde{u}_3$ obtained in the scaling approximation (51). Thus, the scaling approximation for a Fermi-liquid means that all lower multipolarities of a Fermi surface distortion up to $l = 2$ are taken into account. As can be seen from (49), the model independent sum m_1, as it should be, does not depend on the nuclear interaction (Landau's amplitudes F_l). However, the last statement is not correct in the case of specific nuclear excitations, where the sound wave occurs due to the antiphase motion of the neutrons and the protons (isovector vibrations).

3.2. Isovector excitations

Below, we consider the isovector excitations when protons and neutrons move in antiphase. In this case, we rewrite the kinetic equation (21) for the protons and the neutrons separately:

$$\frac{\partial}{\partial t}\delta f_p + \vec{v}\cdot\vec{\nabla}_r\delta f_p - \vec{\nabla}_p f_{p,\rm eq}\cdot\vec{\nabla}_r(\delta U_{p,\rm self} + U_{p,\rm ext}) = 0, \tag{60}$$

$$\frac{\partial}{\partial t}\delta f_n + \vec{v}\cdot\vec{\nabla}_r\delta f_n - \vec{\nabla}_p f_{n,\rm eq}\cdot\vec{\nabla}_r(\delta U_{n,\rm self} + U_{n,\rm ext}) = 0. \tag{61}$$

We neglect the Coulomb interaction and assume $N = Z$. The corresponding corrections are not important on the description of the main characteristics of isovector giant resonances. Subtracting Eq. (60) from Eq. (61) and introducing an isovector variation of the distribution function

$$\delta f' = \delta f_n - \delta f_p, \tag{62}$$

we obtain the kinetic equation for the isovector excitations

$$\frac{\partial}{\partial t}\delta f' + \vec{v}\cdot\vec{\nabla}_r\delta f' - \vec{\nabla}_p\bar{f}_{\rm eq}\cdot\vec{\nabla}_r(\delta U'_{\rm self} + U'_{\rm ext}) = 0. \tag{63}$$

Here, $\bar{f}_{\rm eq}$ is the equilibrium distribution function which is the same for both protons and neutrons according to the above-made assumptions,

$$\int\frac{2d\vec{p}}{(2\pi\,\hbar)^3}\,\bar{f}_{\rm eq}(\vec{r},\vec{p}) = \rho_{n,\rm eq} = \rho_{p,\rm eq} = \frac{p_{\rm F}^3}{3\pi^2\hbar^3}$$

$$\vec{\nabla}_p\bar{f}_{\rm eq}(\vec{r},\vec{p}) = -\vec{v}\frac{m^*}{p_{\rm F}}\delta(p-p_{\rm F}). \tag{64}$$

The variation of the self-consistent field $\delta U'_{\rm self}$ in Eq. (63) has the form which is similar to Eq. (22):

$$\delta U'_{\rm self} = \int\frac{2V\,d\vec{p}'}{(2\pi\hbar)^3}\,v'_{\rm int}(\vec{p},\vec{p}')\,\delta f(\vec{r},\vec{p}';t), \tag{65}$$

where the effective interaction $v'_{\rm int}(\vec{p},\vec{p}')$ for the isovector channel in the Landau approximation is given by [17] (see also (23))

$$v'_{\rm int}(\vec{p},\vec{p}') = \frac{1}{N_{\rm F}}\sum_{l=0}^{\infty} F'_l P_l(\cos\theta_{pp'}). \tag{66}$$

The interaction amplitudes F'_l for the isovector channel differ from the analogous ones F_l for the isoscalar channel in Eq. (23). Thus, in contrast to the amplitude F_0 which determines the nuclear compressibility modulus (see (46)), the similar isovector amplitude F'_0 determines the coefficient of isotopic symmetry $C_{\rm sym}$ in the Weizsäcker mass formula [17, 18]

$$C_{\rm sym} = \frac{2}{3}\varepsilon_{\rm F}\,(1+F'_0) \approx 60\,{\rm MeV}. \tag{67}$$

Below, as it was earlier done for the isoscalar channel in Eq. (25), we assume that

$$F'_0 \neq 0, \quad F'_1 \neq 0, \quad F'_{l\geq 2} = 0. \tag{68}$$

Solving the kinetic equation (63) in the same manner as Eq. (21), we find the isovector response function $\chi'(\omega)$ like $\chi(\omega)$ from (41) as

$$\chi'(\omega) = \frac{\overline{Q}_{00}(s)}{1-\kappa'(s)\overline{Q}_{00}(s)}, \tag{69}$$

where

$$\kappa'(s) = -\frac{1}{N_{\rm F}}\left(F'_0 + \frac{F'_1}{1+F'_1/3}s^2\right).$$

The frequencies of isovector eigenvibrations (the poles of the response function (69)) can be obtained from the dispersion equation

$$1-\kappa'(s)\overline{Q}_{00}(s) = 0. \tag{70}$$

The EWS m_{-1}, m_1, and m_3 (49) for the isovector excitations take the form

$$m'_{-1} = \frac{A}{2}\frac{1}{C_{\rm sym}}, \quad m'_1 = \hbar^2\frac{A}{2m'}q^2,$$

$$m'_3 = \hbar^4\frac{A}{2}\frac{C'_{\rm sym}}{m'^2}q^4. \tag{71}$$

Here, we have introduced the renormalized isotopic symmetry energy $C'_{\rm sym} = C_{\rm sym} + 8\varepsilon_{\rm F}/15$ and the effective mass m'_1 for the isovector channel,

$$m' = \frac{m}{1+\kappa_I},$$

where κ_I is the enhancement factor of the sum rule which is defined by the relation

$$1+\kappa_I = \frac{1+F'_1/3}{1+F_1/3}. \tag{72}$$

Note that, in contrast to the isoscalar sum m_1 (see (49)), the sum m'_1 in (71) is not model independent in sense that it depends on the effective mass m' and thereby on the interaction amplitudes F_1 and F'_1. It is worth noting that the continuity equation (29) for the isovector excitations should be modified as well. Evaluating the zero moment from the kinetic equation (63), we obtain (see also (27)–(29))

$$\frac{\partial}{\partial t}\delta\rho' + \vec{\nabla}(1+\kappa_I)\bar{\rho}\vec{u}' = 0. \tag{73}$$

Here,

$$\bar{\rho} = \frac{1}{2}(\rho_n+\rho_p) = \frac{\rho}{2}, \quad \delta\rho' = \delta\rho_n - \delta\rho_p = \int\frac{2d\vec{p}}{(2\pi\hbar)^3}\delta f',$$

$$\vec{u}' = \vec{u}_n - \vec{u}_p = \frac{1}{\bar{\rho}}\int\frac{2d\vec{p}}{(2\pi\hbar)^3}\frac{\vec{p}}{m}\delta f' \approx \frac{1}{\bar{\rho}_{\rm eq}}\int\frac{2d\vec{p}}{(2\pi\hbar)^3}\frac{\vec{p}}{m}\delta f'.$$

Finally, EWS (71) allow one to calculate the energy centroids of isovector giant resonances for the adiabatic, $\tilde{E}'_1$, and scaling, $\tilde{E}'_3$, approximations as

$$\tilde{E}'_1 = \sqrt{\frac{m'_1}{m'_{-1}}} = \hbar\sqrt{\frac{C_{\rm sym}}{m'}}\,q,$$

$$\tilde{E}'_3 = \sqrt{\frac{m'_3}{m'_1}} = \hbar\sqrt{\frac{C'_{\rm sym}}{m'}}\,q. \tag{74}$$

It is useful to compare relations (74) with the corresponding expressions (50) obtained for the isoscalar excitations.

4. Finite Nuclei. Boundary Conditions

The above-developed approach can be directly applied to the study of the dynamic properties of the infinite nuclear matter, where the distribution function distortion δf has the form of a plane wave in the $\vec{r}$-space. Below, we also apply this approach to the description of the collective excitations in finite nuclei. For heavy nuclei, one can assume a sharp nuclear surface [18].

Then the variation δf of the distribution function in the nuclear interior has the form of the plane wave (32) or its projections on the states with a fixed multipolarity. Moreover, the equation of motion must be supplemented by the boundary conditions at the moving nuclear surface.

To establish the boundary conditions, we introduce the force $\vec{F}$ which is caused by a sound wave and applied to a unit of the nuclear surface S, as well as the surface force $\vec{F}_S$ which is caused by a deformation of the nuclear surface. The general condition of the equilibrium for all forces applied to the free nuclear surface reads

$$\vec{n}\cdot\vec{F}|_S + \vec{n}\cdot\vec{F}_S = 0, \tag{75}$$

where $\vec{n}$ is a unit vector in the normal direction to the nuclear surface. Equation (75) represents the boundary condition for the dispersion equations (43) and (70).

To evaluate the force $\vec{F}$, we calculate the first moment for the kinetic equation (21). Multiplying Eq. (21) by $2d\vec{p}\,p_\nu/(2\pi\hbar)^3$ and integrating over $\vec{p}$, we obtain the Euler equation in the following form [11, 12, 21]:

$$\rho_{\rm eq}\frac{\partial}{\partial t}u_\nu = -\nabla_\mu \delta\Pi_{\nu\mu}, \tag{76}$$

where $\delta\Pi_{\nu\mu}$ is the pressure tensor. For the isovector excitations, we obtain [12]

$$\delta\Pi_{\nu\mu} = \delta\sigma'_{\nu\mu} + \delta P'\,\delta_{\nu\mu}, \tag{77}$$

where

$$\delta P' \equiv \delta P'(\vec{r},t) =$$

$$= \frac{1}{3m}\int\frac{2d\vec{p}}{(2\pi\hbar)^3}p^2\delta f'(\vec{r},\vec{p},t) + \frac{1}{N_{\rm F}}F'_0\bar{\rho}_{\rm eq}\delta\rho'(\vec{r},t) =$$

$$= \frac{2}{3}(1+F'_0)\,\varepsilon_{\rm F}\delta\rho'(\vec{r},t) = C_{\rm sym}\delta\rho'(\vec{r},t). \tag{78}$$

Let us introduce the isovector displacement field $\vec{\chi}'$ which is connected to the corresponding velocity field $\vec{u}'$ through the relation

$$\partial\vec{\chi}'/\partial t = -(1+\kappa_I)\vec{u}'.$$

Taking the continuity equation (73) into account, we find

$$\delta\rho' = \bar{\rho}_{\rm eq}\vec{\nabla}\cdot\vec{\chi}'.$$

Finally, Eq. (78) yields

$$\delta P' = C_{\rm sym}\bar{\rho}_{\rm eq}\vec{\nabla}\cdot\vec{\chi}'. \tag{79}$$

The pressure tensor $\delta\sigma'_{\nu\mu}$ in Eq. (77) is given by

$$\delta\sigma'_{\nu\mu} = \frac{2}{3m^*}\int\frac{d\vec{p}}{(2\pi\hbar)^3}(3p_\nu p_\mu - p^2)\delta f' =$$

$$= \mu'_{\rm F}(\nabla_\nu\chi'_\mu + \nabla_\mu\chi'_\nu - \frac{2}{3}\,\delta_{\nu\mu}\vec{\nabla}\cdot\vec{\chi}'), \tag{80}$$

where

$$\mu'_{\rm F} = \frac{3}{2}\,\bar{\rho}_{\rm eq}\varepsilon_{\rm F}\frac{s^2}{1+F'_1/3}\left[1 - \frac{(1+F'_0)(1+F'_1/3)}{3s^2}\right]. \tag{81}$$

Taking (77), (79), and (80) into account, we obtain

$$\delta\Pi_{\alpha\beta} = \mu'_{\rm F}\left(\nabla_\alpha\chi'_\beta + \nabla_\beta\chi'_\alpha\right) +$$

$$+\left(C_{\rm sym}\bar{\rho}_{\rm eq} - \frac{2}{3}\,\mu'_{\rm F}\right)\vec{\nabla}\cdot\vec{\chi}'\delta_{\alpha\beta} \tag{82}$$

The pressure tensor $\delta\Pi_{\nu\mu}$ determines the force $\vec{F}$ which acts from the side of the sound wave on a unit of the nuclear surface

$$F_\nu = n_\mu\delta\Pi_{\nu\mu}. \tag{83}$$

Using Eqs. (82) and (83), we evaluate the normal component of the force $\vec{F}$ applied to the nuclear surface:

$$\vec{n}\cdot\vec{F}\Big|_S = \frac{1}{r^2}r_\nu r_\mu\delta\Pi_{\nu\mu}\Big|_{r=R_0} =$$

$$= \frac{1}{r^2}\left[r^2\left(C_{\rm sym}\bar{\rho}_{\rm eq}(1+\kappa_I) - \frac{2}{3}\,\mu'_{\rm F}\right)\vec{\nabla}\cdot\vec{\chi}' +\right.$$

$$\left. + 2\mu'_{\rm F}r_\nu r_\mu\chi_\nu\chi_\mu\right]\Big|_{r=R_0} =$$

$$= \left[\left(C_{\rm sym}\bar{\rho}_{\rm eq}(1+\kappa_I) - \frac{2}{3}\,\mu'_{\rm F}\right){\rm div}\vec{\chi}' +\right.$$

$$\left. + 2\mu'_{\rm F}\frac{\partial}{\partial r}(\vec{n}\cdot\vec{\chi}')\right]_{r=R_0}. \tag{84}$$

Let us calculate the normal component $\vec{n}\cdot\vec{F}_S$ of the isovector surface force $\vec{F}_S$ which occurs in Eq. (75). To find the force $\vec{F}_S$, we notice that a shift of protons

against neutrons creates the additional surface energy in the case of isotopic symmetry given by [22]

$$\delta E_{S,\mathrm{sym}} = \frac{1}{3}\rho_{\mathrm{eq}} r_0 \sigma_{\mathrm{sym}} \int \tau^2 dS. \tag{85}$$

Here, r_0 is the mean distance between nucleons ($R_0 = r_0 A^{1/3}$), σ_{sym} is the isovector surface energy which is a parameter of theory, and τ is a shift of the proton sphere against the neutron one. In units of r_0,

$$\tau = \frac{1}{r_0}\left(R_p\left(t\right) - R_n\left(t\right)\right) =$$

$$= \frac{1}{r_0}\left(\left(R_0 + \delta R_1(t)\right) - \left(R_0 - \delta R_1(t)\right)\right) = \frac{2}{r_0}\delta R_1(t), \tag{86}$$

where

$$\delta R_1(t) = R_0 \alpha_S\left(t\right) Y_{10}\left(\hat{r}\right). \tag{87}$$

The amplitude $\alpha_S(t)$ of isovector vibrations of the nuclear surface in Eq. (87) is connected to the corresponding amplitude $\vec{\chi}'$ of the displacement field in a sound wave. To establish this connection, we note that, for a nucleus with sharp edge, the displacement field in the nuclear interior has the form (see Section 6 in [18])

$$\vec{\chi}' = \alpha_1\left(t\right)\frac{1}{q^2}\vec{\nabla}_r\left(j_1\left(qr\right)Y_{10}\left(\hat{r}\right)\right), \tag{88}$$

where $j_1(x)$ is the spherical Bessel function.

Evaluating the normal component of the velocity field $\vec{u}'$ by the use of Eq. (88) and equating it to the surface velocity $\partial\delta R_1(t)/\partial t$, we obtain

$$\alpha_S\left(t\right) = -\alpha_1\left(t\right)\frac{j_1'\left(x\right)}{x(1+\kappa_I)}, \quad x = qR_0. \tag{89}$$

According to the definition of the pressure δP_S caused by a shift of the nuclear surface (see, for example, the appendix to Section 6 in [18]), we obtain the following relation from Eqs. (85) and (86):

$$\delta P_S = \frac{\partial}{\partial \delta R_1}\frac{\delta E_S}{\delta S} = \frac{8}{3}\frac{\rho_{\mathrm{eq}}}{r_0}\,\sigma_{\mathrm{sym}}\delta R_1. \tag{90}$$

Taking into account Eqs. (89) and (90), we can evaluate the normal component $(\vec{n}\cdot\vec{F}_S)$ of the surface force $\vec{F}_S$ in Eq. (75). The result reads

$$(\vec{n}\cdot\vec{F}_S) = -\delta P_S = \frac{8}{3}\frac{\rho_{\mathrm{eq}} j_1'(x)}{qr_0(1+\kappa_I)}\,\sigma_{\mathrm{sym}}\alpha_1\left(t\right)Y_{10}\left(\hat{r}\right). \tag{91}$$

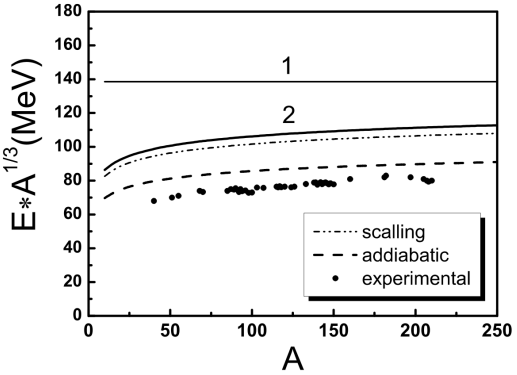

Fig. 2. Dependence of the energy of the isovector giant dipole resonances on the mass number obtained from the dispersion equation (70) (solid curve 2) and from EWS (74) (dashed lines). Solid curve 1 is obtained from the explicit solution of the dispersion equation (70) subsidized by the boundary condition of the Steinwedel—Jensen's model, $j_1'(x) = 0$. For all calculations presented in Fig. 2, we have taken the following parameters: $r_0 = 1.2$ fm, $F_1 = -0.64$, $F_0' = 0.96$, $F_1' = 1$, $\sigma_{\mathrm{sym}} = 17$ MeV. The experimental data were taken from [25]

Finally, from Eqs. (75), (84), (88), and (91), we derive the following secular equation for the wave number q:

$$\left[-\frac{1}{2}C_{\mathrm{sym}}\bar{\rho}_{\mathrm{eq}} - \frac{2}{3}\mu_{\mathrm{F}}' + \frac{2}{x^2}\mu_{\mathrm{F}}'\right] j_1\left(x\right) +$$

$$+\left[-\frac{2}{x}\mu_{\mathrm{F}}' + \frac{4}{3}\frac{\rho_{\mathrm{eq}}}{qr_0(1+\kappa_I)}\sigma_{\mathrm{sym}}\right] j_1'\left(x\right) = 0. \tag{92}$$

We point out that, in the classical limit of the Steinwedel—Jensen's model at $\sigma_{\mathrm{sym}} \to \infty$, the boundary condition (92) coincides with the similar one, $j_1'(x) = 0$, in the traditional liquid drop model [18].

The boundary condition (92) allows us to find the dependence of the wave number q on the mass number A and to evaluate the corresponding excitation energy in finite nuclei. In Fig. 2, we show the dependence of the energy of isovector giant dipole resonances (IGDR) on the mass number A obtained by the use of the explicit solution of the dispersion equation (70) and EWS (74). For both of them, the boundary condition (92) was used. As can be seen from Fig. 2, the lowest energy of IGDR $\sqrt{m_1'/m_{-1}'}$ is obtained with m_{-1}' and corresponds to the first sound regime without distortions of the Fermi surface. The account of a quadrupole Fermi surface deformation in the sum m_3' shifts upward the curve $\sqrt{m_3'/m_1'}$. This is due to the additional contribution to the nuclear stiffness coefficient caused by Fermi-surface distortions. Involving the higher multipolarities of the

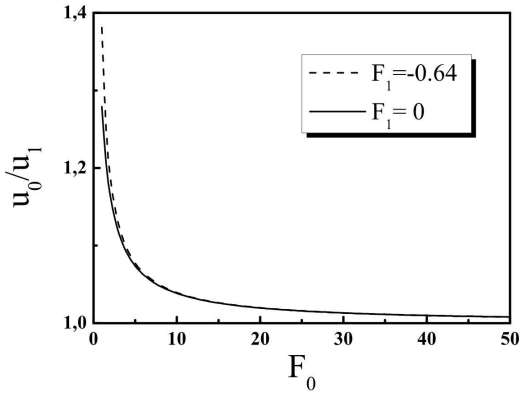

Fig. 3. Dependences of the ratio of the first sound velocity to the zero-sound one on the interaction amplitude F_0 for the isoscalar excitations for two values of the interaction constant F_1

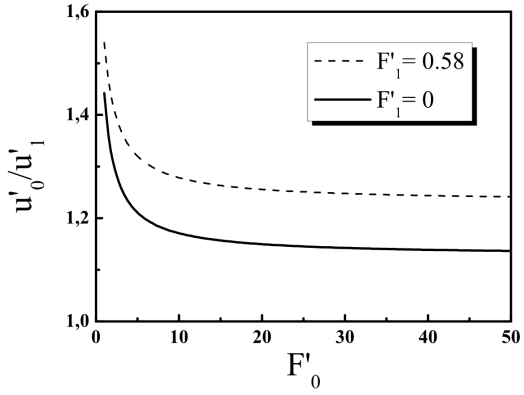

Fig. 4. The same as in Fig. 3, but for isovector excitations

Fermi surface distortions which are present in the dispersion equation (70) leads to the additional increase of the nuclear stiffness and the excitation energy $\hbar\omega_1$– in Fig. 2.

As was noted above, the dependence of the nuclear forces on the nucleon velocities (components with F_1 and F_1' in (23) and (66), respectively) leads to the significant difference between the EWS for the isoscalar and isovector excitations. In particular, the consequence of this difference is the asymptotic behavior of the nuclear stiffness coefficient and the zero-sound velocity at an increase of the internucleon interactions F_0 and F_0'. In Figs. 3 and 4, we show the dependence of the ratio of the zero-sound velocity to the first sound one on the interaction amplitudes F_0 and F_0' for the isoscalar and isovector excitations. The feature of the isoscalar excitations is the fact that the increase of the nucleon-nucleon interaction leads to a shift of the zero-sound velocity towards the first sound one (see Fig. 3). This means that the influence of Fermi surface distortions on the collective motion in the nuclear Fermi-liquid becomes negligible on the increase of the nucleon-nucleon interaction. The behavior of the isovector zero-sound velocity u_0' is qualitatively different (see Fig. 4). With increase in the nucleon-nucleon interaction, the velocity u_0' tends to the asymptotic limit which significantly exceeds the corresponding first sound velocity. This is a consequence of the general enhancement effect of collectivity of the isoscalar zero-sound caused by the dependence of the nuclear forces on the nucleon velocity (see also (72)).

The enhancement factor κ_I for the isovector excitations defined in Eq. (72) depends on the interaction constants F_1 and F_1'. Whereas the isoscalar constant F_1 related to the effective nucleon mass m^* is well studied, the isovector constant F_1' is not much studied, and the experimental investigation of the enhancement factor κ_I (72) can help for its derivation.

The experimental derivation of the enhancement factor in the isovector EWS m_1' is connected to the investigation of the nuclear absorption cross-section $\sigma_{\rm abs}(\omega)$ of γ-quanta with energy $\hbar\omega$. Let us introduce the strength function $S(\omega, q)$ for the density-density response per unit volume V. According to Eqs. (1), (40), and (42), we have

$$S(\omega, q) = \frac{1}{V}\chi^{(d)}(\omega). \tag{93}$$

In the case of the velocity independent forces, in accordance with Eqs. (2), (49), and (71), the strength function $S(\omega, q)$ is normalized by the condition

$$\int\limits_0^\infty d(\hbar\omega)\,\hbar\omega S(\omega, q) = \hbar^2 \frac{1}{2m}\, q^2 \rho_{\rm eq}. \tag{94}$$

In contrast to this, in the case of isovector excitations with the velocity dependent forces, the normalization condition reads

$$\int\limits_0^\infty d(\hbar\omega)\,\hbar\omega S(\omega, q) = \hbar^2 \frac{A}{2m^*}(1 + F_1'/3)\, q^2 \rho_{\rm eq}. \tag{95}$$

The photoabsorption cross-section $\sigma_{\rm abs}(\omega)$ is connected to the strength function by the relation

$$\sigma_{\rm abs}(\omega) = {\rm const} \cdot \omega S(\omega, q). \tag{96}$$

Here, $\vec{q}$ plays the role of the momentum which is transferred to the nucleus at the absorption of a γ-quantum. The constant in Eq. (96) can be

found from the normalization condition for the photoabsorption cross-section $\sigma_{\rm abs}(\omega)$. For the isovector dipole excitations and for the velocity independent forces, this condition reads (Reiche—Thomas—Kuhn rule) [23]

$$\tilde{m}_1 = \int\limits_0^\infty d(\hbar\omega)\ \sigma_{\rm abs}(\omega) = \frac{2\pi^2\hbar e^2}{mc}\frac{NZ}{A}. \tag{97}$$

From Eqs. (96), (94), and (97), one obtains [24]

$$\sigma_{\rm abs}(\omega) = \frac{4\pi^2 e^2}{cq^2\rho_{\rm eq}}\frac{NZ}{A}\omega S(\omega, q). \tag{98}$$

Since the photoabsorption occurs mainly through the giant dipole resonance, the transferred momentum q in (98) can be taken as $q = q_1 = 2.08/R_0$ [24], that corresponds to the classical boundary condition $j_1'(x) = 0$ of the Steinwedel—Jensen's model.

In the case of the velocity dependent forces, the normalization condition $S(\omega, q)$ (94) has to be replaced by condition (95), and the transferred momentum $q = q_1'$ has to be calculated by using the boundary condition (92). As a result, the sum rule for $\sigma_{\rm abs}(\omega)$ takes the form (instead of (97))

$$\tilde{m}_1' = \int\limits_0^\infty d(\hbar\omega)\sigma_{\rm abs}(\omega) = \frac{2\pi^2\hbar e^2}{mc}\frac{NZ}{A}\left(\frac{q_1'}{q_1}\right)^2(1+\kappa_I). \tag{99}$$

In Fig. 5, we demonstrate the dependence of the enhancement factor $\tilde{m}_1'/\tilde{m}_1$ on the mass number for a number of nuclei. We point out that the exceeding of 100% of the sum rule $\tilde{m}_1'$, which is experimentally observed for the isovector giant dipole resonances, is caused by the dependence of the effective nucleon-nucleon interaction on the nucleon velocity. For the values of the isovector amplitude $F_1' \approx 1$, one can adjust (on the average) the results of theoretical calculations of $\tilde{m}_1'$ (solid line in Fig. 5) with the experimental data. The nonmonotonic dependence of the experimental value of $\tilde{m}_1'$ on the mass number A in Fig. 5 is due to the shell effects which are not taken into account within the semiclassical kinetic theory used in this work.

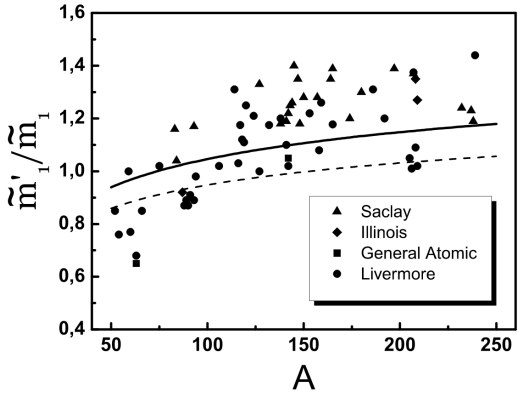

Fig. 5. Dependence of the enhancement factor $\tilde{m}_1'/\tilde{m}_1$ of the EWS m_1' for isovector giant dipole resonances on the mass number A. The results of calculations were obtained by the use of Eqs. (97) and (99) for two values of the isovector amplitude $F_1' = 1.2$ (solid line) and $F_1' = 0.58$ (dashed line) at the constant value of $F_1 = -0.64$; points are the experimental data from [20]

5. Conclusions

Using the Landau—Vlasov kinetic theory, we have studied the linear response function and the EWS m_k for the isoscalar and isovector excitations in heavy nuclei and the nuclear matter. An advantage of our approach is the possibility to derive the explicit analytic expressions and to carry out a detailed analysis for some important nuclear characteristics. One of them is the nuclear stiffness coefficient. The dynamical Fermi-surface distortion influences significantly the formation of the nuclear stiffness. For a slow (adiabatic) nuclear deformation, the nuclear stiffness coefficient is derived by the low-energy sum m_{-1} and coincides with the stiffness coefficient of the classical (non-Fermi) liquid. This stiffness coefficient (adiabatic incompressibility K) causes the propagation of the first sound in a Fermi-liquid which is not accompanied by the Fermi surface distortions [see (10), (47), and (49)]. In the general case of fast motion, the derivation of the nuclear stiffness coefficient requires to solve the dispersion equation (43). A specific role is played here by the scaling approximation. In this last case, the quadrupole Fermi surface distortion is only taken into consideration. We have shown that the stiffness coefficient in the scaling approximation is determined by the high-energy sum m_3 and exceeds significantly the adiabatic incompressibility K. At the same time, the sound velocity approaches that of the Landau's first sound [see (48), (51), and (59)].

In the presence of the velocity dependent nuclear forces, the EWS for the isoscalar and isovector excitations are significantly different [see (49) and (71)]. First of all, the EWS m_1, which is model independent for the isoscalar excitations, becomes model dependent in the case of the isovector excitations.

Another consequence of the mentioned difference of sums (49) and (71) is the different asymptotic behavior of the zero-sound velocity on an increase of the nucleon-nucleon interaction (see Figs. 3 and 4).

A feature of the isoscalar excitations is that the zero-sound velocity approaches the first-sound one with increase in the internucleon interaction. This means that the influence of the Fermi surface distortion on the isoscalar collective motion in the nuclear Fermi-liquid becomes negligible on the increase of the internucleon interaction. In contrast to this, the increase of the internucleon interaction for the isovector mode leads to the asymptotic zero-sound velocity u_0' which exceeds the relevant first-sound velocity. The above-mentioned difference between isoscalar and isovector EWS allows one to explain the fact that, in many cases, the experimental measurement of the EWS m_1' for the isovector giant dipole resonance gives the more than 100 % exhaustion of the corresponding sum rule. According to Eqs. (71) and (72), the dependence of the effective nuclear forces on the nucleon velocities generates the enhancement factor $1 + \kappa_I > 1$, which is absent for the isoscalar excitations, in the sum m_1' for the isovector excitations (see Fig. 5). Note that the enhancement factor (72) depends on the isovector amplitude F_1'. This gives, in principle, the possibility to determine the interaction amplitude F_1' from the fit of the EWS $\tilde{m}_1'$ to the experimental data.

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ЕНЕРГЕТИЧНО ЗВАЖЕНІ СУМИ ДЛЯ КОЛЕКТИВНИХ ЗБУДЖЕНЬ В ЯДЕРНІЙ ФЕРМІ-РІДИНІ

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Р е з ю м е

В рамках кінетичної теорії, адаптованої до опису двокомпонентної ядерної фермі-рідини, досліджено безмодельну m_1, адіабатичну m_{-1} та високоенергетичну m_3 зважені суми для ізоскалярних та ізовекторних збуджень ядра. В адіабатичному і скейлінг-наближеннях встановлено зв'язок енергетично зважених сум (ЕЗС) m_{-1} та m_3 з ядерною жорсткістю та швидкостями першого і нульового звуків. Досліджено коефіцієнт підсилення в ЕЗС для ізовекторних збуджень, який дозволяє пояснити перевищення 100%-вого вичерпування ЕЗС m_1' в експериментах зі збудження ізовекторного гігантського дипольного резонансу. Дано пояснення залежності коефіцієнта підсилення від масового числа A та проаналізовано його залежність від величини ізовекторної амплітуди взаємодії Ландау F_1'.