DETERMINATION OF THE EFFECTIVE PARAMETERS OF PROTON – ^{3}He SCATTERING ON THE BASIS OF THE NEUTRON-TRITON SCATTERING DATA

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We have studied the relations between the neutron-triton scattering lengths and effective ranges and the corresponding quantities for the p –^{3}He scattering in the framework of the potential model with an effective nucleon-nucleus interaction in the form of a δ-shell potential. It is shown that the Coulomb renormalization of the pure nuclear scattering lengths does not change the relation well established for the n + ^{3}H system between the lengths: $A^1 < A^0$. We have predicted the p–^{3}He scattering lengths which give preference to set I of the phase analysis performed by E.A. George et al. (2003), which corresponds to the inequality $A^1_{nc} < A^0_{nc}$ for the scattering lengths.

1. Introduction

The topicality of the determination of parameters of the low-energy scattering in few-nucleon systems consists in the possibility to verify the theoretical approaches and models used for the description of such systems. These parameters are also used as the initial data in the calculations of the more complicated processes with participation of weak forces (for example, for the weak capture of a proton by light nuclei). At present, the low-energy scattering parameters for the physical systems with four nucleons are known with various degrees of accuracy. In some cases, they are so indeterminate that there exist, for example, two essentially different sets of scattering lengths (zero-energy scattering amplitudes). In this connection, of great interest is the problem to study the correlation relations between different characteristics of different four-nucleon systems, which would allow one to obtain the independent information about insufficiently well-measured quantities on the basis of reliable available data.

Here, on the basis of the potential model with an effective nucleon-nucleus interaction in the form of δ-shell potentials, we study the relation between the lengths and effective ranges of the S-wave scattering of a proton by a nucleus ^{3}He (h, in brief) and the corresponding quantities for the scattering of a neutron by a triton ^{3}H (t, in brief).

2. Analysis of the Data on p–^{3}He Scattering

The importance of the consideration of this problem is conditioned by the experimental situation observed now in the determination of low-energy parameters of the scattering of a proton by a ^{3}He nucleus. In Table 1, we present values of the nuclear-Coulomb scattering lengths A^1_{nc} and A^0_{nc}, which are, respectively, a triplet and a singlet by the total spin, and the corresponding effective ranges r^1_{nc} and r^0_{nc} found on the basis of the results of the phase analyses of experimental data on the p–^{3}He scattering [1–6]. The asterisk marks the results obtained by us on the basis of the data of the corresponding works.

The most full phase analysis of all the totality of data for the p–^{3}He scattering performed in the recent work [6] has not given an unambigous set of values for the triplet and singlet scattering lengths of the p-h scattering. In that work, two sets of the scattering lengths were presented.

$$A^1_{nc} = 7.9 \text{ fm}, \quad A^0_{nc} = 15.1 \text{ fm} \quad \text{set I;} \tag{1}$$

$$A^1_{nc} = 10.4 \text{ fm}, \quad A^0_{nc} = 7.2 \text{ fm} \quad \text{set II.} \tag{2}$$

These sets differ qualitatively and quantitatively from each other. Solutions (1) and (2) were obtained on the basis of the dataset in [5], to which the new data on the proton analyzing power [7] and the data on the cross-sections and the analyzing powers at very low energies [8]. Just the last data significantly affected the final result. We note that set I is characterized by a less value of the function $\chi^2 = 0.958$ per data point, than set II ($\chi^2 = 0.973$), which can indicate the higher reliability of the former solution.

It is worth noting that the previous phase analyses [1, 2] have also led to several solutions. From two results related to solutions a) and c) in [1], Table III, our table presents solution c) as such that agrees with the empiric observation $r^1_{nc} < r^0_{nc}$. The values corresponding to work [2] are extracted by us from solution III given in [2].

In [4], all the data available at that time on the p–^{3}He scattering up to 10 MeV (scattering cross-sections, polarizations of a proton and a target, and the polarization transfer coefficients) were analyzed in the framework of the model of the separable potential interaction between a nucleon and a three-nucleon nucleus.

As for the result of work [3], it was obtained by means of the fitting of triplet phase shifts with the help of the expansion of the effective range on the basis of the data on phase shifts [8] and the results of the very old phase analysis [9] which have great errors. As a result, the effective range obtained in [3] seemed to be too small. For this reason, the authors of work [3], with the help of the experimental value of the triplet neutron-triton scattering length $A_n^1 = (3.68 \pm 0.05)$ fm [10] and the well-known Jackson–Blatt formula [11], got a somewhat larger value

$$r_{nc}^1 = (1.30 \pm 0.18)\,\text{fm}. \tag{3}$$

Nevertheless, the value of r_{nc}^1 (3) remains noticeably less than two other values in Table 1.

In order to make an unambiguous choice of one of the two sets of p–^{3}He scattering lengths (1), (2), we will involve the experimental and theoretical data on the charge-symmetric neutron + triton system into the analysis. In the approximation of the charge symmetry of nuclear forces, the system n+^{3}H,which has no Coulomb interaction between a neutron and a triton, can be an additional source of information in studying the characteristics of the nuclear-Coulomb system p+^{3}He.

3. Analysis of the Data on n–^{3}H scattering

The experimental situation for the n–t scattering lengths looks better at least for the reason that these lengths can be directly determine with the help of any pair of such physical quantities as the threshold value of the total cross-section

$$\sigma = \pi({A_n^0}^2 + 3{A_n^1}^2), \tag{4}$$

the coherent length

$$A_c = \frac{1}{4}(A_{nc}^0 + 3A_{nc}^1), \tag{5}$$

and the incoherent length

$$A_i = \frac{\sqrt{3}}{4}(A_n^1 - A_n^0) \tag{6}$$

of the n–t scattering.

In our first calculations [12, 13] of the triplet A_n^1 and singlet A_n^0 pure nuclear n–t scattering lengths, we noted the inconsistency of the experimental data available at that time on σ and A_c (see [13]). From that time, the accuracy of measurements of these quantities is significantly enhanced, and the contemporary experimental situation is presented in Table 2. The available experimental data on the n–t scattering cross-section σ [10] and the coherent scattering length A_c [14, 15] impose, according to relations (4), (5), some empiric restrictions on the lengths A_n^1 and A_n^0. In the plane of the variables A_n^0 and A_n^1, these data correspond to the strips, the intersection region of which gives the expected experimental values of the n–t scattering lengths (two last columns of Table 2).

We note that work [15], besides the recommended value $A_c = (3.59 \pm 0.02)$ fm, gives also

$$A_{c,\text{avr}} = (3.61 \pm 0.02)\,\text{fm}. \tag{7}$$

The last value is obtained by the authors by means of the averaging of three available values of A_c. It turns out however that the small (0.5 %) addition to $A_{c,\text{avr}}$ (7) leads to the very close approach of the upper edge of the strip $A_{c,\text{avr}}$ to the lower edge of the strip related to the experimental value of σ [10]. In this case, a reliable experimental determination of the lengths A_n^1 and A_n^0 on the basis of the data for σ and $A_{c,\text{avr}}$ is quite problematic.

The last row of Table 2 gives the results [16] obtained on the basis of the R-matrix parametrization of the data

T a b l e 1. Values of the nuclear-Coulomb triplet (A_{nc}^1) and singlet (A_{nc}^0) scattering lengths and the corresponding effective ranges r_{nc}^1 and r_{nc}^0 obtained from the data on phase shifts of the p–^{3}He scattering. The asterix marks the results obtained in the present work on the basis of the data of the corresponding phase analyses

Source	A_{nc}^1, fm	r_{nc}^1, fm	A_{nc}^0, fm	r_{nc}^0, fm	
[1]	7.89	1.62	7.94	1.96	
[2]			12.8*	1.7*	
[3]	$10.2 \pm 1,4$	1.02 ± 0.42			
[4]	7.6*	1.8*	16.4*	1.9*	
[5]	8.1 ± 0.5		10.8 ± 2.6		
[6]	7.9 ± 0.2		15.1 ± 0.8		set I
	10.4 ± 0.4		7.2 ± 0.8		set II

T a b l e 2. Experimental data on triplet (A_n^1), singlet (A_n^0), and coherent (A_c) scattering lengths and the total cross-section (σ) of n–t scattering

Source	σ, бн	A_c, fm	A_n^1, fm	A_n^0, fm	
[10]	1.70 ± 0.03		3.6 ± 0.1	3.91 ± 0.12	
[14]		3.82 ± 0.07	3.70 ± 0.21	3.70 ± 0.62	
[15]		3.59 ± 0.02	3.13 ± 0.11	4.98 ± 0.29	set I
			4.05 ± 0.09	2.10 ± 0.31	set II
[16]			3.325 ± 0.016	4.453 ± 0.10	

for p–^{3}He scattering with the approximate account of the Coulomb difference of the p–^{3}He and n–^{3}H systems.

The results of theoretical calculations of the parameters of low-energy n–t scattering for the recent years [17–23] are presented in Table 3. These calculations are mainly based on solutions of the four-particle equations which take strictly the multiparticle dynamics and boundary conditions of the problem into account. As for the interaction between nucleons, the models including even the three-particle forces, in addition to complicated two-particle potentials, have been used in the last years (see, e.g., the third row of the results from [23] in Table 3). The averaging of the theoretical results for scattering lengths and effective ranges, which give the values of the coherent scattering length and total n–t scattering cross-section (Table 2) closely to the experimental data, yields

$$A_n^0 = (4.0 \pm 0.1)\,\text{fm}, \quad r_n^0 = (1.95 \pm 0.05)\,\text{fm}, \tag{8}$$

$$A_n^1 = (3.58 \pm 0.05)\,\text{fm}, \quad r_n^1 = (1.75 \pm 0.05)\,\text{fm}. \tag{9}$$

In this case, for the lengths and the effective ranges (8) and (9), the following relations hold:

$$A_n^1/A_n^0 = 0.895 < 1, \tag{10}$$

$$r_n^1/r_n^0 = 0.897 < 1. \tag{11}$$

An additional source of the information on the parameters of low-energy neutron-triton scattering would be the results of phase analyses of the experimental data in the energy range up to 10 MeV. However, the results of the old phase analysis of n–t scattering [24] were obtained on the basis of a restricted dataset, are characterized large errors, and can be used in order to get the information on the scattering lengths with caution. For the singlet spin channel, the phase shifts from [24] are very much underestimated by modulus and are characterized by great errors. Generally, the two-parameter approximation of the singlet and triplet phase shifts from [24] gives

$$A_n^0 = (2.8 \pm 0.5)\,\text{fm}, \quad r_n^0 \approx 1.6\,\text{fm}, \tag{12}$$

$$A_n^1 = (3.4 \pm 0.5)\,\text{fm}, \quad r_n^1 \approx 2.3\,\text{fm}. \tag{13}$$

The result for the singlet length A_n^0 (12) is significantly less than all available data for this quantity (see Tables 1 and 2). The same conclusion concerns the effective range r_n^0 (12). For the triplet channel, the effective range r_n^1 (13) looks to be very much overestimated. We note that values (12) and (13) do not agree with relations (10) and (11).

In the other phase shift analysis [4], the predictions for the n–t scattering phase shifts were made on the basis of the results of the phase analysis of p–h data in the framework of the model of separable intercluster interaction. After the optimization of fitting parameters of the nuclear p–h potential, the authors of work [4] calculated the phase shifts of p–h scattering and the scattering phase shifts for the charge-symmetric n–t system. The results of work [4] let us to extract the following scattering lengths and effective ranges:

$$A_n^0 = 5.5\,\text{fm}, \quad r_n^0 \approx 2.3\,\text{fm}, \tag{14}$$

$$A_n^1 = 3.2\,\text{fm}, \quad r_n^1 \approx 2.1\,\text{fm}. \tag{15}$$

But, from our viewpoint, just the phase shifts of n–t scattering in [4] are not sufficiently exact, because they are a reflection of the phase shifts of p–h scattering. However, as compared with the later more exact phase analysis [5], the singlet phase shifts [4] in the interval of laboratory energies less than 6 MeV look essentially (by 10°) overestimated. Respectively, lengths (14) and (15) yield the total n–t scattering cross-section $\sigma = 1.92$ b, which exceeds the experimental result [10] by 13% (see Table 2). We note that the effective parameters (14) and (15) satisfy relations (10) and (11).

In what follows as the input data of our analysis of p–h scattering in the triplet spin channel, we use the experimental value of the triplet length of n–t scattering,

$$A_n^1 = (3.6 \pm 0.1)\,\text{fm}, \tag{16}$$

which agrees well with the theoretical prediction (9). For the singlet spin channel, we will consider two following values of the n–t scattering length:

$$A_n^0 = (4.0 \pm 0.1)\,\text{fm}, \tag{17}$$

T a b l e 3. Theoretical date on triplet (A_n^1) and singlet (A_n^0) scattering lengths and the corresponding effective ranges (r_n^1 and r_n^0) and the coherent scattering length (A_c) and the total cross-section (σ) of the scattering of a neutron by a triton

Source	A_n^1, fm	r_n^1, fm	A_n^0, fm	r_n^0, fm	A_c, fm	σ, fm
[17]	3.61		4.09		3.73	175.4
[18]	3.46		4.24		3.66	169
[19]	3.597		3.905		3.68	170
[20]	3.6		4.0		3.7	172.4
[21]	3.80		4.32		3.93	194.7
[22]	3.63		4.10		3.75	177.0
	3.73	1.87	4.13	2.01	3.83	184.7
[23]	3.76		4.31		3.90	191.6
	3.79	1.76	4.31	2.08	3.92	193.7
	3.53	1.71	3.99	1.95	3.65	167.5

which represent the experimental value [10] and the averaged theoretical result (8), and

$$A_n^0 = (4.453 \pm 0.100)\,\text{fm}, \tag{18}$$

being a result of the R-matrix parametrization of the data on $p{-}^3$He scattering [16]. The experimental set I [15] includes the triplet and singlet lengths of n–t scattering which lie outside values (16)–(18).

4. Determination of Effective Parameters of the p–h Scattering from the n–t Data

Under assumption of the charge symmetry of nuclear forces, the available information on the n+t system can be used in the derivation of the scattering lengths and effective ranges for the p+h system. In order to characterize the effective nuclear interaction of a nucleon with a three-nucleon nucleus, we will use the model of δ-shell potential

$$V(r) = -\lambda \frac{\delta(r-R)}{R^2}, \tag{19}$$

where R is the interaction radius, and λ is the potential intensity.

We note that a potential of the form (19) well describes the S-wave phase shifts of NN scattering up to energies $\simeq$250 $\div$ 300 MeV in the laboratory system and ensures a good description of low-energy characteristics of a system of three nucleons [25]. This potential agrees naturally with the fact that low-energy properties of a three-nucleon system turn out insensitive to a detailed behavior of the interaction at small distances and are mainly determined by its intensity at distances about (1.5 $\div$ 2) fm.

Potential (19) was successfully used earlier as NN-forces in the calculation of the characteristics of threshold n–t scattering [18, 26, 27] on the basis of the strict four-particle Faddeev–Yakubovsky integral equations.

For potential (19), the pure nuclear and modified nuclear-Coulomb scattering lengths and effective ranges are expressed through the potential parameters R and λ in the analytic form [28]:

$$\frac{1}{A_n} = \frac{1}{R} - \frac{1}{\lambda}, \tag{20}$$

$$r_n = \frac{2R^2}{3}\left(\frac{1}{R} + \frac{1}{\lambda}\right), \tag{21}$$

$$\frac{1}{A_{nc}} = \frac{2}{a_\text{B} I_1^2}\left[-\frac{R}{\lambda} + 2I_1K_1\right], \tag{22}$$

$$r_{nc} = \frac{2a_\text{B}}{3I_1^2}\left[\frac{R}{\lambda}\frac{x^{3/2}I_2}{I_1} + \frac{1}{2}(I_1^2 - x)\right]. \tag{23}$$

In formulas (22) and (23), $a_\text{B} = \hbar^2/(2\mu e^2)$ is the Bohr radius of the p–h system, μ is the reduced mass, $x = 2R/a_\text{B}$, and I_1, I_2, and K_1 are the modified Bessel function of the variable $z = 2\sqrt{x}$. As $\lambda \to \infty$, formulas (20)–(23) take the form

$$A_n^\infty = R, \tag{24}$$

$$r_n^\infty = \frac{2}{3}R, \tag{25}$$

$$A_{nc}^\infty = \frac{a_\text{B}}{4}\frac{I_1(z)}{K_1(z)}, \tag{26}$$

$$r_{nc}^\infty = \frac{a_\text{B}}{3}\left(1 - \frac{z^2}{4I_1^2(z)}\right), \tag{27}$$

where $z = (8R/a_\text{B})^{1/2}$.

Below, we will find the potential of the pure nuclear interaction of a nucleon with a three-nucleon nucleus (parameters R and λ) from formulas (20) and (21), where we will use, as the input values, the most reliable data for the scattering lengths and effective ranges of a neutron by a triton, A_n and r_n. Then the nuclear potential constructed in such a way will be applied to the determination of the scattering lengths and effective ranges A_{nc} and r_{nc} for the nuclear-Coulomb scattering of a proton by a nucleus ^{3}He.

In order to study the sensitivity of the predictions to the value of the uncertainty of the n–t data, we use the set of δ-shell potentials (19), by varying the range of the potential R and its intensity λ in wide limits at fixed values of the n–t scattering lengths A_n or the effective range r_n. Such a set includes both the attractive and repulsive potentials. Within such an approach, we will clarify the character of the correlation relations between various quantities and use them for the derivation of an important additional information for different physical systems. We note that the use of repulsive potentials can be substantiated by the presence of a strong Pauli repulsion in the interaction of a nucleon with a three-nucleon nucleus in the spin states under consideration. Such a repulsion counteracts the localization of the system in a small region of the configuration space,

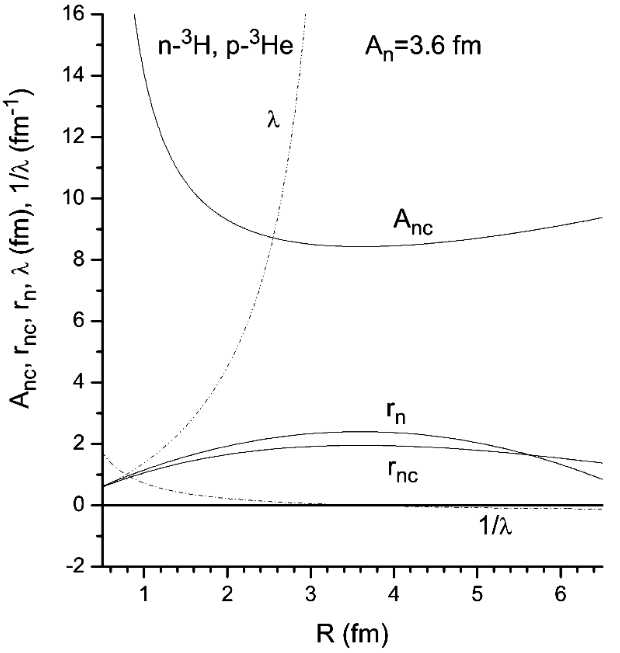

Fig. 1. Effective parameters of triplet p–h scattering A^1_{nc} and r^1_{nc}, the range of n–t scattering r^1_n, and the δ-shell potential intensity λ versus the effective range of the potential R at a fixed experimental value of the n–t scattering length $A^1_n = 3.60$ fm

where the nuclear attraction between nucleons is significant.

For the the nucleon + α-particle system, which is very close to our case as to the character of the intercluster interaction, such S-wave potential of the repulsive form was successfully used in the description of the phase shifts of N–α scattering up to energies $\simeq$ 25 MeV [29]. On the other hand, such systems are traditionally described with potentials of the attractive type [30]. The attractive character of the interaction of a nucleon with an α-particle is also demonstrated by results of the solution of the inverse problem of N–α scattering [31].

In Fig. 1, we present the dependence of the effective parameters of p–^{3}He scattering (A_{nc} and r_{nc}) calculated by us, the effective range of n–t scattering (r_n), and the potential intensity λ on the potential range at a fixed experimental value of the n–t scattering length in the triplet spin channel $A^1_n = 3.6$ fm [10]. The parameters R and λ of the set of attractive ($\lambda > 0$) and repulsive ($\lambda < 0$) δ-shell potentials (19) of the nuclear interaction of a nucleon with a three-nucleon nucleus are varied in a correlated manner so that to ensure the production of a chosen fixed four-nucleon parameter A^1_n [10] with the use of these potentials.

The value of the radius $R_\infty = 3.60$ fm, at which the function $1/\lambda$ crosses the abscissa, corresponds to the scattering by a sphere with the infinite attraction (on the approach to the point from the left) or by the infinite repulsion (on the approach from the right). The last variant is known in the literature as the hard-sphere model which was quite successfully used on the early stages of the studies of the processes of scattering in channels containing no bound or resonant states of the system. At the point R_∞, the plot of the function $A_{nc}(R)$ reaches a minimum, $A^\infty_{nc} = 8.4$ fm, whereas the functions of effective ranges take the maximum values $r^1_{nc} = 1.95$ fm and $r^1_n = 2.40$ fm at this point. The presented numerical values can be obtained directly with the help of formulas (24)–(27) which imply also that the value of R_∞ coincides with the length A^∞_n (24). To the left and to the right from the point R_∞, values of A_{nc}, r_n, and r_{nc} which correspond to, respectively, attractive and repulsive potentials are positioned. We note that the relation

$$r_n > r_{nc} \tag{28}$$

holds for all attractive interactions. For repulsive potentials, inequality (28) at $R \approx 5.63$ fm (for the given specific plot) changes into the inverse one:

$$r_n < r_{nc}. \tag{29}$$

Figure 1 demonstrates clearly how, for example, the choice of a specific value of the effective range r_n of n–t scattering allows one to at once determine a value of R (and, hence, $\lambda(R)$) and to obtain the nuclear-Coulomb quantities $A_{nc}(R)$ and $r_{nc}(R)$ corresponding to these particular pure nuclear quantities A_n and r_n. Analogously, as the second quantity (in addition to $A_n = 3.6$ fm) for the fixation of specific parameters of the potential, R and $\lambda(R)$, we can choose A_{nc} or r_{nc} and to find, respectively, r_n and r_{nc} or A_{nc} and r_n.

In Fig. 2, we present the correlations between the effective parameters of triplet p–h scattering, A^1_{nc} and r^1_{nc}, and the effective range of triplet n–t scattering r^1_n at a fixed experimental value of the triplet length of n–t scattering, $A^1_n = 3.6$ fm [10]. These interrelations directly between four-nucleon characteristics were obtained by the internal parametrization of the calculated quantities in terms of the range of the potential, R. We note that the analytic formulas for the correlation relations shown in the plots follow from (17)–(20):

$$\frac{1}{A_{nc}} = \frac{2}{a_{\rm B} I_1^2}\left[\frac{R}{A_n} + 2I_1K_1 - 1\right], \tag{30}$$

$$\frac{1}{A_{nc}} = \frac{2}{a_{\rm B} I_1^2}\left[1 + 2I_1K_1 - \frac{3}{2R}r_n\right], \tag{31}$$

$$\frac{1}{A_{nc}} = f_1(R) + f_2(R)r_{nc}, \tag{32}$$

$$f_1(R) = \frac{1}{a_{\rm B}}\left[4\frac{K_1}{I_1} + \frac{I_1^2 - x}{x^{3/2}I_1I_2}\right], \quad f_2(R) = \frac{3I_1}{a_{\rm B}^2 x^{3/2} I_2}, \tag{33}$$

$$r_{nc} = \frac{2a_{\rm B}}{3I_1^2}\left[\frac{I_2}{I_1}x^{3/2}(\frac{3}{2R}r_n - 1) + \frac{1}{2}(I_1^2 - x)\right]. \tag{34}$$

In fact, the graphical interpretation of relations (30)–(34) allows us to obtain a numerous additional information in addition to the numerical one.

In Fig. 2, we give also the values of the range R and the intensity λ of potentials (19) which reproduce the above-mentioned experimental value of the nuclear scattering length A_n^1, but give different values of the effective range r_n^1. The extreme right points of the plots in Fig. 2 correspond to the potentials with $\lambda = \pm\infty$ and denote the transition from the set of attractive potentials (branches (+)) to the set of repulsive potentials (branches (–)). The values of A_{nc}, r_{nc} and r_n obtained from this plot as $|\lambda| = \infty$ reconstruct exactly the numerical values given above directly prior to formula (28).

The analysis of the character of the dependence of the quantities A_{nc} and r_{nc} on r_n indicates that the triplet p–^{3}He scattering length for repulsive potentials decreases very weakly in a linear manner from 9.1 to 8.4 fm ($\simeq$8 %) under a quite significant variation of the effective range of n–t scattering r_n in the scope 1.5 ÷ 2.4 fm (=60 %). In this case, the nuclear-Coulomb effective range r_{nc} increases by $\simeq$23 % from 1.6 to 1.95 fm.

For the attractive potentials (19), the nuclear-Coulomb quantities A_{nc} and r_{nc} show a considerably greater sensitivity to the variation of the effective range of n–t scattering. With increase in r_n from 1.5 to 2.4 fm, the length A_{nc} decreases by $\simeq$23 % from 10.97 to 8.43 fm, and the effective range r_{nc} demonstrates a twice greater sensitivity, by growing by 46 % from 1.34 to 1.95 fm.

The above-presented results allow us to conclude that

1) scattering lengths A_{nc} are less (by 2–3 times) sensitive to variations of the effective range of n–t scattering r_n, than the nuclear-Coulomb effective range r_{nc};

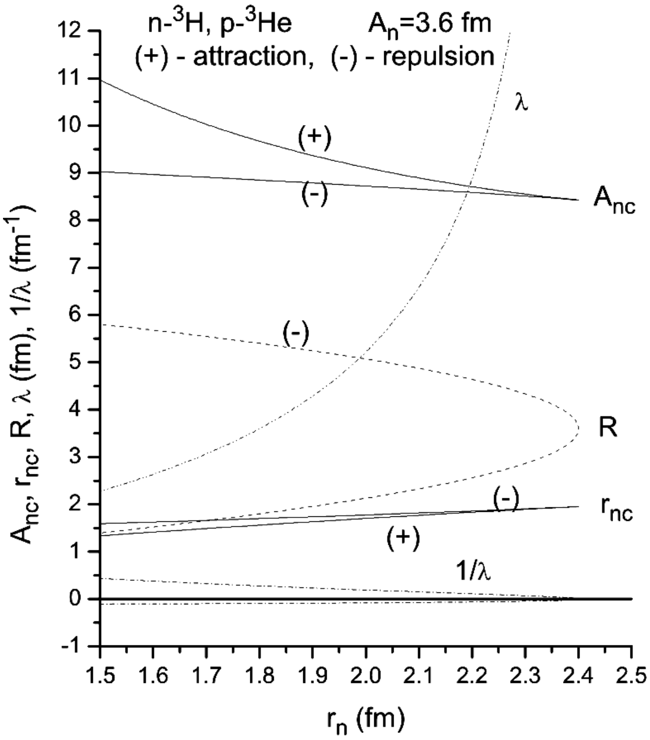

Fig. 2. Correlations between the effective parameters of triplet p–h scattering A_{nc}^1 and r_{nc}^1 and the effective range of triplet n–t scattering r_n^1 at a fixed experimental value of the triplet n–t scattering length A_n^1 = 3.6 fm [10]. The values of the range of the δ-shell potential R and the intensity λ ((+) – attractive, (–) – repulsive), which reproduce the low-energy parameters presented in the plot, are also given

2) in the case of attractive potentials, the sensitivity of the quantities A_{nc} and r_{nc} to the variation of the effective range of n–t scattering r_n is greater, respectively, by $\simeq$3 and $\simeq$2 times, than that for repulsive forces.

In Figs. 3 and 4, we give the correlations between the effective parameters of the singlet p–h scattering A_{nc}^0 and r_{nc}^0 and the effective range of the singlet n–t scattering r_n^0 at fixed values A_n^0 = 4.0 fm (19) and A_n^0 = 4.453 fm [16] given by the R-matrix processing of experimental p–h data. On the whole, the plots give the same pattern as that in Fig. 2. The larger nuclear singlet n–t lengths have led only to a relative increase in the nuclear-Coulomb effective parameters A_{nc}^0 and r_{nc}^0.

In the frame of the developed approach to δ-shell potentials of the repulsive type, we make prediction for the effective parameters of p–h scattering on the basis of the data for the n–t system. For example, for the triplet scattering, the experimental value $A_n^1 = (3.6 \pm 0.1)$ fm

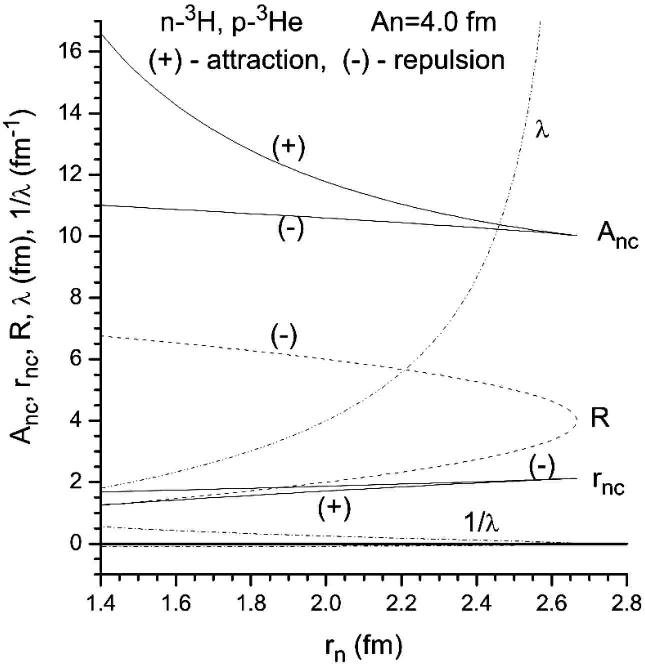

Fig. 3. Correlations between the effective parameters of singlet p–h scattering A^0_{nc} and r^0_{nc} and the effective range of singlet n–t scattering r^0_n at a fixed experimental value of the singlet n–t scattering length $A^0_n = 4.0$ fm (17) (Other designations are the same as in Fig. 2)

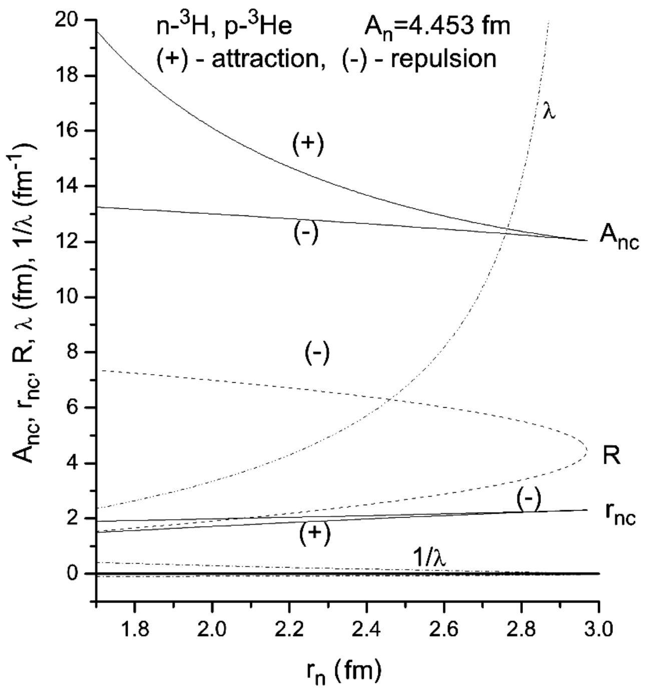

Fig. 4. Correlations between the effective parameters of singlet p–h scattering A^0_{nc} and r^0_{nc} and the effective range of singlet n–t scattering r^0_n at a fixed value of the singlet n–t scattering length $A^0_n = 4.453$ fm which corresponds to the R-matrix processing of the experimental p–h data [16] (R and λ are

[10] and the theoretically calculated value $r^1_n = (1.75 \pm 0.05)$ fm (9) let us to obtain the following values of the p–h parameters:

$$A^1_{nc} = (8.88 \pm 0.48)\,\text{fm}, \quad r^1_{nc} = (1.68 \pm 0.04)\,\text{fm}. \qquad (35)$$

For the scattering in the singlet spin channel, the experimental and theoretical n–t data on $A^0_n = (4.0 \pm 0.1)$ fm (17) and $r^0_n = (1.95 \pm 0.05)$ fm (8) in Fig. 3 correspond to the p–h effective parameters

$$A^0_{nc} = (10.63 \pm 0.52)\,\text{fm}, \quad r^0_{nc} = (1.85 \pm 0.04)\,\text{fm}. \qquad (36)$$

Analogously, for the R-matrix result $A^0_n = (4.453 \pm 0.100)$ fm [16] and $r^0_n = (1.95 \pm 0.05)$ fm (8), we get (see Fig. 4)

$$A^0_{nc} = (13.05 \pm 0.62)\,\text{fm}, \quad r^0_{nc} = (1.97 \pm 0.04)\,\text{fm}. \qquad (37)$$

Result (37) is somewhat overestimated due to the 11% overestimation of the input value $A^0_n = 4.453$ fm as compared with $A^0_n = 4.0$ fm (8).

For the sake of completeness, we present also values of the nuclear-Coulomb scattering lengths and effective ranges which correspond to attractive potentials. For example, the triplet n–t data, $A^1_n = (3.6 \pm 0.1)$ fm and $r^1_n = (1.75 \pm 0.05)$ fm, in Fig. 2 (branch (+)) correspond to the p–h effective parameters

$$A^1_{nc} = (9.90 \pm 0.85)\,\text{fm}, \quad r^1_{nc} = (1.53 \pm 0.04)\,\text{fm}. \qquad (38)$$

Analogously, for the singlet n–t data (8), we get (see Fig. 3)

$$A^0_{nc} = (12.0 \pm 1.0)\,\text{fm}, \quad r^0_{nc} = (1.68 \pm 0.04)\,\text{fm}, \qquad (39)$$

whereas, for $A^0_n = (4.453 \pm 0.100)$ fm and $r^0_n = (1.95 \pm 0.05)$ fm, we have (Fig. 4)

$$A^0_{nc} = (16.7 \pm 1.7)\,\text{fm}, \quad r^0_{nc} = (1.69 \pm 0.05)\,\text{fm}. \qquad (40)$$

Result (40) for A^0_{nc} is somewhat overestimated because, on its determination, we used the value of the effective n–t range r^0_n (8) which is consistent with the calculated value of the n–t length near (4.0 ± 0.1) fm. But, with increase in the n–t scattering length A_n, the corresponding effective ranger_n is also increased, as a rule. Therefore, we may assume that the scattering length $A_n^0 = 4.453$ fm must correspond to the greater effective range $r_n^0 \simeq 2.15$ fm. In this case, the correlation dependence $A_{nc}^0(r_n^0)$ gives a less value, $A_{nc}^0 \simeq 15.0$ fm.

Attractive potentials give the values of scattering lengths which exceed our predictions (35)–(37) for the repulsive effective interaction by 12%. On the other hand, the effective ranges for repulsive potentials exceed the corresponding values for attractive potentials by 10%.

The obtained triplet p–^{3}He scattering length (35) well agrees with the four-particle calculations of this quantity on the basis of the Kohn variational principle and the technique of hyperspheric harmonics with regard for three-particle forces [21]:

$A_{nc}^1 = 9.13\,\text{fm}$ (potential AV18UR).

A somewhat greater value is obtained in work [21] for potential AV18 without regard for three-nucleon forces:

$A_{nc}^1 = 10.1\,\text{fm}.$

The same value is obtained in calculations in the frame of the Monte Carlo method with inclusion of three-particle forces (potential AV18UR) [32]:

$A_{nc}^1 = (10.1 \pm 0.5)\,\text{fm}.$

Our prediction for the singlet p–h scattering length (36) is, again, rather close to the result of calculations in [21] for the interaction including three-particle forces:

$A_{nc}^0 = 11.5\,\text{fm}$ (potential AV18UR).

Without regard for three-particle forces, a somewhat greater value

$A_{nc}^0 = 12.9\,\text{fm}$ (potential AV18)

was obtained in [21].

The efficiency of the applied method of the study of the scattering of a proton and a neutron in charge-symmetric four-nucleon systems can be also demonstrated by the example of the scattering of a proton and a neutron by an α-particle. The experimental data for scattering lengths and effective ranges for these systems are known with a quite high accuracy [33]. Using the data on the scattering of a proton by an α-particle, we determined the attractive δ-potential and the repulsive potential which is characterized by a twice greater effective range and the modulus of the coupling constant (Table 4). The calculations performed with these potentials gave the convergent values of the scattering length and the effective range of the scattering in the $n+\alpha$-particle system. The pure nuclear scattering length obtained from the nuclear-Coulomb experimental data coincides with the experimental values obtained from the phase shifts of the scattering of a neutron by an α-particle. The effective range of the pure nuclear scattering predicted by us is also very close to the experimental result for the n+α system. Thus, the model of δ-shell interaction allows us to get the well consistent description of the nuclear-Coulomb and nuclear charge-symmetric systems p+α and n+α.

5. Conclusions

On the basis of the sufficiently reliable data on the n–t scattering lengths and the obtained correlation relations for repulsive potentials, we have predicted the scattering lengths and effective ranges for the nuclear-Coulomb system p+^{3}He: $A_{nc}^1 = (8.88 \pm 0.48)$ fm, $r_{nc}^1 = (1.68 \pm 0.04)$ fm and $A_{nc}^0 = (10.63 \pm 0.52)$ fm, $r_{nc}^0 = (1.85 \pm 0.04)$ fm.

It is shown that the Coulomb renormalization of pure nuclear lengths does not change the well-established relation $A^1 < A^0$ for the n+t system.

We have established that, on the description of the effective nuclear interaction of a nucleon with a three-nucleon nucleus with repulsive potentials, the nuclear-Coulomb scattering length and the effective range of p–h scattering depend linearly on the effective range of n–t scattering, r_n. For attractive potentials, the relevant dependences differ somewhat from linear ones.

On the whole, our predictions for the p–^{3}He scattering lengths give preference to the results of the phase-shift analysis in [6] corresponding to the inequality $A_{nc}^1 < A_{nc}^0$ (set I). Our consideration shows, however, that the singlet scattering length in [6] is somewhat

Table 4. Scattering length A_n, the effective range r_n, and the shape parameter P of the scattering of a neutron by an α-particle calculated on the basis of the experimental data for the nuclear-Coulomb S-wave scattering of a proton by an α-particle: $A_{nc} = (4.97 \pm 0.12)$ fm, $r_{nc} = (1.30 \pm 0.08)$ fm [33] (λ is the intensity, and R is the range of the δ-shell potential of the interaction of a nucleon and an α-particle)

r_{nc}, fm	λ, fm	R, fm	A_n, fm	r_n, fm	P
1.38	9.8259	1.9895	2.494	1.5948	−0.0693
	−13.8474	3.0471	2.4975	1.5844	−0.0537
1.30	5.3867	1.6919	2.4667	1.4822	−0.0642
	−9.4010	3.3667	2.4789	1.4407	0.0211
1.22	3.7785	1.4812	2.4362	1.3746	−0.0605
	−7.7854	3.5996	2.4615	1.2902	0.1868
Exp. [33]			2.4641±0.0037	1.385±0.041	

overestimated (by $\simeq$ 35%) and that the Coulomb renormalization cannot result in such an increase in the singlet n–t scattering length. We note that the phase analyses performed prior to work [6] also gave not quite reliable results for the singlet phase shift of p–h scattering. In view of the above discussion, we consider the set [6]

$$A^1_{nc} = (8.2 \pm 0.6)\,\text{fm}, \quad A^0_{nc} = (10.3 \pm 2.7)\,\text{fm}$$

as the most reliable phase shift prediction of the p–h scattering lengths, which corresponds to the p–h dataset of [5] supplemented by the experimental values of the proton analyzing power [7].

The performed study indicates the necessity to carry out a new phase analysis of the data on the low-energy p–^{3}He scattering with the use of the scattering lengths predicted by us and the new experimental data for the differential cross-sections and the proton analyzing powers obtained in [34] as the input data.

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ВИЗНАЧЕННЯ ЕФЕКТИВНИХ ПАРАМЕТРІВ РОЗСІЯННЯ ПРОТОНА НА ЯДРІ ^{3}He НА ОСНОВІ ДАНИХ З РОЗСІЯННЯ НЕЙТРОНА НА ТРИТОНІ

В.П. Левашев

Р е з ю м е

На основі потенціальної моделі з ефективною взаємодією нуклона з ядром у вигляді δ-оболонкових потенціалів вивчається зв'язок між довжинами та ефективними радіусами розсіяння нейтрона на тритоні ^{3}H і відповідними величинами для розсіяння протона на ядрі ^{3}He. Показано, що кулонівське перенормування суто ядерних довжин не змінює добре встановленого для системи n + ^{3}H співвідношення між довжинами $A^1 < A^0$. Зроблені нами передбачення для довжин p$-^3$He-розсіяння дозволяють однозначно віддати перевагу набору фазових даних аналізу E.A. George et al. (2003), що відповідає співвідношенню довжин $A^1_{nc} < A^0_{nc}$.