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## FLOWING A DIRECT MICROCURRENT IN THERMAL PLASMAS

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UDC 533.9  
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The flowing of small microcurrents in a thermal complex plasma which exist under the interaction of dust grains or in the probe's measurements has been considered. A change of the ionization degree of the plasma and the formation of nonequilibrium charge carriers during the flowing of a direct microcurrent have been demonstrated. The functional connection between the voltage drops on the plasma layer and on the plasma–dust grain or plasma–probe contact has been established.

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### 1. Introduction

The combustion of a metal powder in the oxygen medium is a perspective method to obtain high-purity submicron metal oxides. The low-temperature plasma with condensed dispersed phase is formed in the region of condensation of the products of combustion of a metal powder cloud. Such a plasma consists of the gas at atmospheric pressure and solid or liquid dust grains resulting from the volume condensation or being the particles of a not-burnt fuel. The absolute temperature of such a plasma is usually about 1500–3500 K (0.1–0.3 eV), and the system is considered isothermal. It usually contains easily ionizable atoms of alkali metals as a natural impurity or in the form the special additional agents which are the basic suppliers of free electrons and singly charged positive ions.

Charged dust grains interact intensely with the plasma and with one another. The interphase interaction or the interaction of charged dust grains are accompanied by the electric current flow. Therefore, the knowledge of the mechanisms of the current flow in a plasma is necessary both for the calculations of parameters of the interaction of dust grains with

a plasma and with one another and for the correct interpretation of the probe's measurements.

The problem of the electric current in a plasma has been studied for a long time since Langmuir who was the first to propose the theory of an electrical probe in the plasma, but for the collisionless plasma within the orbit-limited probe model [1–5]. The current flow in the thermal plasma (the theory of probes at elevated pressures [6–8]) was studied mostly with regard for magneto-hydrodynamic generators, where the basic problem was the increase of the current up to the peak value.

The dependence of the ionization equilibrium on the electric microcurrent in a thermal plasma has been insufficiently studied till now. The present paper is devoted to the study of the influence of the flowing of small direct currents, which do not heat a plasma, on the nonequilibrium ionization in a thermal plasma by the example of the interaction of charged metal planes (probe electrodes).

### 2. Statement of the Problem

#### 2.1. Ionization equilibrium in a thermal plasma

The ionization in the thermal plasma occurs due to the collisions between gas particles. Therefore, such a plasma is strongly collisional unlike a low-pressure gas-discharge plasma. The thermal plasma is characterized by the equality of the temperatures of electrons, ions, and neutral particles. The equilibrium ionization in any microvolume of such a plasma is described by the Saha

equation [9, 10]

$$\frac{n_e n_i}{n_a} = \frac{g_i}{g_a} \nu_e \exp \frac{-I}{T} \equiv K_S, \quad (1)$$

where  $n_e$ ,  $n_i$ , and  $n_a$  are the number densities of electrons, ions, and atoms, respectively;  $\nu_e = 2(m_e T / 2\pi\hbar^2)^{3/2}$  is the effective density of electron states;  $g_i$  and  $g_a$  are the statistical weights of ions and atoms;  $I$  is the potential of ionization of atoms of the additional agent;  $T$  is the equilibrium temperature;  $m_e$  is the electron mass;  $\hbar$  is the Planck constant; and  $K_S$  is the Saha constant.

Here, the conditions of conservation of the mass and the charge

$$n_e = n_i = n_0, \quad n_i + n_a = n_A \quad (2)$$

should be fulfilled, where  $n_0$  is the nonperturbed number density, and  $n_A$  is the number density of the easily ionizable additional agents. In the low-temperature plasma, the ionization degree is so low that  $n_i \ll n_a \sim n_A$ .

The existence of any external perturbation leading to an increase or decrease of the ionization degree of the plasma causes a displacement of the ionization equilibrium that can be described by the thermodynamic parameter of the interphase interaction [11]  $\psi = \mu_e + \mu_i - \mu_a$ , where  $\mu_j$  is the chemical potential of the component  $j$ . In the low-temperature plasma, at a low degree of ionization, the chemical potential of atoms changes a little under the external action as  $n_a \approx n_A$ . Therefore, the parameter  $\psi$  depends only on the changes of the chemical potentials of electrons and ions  $\psi = \delta\mu_e + \delta\mu_i$ .

In the equilibrium case, the nonperturbed number density in the low-temperature plasma is equal, according to Eq. (1), to

$$n_0 = \sqrt{n_A K_S} = \nu_e \exp(\mu_{e0}/T). \quad (3)$$

The presence of a perturbation leads to a change of this number density. In this case, the ionization equilibrium is described by the modernized Saha equation

$$n_q^2/n_A = K_S \exp(\psi/T), \quad (4)$$

where  $n_q = \sqrt{n_e n_i}$  is the quasinonperturbed number density of charge carriers.

Respectively, the quasinonperturbed number density looks as

$$n_q = n_0 \exp(\psi/2T). \quad (5)$$

If the isolated electrodes are the perturbing factor, the ionization equilibrium displacement, being a result of the exchange of charges with the equilibrium electrode (i.e. without current), can be described by the introduction of a concept of bulk plasma potential  $\psi = -e\varphi_{pl}$  [11,12]. The bulk plasma potential characterizes the charge of the whole plasma volume and depends on the potential barriers on the phase boundaries. In particular, the bulk plasma potential can be defined by the expression [12]  $\varphi_{pl} = -2(T/e) \tanh(e\phi_s/4T)$  for semiinfinite plasmas with a potential barrier  $e\phi_s$  on the interface and by

$$\varphi_{pl} = -2\frac{T}{e} \tanh\left(\frac{e\phi_1 + e\phi_2}{4T}\right) \quad (6)$$

for the plasma layer restricted by two flat electrodes with potential barriers  $e\phi_1$  and  $e\phi_2$ . As long as the bulk plasma potential remains constant or is a solution of the Laplace equation  $\Delta\varphi_{pl} = 0$ , the application of the Poisson–Boltzmann theory is possible. Thus, the total (measurable) value of the potential is  $\varphi = \phi + \varphi_{pl}$ , where  $\phi$  is the solution of the Poisson equation.

## 2.2. Balance of currents on the isolated solid surface

The potential barrier on the solid-plasma boundary is determined by the balance of the currents that flow through the surface of a dust grain or a probe electrode. In the thermal plasma, there are the following currents:

(i) For the Richardson–Dushman thermionic emission from the surface,

$$J_e^T = -\frac{4\pi e m_e T^2}{(2\pi\hbar)^3} \exp\left(-\frac{W}{T}\right), \quad (7)$$

where  $J$  is the density of the electric current in the direction from the electrode to the plasma;  $W$  is the electron work function from the metal into the plasma, which is less than the work function into the vacuum  $W = W_m - (1/4)T \ln(m_i/m_e)$  [13].

(ii) The backflow of electrons absorbed by the electrode surface,

$$J_e^{abs} = (1/4)en_{es}v_{Te}, \quad (8)$$

where  $v_{Te} = \sqrt{8T/\pi m_e}$  is the thermal velocity of electrons, and  $n_{es}$  is the electron number density at the electrode surface.

(iii) The current density of the surface recombination of ions,

$$J_i^{\text{rec}} = -(1/4)\gamma_s e n_{is} v_{Ti}, \quad (9)$$

where  $v_{Ti} = \sqrt{8T/\pi m_i}$  is the thermal velocity of ions,  $n_{is}$  is the surface number density of ions,  $\gamma_s$  is the surface recombination coefficient  $\gamma_s = [1 + (g_i/g_a) \exp(-E_s^{\text{ion}}/T)]^{-1}$ ;  $E_s^{\text{ion}} = I - W - \psi_s$  is the surface ionization energy; and  $\psi_s = T \ln(n_{es} n_{is}/n_0^2)$  is the parameter of the interphase interaction at the surface.

(iv) The current density of the surface ionization of atoms,

$$J_a^{\text{ion}} = (1/4)\beta_s e n_{as} v_{Ta}, \quad (10)$$

where  $v_{Ta}$  is the thermal velocity of atoms ( $v_{Ti} \approx v_{Ta}$ ),  $n_{as} \sim n_A$  is the surface number density of atoms, and  $\beta_s$  is the surface ionization coefficient  $\beta_s = [1 + (g_a/g_i) \exp(E_s^{\text{ion}}/T)]^{-1}$ .

In the equilibrium state, the detailed balancing principle is true, i.e. the current densities (7)–(10) in pairs should be equal to

$$J_e^T = -J_e^{\text{abs}} \quad \text{and} \quad J_i^{\text{rec}} = -J_a^{\text{ion}}, \quad (11)$$

which allows us to determine the potential of an isolated electrode  $\phi_{s0}$ . The floating potential of the electrode is defined by the sum

$$\varphi_f = \phi_{s0} + \varphi_{pl}, \quad (12)$$

where the bulk plasma potential  $\varphi_{pl}$  depends on the potentials of both electrodes bounding the plasma layer [Eq. (6)].

The existence of the spatial charge in the plasma at the electrode surface leads to the nonequilibrium ionization. This case is considered in [14,15], where it is shown that the surface number densities of electrons and ions are as follows:

$$n_{es} = n_q \frac{\exp(2e\phi_s/T)}{2 \cosh(e\phi_s/T) - 1}, \quad (13)$$

$$n_{is} = n_q \frac{\exp(-2e\phi_s/T) + 2 \sinh(e\phi_s/T)}{2 \cosh(e\phi_s/T) - 1}. \quad (14)$$

The potential of an isolated electrode with respect to the bulk plasma potential can be determined, by basing on the balance of currents, Eq. (11). In view of Eqs. (13) and (14), this potential is a solution of the equation [13]

$$\frac{n_q}{\nu_e} \exp\left[\frac{W + 2e\phi_{s0}}{T}\right] = 2 \cosh\left(\frac{e\phi_{s0}}{T}\right) - 1. \quad (15)$$

The connection of the electrodes bounding the plasma layer to an external power supply causes the current flow through the layer under the action of a voltage drop on it and due to the spatial inhomogeneity of the ionization degree, which is accompanied by a change of the number densities, Eqs. (13) and (14). In the present paper, we study the influence of the nonequilibrium ionization on the flowing of a direct current through the plasma layer in the limit of small currents which do not heat the plasma.

### 3. Displacement of Ionization Equilibrium by a Current

#### 3.1. Balance of the surface currents and the conduction currents

The flowing of an electric current through the plasma is a complex process which is accompanied by both the injection of electrons into the contact area and a change of the plasma ionization degree. The current of the external circuit  $J_c = J_e + J_i$  is provided by the imbalance of currents (11) on the surface of electrodes,

$$J_e = J_e^T \left(1 - \frac{n_{es}}{n_{es0}}\right) \quad \text{and} \quad J_i = J_a^{\text{ion}} \left(1 - \frac{n_{is}}{n_{is0}}\right), \quad (16)$$

where  $n_{es0}$  and  $n_{is0}$  are, respectively, the surface number density of electrons [Eq. (13)] and ions [Eq. (14)] without current.

The thermionic current (7) remains constant at the surface of the positive electrode, whereas the current owing to the absorption of electrons (8) increases, providing the current of the external circuit. The current of the recombination of ions (9) decreases to the value  $J_i^{\text{rec}} = 0$ , and the current of the external circuit is supported by the current caused by the ionization of atoms, which has a constant value in the low-temperature plasma. Therefore, the current arisen from the ionization of atoms  $J_a^{\text{ion}}$  is the saturation current for the positive electrode.

The current owing to the absorption of electrons decreases to the value  $J_e^{\text{abs}} = 0$  at the surface of the negative electrode, and the current of the external circuit is maintained by the thermionic current. Therefore, the thermionic current  $J_e^T$  is the saturation current for the negative electrode. The current caused by the

recombination of ions increases, which ensures the current flow in the external circuit.

The electroneutrality of plasma is disturbed only in the space charge region at the electrode surface. The rest of the volume of the plasma remains electroneutral. The electroneutrality of plasma means that the following relations should be valid outside of the space charge region:

$$n_e = n_i = n_q, \quad \nabla \cdot \mathbf{E} = 0. \quad (17)$$

In addition, while the current of the external circuit is less than the saturation currents, it is completely provided by the imbalance of currents Eq. (11) and should not cause a spatial change of the electron and ion number densities. Therefore, the current of the external circuit should be provided by the conduction currents

$$J_e = \sigma_e E \quad \text{and} \quad J_i = \sigma_i E, \quad (18)$$

where, for the plasma layer bounded by flat electrodes,  $E = \delta U/d$ ,  $\delta U$  is the voltage drop on the plasma layer,  $d$  is the layer thickness;  $\sigma_{e(i)} = eK_{e(i)}n_q$  is the electron (ion) conductivity, and  $K_{e(i)}$  is the electron (ion) mobility.

### 3.2. Change of the ionization degree of the plasma as a result of the current flow

Let the voltage of the external power supply be set in such a manner that electrode 1 is positive, and electrode 2 is negative. The voltage drops on the contacts make  $\delta\phi_1 > 0$  and  $\delta\phi_2 < 0$ . Thus,  $\phi_1 = \phi_{s0} + \delta\phi_1$  and  $\phi_2 = \phi_{s0} + \delta\phi_2$ . The total voltage  $U = \delta U + \delta\phi_1 - \delta\phi_2$ .

Equations (16) and (18) connect the voltage drop on the contact  $\delta\phi$  and the voltage drop on the plasma layer  $\delta U$ . Let us assign

$$\Omega_e = \frac{\exp\left[\frac{e(\phi_{s0} + \delta\phi)}{T}\right]}{2 \cosh\left[\frac{e(\phi_{s0} + \delta\phi)}{T}\right] - 1},$$

$$\Omega_i = \frac{\exp\left[\frac{-2e(\phi_{s0} + \delta\phi)}{T}\right] + 2 \sinh\left[\frac{e(\phi_{s0} + \delta\phi)}{T}\right]}{2 \cosh\left[\frac{e(\phi_{s0} + \delta\phi)}{T}\right] - 1}.$$

Then, for the positive contact,

$$\frac{\sigma_e}{J_e^T} \frac{\delta U}{d} = 1 - \Omega_e \frac{n_q}{n_{es0}}, \quad (19)$$

$$\frac{\sigma_i}{J_a^{\text{ion}}} \frac{\delta U}{d} = 1 - \Omega_i \frac{n_q}{n_{is0}}. \quad (20)$$

Respectively, for the negative contact, it is necessary to change the sign of the right parts of the equations.

However, these two equations cannot provide for the identical dependence between  $\delta\phi$  and  $\delta U$ , if the quasinonperturbed number density  $n_q$  remains the constant. The voltage drop on the plasma–electrode boundary caused by the electron current  $J_e$  provides for a change of the potential barrier by the value  $\delta\phi$  [Eq. (19)]. At such a change of the barrier, the ion number density caused by the imbalance of the ionization current and the recombination current is more than the one, which is necessary for the flowing of the ionic current  $J_i$ , or it is less. The intensity of the collisional ionization in the thermal plasma is proportional to  $n_e n_A$ , and the volumetric recombination rate is proportional to  $n_e n_i$ . Therefore, the change of the electron concentration does not influence (if one does not consider a change of the volumetric recombination coefficient) the ionization degree of the plasma as it affects the ionization and recombination rates. At the same time, the increase of the ion number density leads to an increase of the recombination rate and a decrease of the ionization degree of plasma. Moreover, the decrease of the ion number density leads to an increase of the ionization degree of the plasma due to the decrease of the recombination rate.

Therefore, in Eqs. (19) and (20) instead of the constant value  $n_q$  which is determined by the bulk plasma potential, it is necessary to use some other value  $n_{qs}$  which provides for the conformity of the currents through the barrier with the currents through the plasma layer. Thus, we obtain the equation for  $n_{qs}(\delta\phi)$ , having excluded the voltage drop on the plasma layer  $\delta U$  from Eqs. (19) and (20):

$$n_{qs} = \frac{J_a^{\text{ion}}/\sigma_i - J_e^T/\sigma_e}{\frac{J_a^{\text{ion}}}{\sigma_i} \frac{\Omega_i}{n_{is0}} - \frac{J_e^T}{\sigma_e} \frac{\Omega_e}{n_{es0}}}. \quad (21)$$

The values of  $n_{qs}(\delta\phi)$  are different for the positive and negative voltage drops  $\delta\phi$ . As an example, Fig. 1 presents the dependence of the ratio of  $n_{qs}$  to the equilibrium value  $n_{q0}$  on the voltage drop on the contact. In this case, the following parameters of the plasma are used: the isothermal temperature  $T = 0.2 \text{ eV}$  (2300 K); the electron work function of a metal into vacuum  $W_m = 4.5 \text{ eV}$ , that provides the thermionic current

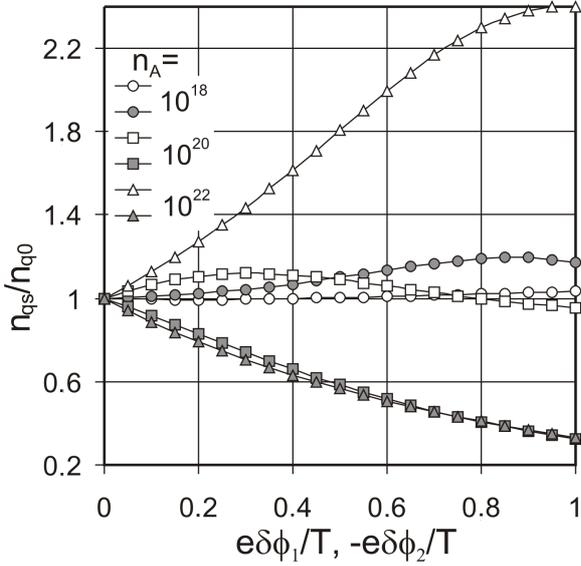


Fig. 1. Dependences of the relative quasinonperturbed number density at the electrode surface on the voltage drop on the contact: light markers for  $\delta\phi_1 > 0$  and dark markers for  $\delta\phi_2 < 0$

density  $J_e^T \approx -14 \text{ kA/m}^2$ . Here, we take three different number densities of the additional agent, potassium, ( $I = 4.34 \text{ eV}$ ):  $n_{A1} = 10^{18} \text{ m}^{-3}$ ,  $n_{A2} = 10^{20} \text{ m}^{-3}$ , and  $n_{A3} = 10^{22} \text{ m}^{-3}$ . This provides the current densities of the ionization of atoms  $J_{a1}^{\text{ion}} \approx 17 \text{ A/m}^2$ ,  $J_{a2}^{\text{ion}} \approx 130 \text{ A/m}^2$ , and  $J_{a3}^{\text{ion}} \approx 2 \text{ kA/m}^2$ .

It should be noted that the ionization degree can simultaneously be incremented at both contacts or be simultaneously decreased. Nevertheless, in any case, there is a gradient of the quasinonperturbed number density which is accompanied by the gradient of the chemical potential [Eq. (3)]. This gradient causes the ambipolar diffusion of electrons and ions which can be directed along the field or against it.

### 3.3. Voltage drop on the plasma layer and on the contact

Any of Eq. (19) or Eq. (20) after the replacement  $n_q$  by  $n_{qs}$  obtained from Eq. (21) allows us to determine the relation between the voltage drops on the contact and the plasma layer. In the case where the potential of the isolated electrodes  $\phi_{s0} \sim 0$ , it is possible to use the following asymptotic expressions:

$$\delta\phi_1 \approx \frac{T}{e} \ln \left( 1 - \frac{\sigma_e}{J_e^T} \frac{\delta U}{d} \right) = -\frac{T}{e} \ln \left( 1 - \frac{\sigma_i}{J_a^{\text{ion}}} \frac{\delta U}{d} \right),$$

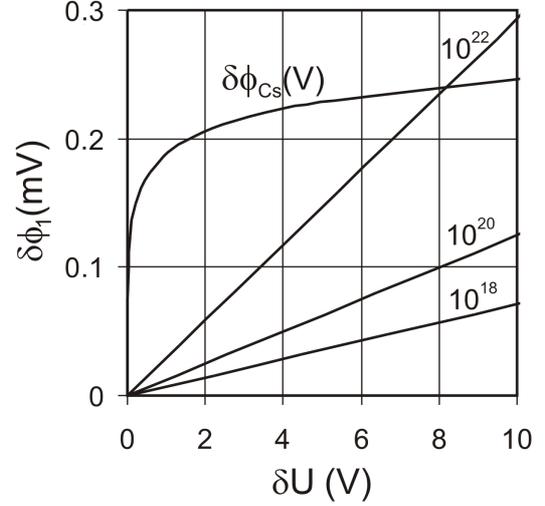


Fig. 2. Dependences of the voltage drop on the contact on the voltage drop on the plasma layer with the additional agent, potassium, at  $T = 0.2 \text{ eV}$ ; and with the additional agent, caesium, with  $n_A = 10^{23} \text{ m}^{-3}$  at the temperature  $T = 0.1 \text{ eV}$

$$\delta\phi_2 \cong \frac{T}{e} \ln \left( 1 + \frac{\sigma_e}{J_e^T} \frac{\delta U}{d} \right) = -\frac{T}{e} \ln \left( 1 + \frac{\sigma_i}{J_a^{\text{ion}}} \frac{\delta U}{d} \right). \quad (22)$$

The limiting of the external current by the saturation currents leads to the limiting of the voltage drop on the plasma layer. As follows from Eq. (22), the current through the positive electrode imposes limiting  $\delta U < J_a^{\text{ion}} d / \sigma_i$ , and the current through the negative electrode imposes the limiting  $\delta U < -J_e^T d / \sigma_e$ .

In Fig. 2, the dependence  $\delta\phi(\delta U)$  is presented for the plasma parameters which are the same as those in Fig. 1. The voltage drop on the contact is usually much less than the voltage drop on the plasma layer. Therefore, the dependence is linear in the limit of the external supply voltages under consideration. This does not hold, when the conduction current of the plasma exceeds the currents through the electrode surface. For example, if the additional agent, caesium, ( $I = 3.9 \text{ eV}$ ) with the large number density  $n_A = 10^{23} \text{ m}^{-3}$  is used, and the temperature is lowered to  $T = 0.1 \text{ eV}$ ,  $\delta\phi > \delta U$  in the range of small values of the external supply voltage.

## 4. Diffusion of Nonequilibrium Charge Carriers

### 4.1. Continuity equations

The imbalance of the surface current densities (11) leads to a change of the ionization degree of the plasma near the electrode surface, when the current flows

through it. The change of the surface value of the quasinonperturbed number density defined by (21) leads to the spatial distribution of  $n_q(r)$ , which causes the ambipolar diffusion of nonequilibrium charge carriers [14].

Let us consider the continuity equations for the electron and ion current densities in the plasma layer [16, 17]

$$\frac{\partial n_e}{\partial t} = \frac{1}{e} \frac{\partial}{\partial r} J_e + \beta_V n_e n_A - \gamma_V n_e n_i,$$

$$\frac{\partial n_i}{\partial t} = -\frac{1}{e} \frac{\partial}{\partial r} J_i + \beta_V n_e n_A - \gamma_V n_e n_i, \quad (23)$$

where  $\beta_V$  is the alkali-metal atom ionization rate, and  $\gamma_V$  is the electron-ion volumetric recombination coefficient. Here, it is considered that the ionization degree in the low-temperature plasma is low enough to assume the atom number density to be equal to the number density of the additional agent  $n_a \sim n_A$ .

Outside the space charge region,  $n_e = n_i = n_q$ . Therefore, the current densities are as follows:

$$J_e = \sigma_e E + e D_e \frac{\partial}{\partial r} n_q, \quad J_i = \sigma_i E - e D_i \frac{\partial}{\partial r} n_q. \quad (24)$$

Then, it follows from Eqs. (23) and (24) in the stationary case that

$$D_e \frac{\partial^2}{\partial r^2} n_q + K_e \frac{\partial}{\partial r} (n_q E) + G = 0, \quad (25)$$

$$D_i \frac{\partial^2}{\partial r^2} n_q - K_i \frac{\partial}{\partial r} (n_q E) + G = 0, \quad (26)$$

where  $G = \beta_V n_q n_A - \gamma_V n_q^2$ .

We now multiply Eq. (25) by  $\sigma_i$  and Eq. (26) by  $\sigma_e$  and combine both equations as

$$\frac{\sigma_i D_e + \sigma_e D_i}{\sigma_e + \sigma_i} \frac{\partial^2}{\partial r^2} n_q + \frac{\sigma_i K_e - \sigma_e K_i}{\sigma_e + \sigma_i} \frac{\partial}{\partial r} (n_q E) + G =$$

$$D \frac{\partial^2}{\partial r^2} n_q + K \frac{\partial}{\partial r} (n_q E) + G = 0, \quad (27)$$

where  $D$  is the ambipolar diffusion coefficient, and  $K$  is the ambipolar drift mobility:

$$D = \frac{n_i D_i D_e + n_e D_e D_i}{n_e D_e + n_i D_i} = 2 \frac{D_e D_i}{D_e + D_i}, \quad (28)$$

$$K = \frac{n_i K_i K_e - n_e K_e K_i}{n_e K_e + n_i K_i} = 0. \quad (29)$$

As follows from Eq. (29), the ambipolar mobility is equal to zero in the electroneutral plasma, i.e. outside the space charge region, and the external electric field does not influence the movement of charge carriers. It is determined by the attractive forces between the electrons and ions which considerably surpass the forces applied to the charges by the external field. Therefore, the action of the field on the electrons and ions, whose number densities are equal, is counterpoised. Without the current,  $G = \beta_V n_{q0} n_A - \gamma_V n_{q0}^2 = 0$ ; therefore, Eq. (27) can be reduced to the form

$$\lambda_R^2 \frac{d^2}{dr^2} \left( \frac{n_q}{n_{q0}} \right) - \left( \frac{n_q}{n_{q0}} \right)^2 + \frac{n_q}{n_{q0}} = 0, \quad (30)$$

where  $\lambda_R = \sqrt{D/\beta_V n_A}$  is the recombination length. In the thermal plasma at atmospheric pressure,  $\lambda_R \sim 0.01 - 0.1 \mu\text{m}$ .

#### 4.2. Space distribution of nonequilibrium carriers caused by the current flow

Let us present the quasinonperturbed number density in the form of a deviation from the equilibrium value  $n_q = n_{q0} + \delta n_q$ . Respectively, the ratio  $n_q/n_{q0} = 1 + \delta n_q/n_{q0} \equiv 1 + \xi$ . Then Eq. (30) is reduced, after the change  $x = r/\lambda_R$ , to

$$\xi'' - \xi^2 - \xi = 0. \quad (31)$$

Having lowered the order of Eq. (31), we obtain the equation with parted variables which can be presented as

$$\int \frac{d\xi}{\sqrt{(2/3)\xi^3 + \xi^2 + C_1}} = \pm x + C_2. \quad (32)$$

We note that our aim is not to find the exact solution to Eq. (31) which can be expressed in terms of the elliptic functions [18], but to define the most simple function describing this solution. At first, let us consider the case  $|\xi| \ll 1$ , when we can neglect the quadratic term in Eq. (31). The solution of the equation  $\xi'' = \xi$  is the function  $\xi = C_1 \exp(x) + C_2 \exp(-x)$ . For the semiinfinite plasma,  $\xi = \xi_1 \exp(-x)$ . The gauge of the solution is defined by the recombination length which is much less than the screening length  $r_D \sim 1 \mu\text{m}$ . The thickness of the plasma layer  $d$  should be much more than the screening length, i.e.  $d \gg \lambda_R$ . Therefore, the solution for  $|\xi| \ll 1$  can be presented as

$$\xi = \xi_1 \exp(-x) + \xi_2 \exp(x - d/\lambda_R). \quad (33)$$

For great values of  $\xi$ , it is possible to use a superposition of solutions for the semiinfinite plasma, which means  $C_1 = 0$  in Eq. (32). Then the solution of Eq. (32),

$$\xi = \frac{3}{2} \frac{1}{\cosh^2(x/2 + C_2)},$$

can be presented, after the definition of the constant from the requirement  $\xi(0) = \xi_1$ , as

$$\frac{x}{2} = \ln \left( \sqrt{\frac{\xi_1}{\xi}} \times \frac{1 + \sqrt{1 + 2\xi/3}}{1 + \sqrt{1 + 2\xi_1/3}} \right).$$

This allows us to put down the superposition of the solutions for each electrode in the form of an equation which similar to Eq. (33),

$$\xi = \xi_1 \exp(-x)\delta_1^2 + \xi_2 \exp(x - d/\lambda_R)\delta_2^2$$

or

$$n_q = n_{q0} + (n_{q1} - n_{q0}) \exp\left(\frac{-r}{\lambda_R}\right) \delta_1^2 + (n_{q2} - n_{q0}) \exp\left(\frac{r-d}{\lambda_R}\right) \delta_2^2 \quad (34)$$

in the dimensional form, where  $n_{q1}$  and  $n_{q2}$  are determined by Eq. (21). The equilibrium value (without current) of  $n_{q0}$  is determined by the bulk plasma potential [Eq. (6)]:  $n_{q0} = n_0 \exp\{\tanh[e(\phi_1 + \phi_2)/4T]\}$ ,

$$\delta_1 \approx \frac{1 + \sqrt{1 + \frac{2}{3} \left(\frac{n_{q1}}{n_{q0}} - 1\right) \exp\left(\frac{-r}{\lambda_R}\right)}}{1 + \sqrt{1 + \frac{2}{3} \left(\frac{n_{q1}}{n_{q0}} - 1\right)}},$$

$$\delta_2 \approx \frac{1 + \sqrt{1 + \frac{2}{3} \left(\frac{n_{q2}}{n_{q0}} - 1\right) \exp\left(\frac{r-d}{\lambda_R}\right)}}{1 + \sqrt{1 + \frac{2}{3} \left(\frac{n_{q2}}{n_{q0}} - 1\right)}}. \quad (35)$$

It should be noted that  $\delta_1 \sim \delta_2 \sim 1$  in most cases. Therefore, we have presented the solution in a form which allows using the solution of the linearized equation [Eq. (33)] with regard for possible small nonlinearities.

The space distribution of nonequilibrium charge carriers (34) defines their ambipolar diffusion and depends on the current flowing through the plasma layer. Thus, the nonequilibrium ionization is promptly damped, because the gauge of the spatial changes of  $\xi$  is defined by the recombination length  $\lambda_R \ll r_D \ll d$ .

## 5. Conclusion

The flowing of a direct microcurrent through a thermal plasma at atmospheric pressure is accompanied by a change of the ionization degree of the plasma and establishes the spatial distribution of nonequilibrium electrons and ions along the streamlines. The nonequilibrium ionization caused by the current flow is promptly damped, while removing from the solid-plasma contact, but influences the relation between the voltage drops on the contact and on the plasma layer.

The presented model of the nonequilibrium ionization of a thermal plasma under the flowing of a direct current is applicable only to the currents which are smaller than the saturation currents that correspond to the limit of the voltage drop on the plasma layer:  $J_e^T d/\sigma_e < \delta U < J_a^{\text{ion}} d/\sigma_i$ . If the current of the external circuit exceeds the saturation current, the additional ionization of the plasma caused by the injection of charge carriers will occur.

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Received 21.08.06

## ПРОТІКАННЯ ПОСТІЙНОГО МІКРОСТРУМУ У ТЕРМІЧНІЙ ПЛАЗМІ

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### Резюме

Досліджено протікання малих мікрострумів у термічній комплексній плазмі, котрі існують за умови взаємодії частинок або у зондовій діагностиці. Продемонстровано зміну ступеня іонізації плазми та утворення нерівноважних носіїв заряду под дією мікроструму. Встановлено функціональний зв'язок між падінням напруги на шарі плазми і на контактній плазма-частинка або плазма-зонд.