t-MATRIX APPROXIMATION IN THE THEORY OF (p,2p) AND (p,np) REACTIONS

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Using the t-matrix approximation for distorted waves, new formulas for the amplitudes of the (p,2p) and (p,np) reactions are obtained. These formulas generalize the McCarthy expressions [8], derived for the symmetric coplanar geometry, to the case of an arbitrary geometry of the momenta of escaping nucleons. The derivation was carried out by the formal methods of the theory of nuclear reactions with account for the direct and indirect mechanisms. The results of calculations and their comparison with the experimental data on the $^{12}\mathrm{C}(p,2p)^{11}\mathrm{B}$ reaction at an energy of 156 MeV in the non-coplanar geometry are presented. A good agreement between the theoretical and experimental data is obtained. It is shown that the exact expression obtained in this work for the reaction amplitude allows the derivation of all known approximations that are used for the description of these reactions.

1. Introduction

One of the effective methods for studying the structure of atomic nuclei and particle interactions is the investigation of the processes of elastic and inelastic scatterings of nucleons and complex particles by nuclei in the range of medium and high energies. The importance of the theoretical investigations of these processes is increasing due to the implementation of complicated correlation experiments.

At high energies of incident particles, the nuclear processes are running in a direct way without the creation of composite nuclei. In these processes, a part of the energy and momentum of the incident particle is transferred to one or several nucleons, which leads to a strongly correlated (relative to the direction of the incident beam) scattering diagram. As an example, we can mention the reactions (e,e′p) [1], (p,pN), (p,pα), (α,2α), etc., whose kinematic diagrams resemble the collision diagrams of free particles. Understanding the reaction mechanisms is necessary for the extraction of a reliable information about the nucleus structure and the mechanisms of interaction between particles.

The most widely used theoretical method for the study of the direct nuclear reactions at energies of more than 100 MeV is the impulse approximation. In this method, the collision of an incident particle with an intranucleus nucleon is assumed to be the same as that with a free nucleon.

The interest to the study of the (p,pN) reactions with the knock-out of nucleons by protons was initially inspired by the assumption about the feasibility of direct measurements of the momentum distribution of nucleons inside a nucleus. The theoretical analysis of the (p,pN) reactions was carried out using the impulse approximation firstly with plane waves (PWIA) and later with distorted waves (DWIA). A comparison of the experimental cross sections for proton energies of more than 100 MeV with the DWIA calculations demonstrated a reasonably good agreement, which indicates the feasibility of obtaining the information about the nuclear structure and the interaction mechanisms. At proton energies of more than 100 MeV, the dominating process is the interaction of quasifree particles. However, the analysis of experimental data for the (p,2p) reactions for proton energies ∼50 MeV shows that the DWIA is not capable to describe this reaction in principle. This makes the energy range 50—150 MeV to be the most interesting one for the determination of the limits of applicability of the impulse approximation.

The comparison of the PWIA calculations with the experimental data showed that it is not possible to ignore the multiple scattering of protons by a residual

neutron [2] even in the ^{2}H(p,2p)n reaction. For the interpretation of the experimental data, it is necessary to use a more complicated theory that would take into account the multiple interaction of the incident nucleon and escaping nucleons inside a nucleus. Such calculations were carried out for a deuteron with the use of the Faddeev equations. However, this method is too complicated for heavier nuclei. The application of the DWIA for the analysis of the (p,2p) reactions is described in works [3—5]. It is shown that, with increase in the energy of incident protons, the distortion effects decrease, and, at sufficiently high energies, the DWIA is equivalent to the PWIA.

At proton energies less than 100 MeV, there are significant discrepancies between the experimental data and calculations based on the DWIA for the (p,2p) reactions, which stimulated the development of new methods of the theoretical analysis of these reactions. In [6], the solution of a three-body problem by the Faddeev method was shown to be applicable to the ^{12}C(p,2p) reaction at 46 MeV. The application of this method to the ^{4}He(p,2p)^{3}H reaction also showed good results. However, for the reaction with a ^{12}C nucleus, it was not possible to reach an eligible agreement with experimental data.

The impulse approximation assumes that the quasifree interaction is dominant. However, there were no attempts to carry out quantitative estimates of the contributions of processes other than the quasifree scattering to the cross section of the (p,2p) reaction. In work [7], the ^{12}C(p,2p) reaction at 50 and 100 MeV was described within the DWIA with regard for the incident proton interaction with the ^{11}B core. The consideration of the quasifree scattering alone did not allow one to describe even roughly the experimental function of the angular correlation at 50 MeV, and therefore it was necessary to take into account the interaction with the core which changes significantly the shape of the theoretical curve. At an energy of 100 MeV, the main contributions come form the quasifree scattering.

The t-matrix method was developed in works [8—12]. In this method, the factorization of the transition matrix element into the distorted momentum distribution and the two-particle matrix element of the interaction of a proton and an intranucleus particle is abandoned. Instead, the operator of NN-interaction is integrated together with the wave functions of particles. The method of the non-local t-matrix was presented in work [8]. This method gives a better agreement with the experiment in comparison to the methods of pseudo-potential [12] and local t-matrix [10, 11]. An essential difference of the t-matrix approximation with distorted waves (DWTA) developed in [8] from the earlier theoretical models is the implementation of the analytic integration in the calculation of the matrix element of the (p,2p) reaction. This was achieved with the use of the Gaussian representation of the intranucleus wave functions, exponential shape of the wave functions of the incident and escaping protons with regard for distortions and the generating functions of spherical harmonics, and the representation of the formfactor of the t-operator of the pp-interaction in the Gaussian form. The DWTA approximation takes into account the processes different from the quasifree scattering, namely, two-stage processes with the excitation and decay of giant multipole resonances, which significantly improves the agreement with experimental data.

However, the formulas obtained in the t-matrix method are suitable only for the symmetric coplanar geometry and do not include the contribution from indirect mechanisms. According to estimates [7], this contribution may be essential at 50 MeV.

The goal of this work is the development of the DWTA theory for the (p,2p) and (p,pN) reactions which is applicable in the regions of low and high energies. The proposed approach takes into account the direct and indirect mechanisms of the reaction and allows the exact calculation of the effects of the descent from the mass surface in contrast to the DWIA formalism, in which these effects are calculated approximately in the asymptotic limit. The developed method is applied to the description of the ^{12}C(p,2p)^{11}B reaction at an energy of 156 MeV. The agreement with the experimental data turns out to be good, which demonstrates the predictive power of the developed approach.

2. Formalism

Consider a transition process from the initial state that consists of the nucleus A and the incident proton (with subscript 0) to the final state with the residual nucleus $(A-1)$ and two escaping nucleons 0 and 1. The simplest interpretation of this reaction involves the one-stage mechanism comprising the direct interaction of an incident particle with an intranucleus nucleon that is then removed from the nucleus. Distorting effects, which are related to the interactions of the incident particle with the nucleus-target and the escaping particles with the residual nucleus, can be easily taken into account with the use of the standard technique of the theory of reactions with distorted waves. By $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$, and $\mathbf{p}_C$, we denote, respectively, the momenta of the incident

nucleon, escaping nucleons, and the residual nucleus in the laboratory system (LS).

To take into account the multiple scattering of the incident and two escaping nucleons, we have to use, instead of plane waves, the waves distorted by the optical potential of the interaction with passive particles which are assumed, for simplicity, to be independent of spin. At medium energies, the distorted waves are calculated by solving the Schrödinger equation with optical potentials which should be known for a wide range of energies. As was shown in [13], at energies ~ 50 MeV and higher, the solutions can be represented by decaying plane waves, because the criteria of validity of the high-energy approximation are fulfilled at such energies [14].

The full Hamiltonian H of the system can be written in two forms:

$$H = H_0 + V_{01} + V_{0C} + V_{1C} = H_i + V_i = H_f + V_f,$$

where $H_0 = T_0 + T_1 + H_C$ is the free Hamiltonian, and T_0, T_1, and T_C are, respectively, the operators of kinetic energy of the nucleons and the residual nucleus. The Hamiltonian of the residual nucleus is $H_C = T_C + h_C(\xi)$, where $h_C(\xi)$ is the internal Hamiltonian of the core that depends on the internal coordinates ξ, T_C is its kinetic energy, V_{01} is the interaction between two nucleons responsible for the direct transition, and V_{0C} and V_{1C} are the operators of interaction of nucleons 0 and 1 with the residual nucleus. The above-given representations of the Hamiltonian H correspond to the configurations of fragments in the input and output channels. The channel Hamiltonians and interactions take the form:

$$H_i = H_0 + V_{1C}, \quad V_i = V_{01} + V_{0C},$$

$$H_f = H_0, \quad V_f = V_{01} + V_{0C} + V_{1C}.$$

The channel wave functions obey the equations

$$H_i\Phi_i = E_i\Phi_i, \quad E_i = M + M_A + \frac{p_0^2}{2M},$$

$$H_f\Phi_f = E_f\Phi_f,$$

$$E_f = 2M + M^*_{A-1} + \frac{p_1^2 + p_2^2}{2M} + \frac{p_C^2}{2M_{A-1}}$$

and take the forms

$$\Phi_i = \frac{1}{(2\pi)^3}\exp(i\mathbf{p}_0\cdot\mathbf{r}_0)\Phi_A(\mathbf{r}_{1C},\xi),$$

$$\Phi_f = \frac{1}{(2\pi)^{9/2}}\exp(i\mathbf{p}_1\cdot\mathbf{r}_0 + i\mathbf{p}_2\cdot\mathbf{r}_1 + i\mathbf{p}_C\cdot\mathbf{R}_C)\Phi_{A-1}(\xi),$$

where Φ_A and Φ_{A-1} are the internal wave functions of the nucleus-target and the residual nucleus.

Let us define the measured energy of the separation of particle 1 as $E_S = M + M^*_{A-1} - M_A = |Q| + E^*$, where $Q < 0$ is the reaction energy, and E^* is the excitation energy of the residual nucleus. The separation energy of a nucleon and the momentum of the recoil nucleus $\mathbf{p}_C$ have a simple physical meaning. The recoil nucleus has a hole in the shell, from which the nucleon escapes, and the separation energy is equal to the energy of this nucleon state. Since the initial nucleus A was at rest, the momentum of the recoil nucleus is equal to the momentum of the intranucleus nucleon before the collision with the opposite sign, $\mathbf{p}_C = -\mathbf{p}$.

Consider now the derivation of the general formula for the reaction amplitude that will be useful for the optical model approximations. The matrix element of the T-operator of a transition on the energy surface is determined by two equivalent expressions,

$$T_{fi} = (\Phi_f, V_f\Psi_i^{(+)}) = (\Psi_f^{(-)}, V_i\Phi_i),$$

depending on the channel interaction serving as a transition operator.

The full wave functions obey the equations

$$\Psi_i^{(+)} = \Phi_i + \frac{1}{E_i - H_i + i\eta}V_i\Psi_i^{(+)},$$

$$\Psi_f^{(-)} = \Phi_f + \frac{1}{E_f - H_f - i\eta}V_f\Psi_f^{(-)}.$$

Let us define the auxiliary distorted wave functions $\chi_i^{(+)}$, $\chi_{f_1}^{(-)}$, and $\chi_{f_2}^{(-)}$ that are generated by the potential of interaction of particles 0 and 1 with the core: V_{0C}, $V_{0C} + V_{1C}$, and V_{1C}, respectively. We have

$$\chi_i^{(+)} = \Phi_i + \frac{1}{E_i - H_i + i\eta}V_{0C}\chi_i^{(+)},$$

$$\chi_{f_1}^{(-)} = \Phi_f + \frac{1}{E_f - H_f - i\eta}(V_{0C} + V_{1C})\chi_{f_1}^{(-)},$$

$$\chi_{f_2}^{(-)} = \Phi_f + \frac{1}{E_f - H_f - i\eta}V_{1C}\chi_{f_2}^{(-)}.$$

These definitions are sufficient for obtaining, after formal transformations, the final representation of the matrix element of the transition,

$$T_{fi} = (\chi_{f_1}^{(-)}, \tau_{01}\chi_i^{(+)}) + (\chi_{f_1}^{(-)}, V_{0C}\Phi_i), \quad (1)$$

where we introduce the operator

$$\tau_{01} = V_{01} + V_{01}\frac{1}{E_i - H + i\eta}V_{01}. \quad (2)$$

The first term in Eq. (1), which contains the two-particle τ-operator, describes the scattering of the incident proton by a single intranucleus nucleon in the presence of all other nucleons and corresponds to the direct knock-out mechanism. We note that the propagator in Eq. (2) contains the full Hamiltonian H. The operator τ_{01} is not a free two-nucleon transition operator, because it includes the effects which are related to the medium and describe how an incident particle and a bound nucleon interact not only with each other by means of the potential V_{01}, but also with the core C by means of the potentials V_{0C} and V_{1C} in the full Hamiltonian. When the influence of the medium is neglected, the operator τ is reduced to the free two-nucleon transition operator t. The latter differs only kinematically from the operator that describes the scattering of particle 0 by free particle 1. In this case, the first term of Eq. (1) gives all the known approximations, which is shown in the Appendix. The second term in Eq. (1), which contains the operator of interaction of a proton with a residual nucleus V_{0C}, is responsible for the indirect reaction mechanism and is interpreted as the amplitude of a hard knock-out of the core. At energies of the order of a hundred MeV, the contribution of this term is negligible.

We note that Eq. (1) for the reaction amplitude is derived by neglecting the effects of antisymmetrization. Following the standard technique accounting the identity of nucleons, we can derive exact formulas that will account for the effects of antisymmetrization and contain all the possible mechanisms of the reaction. However, some of these mechanisms can be insignificant at high energies. A three-particle model of the reaction allows us to separate the main features of the antisymmetrization. In this model, the system in the input channel consists of an incident proton with coordinate $\mathbf{r}_0$ and the nucleus comprising the structureless core with coordinate $\mathbf{R}_C$ and a nucleon bound to it with coordinate $\mathbf{r}_1$. The genealogy of the one-particle bound state should be consistent with the real many-body problem. In the output channel, all three particles are free. Thus, the interaction of incident particle 0 with the nucleus-target leads to the release of bound particle 1. In the considered model, only particles 0 and 1 are identical, and the wave functions should be antisymmetric in the coordinates of these particles. Thus, the final result can be written as

$$(T_{f\,i})_a \approx \sqrt{A}\Big[((1 - P_{01})\chi_{f_1}^{(-)},$$

$$\tau_{01}\chi_i^{(+)}) + ((1 - P_{01})\chi_{f_1}^{(-)}, V_{0C}\Phi_i)\Big]. \quad (3)$$

In the three-particle model, formula (3) is exact. The first term is responsible for the mechanism of the knock-out of a nucleon, and the second term corresponds to the knock-out of the core. Other mechanisms are impossible.

Formula (3) is the main result that serves as the basis of all possible approximations adopted in the theory of (p,pN) reactions at high and medium energies. In the range of intermediate energies ($\sim$1 GeV), the first term in Eq. (3), which alone survives in this case, allows us to develop the diffraction methods of analysis of these reactions [15] which account explicitly the effects of multiple scattering provided that we use the multiparticle distorted waves [16] which are the solutions of a many-center problem.

3. Formula for the Cross Section of Angular Correlation. Direct and Indirect Mechanisms

It is known that, with decrease in the energy of incident protons, the impulse approximation leads to uncertainties due to the off-energy effects, especially at small angles. A more accurate treatment of these effects is possible in the DWTA. This approach was introduced in [8,12], where it was shown that its deviations from the DWIA start at energies less than 100 MeV, and allows the exact consideration of the effects of the finite interaction radius.

The cross section of the considered process in the laboratory system is given by the expression

$$\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dE_1} = (2\pi)^4 \frac{p_1 p_2^3}{p_0}\frac{E_0 E_1 E_2 E_{A-1}}{|E_{A-1}p_2^2 - E_2\mathbf{p}_2\mathbf{p}_{A-1}|}\times$$

$$\times\frac{1}{2}\sum_{\mu_0\mu_1\mu_2}\frac{1}{2J_i+1}\sum_{M_i M_f}|T_{f\,i}|^2, \quad (4)$$

where E is the total energy, $\mathbf{p}$ is the momentum, and μ_i is the spin projection of nucleon i. In accordance to

Eq. (A.10), the matrix element that corresponds to the direct knock-out mechanism takes the form

$$T^d_{fi} = C^{T_i N_i}_{T_f N_f \frac{1}{2}\nu}\sqrt{S_{lj}}\sum_m C^{J_i M_i}_{J_f M_f jm}\times$$

$$\times\int\Big[\psi^{(-)}_{\mathbf{k}_1}(\mathbf{r}_0)\psi^{(-)}_{\mathbf{k}_2}(\mathbf{r}_1)\chi_{\mu_1}(0)\chi_{\mu_2}(1)\zeta_{\nu_1}(0)\zeta_{\nu_2}(1)-$$

$$-(0\leftrightarrow 1)\Big]^{\dagger} t(01,0'1';e)\psi^{(+)}_{\mathbf{k}_0}(\mathbf{r}_0-\frac{\mathbf{r}_1}{A})\chi_{\mu_0}(0')\zeta_{\nu_0}(0')\times$$

$$\times\phi_{jm}(1')\zeta_{\nu}(1')d\mathbf{r}_0 d\mathbf{r}_1 d\mathbf{r}'_0 d\mathbf{r}'_1\,. \qquad (5)$$

In the derivation of Eq. (5), we used the genealogical decomposition of the function of the initial nucleus in the full collection of states of the residual nucleus. Here, we introduced the spectroscopic factor $S_{l\,j}$ of the separated nucleon, which is a result of the overlapping of the functions of the initial and final nuclei. In the sum for the overlapping integral, only the contribution of one value of (nlj) is taken into account. In this representation of the transition matrix element, the function of the bound state of a nucleon ϕ_{jm} is simply the wave function of the shell model with the radial part normalized to unity. The integration variables are reckoned from the center-of-mass of the residual nucleus. The indices 0, 1, $0'$, and $1'$ correspond to the generalized coordinates of nucleons (spatial, spin, and isospin ones). Beyond the energy surface, the energy-dependent two-nucleon $t(01,0'1';e)$-matrix is a solution of the Lippmann—Schwinger equation. In order to calculate the amplitude of the direct transition, we introduce a partial decomposition of the nucleon-nucleon t-matrix:

$$t(01,0'1';e) = \delta(\mathbf{R}-\mathbf{R}')\times$$

$$\times\sum_{JMLST} Y^{JM}_{LS}(\hat{\mathbf{r}},01)t^J_{LST}(r,r';e)\,Y^{JM\,*}_{LS}(\hat{\mathbf{r}}',0'1')P_T. \qquad (6)$$

Here, P_T is the projection operator onto the state with isospin T, $\mathbf{r}=\mathbf{r}_0-\mathbf{r}_1$ and $\mathbf{R}=\frac{1}{2}(\mathbf{r}_0+\mathbf{r}_1)$ are, respectively, the relative coordinate and the coordinate of the center-of-mass of two nucleons in the output channel, $\mathbf{r}'=\mathbf{r}'_0-\mathbf{r}'_1$ and $\mathbf{R}'=\frac{1}{2}(\mathbf{r}'_0+\mathbf{r}'_1)$ are the analogous coordinates of the incident and bound nucleons, and Y^{JM}_{LS} is the spin-angular function.

For calculations of the radial part $t^J_{LST}(r,r')$, we use the potential [17] with the tensor interaction. This potential allows us to properly describe the energy dependence of phase shifts (with the additional account of F-waves) in the channels with isospin $T=0$, 1 and the corresponding parameters of mixing in the energy range 0–500 MeV, and the singlet and triplet scattering lengths and the effective radii. The parameters of the potential were found with the use of the data [18] on the phase shifts in pp- and pn-scatterings. The value of e in t-matrix (6) corresponds to the relative energy of the NN-collision and, strictly speaking, is not exactly known for quasielastic processes. In the considered three-particle problem, the range of this uncertainty is determined by the sum of the separation energy of the knocked nucleon and its kinetic energy inside the nucleus, i.e. by a deviation from the energy surface. In specific calculations of the differential cross sections of the processes of direct knock-out of nucleons by protons, we take $e_i=\varepsilon$ (the initial relative energy) and $e_f=\varepsilon'$ (the final relative energy). The cross sections obtained in these ways differed from one another by at most 3%. This corresponds to the conclusions made in [19], and all our calculations were carried out at the energy $e=e_i$ in accordance with the recommendations of those authors.

The distorted wave functions $\psi^{(\pm)}_{\mathbf{k}}(\mathbf{r})$, that describe the motion of nucleons in the input and output channels, were found by the methods presented in [20,21]. Substituting (6) into (5), passing to the representation of the channel spin, and calculating the spin-isospin matrix elements, we obtain

$$T^d_{fi} = C^{T_i N_i}_{T_f N_f \frac{1}{2}\nu}\sqrt{S_{lj}}\times$$

$$\times\sum_{JMLSTM_T}\sum_{mm_l\mu}\sum_{M_L M'_L M_S M'_S} C^{J_i M_i}_{J_f M_f jm}C^{jm}_{lm_l\frac{1}{2}\mu}\times$$

$$C^{JM}_{LM_L SM_S}C^{JM}_{LM'_L SM'_S}C^{SM_S}_{\frac{1}{2}\mu_1\frac{1}{2}\mu_2}\times$$

$$\times C^{SM'_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu}C^{TM_T}_{\frac{1}{2}\nu_1\frac{1}{2}\nu_2}C^{TM_T}_{\frac{1}{2}\nu_0\frac{1}{2}\nu}\widetilde{Z}^{jlm_l}_{JLSTM_L M'_L}, \qquad (7)$$

where the antisymmetrized matrix element is

$$\widetilde{Z}^{jlm_l}_{JLSTM_L M'_L} = \left[1-(-1)^{L+S+T}\right]Z^{jlm_l}_{JLSTM_L M'_L}, \qquad (8)$$

in which

$$Z^{jlm_l}_{JLSTM_L M'_L} = \int\psi^{(-)*}_{\mathbf{k}_1}(\mathbf{r}_0)\psi^{(-)*}_{\mathbf{k}_2}(\mathbf{r}_1)\times$$

$$\times\delta(\mathbf{R}-\mathbf{R}')Y_{LM_L}(\hat{\mathbf{r}})t^J_{LST}(r,r';e)Y^*_{LM'_L}(\hat{\mathbf{r}}')\times$$

$$\psi^{(+)}_{\mathbf{k}_0}(\mathbf{r}'_0 - \frac{\mathbf{r}'_1}{A})\phi_{lj}(r'_1)Y_{lm_l}(\hat{\mathbf{r}}'_1)d\mathbf{r}d\mathbf{r}'d\mathbf{R}d\mathbf{R}', \quad (9)$$

is the direct spatial matrix element. For the (p,2p) reaction, we set $T = 1$, $\nu_0 = \nu = \nu_1 = \nu_2 = -\frac{1}{2}$ in Eq. (7). Then we obtain

$$T^d_{fi} = C^{T_i N_i}_{T_f N_f \frac{1}{2} -\frac{1}{2}} \sqrt{S_{lj}} \times$$

$$\times \sum_{JMLS} \sum_{mm_l\mu} \sum_{M_L M'_L M_S M'_S} C^{J_i M_i}_{J_f M_f jm} C^{jm}_{lm_l \frac{1}{2}\mu} \times$$

$$C^{JM}_{LM_L SM_S} C^{JM}_{LM'_L SM'_S} C^{SM_S}_{\frac{1}{2}\mu_1 \frac{1}{2}\mu_2} C^{SM'_S}_{\frac{1}{2}\mu_0 \frac{1}{2}\mu} \widetilde{Z}^{jlm_l}_{JLS1M_L M'_L} \quad (10)$$

In the case of the (p, pN) reaction, the permissible values are $T = 1$ and 0, and $\nu_0 = \nu_1 = -\frac{1}{2}$, $\nu = \nu_2 = \frac{1}{2}$. Then we obtain

$$T^d_{fi} = C^{T_i N_i}_{T_f N_f \frac{1}{2} \frac{1}{2}} \sqrt{S_{lj}} \times$$

$$\times \sum_{JMLS} \sum_{mm_l\mu} \sum_{M_L M'_L M_S M'_S} C^{J_i M_i}_{J_f M_f jm} C^{jm}_{lm_l \frac{1}{2}\mu} \times$$

$$C^{JM}_{LM_L SM_S} C^{JM}_{LM'_L SM'_S} C^{SM_S}_{\frac{1}{2}\mu_1 \frac{1}{2}\mu_2} C^{SM'_S}_{\frac{1}{2}\mu_0 \frac{1}{2}\mu} \times$$

$$\times \frac{1}{2} \left(\widetilde{Z}^{jlm_l}_{JLS1M_L M'_L} + \widetilde{Z}^{jlm_l}_{JLS0M_L M'_L} \right). \quad (11)$$

Thus, all the formulas derived below for the (p,2p) reaction will be also valid for the (p,pN) reaction after the substitution

$$\widetilde{Z}^{jlm_l}_{JLS1M_L M'_L} \Rightarrow \frac{1}{2} \left(\widetilde{Z}^{jlm_l}_{JLS1M_L M'_L} + \widetilde{Z}^{jlm_l}_{JLS0M_L M'_L} \right). \quad (12)$$

As can be seen from Eq. (8), the generalized Pauli principle puts the restriction $(-1)^{L+S+T} = -1$ on the summation in both cases.

We now pass to the calculation of the averaged square of the absolute value of the transition matrix element

$$M_{fi} = \frac{1}{2} \sum_{\mu_0\mu_1\mu_2} \frac{1}{2J_i + 1} \sum_{M_i M_f} |T^d_{fi}|^2 \equiv A_{fi} + B_{fi} =$$

$$= \frac{C^2 S_{lj}}{2(2j+1)} \sum_{SM_S m\mu_0} \left| \sum_{JMLM_L M'_L} \sum_{M'_S m_l \mu} C^{JM}_{LM_L SM_S} \times \right.$$

$$\left. \times C^{JM}_{LM'_L SM'_S} C^{SM'_S}_{\frac{1}{2}\mu_0 \frac{1}{2}\mu} C^{jm}_{lm_l \frac{1}{2}\mu} \tilde{Z}^{jlm_l}_{JLS1M_L M'_L} \right|^2. \quad (13)$$

Here, by A_{fi} and B_{fi}, we denote the contributions of the singlet ($S = 0$) and triplet ($S = 1$) states of two protons. By $C^2 S_{lj}$, we denote the product of the square of the isospin coefficient of the vector sum and the spectroscopic factor. From this formula, we obtain the contribution of the singlet state

$$A_{fi} = \frac{C^2 S_{lj}}{4(2j+1)} \sum_{m\mu_0} \left| \sum_{LM_L m_l} C^{jm}_{lm_l \frac{1}{2}\mu_0} \tilde{Z}^{jlm_l}_{LL01M_L M_L} \right|^2 =$$

$$= \frac{C^2 S_{lj}}{4(2l+1)} \sum_{m_l} |H^{m_l}_1|^2, \quad (14)$$

where we have carried out the summation over the projections of moments and defined the quantity

$$H^{m_l}_T = \sum_{LM_L} \tilde{Z}^{jlm_l}_{LL0TM_L M_L}.$$

Note that the expression for the quantity A_{fi} is valid for both values of j, $j = l \pm \frac{1}{2}$. From Eq. (13), we obtain the contribution of the triplet state as

$$B_{fi} = \frac{C^2 S_{lj}}{2(2j+1)} \times$$

$$\times \sum_{mM_S} \left\{ \left| C^{jm}_{lm-\frac{1}{2}\frac{1}{2}\frac{1}{2}} O^{m-\frac{1}{2}}_{1M_S} + \frac{1}{\sqrt{2}} C^{jm}_{lm+\frac{1}{2}\frac{1}{2}-\frac{1}{2}} P^{m+\frac{1}{2}}_{1M_S} \right|^2 + \right.$$

$$\left. + \left| \frac{1}{\sqrt{2}} C^{jm}_{lm-\frac{1}{2}\frac{1}{2}\frac{1}{2}} P^{m-\frac{1}{2}}_{1M_S} + C^{jm}_{lm+\frac{1}{2}\frac{1}{2}-\frac{1}{2}} Q^{m+\frac{1}{2}}_{1M_S} \right|^2 \right\} \quad (15)$$

where we denote

$$O^{m_l}_{TM_S} = \sum_{JMLM_L M'_L} C^{JM}_{LM_L 1M_S} C^{JM}_{LM'_L 11} \tilde{Z}^{jlm_l}_{JL1TM_L M'_L},$$

$$P^{m_l}_{TM_S} = \sum_{JMLM_L M'_L} C^{JM}_{LM_L 1M_S} C^{JM}_{LM'_L 10} \tilde{Z}^{jlm_l}_{JL1TM_L M'_L},$$

$$Q^{m_l}_{TM_S} = \sum_{JMLM_L M'_L} C^{JM}_{LM_L 1M_S} C^{JM}_{LM'_L 1-1} \tilde{Z}^{jlm_l}_{JL1TM_L M'_L}.$$

The obtained expressions correspond to the experimental situation, where the final state of the nucleus is clearly determined. In the opposite case where

the energy resolution is poor, it is necessary to use the sum rule for spectroscopic factors.

As an example, we present the specific formulas for the averaged matrix elements of the transition in the case of the knock-out of protons from the s- and p-states of nuclei.

In the case of the knock-out of s-protons ($l=0,\ j=\frac{1}{2}$), we obtain

$$A_{fi}=\frac{1}{4}C^2S_{0\frac{1}{2}}\left|H_1^0\right|^2,$$

$$B_{fi}=\frac{1}{4}C^2S_{0\frac{1}{2}}\sum_{M_S}\left\{\left|O^0_{1M_S}\right|^2+\left|P^0_{1M_S}\right|^2+\left|Q^0_{1M_S}\right|^2\right\}. \tag{16}$$

In the case of the knock-out of p-protons ($l=1,\ m_l=0,\pm1;\ j=\frac{3}{2},\ \frac{1}{2}$), we get

$$A_{fi}=\frac{1}{12}C^2S_{1j}\sum_{m_l}|H_1^{m_l}|^2, \tag{17}$$

$$B^{(+)}_{fi}=\frac{1}{24}C^2S_{1\frac{3}{2}}\sum_{M_S}\Bigg\{3\left|O^1_{1M_S}\right|^2+\frac{3}{2}\left|P^1_{1M_S}\right|^2+$$

$$+\frac{1}{2}\left|2O^0_{1M_S}+P^1_{1M_S}\right|^2+\left|P^0_{1M_S}+Q^1_{1M_S}\right|^2+$$

$$+\left|O^{-1}_{1M_S}+P^0_{1M_S}\right|^2+$$

$$+\frac{1}{2}\left|P^{-1}_{1M_S}+2Q^0_{1M_S}\right|^2+\frac{3}{2}\left|P^{-1}_{1M_S}\right|^2+3\left|Q^{-1}_{1M_S}\right|^2\Bigg\}, \tag{18}$$

$$B^{(-)}_{fi}=\frac{1}{12}C^2S_{1\frac{1}{2}}\times$$

$$\times\sum_{M_S}\left\{\left|O^0_{1M_S}-P^1_{1M_S}\right|^2+\frac{1}{2}\left|P^0_{1M_S}-2Q^1_{1M_S}\right|^2+\right.$$

$$\left.\frac{1}{2}\left|2O^{-1}_{1M_S}-P^0_{1M_S}\right|^2+\left|P^{-1}_{1M_S}-Q^0_{1M_S}\right|^2\right\}. \tag{19}$$

By the form, formulas (14) and (15) are identical to the analogous formulas of the nonfactorized impulse approximation developed in work [33]. However, the sense of the quantities O, P, and Q is different from that in work [33]. In the plane-wave approximation, the two collections of formulas give the same numerical values of the cross sections. This provides an additional criterion of the correctness of the obtained formulas.

For the symmetric coplanar geometry, formulas (17)– (19) are simplified, because the terms with $m_l=\pm1$ vanish due to the rotational symmetry:

$$A_{fi}=\frac{1}{12}C^2S_{1j}\left|H_1^0\right|^2,$$

$$B^{(\pm)}_{fi}=\frac{1}{12}C^2S_{1j}\sum_{M_S}\left\{\left|O^0_{1M_S}\right|^2+\left|P^0_{1M_S}\right|^2+\left|Q^0_{1M_S}\right|^2\right\}. \tag{20}$$

Expressions (14) and (15) generalize the particular expressions for (20) earlier obtained in [8] to the case of arbitrary orbital moments of a knocked-out nucleon and the asymmetric non-coplanar geometry of the momenta of escaping nucleons. For the (p,np) reaction, formula (11) and all the expressions derived below with substitution (12) are obtained in the t-matrix approximation for the first time. The formulas for the cross sections of the (p,np) reaction presented in [22,23] in the same approximation are erroneous [33].

The contribution from the indirect mechanism of the nucleon release (the knock-out of a core or the hard knock-out) is determined by the second term in Eq. (3) which looks in the expanded form as

$$T^c_{fi}=C^{T_iN_i}_{T_fN_f\frac{1}{2}\nu}\sqrt{S_{lj}}\sum_m C^{J_iM_i}_{J_fM_fjm}\int\Big[\psi^{(-)}_{\mathbf{k}_1}(\mathbf{r}_0)\psi^{(-)}_{\mathbf{k}_2}\times$$

$$\times(\mathbf{r}_1)\chi_{\mu_1}(0)\chi_{\mu_2}(1)\zeta_{\nu_1}(0)\zeta_{\nu_2}(1)-(0\leftrightarrow1)\Big]^\dagger$$

$$\times V_{0C}(r_0)\frac{1}{(2\pi)^{3/2}}\exp\left\{i\mathbf{k}_0\cdot\left(\mathbf{r}_0-\frac{\mathbf{r}_1}{A}\right)\right\}\times$$

$$\times\chi_{\mu_0}(0)\zeta_{\nu_0}(0)\phi_{jm}(1)\zeta_\nu(1)d\mathbf{r}_0d\mathbf{r}_1. \tag{21}$$

We assume that the optical potential of the interaction of a nucleon with the core V_{0C} does not contain the spin operators. Passing in Eq. (21) to the representation of the channel spin and calculating the spin-isospin matrix element, we obtain

$$T^c_{fi}=C^{T_iN_i}_{T_fN_f\frac{1}{2}\nu}\sqrt{S_{lj}}\sum_{mm_l\mu}\sum_{SM_ST\mathcal{M}_T}C^{J_iM_i}_{J_fM_fjm}C^{jm}_{lm_l\frac{1}{2}\mu}\times$$

$$\times C^{SM_S}_{\frac{1}{2}\mu_1\frac{1}{2}\mu_2}C^{SM_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu}C^{T\mathcal{M}_T}_{\frac{1}{2}\nu_1\frac{1}{2}\nu_2}C^{T\mathcal{M}_T}_{\frac{1}{2}\nu_0\frac{1}{2}\nu}\tilde{V}^{m_l}_{ST}, \tag{22}$$

where the antisymmetrized spatial matrix element is

$$\widetilde{V}^{m_l}_{ST}=V^{m_l}(\mathbf{k}_1,\mathbf{k}_2)-(-1)^{S+T}V^{m_l}(\mathbf{k}_2,\mathbf{k}_1), \tag{23}$$

in which

$$V^{m_l}(\mathbf{k}_1,\mathbf{k}_2)=\frac{1}{(2\pi)^{3/2}}\int\psi^{(-)*}_{\mathbf{k}_1}(\mathbf{r}_0)\psi^{(-)*}_{\mathbf{k}_2}(\mathbf{r}_1)V_{0C}(r_0)\times$$

$$\times \exp\left\{i\mathbf{k}_0\left(\mathbf{r}_0 - \frac{\mathbf{r}_1}{A}\right)\right\}\phi_{lj}(r_1)Y_{lm_l}(\hat{\mathbf{r}}_1)d\mathbf{r}_0 d\mathbf{r}_1 \qquad (24)$$

is the direct matrix element. The exchange (second) term in Eq. (23) is obtained from the direct term by the substitution $\mathbf{r}_0 \leftrightarrow \mathbf{r}_1$ in the arguments of distorted waves of the escaping nucleons. All spatial variables in Eq. (24) are reckoned from the center-of-mass of the core. In this case, the contribution of the exchange term cannot be reduced to the contribution of the direct term with a phase factor, as it was done in (8). Therefore, these two terms should be calculated independently.

In the case of the (p,2p) reaction, we obtain

$$T^c_{fi} = C^{T_i N_i}_{T_f N_f \frac{1}{2}-\frac{1}{2}}\sqrt{S_{lj}}\sum_{mm_l\mu}\sum_{SM_S} C^{J_i M_i}_{J_f M_f jm} C^{jm}_{lm_l\frac{1}{2}\mu}\times$$

$$\times C^{SM_S}_{\frac{1}{2}\mu_1\frac{1}{2}\mu_2} C^{SM_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu}\tilde{V}^{m_l}_{S1}. \qquad (25)$$

In the case of the (p,pN) reaction, we have to put $\nu = \frac{1}{2}$ and make change of the variable

$$\tilde{V}^{m_l}_{S1} \Rightarrow \frac{1}{2}\left(\tilde{V}^{m_l}_{S1} + \tilde{V}^{m_l}_{S0}\right) = V^{m_l}(\mathbf{k}_1, \mathbf{k}_2) \qquad (26)$$

in Eq. (25).

The total t-matrix element of the (p,pN) reaction is the sum of the direct and indirect terms. In particular, for the (p,2p) reaction, we obtain

$$T_{fi} = T^d_{fi} + T^c_{fi} = C^{T_i N_i}_{T_f N_f \frac{1}{2}-\frac{1}{2}}\sqrt{S_{lj}}\times$$

$$\times \sum_{mm_l\mu}\sum_{SM_SM'_S} C^{J_i M_i}_{J_f M_f jm} C^{jm}_{lm_l\frac{1}{2}\mu} C^{SM_S}_{\frac{1}{2}\mu_1\frac{1}{2}\mu_2} C^{SM'_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu}\times$$

$$\Big(\sum_{JMLM_LM'_L} C^{JM}_{LM_L SM_S} C^{JM}_{LM'_L SM'_S}\tilde{Z}^{jlm_l}_{JLS1M_LM'_L} +$$

$$+\delta_{M_SM'_S}\tilde{V}^{m_l}_{S1}\Big). \qquad (27)$$

The averaged square of the absolute value of the transition matrix element (13), which determines the differential cross section of the reaction, takes the form

$$M_{fi} = \frac{C^2 S_{lj}}{2(2j+1)}\sum_{SM_Sm\mu_0}\Big|\sum_{M'_S m_l\mu} C^{SM'_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu} C^{jm}_{lm_l\frac{1}{2}\mu}\times$$

$$\Big(\sum_{JMLM_LM'_L} C^{JM}_{LM_L SM_S} C^{JM}_{LM'_L SM'_S}\tilde{Z}^{jlm_l}_{JLS1M_LM'_L} +$$

$$+\delta_{M_SM'_S}\tilde{V}^{m_l}_{S1}\Big)\Big|^2. \qquad (28)$$

Expressions for the contributions of the singlet and triplet states of two protons A_{fi} and B_{fi} are obtained from Eqs. (14) and (15) after the substitutions

$$H^{m_l}_1 \Rightarrow H^{m_l}_1 + \tilde{V}^{m_l}_{01}, \quad O^{m_l}_{1M_S} \Rightarrow O^{m_l}_{1M_S} + \delta_{M_S1}\tilde{V}^{m_l}_{11},$$

$$P^{m_l}_{1M_S} \Rightarrow P^{m_l}_{1M_S} + \delta_{M_S0}\tilde{V}^{m_l}_{11},$$

$$Q^{m_l}_{1M_S} \Rightarrow Q^{m_l}_{1M_S} + \delta_{M_S\,-1}\tilde{V}^{m_l}_{11}.$$

The further calculation of the spatial matrix elements (9) and (24) is carried out by the method of generating functions for spherical harmonics [8].

4. Reaction ^{12}C(p,2p)^{11}B at 156 MeV in a Symmetric Non-coplanar Geometry

A detailed study of the elastic scattering of protons by a ^{12}C nucleus in a wide range of energies was carried out in works [20, 21]. In those works, the collections of parameters for the model distorted wave functions and various optical potentials that provide a good agreement with experimental cross sections were found. It would be interesting to apply these functions to the description of the cross section of the ^{12}C(p,2p)^{11}B reaction in the t-matrix approximation, whose formalism was presented above.

Earlier, the cross section of this reaction was calculated in work [24] using the diproton model [25] at 160 MeV within the factorized DWIA in a symmetric coplanar geometry. At the scattering angle 90° in the center-of-mass system of two nucleons, the cross section of the free pp-scattering was completely on the energy surface. However, the distorting potential for the pair of escaping protons (diproton) was not exactly known. This was the reason for the poor agreement with experimental data [26]. In particular, the right maximum of the p-correlation cross section was two times smaller than the experimental value. In earlier calculations [27], the agreement of the calculated values with experimental data was even worse because the bound state functions were represented by the inadequate oscillator and exponential functions, and the distorting potentials were not known at all. The recent calculations [28] suffer from the same drawbacks.

In our calculations, which are described below, the contribution from the indirect mechanism was found to be small. The distorted waves were taken from [20, 21] for the input channel at 156 MeV and for the output channel at 72 MeV in the case of the optical potential with the bulk and surface absorption. The use

of distorted waves for potentials with a different type of absorption changes mainly the general normalizing factor of the cross section. This leads to a change in the spectroscopic factor C^2S and slightly in the shape of correlation curves. For the curves shown in Fig. 1, we have obtained $C^2S(1p_{3/2}) = 3.68$, $C^2S(1s_{1/2}) = 2.0$. In the case of s-protons, the given value of C^2S corresponds to the value given by the sum rule for spectroscopic factors. In the case of p-protons, the given value corresponds to the contribution of the ground $\frac{3}{2}^-$ and first excited $\frac{1}{2}^-$(2,125 MeV) states of the ^{11}B nucleus. This follows from the calculated values of the spectroscopic factors [29] for these states and agrees with the energy resolution ($\sim 3,5$ MeV) of the analyzed experimental data. Generally speaking, the $p_{3/2}$ distribution can contain the contributions from three states of the residual ^{11}B nucleus. They are the ground state $\frac{3}{2}^-$ and two excited states, $\frac{1}{2}^-$(2.125 MeV) and $\frac{3}{2}^-$(5.020 MeV). According to the data [30] on the (e,e′p) reaction, their relative contributions constitute 79, 12, and 9%, respectively.

For the description of one-particle bound states $1p_{3/2}$ and $1s_{1/2}$ of protons in a ^{12}C nucleus, we used the local shell Woods—Saxon potential [31]. The parameters of this potential were chosen to obtain a good agreement with experimental data on the elastic scattering of electrons and the separation energies obtained from the data on the (p,2p) and (e,e′p) reactions with poor energy resolution. This provided automatically the averaging over the contributions of all final excited states of a residual nucleus. As a result, we obtained the Hartree—Fock energy $E_S = -(E_A^i - \sum_p [T_p^i(lj)]^2 E_{A-1}^p) = |Q| + \sum_p (T_p^i)^2 E_p^*$, where the second terms on the right-hand side determine the average value of the binding energy of final states and the mean excitation energy of a residual nucleus. For a ^{12}C nucleus, we obtain: $E_S(1p_{3/2}) = 16.86$ MeV and $E_S(1s_{1/2}) = 33.89$ MeV [32].

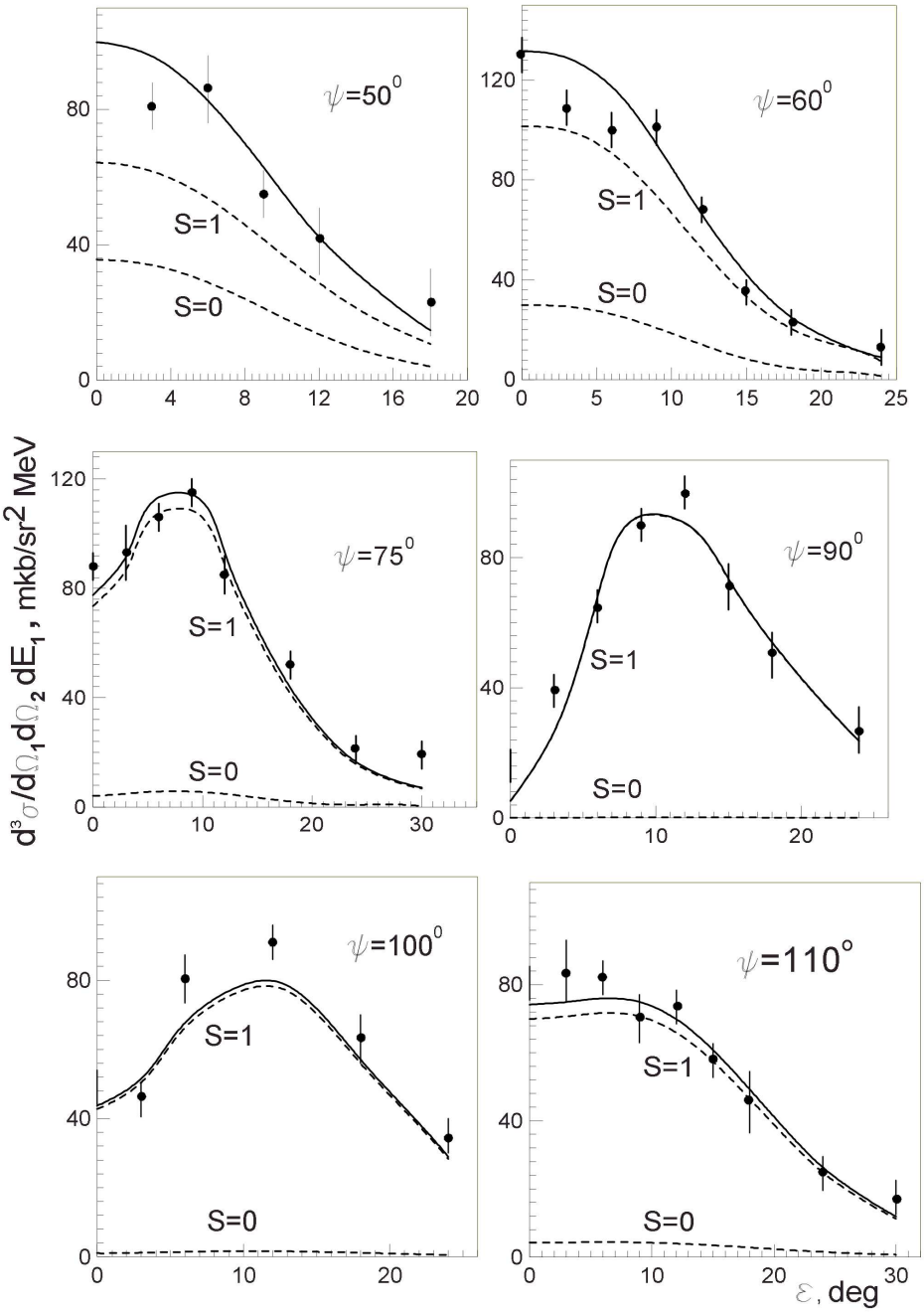
In the case of a high experimental energy resolution, when the final states of the residual nucleus are clearly separated and the distortion of waves is reliably calculated, the theory can be used to get the information on the nucleus structure from the experimental data, namely, the spectroscopic factor C^2S (for the orbital with quantum numbers nlj) and the mean square radius r_{rsm} (with respect to the $A-1$ system) of the wave function of a bound state. In this case, the well depth, for which the parameters of a bound state are calculated, is adjusted to reproduce the separation energy of a nucleon which corresponds to the considered final state in the $A-1$ nucleus.

As usual, the calculated r_{rsm} increases with the excitation energy E_p^* of a residual nucleus [30], which corresponds to the increase in the parameter r_0 in the potential of a bound state and imitates the effects of omitted surface terms in the inhomogeneous differential equation that governs the exact overlapping integral. This is the main idea of one of the methods of solving the inhomogeneous equation by means of a modification of the geometry of the Woods—Saxon well in the homogeneous equation for a one-particle state.

The calculations showed that an increase in the r_0 parameter of the binding potential increases r_{rsm} of the radial function and decreases strongly the value of C^2S which is obtained by the comparison with experimental data. This is explained by the increase in the effective spatial region considerably contributing to integrals (9) and (24) with increase in r_0.

In the symmetric non-coplanar geometry, the angles ϑ_1 and ϑ_2 are the same, $\vartheta_1 = \vartheta_2 = \vartheta$. The configuration of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ is characterized by the correlation angle ψ between these vectors and the non-coplanarity angle ε between directions of the momenta $\mathbf{p}_0$ and $\mathbf{p}_1 + \mathbf{p}_2$. In the coplanar case where the momenta of the incident, scattered, and knocked-out nucleons lie in the same plane, the non-coplanarity angle is zero, $\varepsilon = 0$, and the correlation angle is $\psi = 2\vartheta$. In the non-coplanar case, the polar angles of the momenta of the escaping nucleons are the same and are determined by the relation $\cos\vartheta = \cos\varepsilon \cdot \cos(\psi/2)$. The azimuth angle of one of the nucleons is $\cos\varphi = \sin(\psi/2)\sin\vartheta$, and the other angle is equal to $\pi - \varphi$. At given ψ, the maximum value of the first angle is reached at $\varepsilon = \pi/2$ and is equal to $\varphi_m = (\pi - \psi)/2$.

Figure 1 shows the cross section versus the non-coplanarity angle ε for a number of values of the angle ψ between the vectors $\mathbf{p}_1$ and $\mathbf{p}_2$. In the same figure, we show the experimental data [28] for the knock-out of p-protons from a ^{12}C nuclei at 156 MeV. The dashed lines show the contribution of the singlet ($S = 0$) and triplet ($S = 1$) states of two protons. As seen, the contribution of the triplet state is dominant. The good agreement between the theory and the experiment in this figure is not the direct consequence of a good agreement in the case of a symmetric coplanar geometry [33], because, in the non-coplanar case, the functions g^{m_l} [33] with $m_l \neq 0$ contribute also to the cross section, whereas these functions vanish in the coplanar case. We note also that the t-matrix approximation, which takes accurately the effects of coming off the energy surface into account,



Differential cross section of the ^{12}C(p,2p)^{11}B reaction of the knock-out of $1p_{3/2}$-protons at 156 MeV versus the non-coplanarity angle ε for a number of correlation angles ψ. The solid curves show the results of theoretical calculations in the DWTA, and the dashed curves demonstrate the contributions from the singlet ($S = 0$) and triplet ($S = 1$) states of two protons

gives the results that are identical to those of calculations in the nonfactorized impulse approximation [33], which indicates the insignificance of these effects in the energy range under study.

5. Conclusions

In this work, we have generalize the theory of the t-matrix approximation with distorted waves to the case

of an arbitrary geometry of the momenta of escaping nucleons in the (p,2p) and (p,pN) reactions. Our approach considers the direct and indirect mechanisms of the reactions. The proposed approach allows us to accurately calculate the effects of coming off the mass surface. It is shown that one can derive all possible approximations that are used in modeling the knock-out reactions from the exact expression obtained here for the reaction amplitude.

The comparison of the results obtained on the basis of the developed theory with the data of the non-coplanar experiments for the ^{12}C(p,2p)^{11}B reaction at 156 MeV demonstrates a good agreement, which proves the applicability of the developed approach to the quantitative description of various reactions such as (p,2p) and (p,pn).

APPENDIX

In this appendix, we show how all the possible approximate formulas, which are used for modeling the reactions with the knock-out of nucleons from nuclei under the action of fast protons, can be derived from the exact mathematical expression for the transition matrix element.

According to our definitions, we can write the distorted wave functions which determine the matrix elements of the amplitude as

$$\chi_i^{(+)} = \frac{1}{(2\pi)^{3/2}}\psi_{\mathbf{k}_0}^{(+)}(\mathbf{r}_{0A})\Phi_A(\mathbf{r}_{1C},\xi),$$

$$\chi_{f_1}^{(-)} = \frac{1}{(2\pi)^3}\psi_{\mathbf{k}_1}^{(-)}(\mathbf{r}_{0C})\psi_{\mathbf{k}_2}^{(-)}(\mathbf{r}_{1C})\Phi_{A-1}(\xi), \tag{A.1}$$

where the one-particle distorted wave functions of the continuous spectrum obey the complex equations with the corresponding optical potentials. The momenta $\mathbf{k}_0$, $\mathbf{k}_1$, and $\mathbf{k}_2$ are the momenta of nucleons in the center-of-inertia system (c.i.s). For the sake of simplicity, we neglect all spin and isospin variables and show that the first term of Eq. (1) gives us all the known approximations.

In the three-particle model, the transition operator in the first term of Eq. (1) responsible for the direct mechanism is determined by expression (2), in which the full Hamiltonian of the system takes the form

$$H = T_0 + T_1 + T_C + V_{01} + V_{0C} + V_{1C} =$$

$$= \frac{k^2}{M} + \frac{A+1}{4(A-1)M}q^2 + \frac{P^2}{2(A+1)M} + V_{01}(\mathbf{r}) +$$

$$+V_{0C}(\mathbf{r}_{0C}) + V_{1C}(\mathbf{r}_{1C}). \tag{A.2}$$

The relative positions of these three particles can be determined with the use of two Jacobi coordinates $\mathbf{r} = \mathbf{r}_0 - \mathbf{r}_1$ and $\boldsymbol{\rho} = \frac{1}{2}(\mathbf{r}_0 + \mathbf{r}_1) - \mathbf{R}_C$, where $\mathbf{R}$ is the coordinate of the center of inertia of the whole system. The momenta conjugated to these coordinates are denoted by $\mathbf{k}$, $\mathbf{q}$, and $\mathbf{P}$. The relations between the dynamical Jacobi variables and the variables in the laboratory system are given by the expressions

$$\mathbf{k} = \frac{\mathbf{p}_0 - \mathbf{p}_1}{2},$$

$$\mathbf{q} = \frac{A-1}{A+1}(\mathbf{p}_0 + \mathbf{p}_1) - \frac{2}{A+1}\,\mathbf{p}_C, \quad \mathbf{P} = \mathbf{p}_0 + \mathbf{p}_1 + \mathbf{p}_C\,. \tag{A.3}$$

In the space of three-particle states (plane waves) in the laboratory system (LS), we have

$$\langle \mathbf{p'}_0\mathbf{p'}_1\mathbf{p'}_C|\tau(E)|\mathbf{p}_0\mathbf{p}_1\mathbf{p}_C\rangle \equiv \langle \mathbf{k'}\,\mathbf{q'}\,\mathbf{P'}\,|\tau(E)|\mathbf{kqP}\rangle =$$

$$= \delta(\mathbf{P}-\mathbf{P'})\langle \mathbf{k'}\,\mathbf{q'}\,|\tau \quad E - \frac{P^2}{2M_{A+1}} \quad |\mathbf{kq}\rangle. \tag{A.4}$$

In the coordinate representation, we obtain

$$\langle \mathbf{r'}_0\mathbf{r'}_1\mathbf{R'}_C|\tau(E)|\mathbf{r}_0\mathbf{r}_1\mathbf{R}_C\rangle \equiv \langle \mathbf{r'}\boldsymbol{\rho'}\mathbf{R'}|\tau(E)|\mathbf{r}\boldsymbol{\rho}\mathbf{R}\rangle =$$

$$= \frac{1}{(2\pi)^9}\int d\mathbf{k}\,d\mathbf{k'}d\mathbf{q}d\mathbf{q'}\,d\mathbf{P}\,\langle \mathbf{k'}\,\mathbf{q'}|\tau \quad E - \frac{P^2}{2M_{A+1}} \quad |\mathbf{kq}\rangle\times$$

$$\times \exp[i\mathbf{k'r'} - i\mathbf{kr} + i\mathbf{q'}\boldsymbol{\rho'} - i\mathbf{q}\boldsymbol{\rho} + i\mathbf{P}(\mathbf{R'}-\mathbf{R})]. \tag{A.5}$$

The representations of the three-particle matrix of collision in the LS given by Eqs. (A.4) and (A.5), although being general, are not convenient for the practical use, because they contain explicitly the dependence on the kinetic energy of the center of mass of the whole system. Therefore, we remove the corresponding term from the full Hamiltonian (A.2) and consider three particles in the c.i.s. In the c.i.s., the energy conservation law takes the form

$$\frac{k_0^2}{2M} + \frac{k_A^2}{2M_A} + M + M_A =$$

$$= \frac{k_1^2 + k_2^2}{2M} + \frac{k_C^2}{2M_{A-1}} + 2M + M_{A-1}. \tag{A.6}$$

Then the energy of three particles in the c.i.s. will be $k_0 = \frac{A}{A+1}p_0$

$$E = \frac{k_1^2 + k_2^2}{2M} + \frac{k_C^2}{2(A-1)M} =$$

$$= \frac{A+1}{A}\,\frac{k_0^2}{2M} + Q = \frac{p_0^2}{2M} + Q - \frac{p_0^2}{2(A+1)M}. \tag{A.7}$$

Here, $Q = M_A - M - M_{A-1} < 0$ is the reaction energy (the binding energy of a separated particle). The last term in the third equality in (A.7) represents the center-of-mass energy. It follows from (A.5) that the τ-operator in the coordinate representation in the c.i.s. can be written as

$$\langle \mathbf{r'}\boldsymbol{\rho'}\mathbf{R'}|\tau(E)|\mathbf{r}\boldsymbol{\rho}\mathbf{R}\rangle = \delta(\mathbf{R}-\mathbf{R'})\langle \mathbf{r'}\boldsymbol{\rho'}|\tau(E)|\mathbf{r}\boldsymbol{\rho}\rangle. \tag{A.8}$$

The delta function makes the operator to be translationally invariant and ensures the conservation of the total momentum in the three-particle scattering. The delta function appears as a result of the independence of the propagator in the integral equation for the τ-operator on the momentum of the center-of-mass system of three particles.

After the integration over the internal coordinates of the core and the introduction of the one-particle overlapping integral

$$\Phi(\mathbf{r}_{1C}) = \int d\xi\,\Phi_{A-1}^*(\xi)\Phi_A(\mathbf{r}_{1C},\xi), \tag{A.9}$$

the matrix element responsible for the direct mechanism can be presented as

$$T_{fi}^d = \frac{1}{(2\pi)^{9/2}}\int \psi_{\mathbf{k}_1}^{(-)*}(\mathbf{r'}_{0C})\psi_{\mathbf{k}_2}^{(-)*}(\mathbf{r'}_{1C})\times$$

$$\times\langle \mathbf{r}'\boldsymbol{\rho}'|\tau(E)|\mathbf{r}\boldsymbol{\rho}\rangle\psi^{(+)}_{\mathbf{k}_0}(\mathbf{r}_{0A})\Phi(\mathbf{r}_{1C})d\mathbf{r}d\boldsymbol{\rho}d\mathbf{r}'d\boldsymbol{\rho}'. \quad (A.10)$$

In the Jacobi variables, we have

$$\mathbf{r}_{0C}=\boldsymbol{\rho}+\frac{\mathbf{r}}{2},\quad \mathbf{r}_0=\frac{A-1}{A+1}\boldsymbol{\rho}+\frac{\mathbf{r}}{2}+\mathbf{R},$$

$$\mathbf{r}_{1C}=\boldsymbol{\rho}-\frac{\mathbf{r}}{2},\quad \mathbf{r}_1=\frac{A-1}{A+1}\boldsymbol{\rho}-\frac{\mathbf{r}}{2}+\mathbf{R},$$

$$\mathbf{r}_{0A}=\mathbf{r}_{0C}-\frac{\mathbf{r}_{1C}}{A}=\frac{A-1}{A}\boldsymbol{\rho}+\frac{A+1}{2A}\mathbf{r},$$

$$\mathbf{R}_C=-\frac{2}{A+1}\boldsymbol{\rho}+\mathbf{R}. \quad (A.11)$$

Expressing Eq. (A.10) in terms of the Fourier transforms of the wave functions, we present the matrix element in the form of an integral over momentum variables

$$T^d_{fi}=\frac{1}{(2\pi)^{9/2}}\int d\mathbf{q}_0 d\,\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}\psi^{(-)*}_{\mathbf{k}_1}(\mathbf{q}_1)\psi^{(-)*}_{\mathbf{k}_2}\times$$

$$\times(\mathbf{q}_2)\psi^{(+)}_{\mathbf{k}_0}(\mathbf{q}_0)\Phi(\mathbf{q})d\mathbf{q}_0 d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}\times$$

$$\times\quad \frac{\mathbf{q}_1-\mathbf{q}_2}{2};\mathbf{q}_1+\mathbf{q}_2\,|\tau(E)|\,\frac{A+1}{2A}\mathbf{q}_0-\frac{\mathbf{q}}{2};\frac{A-1}{A}\mathbf{q}_0+\mathbf{q}\quad . \quad (A.12)$$

Formulas (A.10) and (A.12) are exact in the three-particle model, but they are too complicated for the practical use. Let us simplify these formulas.

The impulse approximation consists in the replacement of the three-particle τ-operator of the transition by the term corresponding to the single scattering of two free nucleons described by the two-nucleon t-operator, i.e. $\tau = t$. The two-particle t-operator in the three-particle space is related easily to the two-particle scattering amplitude. The Lippmann—Schwinger equation for an element of the t-matrix in the two-nucleon variables takes the form

$$\langle \mathbf{k}'\mathbf{q}'|t(E)|\mathbf{k}\mathbf{q}\rangle=\langle \mathbf{k}'\mathbf{q}'\,|V_{01}|\mathbf{k}\mathbf{q}\rangle+$$

$$+\int d\mathbf{k}''d\mathbf{q}''\frac{\langle \mathbf{k}'\mathbf{q}'|V_{01}|\mathbf{k}''\mathbf{q}''\rangle\langle \mathbf{k}''\mathbf{q}''|t(E)|\mathbf{k}\mathbf{q}\rangle}{E-\frac{k''^2}{2\mu_{01}}-\frac{q''^2}{2\mu_{BC}}+i\eta}, \quad (A.13)$$

where $\mu_{01}=\frac{M}{2}$ and $\mu_{BC}=\frac{2(A-1)}{A+1}M$ are the corresponding reduced masses. Since the interaction potential V_{01} depends only on the coordinate $\mathbf{r}$ (the corresponding conjugate momentum is $\mathbf{k}$), only the relative momentum of particles 0 and 1 can change during their scattering by each other. The relative momentum of the nucleon pair and the core $\mathbf{q}$ is not changed. Therefore, we have

$$\langle \mathbf{k}'\mathbf{q}'|t(E)|\mathbf{k}\mathbf{q}\rangle=\delta(\mathbf{q}-\mathbf{q}')\quad \mathbf{k}'\ t\quad E-\frac{q^2}{2\mu_{BC}}\quad \mathbf{k}\quad , \quad (A.14)$$

$$\langle \mathbf{r}'\boldsymbol{\rho}'|t(E)|\mathbf{r}\boldsymbol{\rho}\rangle=\frac{1}{(2\pi)^6}\int d\mathbf{k}\,d\mathbf{k}'\mathbf{q}\times$$

$$\times\exp[i\mathbf{k}'\mathbf{r}'-i\mathbf{k}\mathbf{r}+i\mathbf{q}(\boldsymbol{\rho}'-\boldsymbol{\rho})]\quad \mathbf{k}'\ t\quad E-\frac{q^2}{2\mu_{BC}}\quad \mathbf{k}\quad .$$

These formulas establish the required relation between the t-operator and the NN-scattering amplitude. The energy variable of the binary t-matrix is equal to the difference between the total energy of the three-particle system in the c.i.s., (A.7), and the energy of the relative motion of the center of mass of the nucleon pair and the spectator. Thus, this variable represents the relative energy of the binary subsystem (the collision energy) $\varepsilon = E - q^2/2\mu_{BC}$. Substituting (A.14) into (A.12), integrating over $\mathbf{q}$, and passing to three new variables

$$\mathbf{p}=\mathbf{q}_1+\mathbf{q}_2,\quad \mathbf{q}=\mathbf{q}_0-\frac{\mathbf{q}_1+\mathbf{q}_2}{2},\quad \mathbf{q}'=\frac{\mathbf{q}_1-\mathbf{q}_2}{2}, \quad (A.15)$$

we obtain the formula for the exact DTWA:

$$T^d_{fi}=\frac{1}{(2\pi)^{9/2}}\int d\mathbf{p}\,d\mathbf{q}\,d\mathbf{q}'\,\psi^{(-)*}_{\mathbf{k}_1}\quad \frac{\mathbf{p}}{2}+\mathbf{q}'\quad \psi^{(-)*}_{\mathbf{k}_2}\times$$

$$\times\quad \frac{\mathbf{p}}{2}-\mathbf{q}'\quad \mathbf{q}'\ t(E-\frac{p^2}{2\mu_{BC}})\ \mathbf{q}\quad \times$$

$$\times\psi^{(+)}_{\mathbf{k}_0}\quad \frac{\mathbf{p}}{2}+\mathbf{q}\quad \Phi\quad \frac{A+1}{2A}\mathbf{p}-\frac{A-1}{A}\mathbf{q}\quad . \quad (A.16)$$

Thus, in the impulse approximation, T^d_{fi} contains the two-nucleon t-matrix that lies off the energy surface. This approximation corresponds physically to the direct knock-out of a nucleon from the nucleus-target by the incident proton in the effective distorting potential of all other nucleons of the nucleus. The many-body character of this problem is revealed in coming off the two-particle t-matrix from the energy surface. The characteristic feature of expression (A.16) is the integration over the whole space of the momentum $\mathbf{p}$ of the center of mass of particles 0 and 1. This makes impossible to write Eq. (A.16) in the coordinate representation in the usual form of the DWBA.

The next simplification, which will be made below, leads to the energy-dependent McCarthy approximation [8] for the t-matrix. This approximation neglects the collision energy dependence on the pair momentum $\mathbf{p}$ in the argument of the binary t-matrix in formula (A.16) replaces this energy by a certain fixed value ε that corresponds to the energy of the input or output channels or the average of these two values. Generally speaking, this replacement makes the collision energy indeterminate, what is a drawback of this approximation. However, this approximation allows the exact calculation of the coming off from the energy surface in a two-particle collision. This approximation is also more adequate in the average energy region (less than 100 MeV) in comparison with the factorized impulse approximation. As a result, Eq. (A.16) yields the approximate formula of the DWTA that is reduced to that of the DWBA with the non-local operator

$$T^d_{fi}=\frac{1}{(2\pi)^{9/2}}\int d\boldsymbol{\rho}\,d\mathbf{r}\,d\mathbf{r}'\,\psi^{(-)*}_{\mathbf{k}_1}\quad \boldsymbol{\rho}+\frac{\mathbf{r}'}{2}\quad \times$$

$$\times\psi^{(-)*}_{\mathbf{k}_2}\quad \boldsymbol{\rho}-\frac{\mathbf{r}'}{2}\quad \langle \mathbf{r}'|t(\varepsilon)|\mathbf{r}\rangle\times$$

$$\times\psi^{(+)}_{\mathbf{k}_0}\quad \frac{A-1}{A}\boldsymbol{\rho}+\frac{A+1}{2A}\mathbf{r}\quad \Phi\quad \boldsymbol{\rho}-\frac{\mathbf{r}}{2}\quad . \quad (A.17)$$

Formula (A.17) can be formally obtained from Eq. (A.10) if we set

$$\langle \mathbf{r}'\boldsymbol{\rho}'|\tau(E)|\mathbf{r}\boldsymbol{\rho}\rangle\cong\delta(\boldsymbol{\rho}-\boldsymbol{\rho}')\langle \mathbf{r}'|t(\varepsilon)|\mathbf{r}\rangle, \quad (A.18)$$

As can be seen from Eq. (A.14), this is obtained if we neglect the dependence of the collision energy on q. In other words, Eq.(A.18) corresponds to the neglect of all interactions except for V_{01} in the Hamiltonian H in Eq. (A.2) and the replacement $E - T_{BC} = \varepsilon$, which means that the energy of the relative motion of the nucleon

pair and the core is fixed. Since $\mathbf{R}_C = \mathbf{R}'_C$ in this situation, because there are no interactions that can shift the core, the variable $\boldsymbol{\rho} = \frac{1}{2}(\mathbf{r}_0 + \mathbf{r}_1)$ in Eq. (A.17) can be considered identical to the coordinate of the center of mass of the pair.

The last simplification on the integration of Eq.(A.16) is obtained by assuming the small influence of the distorting potentials, which happens if the functions $\psi^{(\pm)}$ are close to plane waves. This assumption is satisfied at sufficiently high energies. In this case, the momentum variables in the t-matrix element can be replaced by their asymptotic values

$$\mathbf{q} = \mathbf{k}_i = \mathbf{k}_0 - \frac{\mathbf{k}_1 + \mathbf{k}_2}{2}, \quad \mathbf{q}' = \mathbf{k}_f = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2},$$

$$\mathbf{p} = \mathbf{k}_1 + \mathbf{k}_2, \tag{A.19}$$

and the element itself can be moved out the integration sign. With regard for (A.7), the energy argument of the t-matrix becomes

$$\varepsilon_f = E - \frac{(\mathbf{k}_1 + \mathbf{k}_2)^2}{2\mu_{BC}} = \frac{1}{M} \quad \frac{\mathbf{k}_1 - \mathbf{k}_2}{2} \quad ^2 = \frac{k_f^2}{M}, \tag{A.20}$$

which corresponds to the collision energy in the output channel. After all the made approximations, the matrix element (A.16) is reduced to a three-dimensional spatial integral. Finally, we obtain the well-known asymptotic factorized impulse approximation [4,5]

$$T_{fi}^d = \langle \mathbf{k}_f | t(\varepsilon_f) | \mathbf{k}_i \rangle \frac{1}{(2\pi)^{3/2}} \times$$

$$\times \int \psi_{\mathbf{k}_1}^{(-)*}(\boldsymbol{\rho}) \psi_{\mathbf{k}_2}^{(-)*}(\boldsymbol{\rho}) \psi_{\mathbf{k}_0}^{(+)} \quad \frac{A-1}{A}\boldsymbol{\rho} \quad \Phi(\boldsymbol{\rho}) d\boldsymbol{\rho}. \tag{A.21}$$

As seen, this matrix element partially deviates from the energy surface to the right. Formally, expression (A.21) follows from Eq.(A.10) by setting

$$\langle \mathbf{r}' \boldsymbol{\rho}' | \tau(E) | \mathbf{r} \boldsymbol{\rho} \rangle \approx$$

$$\approx (2\pi)^3 \delta(\boldsymbol{\rho} - \boldsymbol{\rho}') \delta(\mathbf{r} - \mathbf{r}') \delta(\mathbf{r}) \langle \mathbf{k}_f | t(\varepsilon_f) | \mathbf{k}_i \rangle. \tag{A.22}$$

Sometimes, due to (A.22), formula (A.21) is erroneously called the approximation of zero-radius nuclear forces. This is not true, because the dependence of the t-matrix element on the transferred momentum means that the finite radius of nuclear forces is approximately taken into account, as it is done in the plane-wave approximation. We note that a factorization of the type (A.21) does not happen even in the asymptotic impulse approximation with the explicit account of the dependence of the t-matrix and wave functions on the spin. The approximation obtained in this case was called the non-factorized impulse approximation. This approximation was considered in work [33].

For the sake of completeness, we also present the expression of the transition matrix element (A.10) corresponding to the plane-wave impulse approximation (PWIA). In this case, $\psi_{\mathbf{k}}(\mathbf{p}) = (2\pi)^{3/2}\delta(\mathbf{k} - \mathbf{p})$, and hence

$$T_{fi}^d(PWIA) = \langle \mathbf{k}_f | t(\varepsilon_f) | \mathbf{k}_i \rangle \Phi \quad \mathbf{k}_1 + \mathbf{k}_2 - \frac{A-1}{A}\mathbf{k}_0 \quad . \tag{A.23}$$

It is seen that the argument of the Fourier-transform of the single-particle overlapping integral Φ in Eq. (A.23) is opposite to the momentum of the recoil nucleus in the LS, namely,

$$\mathbf{k}_1 + \mathbf{k}_2 - \frac{A-1}{A}\mathbf{k}_0 = \mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_0 = -\mathbf{p}_C \ . \tag{A.24}$$

This corresponds to the initial idea that the measurement of the reaction cross section as a function of the recoil nucleus momentum allows the determination of the momentum distribution of intranucleus nucleons, because the nucleon momentum in the nucleus is $\mathbf{p} = -\mathbf{p}_C$ prior to the collision.

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t-МАТРИЧНЕ НАБЛИЖЕННЯ В ТЕОРІЇ (p,2p)- ТА (p,np)-РЕАКЦІЙ

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Р е з ю м е

У *t*-матричном наближенні із спотвореними хвилями отримано нові формули для амплітуд (p,2p)- та (p,pn)-реакцій, які узагальнюють вирази МакКарті, одержані для симетричної та компланарної геометрій, на випадок довільної геометрії імпульсів відлітаючих нуклонів. Виведення проведено формальними методами теорії ядерних реакцій і дозволило врахувати і прямий, і непрямий механізми. Виконано розрахунки і порівняння з даними некомпланарних експериментів для реакції ^{12}C(p,2p)^{11}B при енергії 156 МеВ. Отримано задовільне узгодження з експериментом. Показано, як із знайденого точного виразу для амплітуди реакції виходять всі відомі наближення, котрі використовуються для описування цих реакцій.