ON THE KINETICS OF SPATIALLY NON-UNIFORM STATES OF PARTICLES WEAKLY INTERACTING WITH A HYDRODYNAMIC MEDIUM

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Our work considers spatially non-uniform states of particles weakly interacting with a hydrodynamic medium. We have developed a microscopic theory of such systems by using Bogolyubov's reduced description method. It has been shown that such a system has both the kinetic and hydrodynamic stages of evolution. The kinetic stage of evolution for particles interacting with a medium has been considered. At this stage, the one-particle distribution function is a reduced description parameter for particles, and, therefore, a medium is described by five hydrodynamic parameters (density, temperature and velocity). The coupled system of motion equations for the reduced description parameters is obtained on the basis of Bogolyubov's reduced description method. The obtained equations can be used, for example, for the description of neutrons propagating in a hydrodynamic medium without multiplication and capture.

1. Introduction

The development of the kinetic theory of particles weakly interacting with a medium concerns both the general theory of relaxation processes and applied researches. Such a theory can be used in studying the Brownian motion or the transfer of neutrons in different media in a nuclear reactor. The beginning of the theoretical research of such systems dates back to the first works of Einstein and Smoluchowski (see [2]), though many aspects have not been studied yet. First of all, this concerns the construction of a consistent microscopic approach describing the kinetics of particles interacting with a medium.

The reduced description method is the most sequential and promising microscopic approach in modern kinetics. The basic concepts of this method used for the description of classical systems are stated in the book [1] by N.N. Bogolyubov. The extension of this method to quantum many-particle systems is given in book [2]. The reduced description method is based on the concept of a relaxation time hierarchy, the ergodic hypothesis, and the principle of a weakening of spatial correlations. The application of the reduced description method results in a consistent microscopic approach which allows us to obtain kinetic equations (when the system is described by a one-particle distribution function) and hydrodynamic equations (in the case where the system is described with a set of hydrodynamic parameters such as temperature, density, and velocity).

However, such systems may include different subsystems on different stages of their evolution. For example, in a two-component system, one component can be on the kinetic evolution stage, which means that it is described by a one-particle distribution function, while the other component evolves hydrodynamically and is described by a set of hydrodynamic parameters. Such a situation occurs when the system consists of strongly interacting particles of one type (hydrodynamic medium) and particles of the other type which weakly interact with the medium, but do not interact with one another owing their small number. One of the specific examples of such systems are slow neutrons in a hydrodynamic medium.

The consequent microscopic theory describing the spatially homogeneous evolution of particles in a hydrodynamic medium by means of the reduced description method has been developed in [2]. In the present work, we consider spatially inhomogeneous states of particles weakly interacting with a hydrodynamic medium by using the reduced description method. As a result, we obtain a coupled system of equations which describes the evolution of a hydrodynamic medium and particles that weakly interact with this medium.

Prior to the derivation of the system of equations, we will formulate some basic definitions.

2. Basic Description Parameters and Their Properties

We describe the hydrodynamic medium by using a set of parameters $\zeta_\alpha(\mathbf{x})$, $\alpha = 0, i, 4$, where $\zeta_0(\mathbf{x}) \equiv \varepsilon(\mathbf{x})$ is the energy density of the medium, $\zeta_i(\mathbf{x}) \equiv \pi_i(\mathbf{x})$ is the momentum density, and $\zeta_4(\mathbf{x}) \equiv \rho^{(m)}(\mathbf{x})$ is the mass density. We can also introduce the operators $\hat{\zeta}_\alpha(\mathbf{x})$ of hydrodynamic parameters with densities $\zeta_\alpha(\mathbf{x})$, $\alpha = 0, i, 4$, where $\hat{\zeta}_0(\mathbf{x}) \equiv \hat{\varepsilon}(\mathbf{x})$ is the operator of the energy density, $\hat{\zeta}_i(\mathbf{x}) \equiv \hat{\pi}_i(\mathbf{x})$ is operator of the momentum density, and $\hat{\zeta}_4(\mathbf{x}) \equiv \hat{\rho}^{(m)}(\mathbf{x})$ is the operator of the mass density. These operators are expressed in terms of the creation $\varphi^+(\mathbf{x})$ and annihilation $\varphi(\mathbf{x})$ operators for particles of the medium:

$$\hat{\varepsilon}(\mathbf{x}) = \frac{1}{2m_m}\nabla\varphi^+(\mathbf{x})\nabla\varphi(\mathbf{x}) + \frac{1}{2}\int d^3R\times$$
$$\times V_m(\mathbf{R})\varphi^+(\mathbf{x}+\mathbf{R})\varphi^+(\mathbf{x})\varphi(\mathbf{x})\varphi(\mathbf{x}+\mathbf{R}),$$
$$\hat{\pi}_i(\mathbf{x}) = \frac{i}{2}\left(\frac{\partial\varphi^+(\mathbf{x})}{\partial x_i}\varphi(\mathbf{x}) - \varphi^+(\mathbf{x})\frac{\partial\varphi(\mathbf{x})}{\partial x_i}\right), \tag{1}$$
$$\hat{\rho}^{(m)}(\mathbf{x}) = m_m\varphi^+(\mathbf{x})\varphi(\mathbf{x}).$$

Here, m_m is the mass of particles of the medium, and $V_m(\mathbf{R})$ is the pairwise interaction potential of particles of the medium. The mass $\hat{M}$, momentum $\hat{P}_i$, and energy (Hamiltonian) $\mathcal{H}^{(m)}$ of the medium are introduced according to (1) in the following way:

$$\hat{M} = \int d^3x\hat{\rho}^{(m)}(\mathbf{x}),$$
$$\hat{P}_i = \int d^3x\hat{\pi}_i(\mathbf{x}), \tag{2}$$
$$\mathcal{H}^{(m)} = \int d^3x\hat{\varepsilon}(\mathbf{x}).$$

Obviously, these operators commute with one another, hence the densities of the hydrodynamic parameters $\hat{\zeta}_\alpha(\mathbf{x})$, where α varies from 0 to 4, are the densities of the additive integrals of motion of the medium. We note that the interaction of particles of the medium can be more complicated than that specified in (1), although variables (2) play the role of the integrals of motion.

Time derivative operators of additive motion integrals in the Schrödinger representation are given by expressions (see [2, 3])

$$\dot{\hat{\zeta}}_\alpha(\mathbf{x}) = i[\mathcal{H}^{(m)}, \hat{\zeta}_\alpha(\mathbf{x})] = -\frac{\partial\hat{\zeta}_{\alpha k}(\mathbf{x})}{\partial x_k}, \tag{3}$$

where variables $\hat{\zeta}_{\alpha k}(\mathbf{x})$ are the densities of the flow of additive motion integrals; namely, $\hat{\zeta}_{0k}(\mathbf{x}) = \hat{q}_k(\mathbf{x})$ is the operator of the energy flow density, $\hat{\zeta}_{ik}(\mathbf{x}) = \hat{t}_{ik}(\mathbf{x})$ is the operator of the momentum flow density and $\hat{\zeta}_{4k}(\mathbf{x}) = \hat{j}^{(m)}_k(\mathbf{x})$ is the operator of the mass flow density. According to [2], the operators of the additive motion integrals are expressed in terms of the hydrodynamic parameters:

$$\hat{j}^{(m)}_k(\mathbf{x}) = \pi_k(\mathbf{x}) = i\int d^3x' x'_k\int_0^1 d\xi\times$$
$$\times\left[\varepsilon(\mathbf{x}-(1-\xi)\mathbf{x}'), \rho^{(m)}(\mathbf{x}+\xi\mathbf{x}')\right],$$
$$\hat{t}_{kl}(\mathbf{x}) = -\varepsilon(\mathbf{x})\delta_{kl} + i\int d^3x' x'_k\int_0^1 d\xi\times$$
$$\times[\varepsilon(\mathbf{x}-(1-\xi)\mathbf{x}'), \pi_k(\mathbf{x}+\xi\mathbf{x}')], \tag{4}$$
$$\hat{q}_k(\mathbf{x}) = \frac{i}{2}\int d^3x' x'_k\int_0^1 d\xi\times$$
$$\times[\varepsilon(\mathbf{x}-(1-\xi)\mathbf{x}'), \varepsilon(\mathbf{x}+\xi\mathbf{x}')].$$

So, we have defined the operators required for the description of the medium.

Particles, as was mentioned in Introduction, may be on the kinetic evolution stage, where they are described with a one-particle distribution function $f(\mathbf{x}, \mathbf{p})$. So we introduce the operator of Wigner's distribution function (see [2, 4])

$$\hat{f}_\mathbf{p}(\mathbf{x}) \equiv \int d^3x' e^{-i\mathbf{p}\mathbf{x}'}\psi^+\left(\mathbf{x}-\frac{\mathbf{x}'}{2}\right)\psi\left(\mathbf{x}+\frac{\mathbf{x}'}{2}\right), \tag{5}$$

where the variables $\psi^+(\mathbf{x})$ and $\psi(\mathbf{x})$ are the operators of creation and annihilation of particles interacting with the medium. It is essential that the operators of creation and annihilation of particles of the medium and those of the particles interacting with the medium commute:

$$\left[\psi^+(\mathbf{x}), \varphi^+(\mathbf{x})\right] = 0, \quad [\psi(\mathbf{x}), \varphi(\mathbf{x})] = 0,$$
$$\left[\psi(\mathbf{x}), \varphi^+(\mathbf{x})\right] = 0, \quad \left[\psi^+(\mathbf{x}), \varphi(\mathbf{x})\right] = 0.$$

The Hamiltonian of free (i.e. not interacting with one another) particles in terms of the creation and annihilation operators looks as

$$\mathcal{H}_p = \frac{1}{2m}\int d^3x\nabla\psi^+(\mathbf{x})\nabla\psi(\mathbf{x}), \tag{6}$$

where m is the mass of particles. It is obvious that

$$-i\left[\int d^3x\hat{f}_{\mathbf{p}}(\mathbf{x}),\mathcal{H}^{(p)}\right]=0, \tag{7}$$

which means that the Wigner's distribution function operator $\hat{f}_{\mathbf{p}}(\mathbf{x})$ can be treated as an additive motion integral of the subsystem of particles. By the consequent application of expression (6) for the Hamiltonian of free particles $\mathcal{H}_p$ and the definition of the Wigner's distribution function (5), we can show that

$$i\left[\mathcal{H}_p,\hat{f}_{\mathbf{p}}(\mathbf{x})\right]=-\frac{p_k}{m}\frac{\partial}{\partial x_k}\hat{f}_{\mathbf{p}}(\mathbf{x}). \tag{8}$$

The variable

$$\hat{f}_{\mathbf{p}k}(\mathbf{x})\equiv\frac{p_k}{m}\hat{f}_{\mathbf{p}}(\mathbf{x}) \tag{9}$$

can be treated as the flow of the Wigner's distribution function.

We also mention several formulas relevant to the Wigner's distribution function which will be used in future calculations. By using definition (5), we can easily find another expression for the Wigner's distribution function operator:

$$\hat{f}_{\mathbf{p}}(\mathbf{x})=\mathcal{V}\int d^3p_1d^3p_2a^+_{\mathbf{p}_1}a_{\mathbf{p}_2}\times$$
$$\times e^{-i\mathbf{x}(\mathbf{p}_1-\mathbf{p}_2)}\delta\left(\frac{\mathbf{p}_1+\mathbf{p}_2}{2}-\mathbf{p}\right), \tag{10}$$

where $\mathcal{V}$ is the system's volume, $\delta(\mathbf{p})$ is the Dirac's delta function, and $a^+_{\mathbf{p}}$, $a_{\mathbf{p}}$ are the operators of creation and annihilation of particles having momentum $\mathbf{p}$. These operators are introduced with the following expressions:

$$\psi^+(\mathbf{x})=\mathcal{V}^{-\frac{1}{2}}\sum_{\mathbf{p}}a^+_{\mathbf{p}}e^{-i\mathbf{px}},\ \psi(\mathbf{x})=\mathcal{V}^{-\frac{1}{2}}\sum_{\mathbf{p}}a_{\mathbf{p}}e^{i\mathbf{px}}. \tag{11}$$

After integrating (10), we obtain

$$\int d^3x\hat{f}_{\mathbf{p}}(\mathbf{x})\equiv\mathcal{V}a^+_{\mathbf{p}}a_{\mathbf{p}}\equiv\hat{\gamma}_{\mathbf{p}}. \tag{12}$$

In order to formalize further expressions, we introduce a general symbol for the operators used as description parameters. We introduce a generalizing symbol $\hat{\zeta}_A(\mathbf{x})$, where the index "$A$" possesses the value of $A=\{\alpha,\mathbf{p}\}$. For the operators of the additive motion integrals of the medium $\zeta_\alpha(\mathbf{x})$, $\alpha=0,i,4$ and the Wigner's distribution function operator $\hat{f}_{\mathbf{p}}(\mathbf{x})$ (see (1), (5)), we have

$$\hat{\zeta}_A(\mathbf{x})|_{A=\alpha}=\hat{\zeta}_\alpha(\mathbf{x}),\quad \hat{\zeta}_A(\mathbf{x})|_{A=\mathbf{p}}=\hat{f}_{\mathbf{p}}(\mathbf{x}). \tag{13}$$

Similarly, we introduce a generalizing symbol $\hat{\zeta}_{Ak}(\mathbf{x})$ for the flow operators (compare with (9)):

$$\hat{\zeta}_{Ak}(\mathbf{x})|_{A=\alpha}=\hat{\zeta}_{\alpha k}(\mathbf{x}),$$
$$\hat{\zeta}_{Ak}(\mathbf{x})|_{A=\mathbf{p}}=\hat{f}_{\mathbf{p},k}(\mathbf{x})\equiv\frac{p_k}{m}\hat{f}_{\mathbf{p}}(\mathbf{x}). \tag{14}$$

Further, we will need the symmetry properties of the above-inroduced operators in respect to space-time reversal transformations. The unitary space reversal operator $\mathcal{P}$ transforms the field operators of our system $\varphi(\mathbf{x})$ and $\psi(\mathbf{x})$ by the formulas

$$\varphi'(\mathbf{x})=\varphi(-\mathbf{x})=\mathcal{P}\varphi(\mathbf{x})\mathcal{P}^+,$$
$$\psi'(\mathbf{x})=\psi(-\mathbf{x})=\mathcal{P}\psi(\mathbf{x})\mathcal{P}^+. \tag{15}$$

The unitary time reversal operator $\mathcal{T}$ transforms the field operators as

$$\varphi'(\mathbf{x})=\varphi(\mathbf{x})^*=\mathcal{T}\varphi(\mathbf{x})\mathcal{T}^+,$$
$$\psi'(\mathbf{x})=\psi(\mathbf{x})^*=\mathcal{T}\psi(\mathbf{x})\mathcal{T}^+. \tag{16}$$

We have the following expressions for the space and time reversal transformations of the operators of additive motion integrals $\hat{\zeta}_A(\mathbf{x})$ and their flows $\hat{\zeta}_{Ak}(\mathbf{x})$ by using (15) and (16) (see [2] for details):

$$\mathcal{TP}\hat{\zeta}_A(\mathbf{x})(\mathcal{PT})^+=\hat{\zeta}^*_A(-\mathbf{x}),$$
$$\mathcal{TP}\hat{\zeta}_{Ak}(\mathbf{x})(\mathcal{TP})^+=\hat{\zeta}^*_{Ak}(-\mathbf{x}). \tag{17}$$

Thus, we have defined all the quantities and their operators in the secondary quantization representation that are necessary for the description of the medium subsystem and the particle subsystem separately. These two subsystems are joined with the complete Hamiltonian $\mathcal{H}$, which is defined as

$$\mathcal{H}=\mathcal{H}_0+\hat{V},\quad \mathcal{H}_0=\mathcal{H}_m+\mathcal{H}_p, \tag{18}$$

where $\mathcal{H}_0$ describes the non-interacting subsystems of the medium and particles, and $\hat{V}$ describes the interaction of these subsystems. Operators $\mathcal{H}_m$ and $\mathcal{H}_p$ were defined by formulas (2) and (6). Here, we assume that the interaction Hamiltonian has the structure

$$\hat{V}=\sum_{\mathbf{p}_1,\mathbf{p}_2}\hat{\mathcal{J}}(\mathbf{p}_1,\mathbf{p}_2)a^+_{\mathbf{p}_1}a_{\mathbf{p}_2}, \tag{19}$$

where $a^+_{\mathbf{p}_1}$, $a_{\mathbf{p}_2}$ are the operators of creation and annihilation of particles in the momentum representation (see (11)), and the operator $\hat{\mathcal{J}}(\mathbf{p}_1,\mathbf{p}_2)$ contains only the medium's operators. We will not specify the structure of $\hat{\mathcal{J}}(\mathbf{p}_1,\mathbf{p}_2)$. However, the Hermitian property of the Hamiltonian implies that

$$\hat{\mathcal{J}}^+(\mathbf{p}_1,\mathbf{p}_2)=\hat{\mathcal{J}}(\mathbf{p}_2,\mathbf{p}_1). \tag{20}$$

3. Postulates of the Method of Reduced Description in the Kinetic Theory of Particles Interacting with the Hydrodynamic Medium

The physical quantities and their operators defined in the previous section will be used in our study of spatially inhomogeneous states of the medium and particles interacting with the medium. Now, we formulate the basic concepts of the reduced description method and apply them to our system.

At an arbitrary time t, our system can be described with a statistical operator $\rho(t)$ which evolves according to the Liouville equation

$$\frac{\partial\rho(t)}{\partial t} = -i[\mathcal{H}, \rho(t)], \tag{21}$$

where $\mathcal{H}$ is the system's Hamiltonian. For the closed systems, the solution of this equation can be written as

$$\rho(t) = e^{-i\mathcal{H}t}\rho e^{i\mathcal{H}t}, \tag{22}$$

where ρ is the statistical operator of the initial state.

The operator $\rho(t)$ satisfies two fundamental principles according to the reduced description method's concepts (see [2] for details). They are the principle of a weakening of spatial correlations and the ergodic relation. The principle of a weakening of spatial correlations represents a simplification of traces of the statistical operator $\rho(t)$ and the products of quasilocal operators $a(\mathbf{x})$ and $b(\mathbf{y})$ (see [2]) when their arguments are separated:

$$\mathrm{Sp}\rho(t)\, a(\mathbf{x})\, b(\mathbf{y}) \underset{|\mathbf{x}-\mathbf{y}|\gg r_c}{\rightarrow} \mathrm{Sp}\rho(t)\, a(\mathbf{x}) \cdot \mathrm{Sp}\rho(t)\, b(\mathbf{y})\,. \tag{23}$$

Here, r_c is the correlation radius of the state with $\rho(t)$.

The ergodic relation describes the asymptotic form of the statistical operator $\rho(t)$ (and, certainly, traces with this operator) at large times:

$$\rho(t) = e^{-i\mathcal{H}t}\rho e^{i\mathcal{H}} \underset{t\to\infty}{\rightarrow} w. \tag{24}$$

Here, w is the equilibrium Gibbs operator. Actually, relation (24) represents the fact that our system transforms into a state of statistical equilibrium at large time scales described with the Gibbs statistical operator w. The structure of this operator is determined by a collection of physical parameters of the equilibrium state of the system. For our system, the Gibbs statistical operator w is given by expression

$$w(Y) = \exp\{\Omega - Y_A\hat{\gamma}_A\}, \tag{25}$$

where $\hat{\gamma}_A$ are the operators of additive motion integrals

$$\hat{\gamma}_A = \int d^3x \hat{\zeta}_A(\mathbf{x}) \tag{26}$$

commuting with the Hamiltonian $\mathcal{H}_0$ (see (13))

$$[\mathcal{H}_0, \hat{\gamma}_A] = 0. \tag{27}$$

The relation for the thermodynamic potential Ω and generalized thermodynamic forces Y_A in (25) can be derived from the normalization requirement

$$\mathrm{Sp}w(Y) = 1, \tag{28}$$

and the thermodynamic forces Y_A are functions of the additive motion integrals γ_A which are derived from the equations (see also (26))

$$\mathrm{Sp}w(Y)\,\hat{\gamma}_A = \gamma_A. \tag{29}$$

The reduced description method is based on the Bogolyubov's concept of relaxation time hierarchy. According to this concept, a system evolves towards an equilibrium with different sets of the description parameters on different evolution stages. Moreover, as the system approaches the equilibrium, the number of parameters required for the description of the system decreases, and the system's description simplifies (see [2] for more details). A set of parameters which describes the system on one evolution stage is called the set of reduced description parameters of the system.

Now we formulate the concepts of the reduced description method concerning our system.

We suppose that the number of particles is small, so we can neglect the interaction among them. Therefore, the operator $\mathcal{H}_p$ (6) has the structure of a Hamiltonian of free particles. In distinction from this, we assume that the Hamiltonian $\mathcal{H}_m$ describes a strong interaction of particles of the medium with one another. This leads to the fast relaxation of this subsystem to a local equilibrium state. Also the interaction of subsystems is weaker than that of particles of the medium with one another. Therefore, the relaxation time for the medium subsystem is substantially lower than that for the both subsystems τ_0, determined by the inter-subsystem interaction. Consequently, the relaxation time of the whole system is determined by the weaker interaction $\hat{V}$. According to the basic concepts of the reduced description method [2], we accept that, at times $t \gg \tau_0$, the additive integrals densities $\zeta_A(\mathbf{x})$ (see (13)) can be taken as reduced description parameters. This means that the statistical operator $\rho(t)$, which describes our system on the evolution stage when the corresponding time interval is greater than the specific relaxation time τ_0, has the following functional dependence on the additive integrals densities $\zeta_A(\mathbf{x})$:

$$\rho(t) = e^{-i\mathcal{H}t}\rho e^{i\mathcal{H}t} \underset{t\gg\tau_0}{\rightarrow} \sigma(\zeta(\mathbf{x}', t; \rho)). \tag{30}$$

We note that, according to (30), the only dependence of the statistical operator $\rho(t)$ on the initial state (statistical operator ρ) is included in the reduced description parameters $\zeta_A(\mathbf{x}, t; \rho)$. The operator σ is called the coarse-grained statistical operator and has functional dependence on the description parameters $\zeta_A(\mathbf{x})$. The coarse-grained statistical operator must satisfy the relation

$$\zeta_A(\mathbf{x}) \equiv \mathrm{Sp}\sigma(\zeta)\hat{\zeta}_A(\mathbf{x}). \tag{31}$$

It is obvious that, according to (21),

$$e^{-i\mathcal{H}\tau}\sigma(\zeta(\mathbf{x}', t; \rho))e^{i\mathcal{H}\tau} = \sigma(\zeta(\mathbf{x}', t+\tau; \rho)), \tag{32}$$

$$\zeta_A(\mathbf{x}', t; \rho) = \zeta_A\left(\mathbf{x}', t+\tau; e^{-i\mathcal{H}t}\rho e^{i\mathcal{H}t}\right). \tag{33}$$

By differenting (33) with respect to τ and setting $\tau = 0$, we obtain

$$-i[\mathcal{H},\sigma(\zeta_A(\mathbf{x}', t; \rho))] = \frac{\partial}{\partial t}\sigma(\zeta_A(\mathbf{x}', t; \rho)) =$$

$$= \int d^3x \frac{\delta\sigma(\zeta(\mathbf{x}'))}{\delta\zeta_A(\mathbf{x})} L_A(\mathbf{x}, \zeta(\mathbf{x})), \tag{34}$$

$$L_A(\mathbf{x}\,\zeta(\mathbf{x})) \equiv \dot{\zeta}_A(\mathbf{x}). \tag{35}$$

(In relation (34) and further, we assume the summation by repeating indices A.) By multiplying (34) by $\zeta_B(\mathbf{x})$, calculating the trace, and performing some simple transformation, we come to the expression

$$L_A(\mathbf{x}, \zeta(\mathbf{x})) = iSp\sigma(\zeta(\mathbf{x}'))\left[\mathcal{H}, \hat{\zeta}_A(\mathbf{x})\right]. \tag{36}$$

By substituting (18) and using formulas (3), (8), (35), and (36), we come to the following evolution equation for the additive motion integrals:

$$\dot{\zeta}_A(\mathbf{x}) = i\mathrm{Sp}\sigma(\zeta(\mathbf{x}'))\left[\hat{V}, \hat{\zeta}_A(\mathbf{x})\right] -$$

$$-\frac{\partial}{\partial x_k}\mathrm{Sp}\sigma(\zeta(\mathbf{x}'))\hat{\zeta}_{Ak}(\mathbf{x}). \tag{37}$$

In order to obtain an integral equation for the coarse-grained statistical operator $\sigma(\zeta(\mathbf{x}'))$, which can be used in the perturbation theory method for a small interaction, we consider the evolution of the system without interaction $\hat{V}$, i.e. when our system is described by the Hamiltonian $\mathcal{H}_0$. In this case, a spatially inhomogeneous state will be formed after some time. We will describe this state by a coarse-grained statistical operator σ_0

$$e^{-i\mathcal{H}_0\tau}\rho e^{i\mathcal{H}_0\tau} \underset{\tau\to\infty}{\rightarrow} \sigma_0\left(\zeta^0(\mathbf{x}', \tau; \sigma)\right), \tag{38}$$

where (compare with (31))

$$\zeta_A^0(\mathbf{x}) = Sp\sigma_0\left(\zeta^0(\mathbf{x}', \tau; \rho)\right)\hat{\zeta}_A(\mathbf{x}). \tag{39}$$

It is worth noting that any statistical operator ρ which satisfies the principle of a weakening of spatial correlations (23) satisfies (38). Consequently, to define ρ in (38), we introduce an operator $w(Y(\mathbf{x}'))$,

$$w(Y(\mathbf{x}')) = \exp\left\{\Omega(Y(\mathbf{x}')) - \int d^3x' Y_A(\mathbf{x}')\hat{\zeta}_A(\mathbf{x}')\right\}, \tag{40}$$

where $Y_A(\mathbf{x}')$ are arbitrary numeric functions, and $\Omega(Y(\mathbf{x}'))$ is determined by the normalization relation $\mathrm{Sp}w(Y(\mathbf{x}')) = 1$. The choice of the initial statistical operator in the form (40) is explained by the following reasons [2]. First, such an operator satisfies the principle of a weakening of spatial correlations. Secondly, it contains the arbitrary functions $Y_A(\mathbf{x}')$, which allows us to define the operator $\sigma_0\left(\zeta^0(\mathbf{x}', \tau; \rho)\right)$. Moreover, if $Y_A(\mathbf{x}) = Y_A = \mathrm{const}$, then $w(Y(\mathbf{x}')) = \exp\{\Omega - Y_A\hat{\gamma}_A\}$ (see (25)), and, consequently, $[\mathcal{H}_0, w(Y_A)] = 0$. The last fact allows us to apply the perturbation theory method of small gradients of the reduced description parameters $\zeta_A(\mathbf{x})$.

According to (38), we obtain, by using (30):

$$e^{-i\mathcal{H}_0\tau}\sigma(\zeta(\mathbf{x}'))e^{i\mathcal{H}_0\tau} \underset{\tau\to\infty}{\rightarrow} \sigma_0\left(\zeta^0(\mathbf{x}', \tau; \sigma)\right),$$

$$e^{-i\mathcal{H}_0\tau}w(Y(\mathbf{x}'))e^{i\mathcal{H}_0\tau} \underset{\tau\to\infty}{\rightarrow} \sigma_0\left(\zeta^0(\mathbf{x}', \tau; w)\right). \tag{41}$$

The choice of functions $Y_A(\mathbf{x}')$ must satisfy the condition

$$\zeta_A(\mathbf{x}', 0; \sigma) = \zeta_A(\mathbf{x}', 0; w). \tag{42}$$

In this case, the following relation takes place:

$$e^{-i\mathcal{H}_0\tau}(\sigma(\zeta(\mathbf{x}')) - w(Y(\mathbf{x}')))e^{i\mathcal{H}_0\tau} \underset{\tau\to\infty}{\rightarrow} 0. \tag{43}$$

By having in mind that

$$\frac{d}{d\tau}\left\{e^{-i\mathcal{H}_0\tau}(\sigma(\zeta(\mathbf{x}')) - w(Y(\mathbf{x}')))e^{i\mathcal{H}_0\tau}\right\} =$$

$$= -ie^{-i\mathcal{H}_0\tau}[\mathcal{H}_0, \sigma(\zeta(\mathbf{x}')) - w(Y(\mathbf{x}'))]e^{i\mathcal{H}_0\tau}$$

and integrating the formula with respect to τ, we obtain

$$e^{-i\mathcal{H}_0\tau}(\sigma(\zeta(\mathbf{x}')) - w(Y(\mathbf{x}'))e^{i\mathcal{H}_0\tau} = \sigma(\zeta(\mathbf{x}')) - \rho -$$

$$-i\int\limits_0^{\tau_0} d\tau e^{-i\mathcal{H}_0\tau}\left[\mathcal{H}_0,\sigma\left(\zeta\left(\mathbf{x}'\right)\right)-w(Y(\mathbf{x}')\right]e^{i\mathcal{H}_0\tau}. \qquad (44)$$

By turning $\tau\to\infty$ and replacing the integration variable τ by $-\tau$, we come up to the following relation, by taking (43) into account:

$$\sigma\left(\zeta\left(\mathbf{x}'\right)\right)=w(Y(\mathbf{x}')+i\int\limits_{-\infty}^{0}d\tau\times$$

$$\times e^{i\mathcal{H}_0\tau}\left\{\left[\mathcal{H}_0,\sigma\left(\zeta\left(\mathbf{x}'\right)\right)\right]-\left[\mathcal{H}_0,w(Y(\mathbf{x}')\right]\right\}e^{-i\mathcal{H}_0\tau}. \qquad (45)$$

We have obtained the integral equation for $\sigma\left(\zeta\left(\mathbf{x}'\right)\right)$ in the case where the subsystems do not interact, i.e. $\hat{V}=0$. However, according to (41), we can obtain a similar equation for the interacting subsystems. By substituting (34), (37), and (18) into (44), we obtain an integral equation for the coarse-grained statistical operator:

$$\sigma\left(\zeta\left(\mathbf{x}'\right)\right)=w(Y(\mathbf{x}')-i\int\limits_{-\infty}^{0}d\tau e^{i\mathcal{H}_0\tau}\left\{\left[\mathcal{H}_0,w(Y(\mathbf{x}')\right]-\right.$$

$$-\int d^3x\frac{\delta\sigma\left(\zeta\left(\mathbf{x}'\right)\right)}{\delta\zeta_A\left(\mathbf{x}\right)}\frac{\partial}{\partial x_k}\mathrm{Sp}\sigma\hat{\zeta}_{Ak}\left(\mathbf{x}\right)+\left[\hat{V},\sigma\left(\zeta\left(\mathbf{x}'\right)\right)\right]+$$

$$\left.+\,i\int d^3x\frac{\delta\sigma\left(\zeta\left(\mathbf{x}'\right)\right)}{\delta\zeta_A\left(\mathbf{x}\right)}\mathrm{Sp}\sigma\left[\hat{V},\hat{\zeta}_A\left(\mathbf{x}\right)\right]\right\}e^{-i\mathcal{H}_0\tau}. \qquad (46)$$

4. Perturbation Theory for a Coarse-Grained Statistical Operator

The integral equation (46) and Eq. (37) formally define the evolution of the system when $t\gg\tau_0$. We use the perturbation theory method to solve this equation and to obtain the equation of motion for the reduced description parameters $\zeta_A\left(\mathbf{x}\right)$. We suppose that the free paths of particles of both subsystems are much less than the specific inhomogeneity length [2]. Thats why the perturbation theory method can be applied owing to small gradients of the reduced description parameters and the low inter-subsystem interaction $\hat{V}$.

We note that the following translation relations are valid for the creation and the annihilation operators of the medium $\varphi^+\left(\mathbf{x},\right)$ $\varphi\left(\mathbf{x}\right)$ and those for particles $\psi^+\left(\mathbf{x},\right)$ $\psi\left(\mathbf{x}\right)$:

$$e^{-i\mathbf{Px}}\varphi\left(\mathbf{x}'\right)e^{i\mathbf{Px}}=\varphi\left(\mathbf{x}'+\mathbf{x}\right),$$

$$e^{-i\mathbf{Px}}\varphi^+\left(\mathbf{x}'\right)e^{i\mathbf{Px}}=\varphi^+\left(\mathbf{x}'+\mathbf{x}\right),$$

$$e^{-i\mathbf{Px}}\psi\left(\mathbf{x}'\right)e^{i\mathbf{Px}}=\psi\left(\mathbf{x}'+\mathbf{x}\right), \qquad (47)$$

$$e^{-i\mathbf{Px}}\psi^+\left(\mathbf{x}'\right)e^{i\mathbf{Px}}=\psi^+\left(\mathbf{x}'+\mathbf{x}\right).$$

In relations (47), the total momentum operator $\mathbf{P}$

$$\mathbf{P}_i=\int d^3x\pi_i(\mathbf{x})+\sum_{\mathbf{p}}p_ia^+_{\mathbf{p}}a_{\mathbf{p}} \qquad (48)$$

acts as a translation operator. The first term in (48) represents the total momentum of medium (see (1), (2)), and the second addend represents the total momentum of particles. It is obvious that, according to definitions (13) and (14), the relations similar to (47) take place for the additive motion integrals $\hat{\zeta}_A\left(\mathbf{x}\right)$ and their flows $\hat{\zeta}_{Ak}\left(\mathbf{x}\right)$:

$$e^{-i\mathbf{Px}}\hat{\zeta}_A\left(\mathbf{x}\right)e^{i\mathbf{Px}}=\hat{\zeta}_A\left(\mathbf{x}'+\mathbf{x}\right),$$

$$e^{-i\mathbf{Px}}\hat{\zeta}_{Ak}\left(\mathbf{x}\right)e^{i\mathbf{Px}}=\hat{\zeta}_{Ak}\left(\mathbf{x}'+\mathbf{x}\right). \qquad (49)$$

We find consequently that the following equality is valid:

$$e^{i\mathbf{Px}}w\left(Y\left(\mathbf{x}'\right)\right)e^{-i\mathbf{Px}}=w\left(Y\left(\mathbf{x}+\mathbf{x}'\right)\right). \qquad (50)$$

By using equalities (47) and (50), we find that

$$e^{i\mathbf{Px}}\sigma\left(\zeta\left(\mathbf{x}'\right)\right)e^{-i\mathbf{Px}}=\sigma\left(\zeta\left(\mathbf{x}+\mathbf{x}'\right)\right). \qquad (51)$$

Hence, the equality

$$\mathrm{Sp}\sigma\left(\zeta\left(\mathbf{x}'\right)\right)a\left(\mathbf{x}\right)=\mathrm{Sp}\sigma\left(\zeta\left(\mathbf{x}+\mathbf{x}'\right)\right)a\left(0\right) \qquad (52)$$

takes place for any translation invariant operator $a\left(\mathbf{x}\right)$ (i.e. the operator satisfying the relation $i\dfrac{\partial a\left(\mathbf{x}\right)}{\partial x_k}=\left[P_k,a\left(\mathbf{x}\right)\right]$).

The right-hand side of (52) contains the operator $a\left(\mathbf{x}\right)$ with zero argument $\mathbf{x}=0$. Owing to low gradients, only $\zeta\left(\mathbf{x}+\mathbf{x}'\right)$ with the argument $\mathbf{x}'\approx0$ makes a significant contribution to the mean value of $a\left(\mathbf{x}\right)$ in (52). The last fact allows us to use the series expansion

$$\zeta\left(\mathbf{x}+\mathbf{x}'\right)=\zeta\left(\mathbf{x}\right)+x'_{\mathbf{k}}\frac{\partial\zeta\left(\mathbf{x}\right)}{\partial x_k}+... \qquad (53)$$

to build a theory of corrections to the coarse-grained statistical operator σ under small gradients of the reduced description parameters $\zeta_A\left(\mathbf{x}\right)$:

$$\sigma\left(\zeta\left(\mathbf{x}\right)\right)=\sigma^{(0)}\left(\mathbf{x}\right)+\sigma^{(1)}\left(\mathbf{x}\right)...,$$

where

$$\sigma^{(0)}\left(\mathbf{x}\right)=\left.\sigma\left(\zeta\left(\mathbf{x}'\right)\right)\right|_{\zeta(\mathbf{x}')=\zeta(\mathbf{x})}, \qquad (54)$$

$$\sigma^{(1)}(\mathbf{x}) = \frac{\partial \zeta_A(\mathbf{x})}{\partial x_k} \int d^3x' x'_k \frac{\delta\sigma(\zeta(\mathbf{x}'))}{\delta\zeta_A(\mathbf{x}')}\Big|_{\zeta(\mathbf{x}')\to\zeta(\mathbf{x})}.$$

According to (31), the condition $\mathrm{Sp}\sigma(\zeta(\mathbf{x}+\mathbf{x}'))\hat{\zeta}_A(0) = \zeta_A(\mathbf{x})$ is held, so we find that

$$\mathrm{Sp}\sigma^{(k)}(\zeta(\mathbf{x}+\mathbf{x}'))\hat{\zeta}_A(0) = \zeta_A(\mathbf{x})\delta_{k0}, \quad k=0,1. \tag{55}$$

In order to find the operators $\sigma^{(0)}(\mathbf{x})$, $\sigma^{(1)}(\mathbf{x})$, we expand the operator $w(Y(\mathbf{x}+\mathbf{x}'))$ into series in the gradients of the functions $Y_A(\mathbf{x})$ (see (2), (51)):

$$w(Y(\mathbf{x}+\mathbf{x}')) = w^{(0)}(\mathbf{x}) + w^{(1)}(\mathbf{x}) + ... \tag{56}$$

We use the following expansion of the operator $\exp\left(\hat{A}+\hat{B}\right)$ in a polynomial series in a small operator $\hat{B}$ (see [2]) in order to obtain the operators $w^{(0)}(\mathbf{x})$ and $w^{(1)}(\mathbf{x})$ from expression (40):

$$\exp\left(\hat{A}+\hat{B}\right) = e^{\hat{A}}\left\{1 + \int_0^1 d\lambda e^{-\lambda\hat{A}}\hat{B}e^{\lambda\hat{A}} + ...\right\}. \tag{57}$$

Finally, we obtain

$$w^{(0)}(\mathbf{x}) = \exp\{\Omega(\mathbf{x}) - Y_A(\mathbf{x})\hat{\gamma}_A\}, \tag{58}$$

$$w^{(1)}(\mathbf{x}) = -\frac{\partial Y_A(\mathbf{x})}{\partial x_k} w^{(0)}(\mathbf{x}) \times$$

$$\times \int_0^1 d\lambda \int d^3x' x'_k \left(\hat{\zeta}_A(\mathbf{x}',\lambda) - \left\langle\hat{\zeta}_A\right\rangle\right). \tag{59}$$

Here, some new symbols have been introduced:

$$a(\mathbf{x}',\lambda) = w^{(0)-\lambda} a(\mathbf{x}') w^{(0)\lambda}, \quad \langle a\rangle \equiv \mathrm{Sp}w^{(0)}a, \tag{60}$$

which simplifies the expressions.

The relations $\left[\mathcal{H}_0, \hat{\zeta}_A(\mathbf{x}',\lambda)\right] = i\frac{\partial\hat{\zeta}_{Ak}(\mathbf{x}',\lambda)}{\partial x_k}$ and $\left[\mathcal{H}_0, w^{(0)}(\mathbf{x})\right] = 0$ follow from Eqs. (25), (26), (3), and (60). Hence, the first-order approximation for$[\mathcal{H}_0, w(Y(\mathbf{x}')]$ in Eq. (46) is equal to

$$\left[\mathcal{H}_0, w^{(1)}(\mathbf{x})\right] = -i\frac{\partial Y_A(\mathbf{x})}{\partial x_j} w^{(0)}(\mathbf{x}) \times$$

$$\times \int_0^1 d\lambda \int d^3x' x'_j \frac{\partial}{\partial x_k}\hat{\zeta}_{Ak}(\mathbf{x}',\lambda).$$

By integrating the right-hand side of this equality by parts and taking the principle of a weakening of spatial correlations into account, we get

$$\left[\mathcal{H}_0, w^{(1)}(\mathbf{x})\right] = i\frac{\partial Y_A(\mathbf{x})}{\partial x_k} w^{(0)}(\mathbf{x}) \times$$

$$\times \int_0^1 d\lambda \int d^3x' \left(\hat{\zeta}_{Ak}(\mathbf{x}',\lambda) - \left\langle\hat{\zeta}_{Ak}\right\rangle\right). \tag{61}$$

Equations (54)—(61) allow us to construct the series expansion for the coarse-grained statistical operator σ in small gradients of the reduced description parameters $\zeta_A(\mathbf{x})$ and a small inter-subsystem interaction $\hat{V}$:

$$\sigma(\mathbf{x}) = \sigma^{(0,0)}(\mathbf{x}) + \sigma^{(0,1)}(\mathbf{x}) + \sigma^{(1,0)}(\mathbf{x}) + ... \tag{62}$$

In what follows, we use the symbol $D^{(n,m)}$ to label the terms of the perturbation series in the variable D. The term $D^{(n,m)}$ is derived in the n-th order in the gradients of $\zeta_A(\mathbf{x})$ and in the m-th order in the interaction $\hat{V}$. By taking Eqs. (54)—(61) and (46) into account, we get

$$\sigma^{(0,0)}(\mathbf{x}) = w^{(0)}(\mathbf{x}). \tag{63}$$

In accordance with (55), the thermodynamic potential $\Omega(\mathbf{x}) \equiv \Omega(Y(\mathbf{x}))$ and the thermodynamic forces $Y_A(\mathbf{x})$ follow from the relations

$$\mathrm{Sp}w^{(0)}(\mathbf{x}) = 1, \quad \mathrm{Sp}w^{(0)}(\mathbf{x})\hat{\zeta}_A(0) = \zeta_A(\mathbf{x}). \tag{64}$$

We emphasize that, according to (58) and (63), the expression for the statistical operator $\sigma^{(0,0)}(\mathbf{x})$ coincides with the Gibbs equilibrium operator (25) where the parameter Y_A is replaced by the function $Y_A(\mathbf{x})$. Therefore, the statistical operator defined by (58) is called the locally equilibrium Gibbs distribution $w^{(0)}(\mathbf{x})$.

For $\sigma^{(0,1)}(\mathbf{x})$, we easily obtain

$$\sigma^{(0,1)}(\mathbf{x}) = -\int_{-\infty}^0 d\tau e^{i\mathcal{H}_0\tau}\left\{i\left[\hat{V}, w^{(0)}(\mathbf{x})\right] + \right.$$

$$\left. +i\int d^3x' \frac{\delta w^{(0)}(\mathbf{x})}{\delta\zeta_A(\mathbf{x}')}\mathrm{Sp}w^{(0)}(\mathbf{x}')\left[\hat{V}, \hat{\zeta}_A(0)\right]\right\} e^{-i\mathcal{H}_0\tau}. \tag{65}$$

By using (46) and (58) and making some transformations, we derive (see [5] for details) the expression for $\sigma^{(1,0)}(\mathbf{x})$

$$\sigma^{(1,0)}(\mathbf{x}) = w^{(1)}(\mathbf{x}) + \int_{-\infty}^0 d\tau \int_0^1 d\lambda \int d^3x' \left\{\frac{\partial Y_A(\mathbf{x})}{\partial x_k} \times\right.$$

$$\times e^{i\mathcal{H}_0\tau} w^{(0)}(\mathbf{x})\left(\hat{\zeta}_{Ak}(\mathbf{x}',\lambda) - \left\langle\hat{\zeta}_{Ak}\right\rangle\right) e^{-i\mathcal{H}_0\tau} +$$

$$+\frac{\delta w^{(0)}(\mathbf{x})}{\delta\zeta_A(\mathbf{x}')}\frac{\partial}{\partial x_j}\mathrm{Sp}w^{(0)}(\mathbf{x}')\hat{\zeta}_{Aj}(0)\Big\}-$$

$$-\frac{\partial w^{(0)}(\mathbf{x})}{\partial\zeta_A(\mathbf{x})}\mathrm{Sp}w^{(1)}(\mathbf{x})\hat{\zeta}_A(0), \tag{66}$$

where

$$\langle...\rangle\equiv\mathrm{Sp}w^{(0)}(\mathbf{x})... \quad . \tag{67}$$

The above-obtained expressions can be simplified if we use the fact that the locally equilibrium statistical operator $w^{(0)}(\mathbf{x})$ commutes with the additive motion integrals $\hat{\gamma}_A=\int d^3x\hat{\zeta}_A(\mathbf{x})$ (see (26), (58)),

$$\left[w^{(0)}(\mathbf{x}),\hat{\gamma}_A\right]=0. \tag{68}$$

So, for example, the quantity $\mathrm{Sp}w^{(0)}(\mathbf{x}')\left[\hat{V},\hat{\zeta}_A(0)\right]$ appearing on the right-hand side of equality (65) can be reduced to the expression

$$\mathrm{Sp}w^{(0)}(\mathbf{x}')\left[\hat{V},\hat{\zeta}_A(0)\right]=$$

$$=\frac{1}{\mathcal{V}}\int d^3\mathbf{x}\mathrm{Sp}w^{(0)}(\mathbf{x}')\left[\hat{V},\hat{\zeta}_A(\mathbf{x})\right]=$$

$$=\frac{-1}{\mathcal{V}}\mathrm{Sp}\hat{V}\left[w^{(0)}(\mathbf{x}'),\hat{\gamma}_A\right]=0, \tag{69}$$

and the further simplification of (65) becomes obvious:

$$\sigma^{(0,1)}(\mathbf{x})=-i\int\limits_{-\infty}^{0}d\tau e^{i\mathcal{H}_0\tau}\left[\hat{V},w^{(0)}(\mathbf{x})\right]e^{-i\mathcal{H}_0\tau}. \tag{70}$$

Operations similar to (69) allow us to replace the mean values of $\hat{\zeta}_A(0)$ with more simple mean values of $\hat{\gamma}_A$. Such a method is widely used below.

A further simplification of expression (66) is also possible. Some terms in (66) are equal to zero owing to their symmetry. According to the definitions of the space-time reverse transformation operators (15)-(17), it is obvious that $\mathcal{TP}\hat{\gamma}_A(\mathcal{TP})^{-1}=\hat{\gamma}_A^*$. So, by using (58), we obtain

$$\mathcal{TP}w^{(0)}(\mathbf{x})(\mathcal{TP})^{-1}=w^{(0)}(\mathbf{x})^*. \tag{71}$$

In view of the equality $\left\langle\hat{\zeta}_A\right\rangle=\left\langle\hat{\zeta}_A\right\rangle^*$, we obtain

$$\mathcal{TP}w^{(1)}(\mathbf{x})(\mathcal{TP})^{-1}=-\frac{\partial Y_A(\mathbf{x})}{\partial x_k}\int\limits_0^1 d\lambda\int d^3x'x'_k\times$$

$$\times w^{(0)}(\mathbf{x})^{(1-\lambda)*}\left(\hat{\zeta}_A(-\mathbf{x}')-\left\langle\hat{\zeta}_A\right\rangle\right)^*w^{(0)}(\mathbf{x})^{\lambda*},$$

which leads finally to

$$\mathcal{TP}w^{(1)}(\mathbf{x})(\mathcal{TP})^{-1}=-w^{(1)}(\mathbf{x})^*. \tag{72}$$

It is obvious from expression (17) which defines the space-time reverse transformations of additive integrals densities and their flows that

$$\mathrm{Sp}w^{(1)}(\mathbf{x})\hat{\zeta}_A(0)=0,\quad \mathrm{Sp}w^{(1)}(\mathbf{x})\hat{\zeta}_{Ak}(0)=0. \tag{73}$$

By using the last two equations and the equality $\dfrac{\delta w^{(0)}(\mathbf{x})}{\delta\zeta_A(\mathbf{x}')}=\delta(\mathbf{x}'-\mathbf{x})\dfrac{\partial w^{(0)}(\mathbf{x})}{\partial\zeta_A(\mathbf{x}')}$, we simplify expression (66):

$$\sigma^{(1,0)}(\mathbf{x})=w^{(1)}(\mathbf{x})+\int\limits_{-\infty}^{0}d\tau\int\limits_0^1 d\lambda\int d^3x'\left\{\frac{\partial Y_A(\mathbf{x})}{\partial x_k}\times\right.$$

$$\left.\times e^{i\mathcal{H}_0\tau}w^{(0)}(\mathbf{x})\left(\hat{\zeta}_{Ak}(\mathbf{x}',\lambda)-\left\langle\hat{\zeta}_{Ak}\right\rangle\right)e^{-i\mathcal{H}_0\tau}\right\}+$$

$$+\int\limits_{-\infty}^{0}d\tau\int\limits_0^1 d\lambda\frac{\partial w^{(0)}}{\partial\zeta_A}\frac{\partial\left\langle\hat{\zeta}_{Aj}\right\rangle}{\partial x_j}. \tag{74}$$

As the next step, we turn to formulas obtained in [2]:

$$\frac{\partial\left\langle\hat{\zeta}_A\right\rangle}{\partial Y_B}=\frac{\partial\left\langle\hat{\zeta}_B\right\rangle}{\partial Y_A},\quad \frac{\partial\left\langle\hat{\zeta}_{Ak}\right\rangle}{\partial Y_B}=\frac{\partial\left\langle\hat{\zeta}_{Bk}\right\rangle}{\partial Y_A}, \tag{75}$$

$$\frac{\partial w^{(0)}}{\partial Y_B}=-w^{(0)}\int\limits_0^1 d\lambda\int d^3x'\left(\hat{\zeta}_B(\mathbf{x}',\lambda)-\left\langle\hat{\zeta}_B\right\rangle\right). \tag{76}$$

By using these two formulas, the following equality can be easily proved:

$$\frac{\partial w^{(0)}}{\partial\zeta_A}\frac{\partial\langle\zeta_{Aj}\rangle}{\partial x_j}=-\frac{\partial Y_A}{\partial x_j}\frac{\partial\langle\zeta_{Aj}\rangle}{\partial\zeta_B}w^{(0)}\int\limits_0^1 d\lambda\int d^3x'\times$$

$$\times\left(w^{(0)-\lambda}\frac{\partial\langle\zeta_{Aj}\rangle}{\partial\zeta_B}\hat{\zeta}_A(\mathbf{x}')w^{(0)\lambda}-\left\langle\hat{\zeta}_B\frac{\partial\langle\zeta_{Aj}\rangle}{\partial\zeta_B}\right\rangle\right). \tag{77}$$

In accordance with (77) after introducing a new variable

$$\hat{\zeta}'_{Aj}(\mathbf{x}')=\hat{\zeta}_{Aj}(\mathbf{x}')-\frac{\partial\left\langle\hat{\zeta}_{Aj}\right\rangle}{\partial\zeta_B}\hat{\zeta}_B, \tag{78}$$

expression (74) is transformed into

$$\sigma^{(1,0)}(\mathbf{x})=w^{(1)}(\mathbf{x})+\frac{\partial Y_A(\mathbf{x})}{\partial x_k}w^{(0)}(\mathbf{x})\int\limits_{-\infty}^{0}d\tau\int\limits_0^1 d\lambda\times$$

$$\times \int d^3x' \left\{ e^{i\mathcal{H}_0\tau} \left(\hat{\zeta}'_{Ak} (\mathbf{x}', \lambda) - \left\langle \hat{\zeta}'_{Ak} \right\rangle \right) e^{-i\mathcal{H}_0\tau} \right\}. \quad (79)$$

In what follows, we use two new operators

$$w_p^{(0)} (\mathbf{x}) = \exp \left\{ \Omega_p (\mathbf{x}) - Y_{\mathbf{p}} (\mathbf{x}) \hat{\gamma}_{\mathbf{p}} \right\},$$

$$w_m^{(0)} (\mathbf{x}) = \exp \left\{ \Omega_m (\mathbf{x}) - Y_\alpha (\mathbf{x}) \hat{\gamma}_\alpha \right\}. \quad (80)$$

We note that definition (58) and the expressions

$$\left[\hat{\zeta}_\alpha (\mathbf{x}), \hat{f}_{\mathbf{p}} (\mathbf{x}) \right] = 0, \quad [\hat{\gamma}_\alpha, \hat{\gamma}_{\mathbf{p}}] = 0$$

yield (see also (26))

$$\Omega (\mathbf{x}) = \Omega_p (\mathbf{x}) + \Omega_m (\mathbf{x}),$$

$$w^{(0)} (\mathbf{x}) = w_p^{(0)} (\mathbf{x}) \, w_m^{(0)} (\mathbf{x}). \quad (81)$$

The dependence of the thermodynamic potentials $\Omega_m (\mathbf{x})$ and $\Omega_p (\mathbf{x})$ on $Y_\alpha (\mathbf{x})$ and $Y_{\mathbf{p}} (\mathbf{x})$ can be clarified by using the normalization requirements

$$\mathrm{Sp} w_m^{(0)} (\mathbf{x}) = 1, \quad \mathrm{Sp} w_p^{(0)} (\mathbf{x}) = 1. \quad (82)$$

The thermodynamic forces $Y_\alpha (\mathbf{x})$ and $Y_{\mathbf{p}} (\mathbf{x})$ are functions of the reduced description parameters $\zeta_\alpha (\mathbf{x})$, $f_{\mathbf{p}} (\mathbf{x})$. The relevant functional dependence is defined by the relations

$$\mathrm{Sp} w_m^{(0)} (\mathbf{x}) \, \hat{\zeta}_\alpha (0) = \zeta_\alpha (\mathbf{x}),$$

$$\mathrm{Sp} w_p^{(0)} (\mathbf{x}) \, \hat{f}_{\mathbf{p}} (0) = f_{\mathbf{p}} (\mathbf{x}). \quad (83)$$

Thus, we have separated the operator $w^{(0)} (\mathbf{x})$ into a product of two terms. One term contains only the operators related to the medium, and the another term accounts for the particles only. Consequently in accordance with (81), the operators $w^{(1)} (\mathbf{x})$ and $\sigma^{(1,0)} (\mathbf{x})$ can be transformed into

$$w^{(1)} (\mathbf{x}) = w_m^{(0)} (\mathbf{x}) \, w_p^{(1)} (\mathbf{x}) + w_p^{(0)} (\mathbf{x}) \, w_m^{(1)} (\mathbf{x}),$$

$$w_m^{(1)} (\mathbf{x}) = - \frac{\partial Y_\alpha (\mathbf{x})}{\partial x_k} w_m^{(0)} (\mathbf{x}) \times$$

$$\times \int_0^1 d\lambda \int d^3x' x'_k \left(w_m^{(0)-\lambda} \hat{\zeta}_\alpha (\mathbf{x}') \, w_m^{(0)\lambda} - \left\langle \hat{\zeta}_\alpha \right\rangle_m \right),$$

$$w_p^{(1)} (\mathbf{x}) = - \frac{\partial Y_{\mathbf{p}} (\mathbf{x})}{\partial x_k} w_p^{(0)} (\mathbf{x}) \times$$

$$\times \int_0^1 d\lambda \int d^3x' x'_k \left(w_p^{(0)-\lambda} \hat{\zeta}_{\mathbf{p}} (\mathbf{x}') \, w_p^{(0)\lambda} - \left\langle \hat{\zeta}_{\mathbf{p}} \right\rangle_p \right), \quad (84)$$

$$\sigma^{(1,0)} (\mathbf{x}) = w_m^{(0)} (\mathbf{x}) \, \sigma_p^{(1,0)} (\mathbf{x}) + w_p^{(0)} (\mathbf{x}) \, \sigma_m^{(1,0)} (\mathbf{x}),$$

$$\sigma_m^{(1,0)} (\mathbf{x}) = w_m^{(1)} (\mathbf{x}) + \frac{\partial Y_\alpha (\mathbf{x})}{\partial x_k} w_m^{(0)} (\mathbf{x}) \int_{-\infty}^0 d\tau \int_0^1 d\lambda \times$$

$$\times \int d^3x' \left\{ e^{i\mathcal{H}_0\tau} \left(\hat{\zeta}'_{\alpha k} (\mathbf{x}', \lambda) - \left\langle \hat{\zeta}'_{\alpha k} \right\rangle_m \right) e^{-i\mathcal{H}_0\tau} \right\},$$

$$\sigma_p^{(1,0)} (\mathbf{x}) = w_p^{(1)} (\mathbf{x}) + \frac{\partial Y_{\mathbf{p}} (\mathbf{x})}{\partial x_k} w_p^{(0)} (\mathbf{x}) \int_{-\infty}^0 d\tau \int_0^1 d\lambda \times$$

$$\times \int d^3x' \left\{ e^{i\mathcal{H}_0\tau} \left(\hat{\zeta}'_{\mathbf{p}k} (\mathbf{x}', \lambda) - \left\langle \hat{\zeta}'_{\mathbf{p}k} \right\rangle_p \right) e^{-i\mathcal{H}_0\tau} \right\}. \quad (85)$$

We have introduced the following new symbols for traces:

$$\langle a \rangle_m = \mathrm{Sp} w_m^{(0)} a, \quad \langle a \rangle_p = \mathrm{Sp} w_p^{(0)} a. \quad (86)$$

It is important that the operator $\sigma_p^{(1,0)} (\mathbf{x})$ does not contain operators relevant to the medium, and the operator $\sigma_m^{(1,0)} (\mathbf{x})$ does not contain the operators related to particles. Moreover, $\mathrm{Sp}\sigma_m^{(1,0)} (\mathbf{x}) = 0$ and $\mathrm{Sp}\sigma_p^{(1,0)} (\mathbf{x}) = 0$.

It is obvious from (9) that $\frac{\partial \langle \hat{\zeta}_{\mathbf{p}j} \rangle}{\partial \zeta_{\mathbf{p}'}} = \delta_{\mathbf{p}\mathbf{p}'} \frac{p_j}{m}$, and, hence, we get

$$\hat{\zeta}'_{\mathbf{p}j} (\mathbf{x}') = \frac{p_j}{m} \hat{f}_{\mathbf{p}} (\mathbf{x}') - \delta_{\mathbf{p}\mathbf{p}'} \frac{p_j}{m} \hat{f}_{\mathbf{p}'} (\mathbf{x}') = 0. \quad (87)$$

As a consequence, the integrand in expression (85) for $\sigma_p^{(1,0)} (\mathbf{x})$ is equal to zero, so that

$$\sigma_p^{(1,0)} (\mathbf{x}) = w_p^{(1)} (\mathbf{x}). \quad (88)$$

Expressions (70), (84), (85), and (88) determine the coarse-grained statistical operator in the first order of perturbation theory in gradients and interactions. Hence, we can consider the equations for the reduced description parameters in the second-order of the approximation:

$$\frac{\partial \zeta_A (\mathbf{x})}{\partial t} = L_A^{(1,0)} (\mathbf{x}) + L_A^{(0,1)} (\mathbf{x}) +$$

$$+ L_A^{(1,1)} (\mathbf{x}) + L_A^{(0,2)} (\mathbf{x}) + L_A^{(2,0)} (\mathbf{x}),$$

$$L_A^{(1,0)} (\mathbf{x}) = - \frac{\partial}{\partial x_k} \mathrm{Sp} w^{(0)} (\mathbf{x}) \, \hat{\zeta}_{Ak} (0),$$

$$L_A^{(2,0)} (\mathbf{x}) = - \frac{\partial}{\partial x_k} \mathrm{Sp} \sigma^{(1,0)} (\mathbf{x}) \, \hat{\zeta}_{Ak} (0),$$

$$L_A^{(0,1)}(\mathbf{x}) = i\mathrm{Sp}w^{(0)}(\mathbf{x})\left[\hat{V},\hat{\zeta}_A(0)\right],$$

$$L_A^{(0,2)}(\mathbf{x}) = i\mathrm{Sp}\sigma^{(0,1)}(\mathbf{x})\left[\hat{V},\hat{\zeta}_A(0)\right],$$

$$L_A^{(1,1)}(\mathbf{x}) = -\frac{\partial}{\partial x_k}\mathrm{Sp}\sigma^{(0,1)}(\mathbf{x})\hat{\zeta}_{Ak}(0) +$$

$$+i\mathrm{Sp}\sigma^{(1,0)}(\mathbf{x})\left[\hat{V},\hat{\zeta}_A(0)\right]. \qquad (89)$$

The derivation of $L_A^{(i,k)}(\mathbf{x})$ is given in the next section.

5. Equations of Motion

In this section, expressions (89) are analyzed, and some rearrangements are made, which result in obtaining the coupled motion equations. The equations have structure similar to that of the kinetic equation for particles and the hydrodynamic equations for a medium.

It is important that the explicit dependence of the Wigner's distribution function $f(\mathbf{p},\mathbf{x})$ on the thermodynamic forces $Y_{\mathbf{p}}(\mathbf{x})$ can be expressed as

$$f(\mathbf{p},\mathbf{x}) = \mathrm{Sp}w_p^{(0)}(\mathbf{x})\,a_{\mathbf{p}}^{+}a_{\mathbf{p}} = \frac{1}{e^{Y_{\mathbf{p}}(\mathbf{x})}+1}. \qquad (90)$$

We concentrate on calculating the functions $L_A^{(i,k)}(\mathbf{x})$ defined in (89). The transformations similar to (69) lead to the expressions

$$L_A^{(0,1)}(\mathbf{x}) = i\mathrm{Sp}w^{(0)}(\mathbf{x})\left[\hat{V},\hat{\zeta}_A(0)\right] = 0, \qquad (91)$$

$$L_A^{(0,2)}(\mathbf{x}) = \frac{i}{\mathcal{V}}\mathrm{Sp}\sigma^{(0,1)}(\mathbf{x})\left[\hat{V},\hat{\gamma}_A\right]. \qquad (92)$$

According to (70), we obtain

$$L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) =$$

$$= \int_{-\infty}^{0} d\tau \mathrm{Sp}e^{i\mathcal{H}_0\tau}\left[\hat{V},w^{(0)}(\mathbf{x})\right]e^{-i\mathcal{H}_0\tau}\left[\hat{V},a_{\mathbf{p}}^{+}a_{\mathbf{p}}\right]. \qquad (93)$$

We introduce a new operator

$$\hat{V}(\tau) \equiv e^{i\mathcal{H}_0\tau}\hat{V}e^{-i\mathcal{H}_0\tau}. \qquad (94)$$

After a cyclic rearrangement of the operators in (93) in accordance to the relations $[\mathcal{H}_0,\hat{\gamma}_A] = 0$ and $[\mathcal{H}_0, w_0(\mathbf{x})] = 0$, we obtain

$$L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) = \int_{-\infty}^{0} d\tau \mathrm{Sp}w^{(0)}(\mathbf{x})\left[\left[\hat{V}(-\tau),a_{\mathbf{p}}^{+}a_{\mathbf{p}}\right],\hat{V}\right]. \qquad (95)$$

By using the Jacobi identity and the relation

$$\mathrm{Sp}w^{(0)}(\mathbf{x})\left[\left[\hat{V}(-\tau),a_{\mathbf{p}}^{+}a_{\mathbf{p}}^{+}\right],\hat{V}\right] =$$

$$= \mathrm{Sp}w^{(0)}(\mathbf{x})\left[\left[\hat{V}(\tau),a_{\mathbf{p}}^{+}a_{\mathbf{p}}^{+}\right],\hat{V}\right],$$

we expand the integration limits in (95) towards $-\infty$ and ∞:

$$L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) =$$

$$= -\frac{1}{2}\int_{-\infty}^{+\infty} d\tau \mathrm{Sp}w^{(0)}(\mathbf{x})\left[\hat{V},\left[\hat{V}(\tau),a_{\mathbf{p}}^{+}a_{\mathbf{p}}^{+}\right]\right]. \qquad (96)$$

It is more convenient to introduce a new operator $\hat{\mathcal{J}}(1,2;\tau)$ similarly to (94) as

$$\hat{V}(\tau) = \sum_{1,2}\hat{\mathcal{J}}(1,2;\tau)\,a_1^{+}a_2 e^{i(\varepsilon_1-\varepsilon_2)\tau},$$

$$\hat{\mathcal{J}}(1,2;\tau) = e^{i\mathcal{H}_m\tau}\hat{\mathcal{J}}(1,2)\,e^{-i\mathcal{H}_m\tau}, \qquad (97)$$

where the indices "1", "2" label the momentum vectors $\mathbf{p}_1$ and $\mathbf{p}_2$. Let us return to commutators in the collision integral (96). Owing to the anticommutation of the fermion operators a and a^{+}, we find that

$$\left[\hat{V},a_{\mathbf{p}}^{+}a_{\mathbf{p}}\right] = \sum_{1,2}\hat{\mathcal{J}}(1,2)\left[a_1^{+}a_2,a_{\mathbf{p}}^{+}a_{\mathbf{p}}\right] =$$

$$= \sum_{1,2}\hat{\mathcal{J}}(1,2)\,a_1^{+}a_2\,(\delta_{2\mathbf{p}}-\delta_{1\mathbf{p}}). \qquad (98)$$

The operator $\hat{\mathcal{J}}(1,2)$ does not contain operators relevant to particles, hence, we have

$$L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) = -\frac{1}{2}\int_{-\infty}^{+\infty} d\tau \sum_{1,2,1',2'}(\delta_{2\mathbf{p}}-\delta_{1\mathbf{p}})\times$$

$$\times\left\{\left\langle\hat{\mathcal{J}}(1',2';\tau)\,\hat{\mathcal{J}}(1,2)\right\rangle_m\left\langle a_{1'}^{+}a_{2'}a_1^{+}a_2\right\rangle_p -\right.$$

$$\left. - \left\langle\hat{\mathcal{J}}(1,2)\,\hat{\mathcal{J}}(1',2';\tau)\right\rangle_m\left\langle a_1^{+}a_2a_{1'}^{+}a_{2'}\right\rangle_p\right\}. \qquad (99)$$

After calculating the mean values in (99), we obtain

$$L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) = \sum_{1,2}(\delta_{1\mathbf{p}}-\delta_{2\mathbf{p}})\,f_2(\mathbf{x})\,(1-f_1(\mathbf{x}))\times$$

$$\times\int_{-\infty}^{+\infty} d\tau\left\langle\hat{\mathcal{J}}(2,1;\tau)\,\hat{\mathcal{J}}(1,2)\right\rangle_m e^{i(\varepsilon_1-\varepsilon_2)\tau}. \qquad (100)$$

Here, we use a shortened labeling for the distribution function $f_i(\mathbf{x}) \equiv f(\mathbf{p}_i, \mathbf{x})$.

Then we define the correlation function

$$I_{1,2}(\tau, \mathbf{x}) \equiv \left\langle \hat{\mathcal{J}}(2,1;\tau)\, \hat{\mathcal{J}}(1,2) \right\rangle_m, \tag{101}$$

and the spectral function

$$I_{1,2}(\omega, \mathbf{x}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau I_{1,2}(\tau, \mathbf{x})\, e^{i\omega\tau}, \tag{102}$$

which is the Fourier transform of the correlation function.

By substituting relation (102) into the collision integral (100), we obtain finally

$$L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) = 2\pi \sum_{1,2} \delta_{2\mathbf{p}} \{ f_1(\mathbf{x})(1 - f_2(\mathbf{x}))\, I_{2,1}(\varepsilon_1 - \varepsilon_2, \mathbf{x}) -$$

$$- f_2(\mathbf{x})(1 - f_1(\mathbf{x}))\, I_{1,2}(\varepsilon_2 - \varepsilon_1, \mathbf{x}) \}. \tag{103}$$

Let us return back to the commuting operators $\hat{\gamma}_\alpha$ (see (2)), where $\hat{\gamma}_0 = \mathcal{H}_m$ is the total energy operator of the medium, $\hat{\gamma}_i = \hat{P}_i^{(m)}$ is the momentum operator of the medium, $\hat{\gamma}_4 = \hat{M}^{(m)}$ is the mass operator of the medium. The eigenvalues of these operators can be labeled as $(\gamma_0)_n = \mathrm{E}_n^{(m)}$,$(\gamma_i)_n = \left(P_i^{(m)}\right)_n$, $(\gamma_4)_n = M_n^{(m)}$. According to expressions (101) and (102), we obtain

$$\left\langle \hat{\mathcal{J}}(2,1;\tau)\, \hat{\mathcal{J}}(1,2) \right\rangle_m =$$

$$= \sum_{n,m} \left| \hat{\mathcal{J}}_{m,n}(1,2) \right|^2 e^{i\tau(\mathrm{E}_n - \mathrm{E}_m)} \left(w_0^{(m)}(\mathbf{x}) \right)_n,$$

$$I_{1,2}(\omega, \mathbf{x}) =$$

$$= \sum_{n,m} \delta(\omega + \mathrm{E}_n - \mathrm{E}_m) \left| \hat{\mathcal{J}}_{m,n}(1,2) \right|^2 \left(w_0^{(m)}(\mathbf{x}) \right)_n. \tag{104}$$

It is seen that $I_{1,2}(\omega, \mathbf{x}) \geqslant 0$. It is important that the interaction operator $\hat{V}$ commutes with the total momentum operator

$$\left[V, \hat{\mathbf{P}}^{(m)} + \sum_{\mathbf{p}} \mathbf{p} a_{\mathbf{p}}^{+} a_{\mathbf{p}} \right] = 0. \tag{105}$$

Consequently, the coefficient functions $\hat{\mathcal{J}}_{m,n}(1,2)$ are not equal to zero only if the equality $\mathbf{P}_n + \mathbf{p}_1 - \mathbf{P}_m - \mathbf{p}_2 = 0$ takes place. Hence, according to (20) and (104), we obtain a symmetry relation for $I_{1,2}(\omega, \mathbf{x})$ in the form

$$I_{2,1}(-\omega, \mathbf{x}) =$$

$$= I_{1,2}(\omega, \mathbf{x}) \exp\left(-Y_0(\mathbf{x})\, \omega - Y_i(\mathbf{x})(\mathbf{p}_2 - \mathbf{p}_1)_i \right) \tag{106}$$

Substituting (106) in (103), we find that the collision integral looks as

$$L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) = 2\pi \sum_{1,2} \delta_{2\mathbf{p}} I_{1,2}(\varepsilon_2 - \varepsilon_1, \mathbf{x}) \{ (1 - f_2(\mathbf{x})) \times$$

$$\times f_1(\mathbf{x}) \exp\left(-Y_0(\mathbf{x})(\varepsilon_2 - \varepsilon_1) - Y_i(\mathbf{x})(\mathbf{p}_2 - \mathbf{p}_1)_i \right) -$$

$$- f_2(\mathbf{x})(1 - f_1(\mathbf{x})) \}. \tag{107}$$

The equilibrium condition for the medium and particles can be easily derived by using (107). The collision integral $L_{\mathbf{p}}^{(0,2)}(\mathbf{x})$ vanishes, $L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) = 0$, when the above-mentioned distribution function in the collision integral is determined by equality (90), where the function $Y_{\mathbf{p}}(\mathbf{x})$ is given by the expression

$$Y_{\mathbf{p}}(\mathbf{x}) = Y_0(\mathbf{x})\, \varepsilon_{\mathbf{p}} + Y_i(\mathbf{x})\, p_i + c(\mathbf{x}). \tag{108}$$

Here, $c(\mathbf{x})$ is an arbitrary numerical function independent of $\mathbf{p}$. Relation (108) claims that particles are in equilibrium with medium when their temperature and mean velocity are equal to those of the medium. By using (107), we can also find that the following equality takes place for an arbitrary distribution function $f_{\mathbf{p}}(\mathbf{x})$:

$$\sum_{\mathbf{p}} L_{\mathbf{p}}^{(0,2)} = 0. \tag{109}$$

The last fact represents the conservation law for the number of particles.

Further, we find the relation for the collision integral $\mathrm{L}_{\mathbf{p}}^{(0,2)}(\mathbf{x})$ and the functions $L_\alpha^{(0,2)}(\mathbf{x})$ defined by expression (92), where $A = \alpha$. As long as the interaction $\hat{V}$ does not affect the mass of our system, the equality $\left[\hat{V}, \hat{\gamma}_4\right] = 0$ takes place. Hence, by using (92) and assuming $\alpha = 4$, we obtain

$$L_4^{(0,2)}(\mathbf{x}) = 0. \tag{110}$$

According to (105), we have $[V, \hat{\gamma}_i] = -\sum_{\mathbf{p}} p_i a_{\mathbf{p}}^{+} a_{\mathbf{p}}$. Substituting this expression into (92) and assuming $\alpha = i$, we find

$$L_i^{(0,2)}(\mathbf{x}) = -\sum_{\mathbf{p}} p_i L_{\mathbf{p}}^{(0,2)}(\mathbf{x}). \tag{111}$$

It is easy to prove that the following equality takes place:

$$\mathrm{Sp} w^{(0)}(\mathbf{x}) \left[V(\tau), \left[\hat{V}, H_m + H_p \right] \right] =$$

$$= \mathrm{Sp}w^{(0)}(\mathbf{x})\left[\hat{V},[V(-\tau),\mathcal{H}_0]\right] =$$

$$= -i\mathrm{Sp}w^{(0)}(\mathbf{x})\left[\hat{V},\frac{\partial\hat{V}(-\tau)}{\partial\tau}\right].$$

By using this equality together with (105) and (92) and by assuming $\alpha = 0$, we obtain

$$L_0^{(0,2)}(\mathbf{x}) = -\sum_{\mathbf{p}}\varepsilon_{\mathbf{p}}L_{\mathbf{p}}^{(0,2)}(\mathbf{x}). \tag{112}$$

The introduction of the new functions

$$\chi_0(\mathbf{p}) = \frac{p^2}{2m}, \quad \chi_i(\mathbf{p}) = p_i, \quad \chi_4(\mathbf{p}) = m \tag{113}$$

allows us to combine equalities (110)-(112) in a single formula

$$L_\alpha^{(0,2)}(\mathbf{x}) = -\sum_{\mathbf{p}}\chi_\alpha(\mathbf{p})L_{\mathbf{p}}^{(0,2)}(\mathbf{x}). \tag{114}$$

Next, we will calculate $L_A^{(1,1)}(\mathbf{x})$. By using (85), we split (89) into three terms:

$$L_A^{(1,1)}(\mathbf{x}) = L_{1A}^{(1,1)}(\mathbf{x}) + L_{2A}^{(1,1)}(\mathbf{x}) + L_{3A}^{(1,1)}(\mathbf{x}). \tag{115}$$

Here,

$$L_{1A}^{(1,1)}(\mathbf{x}) = -\frac{\partial}{\partial x_k}\mathrm{Sp}\sigma^{(0,1)}(\mathbf{x})\hat{\zeta}_{Ak}(0),$$

$$L_{2A}^{(1,1)}(\mathbf{x}) = +i\mathrm{Sp}w_p^{(0)}(\mathbf{x})\sigma_m^{(1,0)}(\mathbf{x})\left[\hat{V},\hat{\zeta}_A(0)\right],$$

$$L_{3A}^{(1,1)}(\mathbf{x}) = +i\mathrm{Sp}w_m^{(0)}(\mathbf{x})w_p^{(1)}(\mathbf{x})\left[\hat{V},\hat{\zeta}_A(0)\right].$$

It is easy to find that $L_{1\mathbf{p}}^{(1,1)}(\mathbf{x}) = 0$. Indeed,

$$L_{1\mathbf{p}}^{(1,1)}(\mathbf{x}) = \frac{i}{\mathcal{V}}\frac{\partial}{\partial x_k}\int_{-\infty}^{0}d\tau\frac{p_k}{m}\mathrm{Sp}\left[\hat{V},w^{(0)}(\mathbf{x})\right]\hat{\gamma}_{\mathbf{p}} = 0. \tag{116}$$

The equality $\left[\sigma_m^{(1,0)}(\mathbf{x}),\hat{\gamma}_{\mathbf{p}}\right] = 0$ is valid since the operator $\sigma_m^{(1,0)}(\mathbf{x})$ does not contain any operators relevant to the particles. Therefore, the following equality is satisfied:

$$L_{2\mathbf{p}}^{(1,1)}(\mathbf{x}) = \frac{i}{\mathcal{V}}i\mathrm{Sp}\sigma_m^{(1,0)}(\mathbf{x})\hat{V}\left[\hat{\gamma}_{\mathbf{p}},w_p^{(0)}(\mathbf{x})\right] = 0. \tag{117}$$

The operator $\hat{V}$ has the symmetry properties

$$\mathcal{TP}\hat{V}(\mathcal{TP})^{-1} = \hat{V}^*, \tag{118}$$

which is seen from the definitions of the translation operators (15)—(17). By using the last relation, we obtain

$$L_{3\mathbf{p}}^{(1,1)}(\mathbf{x}) = 0. \tag{119}$$

Thus, we have found that $L_p^{(1,1)}(\mathbf{x}) = 0$. However, the function $L_\alpha^{(1,1)}(\mathbf{x}) = 0$ can be nonzero now. The equalities

$$L_{2\alpha}^{(1,1)}(\mathbf{x}) = -\sum_{\mathbf{p}}\chi_\alpha(\mathbf{p})L_{2\mathbf{p}}^{(1,1)}(\mathbf{x}) = 0,$$

$$L_{3\alpha}^{(1,1)}(\mathbf{x}) = -\sum_{\mathbf{p}}\chi_\alpha(\mathbf{p})L_{3\mathbf{p}}^{(1,1)}(\mathbf{x}) = 0$$

are obtained in the same way as equalities (110)–(112). Although the equality $L_{1\alpha}^{(1,1)}(\mathbf{x}) = 0$ takes place only if the operator $\hat{\mathcal{J}}(1,2)$ depends exclusively on the density operator of the medium. In the present work, we do not consider the structure $L_{1\alpha}^{(1,1)}(\mathbf{x})$. Nevertheless, in the equations of motion, we neglect the function $L_A^{(1,1)}(\mathbf{x})$ [see (89) and (115)].

Next, we introduce the rest of the functions defined in (89). We have the following expressions related to the particles:

$$L_{\mathbf{p}}^{(1,0)}(\mathbf{x}) = -\frac{\partial}{\partial x_k}\mathrm{Sp}w^{(0)}(\mathbf{x})\hat{\zeta}_{Ak}(0) =$$

$$= -\frac{p_k}{m}\frac{\partial}{\partial x_k}f(\mathbf{p},\mathbf{x}), \tag{120}$$

$$L_{\mathbf{p}}^{(2,0)}(\mathbf{x}) = -\frac{\partial}{\partial x_k}\frac{p_k}{m}\Big(\mathrm{Sp}w_p^{(0)}(\mathbf{x})\sigma_m^{(1,0)}(\mathbf{x})\hat{\zeta}_{\mathbf{p}}(0) +$$

$$+ \mathrm{Sp}w_p^{(1)}(\mathbf{x})w_m^{(0)}(\mathbf{x})\hat{\zeta}_{\mathbf{p}}(0)\Big) =$$

$$= -\frac{\partial}{\partial x_k}\frac{p_k}{m}\Big(\mathrm{Sp}^{(m)}\sigma_m^{(1,0)}(\mathbf{x})\mathrm{Sp}^{(p)}w_p^{(0)}(\mathbf{x})\hat{\zeta}_{\mathbf{p}}(0) +$$

$$+ \mathrm{Sp}^{(m)}w_m^{(0)}(\mathbf{x})\mathrm{Sp}^{(p)}w_p^{(1)}(\mathbf{x})\hat{\zeta}_{\mathbf{p}}(0)\Big) = 0. \tag{121}$$

The functions $L_\alpha^{(1,0)}(\mathbf{x})$ and $L_\alpha^{(2,0)}(\mathbf{x})$ determine the hydrodynamic equations. Obviously,

$$L_\alpha^{(1,0)}(\mathbf{x}) =$$

$$= -\frac{\partial}{\partial x_k}\mathrm{Sp}^{(m)}w_m^{(0)}(\mathbf{x})\hat{\zeta}_{\alpha k}(0) = -\frac{\partial}{\partial x_k}\zeta_{\alpha k}^{(0)}(\mathbf{x}). \tag{122}$$

In view of the relation $\mathrm{Sp}\sigma_p^{(1,0)}(\mathbf{x}) = 0$ and (84), we obtain

$$L_\alpha^{(2,0)}(\mathbf{x}) =$$

$$= -\frac{\partial}{\partial x_k}\mathrm{Sp}^{(m)}\sigma_m^{(1,0)}(\mathbf{x})\,\hat{\zeta}_{\alpha k}(0) = -\frac{\partial}{\partial x_k}\zeta_{\alpha k}^{(1)}(\mathbf{x})\,. \quad (123)$$

Expressions (122) and (123) include no operators related to the subsystem of particles. However, the direct calculation of traces (122) and (123) is rather complicated. Moreover, the calculation of (123) requires to know the interaction of particles of the medium introduced into the Hamiltonian $\mathcal{H}_m$. The structure of the hydrodynamic equations can be obtained by using the symmetry of the operators $\hat{\zeta}_\alpha(\mathbf{x})$ and $\hat{\zeta}_{\alpha k}(\mathbf{x})$ in respect to the Galilei transformations (see [2]). By using the symmetry properties, we find the expressions for the flows of the hydrodynamic parameters $\zeta_{\alpha k}^{(0)}(\mathbf{x})$:

$$\zeta_{4k}^{(0)}(\mathbf{x}) \equiv j_k^{(0)}(\mathbf{x}) = \rho(\mathbf{x})\,u_k(\mathbf{x})\,,$$

$$\zeta_{ik}^{(0)}(\mathbf{x}) \equiv t_{ik}^{(0)}(\mathbf{x}) = p(\mathbf{x})\,\delta_{ik} + \rho(\mathbf{x})\,u_i(\mathbf{x})\,u_k(\mathbf{x})\,,$$

$$\zeta_{0k}^{(0)}(\mathbf{x}) \equiv q_k^{(0)}(\mathbf{x}) = p(\mathbf{x})\,u_k(\mathbf{x}) + \varepsilon_0(\mathbf{x})\,u_k(\mathbf{x}) +$$

$$+\frac{1}{2}\rho(\mathbf{x})\,u^2(\mathbf{x})\,u_k(\mathbf{x})\,. \quad (124)$$

Here, $\beta(\mathbf{x}) = Y_0(\mathbf{x}) = 1/T(\mathbf{x})$ is the inverse temperature $T(\mathbf{x})$, and $\pi_k(\mathbf{x}) = -\frac{Y_k(\mathbf{x})}{Y_0(\mathbf{x})}\rho(\mathbf{x})$, so that $u_k(\mathbf{x}) = -\frac{Y_k(\mathbf{x})}{Y_0(\mathbf{x})} = \frac{\pi_k(\mathbf{x})}{\rho(\mathbf{x})}$ is the local velocity of the medium; $\varepsilon_0(\mathbf{x}) = \mathrm{Sp}^{(m)}w_m^{(0)}(\mathbf{x})\,\hat{\varepsilon}(0)$ is the density of internal energy of the medium, and $p(\mathbf{x})$ is the medium pressure defined by the formula

$$p(\mathbf{x}) = \mathrm{Sp}w_m^{(0)}(\mathbf{x})\,\hat{t}_{kl}(0)\,.$$

The functions $\zeta_{\alpha k}^{(1)}(\mathbf{x})$ are given by the following expressions (see [2] for details):

$$\zeta_{4k}^{(1)}(\mathbf{x}) \equiv j_k^{(1)}(\mathbf{x}) = 0,$$

$$\zeta_{ik}^{(1)}(\mathbf{x}) \equiv t_{ik}^{(1)}(\mathbf{x}) = -\zeta\delta_{ik}\frac{\partial u_l(\mathbf{x})}{\partial x_l} -$$

$$-\eta\left(\frac{\partial u_i(\mathbf{x})}{\partial x_k} + \frac{\partial u_k(\mathbf{x})}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial u_l(\mathbf{x})}{\partial x_l}\right),$$

$$\zeta_{0k}^{(1)}(\mathbf{x}) \equiv q_k^{(1)}(\mathbf{x}) = u_i(\mathbf{x})\,t_{ik}^{(1)}(\mathbf{x}) + \frac{\bar{\kappa}}{\beta^2(\mathbf{x})}\frac{\partial\beta(\mathbf{x})}{\partial x_k}. \quad (125)$$

Here, η, ζ are the first and second viscosity factors, respectively, and $\bar{\kappa} = \beta^2\kappa$ is the thermal conductivity factor. These factors are defined in terms of the correlation function $\langle\hat{a}\hat{b}\rangle_{x,t}$ as

$$\eta = \frac{1}{2}\beta\int\limits_{-\infty}^{\infty}d\tau\int d^3x\,\langle\hat{t}_{12}\hat{t}_{12}\rangle_{x,\tau}\,,$$

$$\zeta = \frac{1}{2}\beta\int\limits_{-\infty}^{\infty}d\tau\int d^3x\,\langle\hat{t}_{ik}\hat{t}_{ik}\rangle_{x,\tau}\,,$$

$$\bar{\kappa} = \frac{\beta^2}{2}\int\limits_{-\infty}^{\infty}d\tau\int d^3x\,\langle\hat{q}'_l\hat{q}'_l\rangle_{x,\tau}\,,$$

where

$$\hat{q}'_l(\mathbf{x}) = \hat{q}_l(\mathbf{x}) - \frac{\partial\langle\hat{q}_l\rangle}{\partial\zeta_\alpha}\hat{\zeta}_\alpha(\mathbf{x})\,.$$

Finally, we obtain the system of equations of motion for our system:

$$\frac{\partial f(\mathbf{p},\mathbf{x})}{\partial t} + \frac{p_k}{m}\frac{\partial f(\mathbf{p},\mathbf{x})}{\partial x_k} = L_{\mathbf{p}}^{(0,2)}(\mathbf{x})\,,$$

$$\frac{\partial\zeta_a(\mathbf{x})}{\partial t} + \frac{\partial}{\partial x_k}\zeta_{\alpha k}^{(0)}(\mathbf{x}) + \frac{\partial}{\partial x_k}\zeta_{\alpha k}^{(1)}(\mathbf{x}) = L_\alpha^{(0,2)}(\mathbf{x})\,,$$

$$L_{\mathbf{p}}^{(0,2)}(\mathbf{x}) = 2\pi\sum_{1,2}\delta_{2\mathbf{p}}\left\{I_{2,1}(\varepsilon_1-\varepsilon_2,\mathbf{x})\,f_1(\mathbf{x})\,\times\right.$$

$$\left.\times\,(1-f_2(\mathbf{x})) - I_{1,2}(\varepsilon_2-\varepsilon_1,\mathbf{x})\,f_2(\mathbf{x})\,(1-f_1(\mathbf{x}))\right\},$$

$$L_\alpha^{(0,2)}(\mathbf{x}) = -\sum_{\mathbf{p}}\chi_\alpha(\mathbf{p})L_{\mathbf{p}}^{(0,2)}(\mathbf{x})\,. \quad (126)$$

Here, the variables $\zeta_{\alpha k}^{(0)}(\mathbf{x})$, $\zeta_{\alpha k}^{(1)}(\mathbf{x})$ are defined by formulas (124) and (125). This system of coupled equations of motion describes the kinetics of spatially inhomogeneous states of particles that weakly interact with the hydrodynamic medium. The obtained equations describe, for example, neutrons propagating in a medium without multiplication and capture.

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ДО КІНЕТИКИ ПРОСТОРОВО НЕОДНОРІДНИХ СТАНІВ ЧАСТИНОК, ЩО ВЗАЄМОДІЮТЬ ІЗ ГІДРОДИНАМІЧНИМ СЕРЕДОВИЩЕМ

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Р е з ю м е

Побудовано кінетичну теорію просторово неоднорідних станів частинок, що слабо взаємодіють із гідродинамічним середовищем. Для побудови мікроскопічної теорії такої системи використано метод скороченого опису. Розглянуто випадок, коли підсистема частинок, що взаємодіють із середовищем, перебуває на кінетичному етапі еволюції й описується одночастинковою функцією розподілу. У рамках методу скороченого опису одержано зв'язані рівняння руху для параметрів скороченого опису: одночастинкової функції розподілу для частинок, що взаємодіють із середовищем, та гідродинамічних характеристик середовища (його густини, температури та середньої швидкості частинок середовища). Прикладом фізичного об'єкта, еволюція якого описується отриманими рівняннями, можуть служити нейтрони, які поширюються у гідродинамічному середовищі без поглинання та розмноження.