STUDIES OF ELECTRO-ACOUSTIC VIBRATIONS IN $LiNBO_3$ PLATE-RESONATORS

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The variational formulation of the problem concerning three-dimensional linear electro-acoustic vibrations that are electrically induced in a piezoelectric crystal resonator belonging to an arbitrary crystallographic class and cut is presented. The discrepancies between the experimentally measured and theoretical values of the eigenfrequencies for an extensional vibration mode of 128°-rotated Y-cut $LiNbO_3$ rectangular plates have been shown to fall within a few percent. For that vibration mode, the dependences of the eigenfrequencies on the plate's linear dimensions (the frequency plans) have been calculated for several lower overtones.

1. Introduction

Confined piezoelectric bodies (plates, disks, rings, and so on) are widely used as piezoceramic elements in modern solid state devices such as ultrasonic transducers, acoustic resonators, sensors, piezoelectric transformers, piezomotors, piezogyroscopes, etc. In a lot of cases, the physical and technical characteristics of such devices are determined by the features of spatial (in particular, superficial) distributions of the elastic and piezoelectric field components which occur in piezoceramic elements functioning at operating frequencies. Therefore, the parameters of vibrations of piezoelements with regular geometric shapes have been widely studied in recent years both experimentally and using theoretical methods [1–4].

Typical single-crystal materials for piezoelements are quartz and lithium niobate. Note that, despite a plenty of works in this domain, the detailed characteristics of three-dimensional electroelastic vibrations of rectangular-shaped piezocrystals have not been established yet. This situation is associated, in particular, with the fact that the boundary problems dealing with vibrations of three-dimensional piezoelectric resonators have no exact analytical solutions, even for bodies with simple geometrical shapes [5].

This work aimed at analyzing the extensional vibrations making use of the Rayleigh–Ritz variational method [6]. We report the results of numerical calculations obtained for the eigenfrequency spectra of extensional vibrations in rectangular $LiNbO_3$ plates with various dimensions and compare them with experimentally measured eigenfrequency values.

2. Theoretical Consideration

The geometry of the problem is shown in Fig. 1. Metal electrodes *2* and *3* are deposited onto a rectangular piezoelectric crystal *1* with linear dimensions L_X, L_Y, and L_Z along the coordinate axes X, Y, and Z, respectively. Suppose that, if electroelastic vibrations with frequency f are excited in this plate, the potential of electrode *3* is zero (i.e. the electrode is grounded) and that of electrode *2* is supplied by an external generator and equals $V_G \exp(j2\pi ft)$.

Three-dimensional electroelastic vibrations of confined piezoelectric bodies are described by the spatial dependences of the piezoelectric potential Φ and the components U_X, U_Y, and U_Z of the vector of elastic displacements $\vec{U}$. By the mode of vibrations of an object under investigation is traditionally meant [4] the distributions of the components of the elastic and electric fields on the object's surface and in the direction normal to it.

In the case of a linear piezoelectric insulator, the differential formulation of a boundary problem

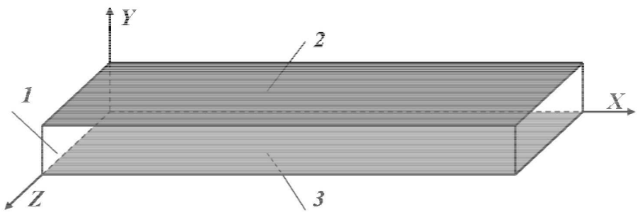

Fig. 1. Geometry of the problem

concerning three-dimensional electroelastic vibrations of a plate like shown in Fig. 1 includes the following equations [8]:
the equation of motion for particles of the piezoelectric medium

$$\rho\frac{\partial^2 U_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j}; \quad (1)$$

the Maxwell equation

$$\frac{\partial D_m}{\partial x_m} = 0; \quad (2)$$

mechanical boundary conditions on the whole surface

$$N_j T_{ij} = 0; \quad (3)$$

electrical boundary conditions on that part of the surface that is free from electrodes

$$N_m D_m = 0; \quad (4)$$

on the surface $y = L_Y$,

$$\Phi = V_G; \quad (5)$$

on the surface $y = 0$,

$$\Phi = 0; \quad (6)$$

material relations for the components of the electric induction vector $\vec{D}$,

$$D_m = e_{mkl}U_{k,l} - \varepsilon_0\varepsilon^S_{mn}\Phi_n + \varepsilon_0\varepsilon^S_{m2}V_G/L_Y; \quad (7)$$

and for the components of the mechanical stress tensor $\hat{T}$,

$$T_{ij} = c^E_{ijkl}U_{k,l} + e_{nij}\Phi_{,n} - e_{2ij}V_G/L_Y. \quad (8)$$

Here, the fact that the electric field is potential in the quasielectrostatic approximation, i.e. $\vec{E} = -\vec{\nabla}\Phi$, was taken into account. Equation (3) was written down in the assumption that the electrodes are infinitesimally thin and do not exert mass loading onto those sections of the surface, where they are located. In the system of equations (1)–(8), the following notations are used: ρ is the mechanical density; c^E_{ijkl} are the components of the tensor of elastic constants which are determined provided that the electric field strength is constant; e_{kij} are the components of the tensor of piezoelectric constants; ε^S_{ij} are the components of the tensor of dielectric constants which are determined provided that the deformation is constant; and N_j are the components of the vector that is normal to the surface. All the indices run from 1 to 3, the comma means the differentiation with respect to that coordinate, whose index follows the comma, and the summation over identical indices is implied.

In our case, we examine stationary harmonic vibrations with the circular frequency $\omega = 2\pi f$, so that all physical quantities that are dependent on time contain the multiplier $\exp(j\omega t)$; therefore, we may write down that $\partial U_i/\partial t = j\omega U_i$. While analyzing characteristic vibrations, one has to put $V_G = 0$ in Eqs. (7) and (8) and make the right-hand sides of Eqs. (5) and (6) equal to V_2 and V_3, respectively, considering those two as the quantities to be determined.

It is known [9] that the following simple types of vibrations can be excited in a confined solid: (i) extensional, (ii) bending, and (iii) torsion vibrations. For each type of vibrations, the basic mode (the tone) and the overtones are distinguished. Each of those simple types of vibrations is described by a specific equation of motion [10]. The equation of motion (1) describes just extensional vibrations.

Since the formulated boundary-value problem (1)–(8) has no exact analytical solution, we carried out numerical calculations in the framework of the Rayleigh–Ritz variational method [6]. The variational formulation of the boundary-value problem (1)–(8) following the Ostrogradsky–Gauss principle consists in finding the functional of electroelastic interactions $F(U_X, U_Y, U_Z, \Phi, V_2, V_3)$, the stationary value of which is attained for such functions $U_X(x, y, z)$, $U_Y(x, y, z)$, $U_Z(x, y, z)$, and $\Phi(x, y, z)$ that must satisfy the system of equations (1)–(8) [5]. Eer Nisse and Holland were the first who proposed a functional of the form $F(U_X, U_Y, U_Z, \Phi, V(q))$, where $V(q)$ is the unknown potential of the q-th electrode, for the analysis of free vibrations in a three-dimensional piezoceramic resonator with an arbitrary configuration of electrodes on its surface [8, 11]. Nevertheless, the application of this functional is correct only if free vibrations are considered. Therefore, we have modified this functional for the case of forced vibrations that are electrically

excited in a confined piezoelectric crystalline body with an arbitrary crystallographic orientation. If the geometry of the problem is identical to that illustrated in Fig. 1, the functional looks like

$$F(U_i;\Phi) = \frac{1}{2}\iiint\limits_{\Omega_R} n_{i\alpha}n_{j\beta}n_{k\gamma}n_{l\delta}c^E_{\alpha\beta\gamma\delta}U_{i,j}U_{k,l}d\Omega -$$

$$-\frac{1}{2}\omega^2\iiint\limits_{\Omega_R}\rho U_iU_id\Omega +$$

$$+\iiint\limits_{\Omega_R} n_{m\alpha}n_{k\beta}n_{l\gamma}e_{\alpha\beta\gamma}\Phi_{,m}U_{k,l}d\Omega -$$

$$-\frac{1}{2}\iiint\limits_{\Omega_R} n_{m\alpha}n_{n\beta}\varepsilon^S_{\alpha\beta}\Phi_{,m}\Phi_{,n}d\Omega -$$

$$-\frac{V_G}{L_Y}\iiint\limits_{\Omega_R} n_{m2}n_{k\beta}n_{l\gamma}e_{2\beta\gamma}U_{k,l}d\Omega +$$

$$+\frac{V_G}{2L_Y}\iiint\limits_{\Omega_R}(n_{2\alpha}n_{n\beta}\varepsilon^S_{\alpha\beta}\Phi_{,n} + n_{m\alpha}n_{2\beta}\varepsilon^S_{\alpha\beta}\Phi_{,m})d\Omega -$$

$$-\frac{V_G^2}{2L_Y}\iiint\limits_{\Omega_R} n_{2\alpha}n_{2\beta}\varepsilon^S_{\alpha\beta}d\Omega +$$

$$+\sum_{p=1}^{2}\iint\limits_{S(p)} N_m(p)n_{m\alpha}n_{k\beta}n_{l\gamma}e_{\alpha\beta\gamma}U_{k,l}[\Phi - V_G(p)]dS -$$

$$-\sum_{p=1}^{2}\iint\limits_{S(p)} N_m(p)n_{m\alpha}n_{m\beta}\varepsilon^S_{\alpha\beta}\Phi_{,n}[\Phi - V_G(p)]dS, \qquad (9)$$

where $N_m(p)$ are the components of the vectors which are normal to either exciting electrode *2* ($p = 1$) or *3* ($p = 2$). In functional (9), n_{ij} designates the components of the transition matrix from the crystallographic coordinate system $\{X_C, Y_C, Z_C\}$ to a new coordinate system $\{X, Y, Z\}$; this matrix makes allowance for the variation of the components of the tensors of material constants, which takes place at that:

$$[n] = \begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\alpha_2\cos\alpha_3 & -\cos\alpha_2\sin\alpha_3 & \sin\alpha_2 \\ \sin\alpha_1\sin\alpha_2\cos\alpha_3 + \cos\alpha_1\sin\alpha_3 & -\sin\alpha_1\sin\alpha_2\sin\alpha_3 + \cos\alpha_1\cos\alpha_3 & -\sin\alpha_1\cos\alpha_2 \\ -\cos\alpha_1\sin\alpha_2\cos\alpha_3 + \sin\alpha_1\sin\alpha_3 & \cos\alpha_1\sin\alpha_2\sin\alpha_3 + \sin\alpha_1\cos\alpha_3 & \cos\alpha_1\cos\alpha_2 \end{pmatrix}. \qquad (10)$$

Here, α_1, α_2, and α_3 are the rotation angles around the coordinate axes X_C, Y_C, and Z_C, respectively. While analyzing the characteristic vibrations, one should put $V_G(1) = V_2$ and $V_G(2) = V_3 = 0$ in functional (9). The integrals over the volume of the plate Ω_R and over its surfaces $S(p)$ are reduced to the repeated ones:

$$\iiint\limits_{\Omega_R} d\Omega = \int_0^{L_X}dx\int_0^{L_Y}dy\int_0^{L_Z}dz, \quad \iint\limits_{S(p)} dS = \int_0^{L_X}dx\int_0^{L_Z}dz.$$

In order to minimize functional (9), the unknown functional dependences $U_X(x,y,z)$, $U_Y(x,y,z)$, $U_Z(x,y,z)$, and $\Phi(x,y,z)$ are tried in the form of expansions into the functional series of the trial functions $BF_1(\alpha,x,y,z)$, where $\alpha = 1\ldots Q_U$, $BF_2(\alpha,x,y,z)$, where $\alpha = Q_U + 1\ldots 2Q_U$, $BF_3(\alpha,x,y,z)$, where $\alpha = 2Q_U + 1\ldots 3Q_U$, and $BF_4(\beta,x,y,z)$, where $\beta = 1\ldots Q_\Phi$, respectively, and where the integers Q_U and Q_Φ determine the number of trial functions:

$$U_{X,Y,Z}(x,y,z) = \sum_{\alpha=1}^{Q_U} B(\alpha)\cdot BF_n(\alpha,x,y,z); \qquad (11)$$

$$\Phi(x,y,z) = \sum_{\beta=1}^{Q_\Phi} C(\beta)\cdot BF_4(x,y,z). \qquad (12)$$

In mathematical physics, when solving – by variational methods – the problem of characteristic

vibrations for parallelepiped-shaped physical objects, the following three-dimensional trigonometric basic functions are widely used [6]:

$$BF_n(\alpha, \mathrm{x}, \mathrm{y}, \mathrm{z}) = g_{n1}[\pi M_n(\alpha)x/L_X]\times$$

$$\times g_{n2}[\pi N_n(\alpha)y/L_Y]\cdot g_{n3}[\pi P_n(\alpha)z/L_Z]\ , \qquad (13)$$

where, each of the functions g_{n1}, g_{n2}, and g_{n3} is either sine or cosine; and $M_n(\alpha)$, $N_n(\alpha)$, and $P_n(\alpha)$ are positive integers which will be referred to as "coordinate characteristic numbers" (CCNs) below. The scheme how the basic sines and cosines are distributed among the trial functions $BF_n(\alpha, x, y, z)$ is quoted in Table 1. This basis differs from the basis that was proposed earlier by Holland [12] for the analysis of characteristic vibrations of a piezoceramic parallelepiped with short-circuited electrodes; the difference consists in that we use two components $\sin[\pi M_4(\beta)x/L_X]\ \cos[\pi N_4(\beta)y/L_Y]\ \cos[\pi P_4(\beta)z/L_Z]$ and $\cos[\pi M_4(\beta)x/L_X]\sin[\pi N_4(\beta)y/L_Y]\cos[\pi P_4(\beta)z/L_Z]$ instead of a single component $\cos[\pi M_4(\beta)x/L_X]\times\cos[\pi N_4(\beta)y/L_Y]\sin[\pi P_4(\beta)z/L_Z]$ to approximate Φ. Such an approach allows us to take into account, as the first approximation, the effect of the piezoelectric field penetration beyond the boundaries of the plate at those sections of its surface which are free from electrodes.

In the course of finding the solution for the characteristic vibration problem, the standard procedure of the determination of the unknown factors $B(\alpha)$, $C(\beta)$ and the potential V_2 is used [8]. But, before its application to carrying out specific numerical calculations, it is necessary to preset CCNs (27 sets of parameters for our basis). In this case, every excited mode of vibrations is characterized by the parities (evenness or oddness) of those CCNs [11]. An even (odd) set of CCNs describes an asymmetric (symmetric) distribution of the corresponding physical function ($U_X(x,y,z)$, $U_Y(x,y,z)$, $U_Z(x,y,z)$, or $\Phi(x,y,z)$) over the given coordinate. Since the conditions of electric excitation of three-dimensional modes of extensional vibrations had not been studied before, we made the theoretical analysis of this issue in the framework of the Rayleigh–Ritz variational approach.

The parities of CCNs, which should be used for the description of electrically excited vibration modes, are determined from the following relations:
for BF_1, BF_2, and BF_3,

$$\iiint\limits_{\Omega_R} n_{m\alpha}n_{k\beta}n_{l\gamma}e_{\alpha\beta\gamma}U_{k,l}d\Omega\ \neq\ 0, \qquad (14)$$

where $\alpha = 1, 2, 3$ for YZ, XZ, and XY faces, respectively; for BF_4,

$$\iiint\limits_{\Omega_R} (n_{k\alpha}n_{n\beta}\varepsilon^S_{\alpha\beta}\Phi_{,n}\ +\ n_{m\alpha}n_{k\beta}\varepsilon^S_{\alpha\beta}\Phi_{,m})d\Omega\ \neq\ 0, \quad (15)$$

where $k = 1, 2, 3$ for YZ, XZ, and XY faces, respectively. Note that these relations are applicable to any piezocrystal belonging to an arbitrary crystallographic class and having an arbitrary crystallographic cut.

The analysis of these relations shows that, in the case of a symmetric arrangement of exciting electrodes on the opposite XZ faces of 128°-rotated Y-cut $LiNbO_3$ plates (see Fig. 1), only one mode (conditionally designated as 5676) can be excited; its CCN parities are quoted in Table 2. The problem how many different modes of extensional vibrations can be excited in a rectangular piezoelectric plate belonging to a definite crystallographic class – in particular, in a $LiNbO_3$ plate – requires a separate consideration which goes beyond the scope of this work.

3. Experimental Results, Numerical Calculations, and Discussion

Experimental researches and numerical calculations were executed for 128°-rotated Y-cut lithium niobate ($LiNbO_3$) rectangular plates. Such plates are widely used in acoustoelectronics owing to a large value of the electromechanical coupling constant [13]. The linear

T a b l e 1. Scheme of distribution of basic sines and cosines (g_{n1}, g_{n2}, g_{n3}) among the functions $BF_n(\alpha, x, y, z)$

	$BF_{\mathrm{T}}(\alpha)$		$BF_2(\alpha)$		$BF_3(\alpha)$		$BF_4(\alpha)$		
	$\alpha = 1, ..., 8$	$\alpha = 9, ..., 33$	$\alpha = 1, ..., 8$	$\alpha = 9, ..., 33$	$\alpha = 1, ..., 8$	$\alpha = 9, ..., 33$	$\alpha = 1, ..., 8$	$\alpha = 9, ..., 16$	$\alpha = 17, ..., 33$
g_{nl}	cos	cos	sin	cos	sin	cos	sin	cos	sin
g_{n2}	sin	cos	cos	cos	sin	cos	cos	sin	sin
g_{n3}	sin	cos	sin	cos	cos	cos	cos	cos	sin

T a b l e 2. Parities of coordinate characteristic numbers M_n, N_n, and P_n for the vibration mode 5676

	For $BF_{\rm T}(\alpha)$		For $BF_2(\alpha)$		For $BF_3(\alpha)$		For $BF_4(\alpha)$		
	$\alpha = 1, ..., 8$	$\alpha = 9, ..., 33$	$\alpha = 1, ..., 8$	$\alpha = 9, ..., 33$	$\alpha = 1, ..., 8$	$\alpha = 9, ..., 33$	$\alpha = 1, ..., 8$	$\alpha = 9, ..., 16$	$\alpha = 17, ..., 33$
M_n	odd	odd	odd	even	odd	even	odd	even	odd
N_n	odd	even	odd	odd	odd	even	odd	even	even
P_n	odd	even	odd	even	odd	odd	even	even	odd

T a b l e 3. Experimentally measured ($f_{a,nE}$) and calculated in the framework of the Rayleigh–Ritz method ($f_{a,nT}$) vibration eigenfrequencies for various specimens

	Overtone number, n			
	$n = 1$	$n = 2$	$n = 3$	$n = 4$
	Specimen LNO-4a			
f_{anE}, kHz	160.25	462.01	738.95	782.36
$f_{a,nT}$, kHz	152.43	451.78	698.65	773.88
Δf_n, %	4.8	2.2	5.4	1.0
	Specimen LNO-4b			
$f_{a,nE}$, kHz	213.64	610.07	751.20	813.39
$f_{a,nT}$, kHz	207.98	596.37	765.75	787.59
Δf_n, %	2.6	2.2	1.9	3.1

dimensions $L_X L_Y L_Z$ of the specimens under investigation were $20.12 \times 0.786 \times 3.74$ (specimen LNO-4a) and $14.71 \times 0.785 \times 3.75$ mm^3 (specimen LNO-4b), with the variability along each dimension ($\Delta L_X, \Delta L_Y, \Delta L_Z$) not exceeding 0.05 mm. In Table 3, the measured ($f_{a,nE}$) and calculated ($f_{a,nT}$) values of the lowest eigenfrequencies $f_{a,n}$ ($n = 1, 2, 3, 4$) for specimens LNO-4a and LNO-4b are compared. The frequencies $f_{a,nE}$ were determined as antiresonance ones [14] by measuring the frequency dependences of the absolute value of the total specimen conductance following a standard technique [15]. The frequencies $f_{a,nT}$ were calculated as antiresonance eigenfrequencies, i.e. the situation, where both electrodes were open [16], was simulated. In so doing, the potential of the lower electrode was considered to be zero, $V_3 = 0$ (see Fig. 1). The calculations were carried out for various numbers Q_U and Q_Φ of basic functions in expansions (11) and (12), respectively.

The values of material constants for $LiNbO_3$ were taken from work [17] and recalculated for a 128°-rotated Y-cut making use of the rotation matrix $[n]$ components (relation (10)), where the specific values $\alpha_1 = 128°$ and $\alpha_2 = \alpha_3 = 0°$ were taken into account. The quantities Δf_n, which characterize a discrepancy between the measured and the calculated values of characteristic frequencies, were estimated as $\Delta f_n = [(f_{a,nE} - f_{a,nT})/f_{a,nE}] \times 100\%$.

The data quoted in Table 3 demonstrate that Δf_n does not exceed a few percent for the lowest overtones. We emphasize that the main factors, which cause the discrepancy between the experimentally measured and the theoretically calculated values of eigenfrequencies, are (i) the accuracy of determination of the material constants that were involved into calculations (for mode 5676, these are c^E_{11}, c^E_{12}, c^E_{13}, c^E_{14}, c^E_{22}, c^E_{23}, c^E_{24}, c^E_{33}, c^E_{34}, c^E_{44}, c^E_{55}, c^E_{56}, c^E_{66}, e_{15}, e_{16}, e_{21}, e_{22}, e_{23}, e_{24}, e_{31}, e_{32}, e_{33}, e_{34}, ε^S_{11}, ε^S_{22}, ε^S_{23}, and ε^S_{33}), (ii) the variability of linear dimensions of the plate, and (iii) non-parallelism of its opposite faces. The latter two factors arise owing to specimen polishing, and they may be taken as a reason which explains the nonmonotonous dependence of Δf_n on n.

We also note that the frequencies $f_{a,nT}$ become somewhat lower as the numbers Q_U and Q_Φ increase, which is characteristic of the variational method of calculations [6].

At last, it should be noted that the minimal values for Δf_n were obtained just for mode 5676 which has been distinguished above among other modes from theoretical considerations.

In Fig. 2, the dependences of eigenfrequencies f_n for the tone ($n = 1$) and three lowest overtones ($n = 2, 3, 4$) on the dimension L_X of a 128°-rotated Y-cut $LiNbO_3$ rectangular plate with the other dimensions $L_Y = 0.785$ mm and $L_Z = 3.75$ mm are depicted. The points correspond to the results of measurements. From this frequency plan, it is evident that the ratios between various eigenfrequencies depend in a complicated manner on the ratios between the linear dimensions of the plate. These dependences make it also possible to trace the asymptotics of the eigenfrequency spectrum in the cases $L_X \to 0$ (Fig. 2,*a*) and $L_X \to \infty$ (Fig. 2,*b*). For thin plates ($L_X \ll L_Y, L_Z$), low-frequency vibrations become two-dimensional, i.e., the eigenfrequency values do not depend on L_X and depend only on the ratio between the linear dimensions L_Y and L_Z. In the case of a long bar ($L_X \gg L_Y, L_Z$), the spectrum also changes, because its every frequency f_n tends to the corresponding limit.

In Fig. 3, the dependences of eigenfrequencies f_n for the tone ($n = 1$) and three lowest overtones ($n = 2, 3, 4$) on the transversal dimension L of the plate are exhibited

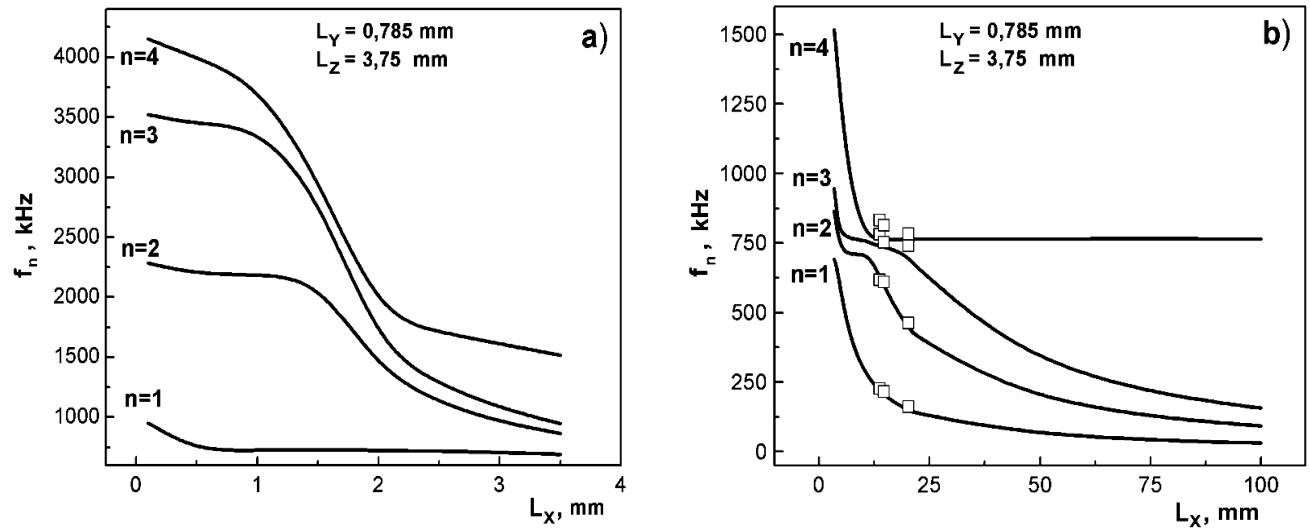

Fig. 2. Dependences of eigenfrequencies f_n $(n = 1 - 4)$ for a 128°-rotated Y-cut $LiNbO_3$ rectangular plate on its dimension L_X

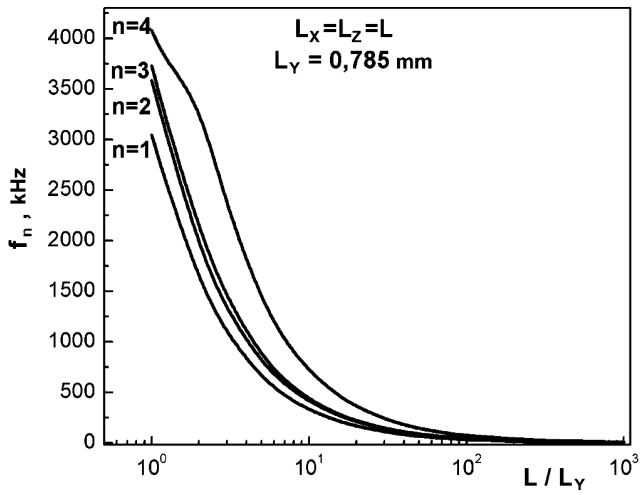

Fig. 3. Dependences of eigenfrequencies f_n $(n = 1 - 4)$ for a 128°-rotated Y-cut $LiNbO_3$ square plate on its transverse dimension L $(L_X = L_Z = L)$ in the case $L_Y = 0.785$ mm

for a 128°-rotated Y-cut $LiNbO_3$ square ($L_X = L_Z = L$) plate with $L_Y = 0.785$ mm = const. These dependences are qualitatively similar to those presented in Fig. 2.

The results of numerical calculations also testify that the ratio f_n/f_1 between the eigenfrequencies does not change for a three-dimensional piezoelectric crystal plate, if the ratios between its linear dimensions remain constant, i.e. $L_X : L_Y : L_Z = \text{const}$. The results of these calculations are quoted in Table 4.

4. Conclusions

In general, the theoretical and experimental results that were obtained in this work bring about the following conclusions.

1. The Rayleigh–Ritz variational method was demonstrated to be valid for the analysis of excited vibration modes of the extensional type and for the calculations of eigenfrequency spectra of three-dimensional rectangular lithium niobate piezoelectric crystal resonators.

2. Only one mode of extensional vibrations can be excited in a 128°-rotated Y-cut $LiNbO_3$ rectangular

T a b l e 4. Dependences of the ratio $f_{a,nT}/f_{a,1T}$ between the frequencies of a rectangular plate on its linear dimensions L_X, L_Y, and L_Z, provided $L_X : L_Y : L_Z = \text{const}$ ($LiNbO_3$, a 128°-rotated Y-cut)

	$L_X = 14.71$ mm, $L_Y = 0.785$ mm, $L_Z = 3.75$ mm					
n	$L_X : L_Y : L_Z$		$0.99L_X : 0.99L_Y : 0.99L_Z$		$0.9L_X : 0.9L_Y : 0.9L_Z$	
	$f_{a,nT}$, kHz	$f_{a,nT}/f_{a,lT}$	$f_{a,nT}$, kHz	$f_{a,nT}/f_{a,lT}$	$f_{a,nT}$, kHz	$f_{a,nt}/f_{a,lT}$
1	207.98	1.00	210.08	1.00	231.09	1.09
2	596.37	2.82	602.40	2.82	662.64	2.82
3	765.75	3.49	773.48	3.49	850.83	3.49
4	787.59	3.63	795.55	3.63	875.10	3.63

plate-resonator with a symmetric arrangement of electrodes on the opposite XZ-faces of the plate.

3. In a rectangular $LiNbO_3$ plate-resonator, the frequencies of characteristic extensional vibrations are the complicated functions of its linear dimensions (L_X, L_Y, and L_Z). However, provided the condition $L_X : L_Y : L_Z = \text{const}$ is fulfilled, the ratios between eigenfrequencies also remain constant.

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ВИВЧЕННЯ ЕЛЕКТРО-АКУСТИЧНИХ КОЛИВАНЬ ПЛАСТИН-РЕЗОНАТОРІВ $LiNbO_3$

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Р е з ю м е

Наведено варіаційне формулювання задачі про лінійні електрично збуджувані тривимірні електроакустичні коливання п'єзокристалічного резонатора довільного кристалографічного класу та зрізу. Для однієї з мод коливань типу розтягу-стиску прямокутної пластини $LiNbO_3$ 128°-Y-повернутого зрізу показано, що різниця між експериментально виміряними та чисельно розрахованими значеннями власних частот не перевищує кількох відсотків. Для кількох нижчих обертонів цієї моди розраховано плани частот, тобто їх залежності від співвідношення лінійних розмірів пластини.