REGULARITIES OF MANIFESTATION OF THE BREWSTER AND PSEUDO-BREWSTER ANGULAR CONDITIONS IN SPECTRA OF LIGHT REFLECTION FROM A THIN TRANSPARENT LAYER

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UDC 53.082.54:563.5

In the present work, a computer simulation technique is used for the analysis of regularities that determine the formation of the spectra of the oblique light reflection from a transparent one-film structure. It is substantiated that, at the Brewster angle for a single interface, the envelopes of the Fabry—Perot spectra touch one another, while, at the angle of the pseudo-Brewster condition, the absolute values of the amplitudes of Fresnel factors for the light reflection from opposite surfaces of the film are equal to each other.

One of the techniques used to determine the parameters of a thin or ultrathin film is the investigation of the regularities that determine the formation of the oblique reflection profile $R_p(\alpha)$ for an electromagnetic wave of p-polarization, especially in the region of the formation of a minimum, which is associated in the literature with the manifestation of the Brewster [1,2] and pseudo-Brewster angular conditions [3].

Inherently, the Brewster angular condition is specific of a single interface of two insulators, where, in the p-component, there exists an angle $\alpha_{p\mathrm{Br}}$ (the Brewster one), for which the energy coefficient in the reflection minimum is equal to zero, $R_p(\alpha_{\mathrm{Br}}) = 0$. If $R_p(\alpha_{\mathrm{Br}}) \neq 0$ in the minimum, such an angle is considered to be a quasi-Brewster one [2], or it is also called pseudo-Brewster [4,5].

The angular position of the minimum of the profile of the light reflection from a binary plane-parallel interface (film) $R_p(\alpha)$ depends on the relation between the optical characteristics of media and the layer depth d. Moreover, the minimum of the profile of the reflection from a binary interface can also manifest itself in s-polarization [6,7], which is theoretically forbidden for a single interface by the Fresnel formulae.

Thus, the basic purpose of the given paper was in the establishment of the regularities of manifestations of the Brewster and pseudo-Brewster angular conditions in the spectra of the light reflection from a thin transparent layer. It is shown that, at the Brewster angles for both single interfaces of the layer, the envelopes of the Fabry—Perot spectra touch each other, while the pseudo-Brewster angular condition for the light reflection from a one-layer parallel-sided structure manifests itself in the part of the angular dependence of the envelopes $R_{\min}(\alpha)$, in which the condition of equality of the absolute values of the Fresnel amplitudes for opposite surfaces of the film $\tilde{r}_{12,23}$ in the p- and s-components of polarization is satisfied.

Indeed, according to [8,9], the energy reflection factor can be expressed in terms of the so-called envelopes of the extrema $R_{\max,\min}$ of spectra in the following way:

$$R_{\max,\min} = \left(\frac{\sigma_{12} \pm \sigma_{23}}{1 \pm \sigma_{12}\sigma_{23}} \right)^2 . \qquad (1)$$

Here, $\tilde{r}_{12,23} = \sigma_{12,23} \exp\left(i\phi_{12,23}\right)$ denotes the amplitudes of the coefficients of the reflection from the interfaces: the medium with refractive index n_1– a film with refractive index n_2 (index 12) and a film – the medium (mainly substrate) with refractive index n_3 (index 23).

At the Brewster angle $\alpha_{p\mathrm{Br}}$, the multiple-beam interference is absent. That's why in this case, the envelopes $R_{p,\max,\min}$ touch each other:

$$R_{p,\max} = R_{p,\min}. \qquad (2)$$

According to (1), the difference of $R_{\max}$ and $R_{\min}$ amounts to:

$$\Delta R_p = R_{p,\max} - R_{p,\min} =$$

$$= 4\sigma_{p,12}\sigma_{p,23}\frac{\left(1-\sigma^2_{p,12}\right)\left(1-\sigma^2_{p,23}\right)}{\left(1-\sigma^2_{p,12}\sigma^2_{p,23}\right)^2}, \qquad (3)$$

which indicates that equality (2) holds true in three cases:
in asymmetric structures ($\sigma_{12} \neq \sigma_{23}$):
a) at the angle $\alpha_{12\mathrm{Br}}$, where

$$\sigma_{p12} = 0, \sigma_{p23} \neq 0; \qquad (4)$$

b) at the angle $\alpha_{23\mathrm{Br}}$, where

$$\sigma_{p12} \neq 0, \quad \sigma_{p23} = 0; \qquad (5)$$

c) in symmetric structures ($\sigma_{12} = \sigma_{23}$)
at the common angle $\alpha_{12\mathrm{Br}} = \alpha_{23Br}$, where

$$\sigma_{p12} = 0, \quad \sigma_{p23} = 0. \qquad (6)$$

The angles $\alpha_{p,12,23}$ are the known Brewster angles for single interfaces 12 and 23. Their values are determined in terms of the optical characteristics of media [10]. At the Brewster angles, the energy reflection factors are equal to $R(\alpha_{12\mathrm{Br}}) = \sigma^2_{p23}$ and $R(\alpha_{23\mathrm{Br}}) = \sigma^2_{p12}$ and don't depend on the thickness of the layer. That's why $\alpha_{p,12,23}$ can be experimentally determined as the intersection angle of the reflection profiles $R_p(\alpha)$ obtained for several wavelengths.

As follows from (1), the envelope $R_{\min}(\alpha)$ of the minima of the reflection spectra under a multiple-beam interference can possess its minimum at the angle $\alpha_{ps\mathrm{Br}}$, at which the following condition is satisfied:

$$\sigma_{12} = \sigma_{23}, \quad \text{where} \quad R_{\min}(\alpha_{psBr}) \to 0. \qquad (7)$$

To our mind, it is the value of the angle $\alpha_{ps\mathrm{Br}}$ that can be considered as the one at which the pseudo-Brewster angular condition for a parallel-sided layer is fulfilled. Hence, the reflection factor will be equal to zero in the minimum of the profile $R_p(\alpha)$ only in the case where its angular position coincides with the value of $\alpha_{ps\mathrm{Br}}$.

On the other hand, the method of envelope functions correctly describes the regularities of the formation of Fabry–Perot spectra in the both polarizations [8,9]. That's why the pseudo-Brewster behavior of the envelope of minima will manifest itself both in the s- and in p-components depending on the relation between the refractive indices. In the p-polarization, there can be several such angles $\alpha_{ps\mathrm{Br}}$:

$$\alpha_{ps\mathrm{Br1}} = \mathrm{arctg}(n_{31}) \qquad (8.a)$$

and

$$\alpha_{ps\mathrm{Br2}} = \mathrm{arctg}\sqrt{\frac{2n^2_{21}n^2_{31}\left(n^4_{21}-n^2_{31}\right)}{2n^4_{31}\left(n^2_{21}-1\right)-n^6_{21}\left(n^2_{31}-1\right)+\sqrt{z}}}, \qquad (8.b)$$

if the condition $n^2_{21} > n_{31}$ is satisfied. Here, the relative values of the refractive indices are $n_{21} = \frac{n_2}{n_1}$, $n_{31} = \frac{n_3}{n_1}$,

$$z = \left[2n^4_{31}\left(n^2_{21}-1\right)-n^6_{21}\left(n^2_{31}-1\right)\right]^2 +$$

$$+4n^6_{31}\left(n^2_{21}-1\right)^2\left(n^4_{21}-n^2_{31}\right).$$

If the relative refractive indices lie in the intervals

$$1 < n_{21} < 1.0525, \quad \text{where} \quad n'_{31} < n_{31} < n''_{31}, \qquad (9a)$$

$$n_{21} < 1, \quad \text{where} \quad n''_{31} < n_{31} < n_{21}, \qquad (9b)$$

condition (7) holds true at the angles $\alpha_{ps\mathrm{Br1}}$, $\alpha_{ps\mathrm{Br2}}$, and

$$\alpha_{ps\mathrm{Br3}} = \mathrm{arctg}\sqrt{\frac{2n^2_{21}n^2_{31}\left(n^4_{21}-n^2_{31}\right)}{2n^4_{31}\left(n^2_{21}-1\right)-n^6_{21}\left(n^2_{31}-1\right)-\sqrt{z}}}, \qquad (10)$$

as depicted in Fig. 1. In the case where

$$n_{21} < 1, \quad \text{but} \quad n_{21} < n_{31} < n'''_{31}, \qquad (11)$$

the minimum of the envelope, for which the pseudo-Brewster angular condition is satisfied, will manifest itself for the angles $\alpha_{ps\mathrm{Br1}}$ and $\alpha_{ps\mathrm{Br3}}$. Here,

$$n'_{31} = \sqrt{2\sqrt[3]{S}\cos\left(\frac{A+4\pi}{3}\right)+H},$$

$$n''_{31} = \sqrt{2\sqrt[3]{S}\cos\left(\frac{A}{3}\right)+H},$$

$$n'''_{31} = \frac{n_{21}}{\sqrt{1-n^2_{21}}}, \quad \cos A = -\frac{q}{S},$$

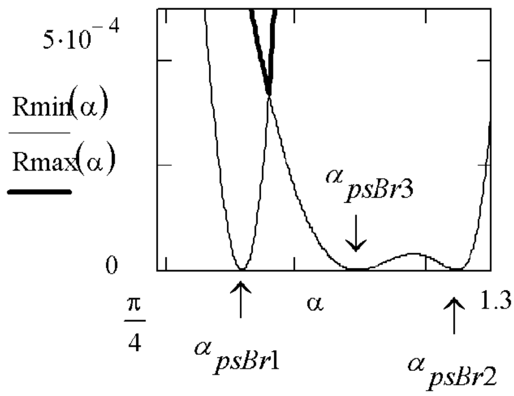

Fig. 1. Angular dependence of the envelopes $R_{\min,\max}(\alpha)$ for structures with the relation of refractive indices (9a) and (9b)

$$S=\sqrt{-\left[-\frac{[n_{21}^8+4n_{21}^2(n_{21}^2-1)]^2}{12^2(n_{21}^2-1)^2}+\frac{n_{21}^8}{6(n_{21}^2-1)}\right]^3},$$

$$H=\frac{n_{21}^8+4n_{21}^2(n_{21}^2-1)}{12(n_{21}^2-1)},$$

$$q=-\frac{[n_{21}^8+4n_{21}^2(n_{21}^2-1)]^3}{12^3(n_{21}^2-1)^3}+$$

$$+\frac{n_{21}^8[n_{21}^8+4n_{21}^2(n_{21}^2-1)]}{48(n_{21}^2-1)^2}-\frac{n_{21}^8}{8(n_{21}^2-1)}.$$

If the relative refractive indices lie in the intervals

$$n_{21}<1 \quad \text{and} \quad n_{31}>n_{31}''', \qquad (12)$$

condition (7) holds true only for the angle $\alpha_{ps\mathrm{Br}3}$. In the case where $n_{21}^2<n_{31}$, but conditions (9a), (9b), (11), and (12) are not satisfied, the minimum of the envelope manifests itself only for the angle of incidence $\alpha_{ps\mathrm{Br}1}$. As one can see from (8a), at this angle, the classical Brewster angular condition for a substrate without a film is fulfilled.

In the s-polarization, the pseudo-Brewster angular condition (7) is also not forbidden. The corresponding angle amounts to

$$\alpha_{ss\mathrm{Br}}=\mathrm{arctg}\left(\sqrt{\frac{n_{31}^2-n_{21}^4}{(n_{21}^2-1)^2}}\right), \qquad (13)$$

if the relative refractive indices lie in the intervals $n_{31}>n_{21}^2$, where $n_{21}>1$ or

$$n_{21}>n_{31}>n_{21}^2, \quad \text{where} \quad n_{21}<1.$$

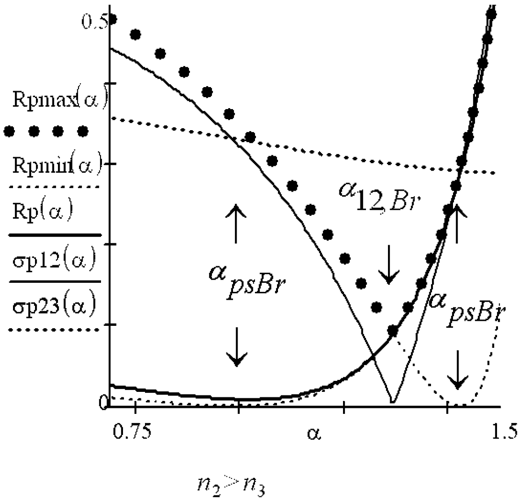

Fig. 2. Calculated spectra for the oblique incidence of a beam on a film with the thickness $d = 3$ nm.

As one can see from Fig. 2, the angles $\alpha_{ps\mathrm{Br}}$ can manifest themselves on either side of the Brewster angle $\alpha_{p\mathrm{Br}}$ depending on the values of $n_{1,2,3}$. Provided that the film is characterized with a higher optical density than that of the substrate, then $\alpha_{ps\mathrm{Br}}<\alpha_{\mathrm{Br}}$. If the optical density of the substrate is higher, $\alpha_{ps\mathrm{Br}}>\alpha_{\mathrm{Br}}$.

In conclusion, it's worth noting the following. With decrease in the thickness of the layer $d\to 0$, especially in the region of small values, the dynamics of variation of the profiles $R_{p,s}(\alpha)$ is fundamentally different. In the $p-$polarization, a decrease in the layer thickness results in the formation of the minimum of $R_p(\alpha)$ in the neighborhood of the angle $\alpha_{ps\mathrm{Br}}$, so that, in the limiting case $d\to 0$, the reflection profile in this region of the spectrum coincides with the envelope of minima. The minimum of this envelope function coincides with the value of the Brewster angle for the substrate which is determined by formula (8a).

In the s-component, the form of the spectral profile $R_s(\alpha)$ in the limiting case $d\to 0$ tends to the corresponding one allowed by Fresnel formulae in the case of the light reflection from the pure surface of the substrate.

The basic conclusions of the work can be formulated as follows.

1. At the Brewster angle for single interfaces on the both sides of the film, the envelopes of the Fabry–Perot interference bands touch each other, $R_{\max}=R_{\min}$.

2. The angular position of the minimum of the envelope of the minima of Fabry–Perot interference

bands in the geometry of light reflection from a film, which is conditioned by the equality of the absolute values of the amplitudes of Fresnel factors for the opposite faces of the film, doesn't depend on the thickness of the film and is determined by the relation between the refractive indices of the media that form the one-film structure; from the viewpoint of the Brewster effect for a single interface, it represents the pseudo-Brewster effect for a binary interface that is not forbidden for both p- and s-polarizations.

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Received 07.07.06.
Translated from Ukrainian by H. Kalyuzhna

ЗАКОНОМІРНОСТІ ПРОЯВУ БРЮСТЕРІВСЬКОЇ І ПСЕВДОБРЮСТЕРІВСЬКОЇ КУТОВИХ УМОВ В СПЕКТРАХ ВІДБИТТЯ СВІТЛА ТОНКИМ ПРОЗОРИМ ШАРОМ

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Р е з ю м е

Методом комп'ютерного моделювання проаналізовано закономірності формування спектрів похилого відбиття світла прозорою одноплівковою структурою. Показано, що під кутом Брюстера для одинарної межі обвідні спектрів Фабрі—Перо дотикаються між собою, тоді як під кутом, що відповідає псевдобрюстерівській умові, модулі амплітуд коефіцієнтів Френеля відбиття світла протилежними поверхнями плівки рівні між собою.