ABOUT BINDING CONDITIONS FOR A SYSTEM OF THREE FERMIONS AND IMPOSSIBILITY OF EXISTENCE OF A TRINEUTRON

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Conditions of the existence of a three-fermion bound state are investigated for the states with nonzero angular momentum $L = 1$ and a wide class of interaction potentials. It is shown that the binding of three fermions with purely attractive potentials or the standard attractive potentials at high distances and a short-range repulsion is impossible if two of the fermions are unbound. Moreover, this system is even farther from a bound state when the parameters of potentials are in agreement with the low-energy two-neutron data. The possibility for a bound state of a three-fermion system with $L = 1$ without bound subsystems to exist is demonstrated for the potentials with two attractive regimes. But there exists no bound trineutron for the realistic neutron-neutron interaction even with two regimes of attractions, in contrast to a tetraneutron considered in [9].

1. Introduction

The problem of the existence of multineutrons is of a great interest in nuclear physics from both the experimental and theoretical points of view [1–3]. It remains relevant for a long time, and there is still no its satisfactory solution. A great number of papers, both theoretical and experimental ones, were devoted to the investigation of this question.

The recent experiments [4, 5] and their interpretation as the manifestation of a tetraneutron or a four-neutron cluster induce the necessity of a deeper theoretical analysis of all possibilities of the formation of multineutron systems from a few neutrons and/or resonances in the systems containing many neutrons and the clarification of structural peculiarities of such hypothetical formations [5–9]. Up to now, the full theoretical analysis is possible for a hypothetical trineutron and a tetraneutron. In [9], a new class of $n - n$ interaction potentials with two regimes of attraction was proposed. With such potentials, it is possible to construct the bound state of a hypothetical tetraneutron without any contradiction with the low-energy neutron-neutron scattering experimental data.

In this paper, we focus on the analysis of the binging conditions of three-fermion systems with angular momentum $L = 1$ and investigate the possibilities for the existence of a hypothetical trineutron. All calculations are performed with the use of high-precision variational schemes, and their comparison with other rougher estimates is made.

2. Statement of the Problem

Consider a system of three fermions with central pairwise interaction. In what follows, we use the obtained results in the analysis of the binding conditions of three neutrons. The Schrödinger equation for three fermions with central pairwise interaction,

$$\left\{\sum_{i=1}^{3}\frac{\hat{p}_i^2}{2m}+\sum_{i<j=1}^{3}(V_s^+(r_{ij})\hat{P}_s(ij)+ \right.$$

$$\left.+V_t^-(r_{ij})\hat{P}_t(ij))\right\}\Psi_a = E\Psi_a, \qquad (1)$$

contains the singlet $V_s^+(r_{ij})$ and triplet $V_t^-(r_{ij})$ components of the interaction potential in the even

and odd (with respect to the angular momentum) states of two particles. $\hat{P}_s(ij)$ and $\hat{P}_t(ij)$ are the projection operators on the singlet and triplet states of two particles, respectively. If we consider the spin functions of three particles as two-component vectors

$$\xi' \to \begin{pmatrix} \xi' \\ 0 \end{pmatrix}, \qquad \xi'' \to \begin{pmatrix} 0 \\ \xi'' \end{pmatrix},$$

then we can present the projection operators written in terms of the Pauli matrices

$$\hat{P}_s(ij) = \frac{1}{4}(1 - \boldsymbol{\sigma}_i\boldsymbol{\sigma}_j) \qquad \hat{P}_t(ij) = \frac{1}{4}(3 + \boldsymbol{\sigma}_i\boldsymbol{\sigma}_j),$$

as 2×2 matrices:

$$\hat{P}_s(12) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{P}_t(12) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\hat{P}_s(13) = \begin{pmatrix} \frac{1}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{3}{4} \end{pmatrix}, \hat{P}_t(13) = \begin{pmatrix} \frac{3}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{1}{4} \end{pmatrix},$$

$$\hat{P}_s(23) = \begin{pmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{pmatrix}, \hat{P}_t(23) = \begin{pmatrix} \frac{3}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} \end{pmatrix}. \qquad (2)$$

In the lowest energy state, the spin of the three-particle system is $S = 1/2$, and the full antisymmetric wavefunction in a doublet spin state is

$$\Psi_a = \frac{1}{\sqrt{2}}\left(\xi'\phi''(\mathbf{r}_{12}, \mathbf{r}_{13}, \mathbf{r}_{23}) - \xi''\phi'(\mathbf{r}_{12}, \mathbf{r}_{13}, \mathbf{r}_{23})\right), \qquad (3)$$

where the spatial components $\phi'(\mathbf{r}_{12}, \mathbf{r}_{13}, \mathbf{r}_{23})$ and $\phi''(\mathbf{r}_{12}, \mathbf{r}_{13}, \mathbf{r}_{23})$ and the spin components ξ', ξ'' possess the symmetry $[2, 1]$. The projection of Eq. (1) on different spin states produces a system of two equations for the spatial components:

$$\left(\sum_{i=1}^{3} \frac{\hat{p}_i^2}{2m} + V_s^+(r_{ij}) - E\right)\phi'' +$$

$$+\frac{1}{4}\sum_{(ij)\neq(12)} \left(3V_t^-(r_{ij}) + V_s^+(r_{ij})\right)\phi'' +$$

$$+\frac{\sqrt{3}}{4}\sum_{(ij)\neq(12)} (-1)^{i+j}\left(V_s^+(r_{ij}) - V_t^-(r_{ij})\right)\phi' = 0,$$

$$\left(\sum_{i=1}^{3} \frac{\hat{p}_i^2}{2m} + V_t^-(r_{ij}) - E\right)\phi' +$$

$$+\frac{1}{4}\sum_{(ij)\neq(12)} \left(V_t^-(r_{ij}) + 3V_s^+(r_{ij})\right)\phi' +$$

$$+\frac{\sqrt{3}}{4}\sum_{(ij)\neq(12)} (-1)^{i+j}\left(V_s^+(r_{ij}) - V_t^-(r_{ij})\right)\phi'' = 0. \qquad (4)$$

The spatial components ϕ' and ϕ'' correspond to an irreducible representation of the permutation group with the Young table [1, 2] and are not independent. We choose one spatial component ϕ'' symmetric with respect to the permutation of the coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ as an independent one, and the other one can be related with it in the next way:

$$\phi' = \frac{1}{\sqrt{3}}(2\hat{P}_{23} + 1)\phi'', \qquad (5)$$

where $\hat{P}_{23}$ is the permutation operator for particles 2 and 3. Then we can rewrite the Schrödinger equation for only one independent spatial component of the wavefunction ϕ'' (here and later on, we use the notation $\Phi \equiv \phi''$) which is symmetric with respect to the permutation of particles 1 and 2:

$$\left\{\sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + \frac{1}{2}\sum_{i>j=1}^{3} \left(V_s^+(r_{ij}) + V_t^-(r_{ij})\right) +\right.$$

$$+\frac{1}{2}\sum_{(ij)\neq(23)} (-1)^{i+j}\left(V_s^+(r_{ij}) - V_t^-(r_{ij})\right) -$$

$$\left. -\frac{1}{2}\sum_{(ij)\neq(12)} (-1)^{i+j}\left(V_s^+(r_{ij}) - V_t^-(r_{ij})\right)\hat{P}_{23}\right\}\Phi = E\Phi. \qquad (6)$$

The equation for three fermions (6) in the doublet spin state is already quite convenient to be used in numerical calculations based on the Bubnov—Galerkin variational method. We perform all our numerical calculations using the Gaussian basis similar to that in [9].

3. Matrix Elements of the Hamiltonian for Angular Momentum $L = 1$

For the complete analysis of possibilities to bind three fermions (and a trineutron) by using the wave equation (6), we investigate the lowest energy states of three particles with the total angular momenta $L = 0$ and $L = 1$. The analysis of these states can be essential also for the investigation of a hypothetical tetraneutron and its $^4\mathrm{n} \to {}^3\mathrm{n} + \mathrm{n}$ channel of dissociation.

The variational wavefunction corresponding to zero angular momentum must depend only on the absolute values of interparticle distances and, for the Gaussian basis, is a properly antisymmetrized linear combination of the basis functions:

$$\Phi_{L=0}(r_{12}, r_{13}, r_{23}) = \hat{S} \sum_{i=1}^{K} C_i \exp\left(- \sum_{k<l=1} 3a_{kl}^i r_{kl}^2 \right) \equiv$$

$$\equiv \hat{S} \sum_{i=1}^{K} C_i \psi_i. \qquad (7)$$

Here, $\hat{S}$ is the operator of antisymmetrization with respect to the permutation of particles 1 and 2: $\hat{S} = 1 - \hat{P}_{12}$. When we use the Bubnov—Galerkin scheme, all matrix elements of Eq. (6) on functions (7) are standard enough [9]. Somewhat more difficult is the situation with angular momentum $L = 1$, when the wavefunction contains, in addition to the absolute values of interparticle distances, spherical functions of the first order. We can build the most general spatial wavefunction of the system of three particles that describe odd states with angular momentum $L = 1$ and its projection $M = 0$ in the following form:

$$\Phi_{\substack{L=1,\\ M=1}} = q_{1z}\varphi_1(r_{12}, r_{13}, r_{23}) + q_{2z}\varphi_2(r_{12}, r_{13}, r_{23}). \qquad (8)$$

Here, q_{1z} and q_{2z} are the z-components of the relative Jacobi coordinates,

$$\mathbf{q}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \quad \mathbf{q}_2 = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3), \qquad (9)$$

φ_1 and φ_2 are symmetric and antisymmetric functions with respect to permutation of particles 1 and 2 (symmetrized to correspond to thye Young table [2,1]). To perform variational calculations within the Bubnov—Galerkin method, we choose variational trial functions φ_1 and φ_2 in the form of an independent properly symmetrized superposition of Gaussian functions.

Prior to our numerical calculations, the important step is the computation of the overlapping matrix elements and the matrix elements of the potential and kinetic energies on various basis functions. All these matrix elements can be calculalted explicitly when we use the Gaussian basis even for a very complex basis function with nonzero angular momentum (8). In the case of the angular momentum $L = 1$ and the variational function (8), we need to calculate matrix elements on the basis functions of two types:

$$\phi_i^{(1)} = q_{1z} \exp\left(- \sum_{k<l=1} 3a_{kl}^{i(1)} r_{kl}^2 \right),$$

$$\phi_i^{(2)} = q_{2z} \exp\left(- \sum_{k<l=1} 3a_{kl}^{i(2)} r_{kl}^2 \right). \qquad (10)$$

To express all matrix elements in the universal form for both types of basis functions (10), we introduce a matrix that consists of the linear combinations of nonlinear variational parameters:

$$\mathbf{B} = \begin{pmatrix} \frac{1}{2}(4a_{12} + a_{13} + a_{23}) & \frac{\sqrt{3}}{2}(a_{13} - a_{23}) \\ \frac{\sqrt{3}}{2}(a_{13} - a_{23}) & \frac{3}{2}(a_{13} + a_{23}) \end{pmatrix}. \qquad (11)$$

Here, $a_{kl} = a_{kl}^i + akl^j$. The matrix $\mathbf{B}$ defines the transition to Jacobi coordinates in the exponent

$$\sum_{k<l=1}^{3} a_{kl} r_{kl}^2 = \sum_{n,m=1}^{3} B_{nm}(\mathbf{q}_n \mathbf{q}_m).$$

Also we introduce the matrix inverse to $\mathbf{B}$,

$$\mathbf{C} \equiv \mathbf{B}^{-1} \cdot \det(\mathbf{B}),$$

$$\mathbf{C} = \begin{pmatrix} \frac{3}{2}(a_{13} + a_{23}) & \frac{\sqrt{3}}{2}(a_{23} - a_{13}) \\ \frac{\sqrt{3}}{2}(a_{23} - a_{13}) & \frac{1}{2}(4a_{12} + a_{13} + a_{23}) \end{pmatrix}. \qquad (12)$$

Using the matrices $\mathbf{B}$ and $\mathbf{C}$, we can write the overlapping matrix element in a simple form

$$\left\langle \phi_i^{(n)} \middle| \phi_j^{(m)} \right\rangle = \frac{\pi^3}{D^{5/2}} \frac{C_{nm}}{2}, \qquad n, m = 1, 2, \qquad (13)$$

and a matrix element of an arbitrary function depending only on the absolute value of the interparticle distance r_{kl} in the form

$$\left\langle \phi_i^{(n)} \middle| f(r_{kl}) \middle| \phi_j^{(m)} \right\rangle = \left\langle \phi_i^{(n)} \middle| \phi_j^{(m)} \right\rangle \frac{8}{3\sqrt{\pi}} \times$$

$$\times \int_0^\infty f(t\sqrt{D_{kl}/D}) \left[\left(\frac{3}{2} - t^2 \right) \frac{D}{D_{kl} C_{nm}} \frac{\partial C_{nm}}{\partial a_{kl}} + t^2 \right] \times$$

$$\times t^2 e^{-t^2} dt. \quad (14)$$

Here, we introduce the notation

$$D = \det(B) = 3(a_{12}a_{13} + a_{12}a_{23} + a_{13}a_{23}),$$

$$D_{kl} = \frac{\partial D}{\partial a_{kl}}. \quad (15)$$

It is more useful to write down the matrix elements of the kinetic energy $K = -\sum_{i=1}^{3} \Delta_i/2$ (for simplicity, we use the $\hbar = m = 1$ unit system) separately for different combinations of the basis functions, because the universal expression is cumbersome and not informative:

$$\left\langle \phi_i^{(1)} \right| K \left| \phi_j^{(1)} \right\rangle =$$

$$= \frac{\pi^3}{D^{5/2}} \left[-C_{12}^2 + \left(3 + \frac{5C_{12}^2}{D} \right) Y_{22} + \frac{5C_{11}^2}{D} Y_{11} \right],$$

$$\left\langle \phi_i^{(2)} \right| K \left| \phi_j^{(2)} \right\rangle =$$

$$= \frac{\pi^3}{D^{5/2}} \left[-C_{12}^2 + \left(3 + \frac{5C_{12}^2}{D} \right) Y_{22} + \frac{5C_{11}^2}{D} Y_{11} \right],$$

$$\left\langle \phi_i^{(1)} \right| K \left| \phi_j^{(2)} \right\rangle = \left\langle \phi_i^{(2)} \right| K \left| \phi_j^{(1)} \right\rangle =$$

$$= \frac{\pi^3}{D^{5/2}} \Big[\left(B_{11}^j - B_{22}^i \right) \left(B_{12}^i - B_{12}^j \right) +$$

$$+ \frac{5C_{12}^2}{3D} \left(Y_{12} + Y_{12} \right) \Big]. \quad (16)$$

Here, we introduce the matrices $\mathbf{B}^i$ and $\mathbf{B}^j$ that are built similarly to the matrix $\mathbf{B}$ with one difference: instead of the parameters a_{kl}, a_{kl}^i and a_{kl}^j are used. The matrix product $\mathbf{Y} = B^i \cdot B^j$ is also used in these expressions. "Diagonal" matrix elements on the functions $\phi^{(1)}$ and $\phi^{(2)}$ can be also simply rewritten in terms of the matrix $\mathbf{C}$ only:

$$\langle \phi_i^{(1)} | K | \phi_j^{(1)} \rangle = \frac{\pi^3}{D^{5/2}} \left[-C_{12}^2 + \left(3 + \frac{5C_{12}^2}{D} \right) \times \right.$$

$$\left. \times (C_{12}^i C_{12}^j + C_{11}^i C_{11}^j) + \frac{5C_{11}^2}{D} (C_{12}^i C_{12}^j + C_{22}^i C_{22}^j) \right],$$

$$\langle \phi_i^{(2)} | K | \phi_j^{(2)} \rangle = \frac{\pi^3}{D^{5/2}} \left[-C_{12}^2 + \left(3 + \frac{5C_{12}^2}{D} \right) \times \right.$$

$$\left. \times (C_{12}^i C_{12}^j + C_{22}^i C_{22}^j) + \frac{5C_{11}^2}{D} (C_{12}^i C_{12}^j + C_{11}^i C_{11}^j) \right]. \quad (17)$$

Here, the matrices $\mathbf{C}^i$ and $\mathbf{C}^i$ are built using the same rule as for $\mathbf{B}^i$ and $\mathbf{B}^i$. Using the obtained expressions together with special optimization schemes on nonlinear parameters lets us to perform calculations with a very high accuracy that can be controlled.

4. Analysis of Binding Conditions of Three Fermions with Different Interaction Potentials

First, we consider a simple problem with the interaction in (6) which is independent of spin. If the triplet potential is equal to the singlet one, we obtain the amplification of attraction in a three-fermion system, and this is the most favorable case for the binding of three fermions in the doublet spin state. Moreover, to compare with the four-particle system, we choose the same potentials for calculations of a trineutron as those for four fermions in [9]: the simple attractive Gaussian potential $V_1(r) = -g\exp(-r^2)$, the Volkov potential with variable strength $V_2(r) = -g(144.86\exp(-(r/0.82)^2) - 83.34\exp(-(r/1.6)^2))$ as a typical nuclear potential with short-range repulsion (all potentials are in dimensionless units like in [9]), the optimal Volkov-type potential giving the best binding conditions $V_3(r) = -g(\exp(-r^2) - 1.5\exp(-(r/0.9)^2))$, and the potential with two regimes of attraction,

$$V_s^+(r) = g(0.43\exp(-(r/0.6)^2) - \exp(-r^2) +$$

$$+1.085\exp(-(r/1.3)^2) - 0.42\exp(-(r/1.5)^2)), \quad (18)$$

proposed in [9] (here the distance scale is $r_0 = 0.488519$ fm). The results of calculations of the critical coupling constants, when a bound state of the three-fermion system appears on the threshold with rise in the coupling constant g, are presented in the first three rows of Table 1 for the simple potentials and in the fourth row for the potential proposed in [9] with two regimes of attraction.

As expected, for all the considered potentials, the energy level of three fermions with angular momentum $L = 1$ is essentially lower than the level with $L = 0$. For all potentials, it is seen that the critical coupling constant of three particles with angular momentum $L = 1$ is very close to the critical constant of a four-particle system (considerably closer, than the level with $L = 0$).

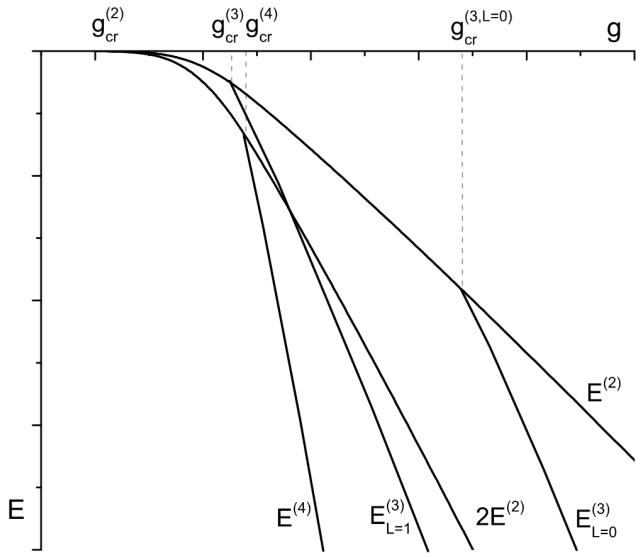

Fig. 1. Energy levels scheme for two, three, and four fermions with simple short-range (nuclear) interaction potentials

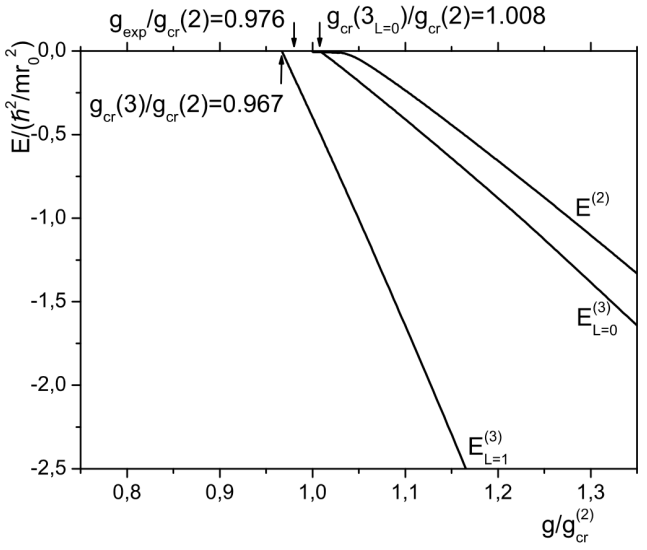

Fig. 2. Dependence of the lowest state energies on the coupling constant for two- and three-fermion systems (18)

Therefore, for the four-fermion system with symmetry [2, 2], not only the threshold $4 \to 2+2$ is important, but also $4 \to 3_{L=1}+1$. As seen from Table 1, $g_{\rm cr}(3) > g_{\rm cr}(4)$ for some potentials, while $g_{\rm cr}(3) < g_{\rm cr}(4)$ for others. We emphasize that, for all the considered simple potentials, the four-particle level disappears on the $2+2$ threshold with decrease in the coupling constant, rather than on the threshold $E_{L=1}(3)$. We also note that there is no possibility with simple potentials to obtain a bound state in the three-fermion system with two unbound particles (always $g_{\rm cr}(2) < g_{\rm cr}(3)$). This result is mostly the same as obtained in [9] for the system of four neutrons. Figure 1 schematically shows the dependence of binding energies of the considered systems on the coupling constant; this energy level scheme remains almost the same for all simple potentials.

More interesting, of course, is the analysis of binding conditions of the system of three fermions with potential (18) with two regimes of attraction that gives rise to the binding of a tetraneutron [9]. The results of our calculations of the critical coupling constants for this potential in the "spinless" case is presented in the last row of Table 1. We emphasize that it is possible with this interaction potential to obtain bound states of four fermions with zero angular momentum [9] and those of three fermions with angular momentum $L+1$ with the unbound two-fermion subsystem. Moreover, both these systems are also bound with the coupling constant $g_{\rm exp} = 322.4$, which allows one to reproduce the low-energy neutron-neutron data (the scattering length $a_{s(nn)} = -18.9$, and the effective interaction radius $r_{0s(nn)} = 2.75$ fm).

Figure 2 gives a clear evidence that a bound state appears in the three-fermion system with potential (18) with two regimes of attraction in the $L = 1$ case. But the binding is absent for zero angular momentum in the area, where a two-particle system is unbound. This behavior takes place, because the different parts of the attractive potential dominate in the two- and three-fermion systems in the region, where a two-particle bound state appears. In the three-particle system, the short-range attraction is significant, while longer ranges and the second attractive well play the crucial role for two particles. We consider this class of potentials with two regimes of attraction (18) to be useful in investiga-

T a b l e 1. Critical coupling constants of three- and four-fermion systems

Potential	$g_{\rm cr}(3_{L=1})$	$g_{\rm cr}(3_{L=1})/g_{\rm cr}(2)$	$g_{\rm cr}(3_{L=0})/g_{\rm cr}(2)$	$g_{\rm cr}(4)/g_{\rm cr}(2)$ [9]
$-g\exp(-r^2)$	3.84	1.43	3.3	1.46
Volkov potential	0.0265	1.39	3.45	1.44
$-g(\exp(-r^2) - 1.5\exp(-(r/0.9)2))$	170	1.278	2.18	1.27
Potential (18)	319.6	0.967	1.008	0.954

ting the binding conditions of three neutrons under the real conditions with spin-dependent potential.

Consider now a spin-dependent potential, being the same as that in [9]. Namely, we choose the singlet potential as (18) and the triplet potential as

$$V_t^-(r) = g(2.212\exp(-(r/2)^2) -$$

$$-2.334\exp(-(r/3)^2) + \exp(-(r/4)^2)). \qquad (19)$$

The results of calculations based on the Schrödinger equation (6) using optimized schemes of the variational method with a Gaussian basis showed that there is no bound states in the system of three neutrons when two neutrons are unbound. Notice that the system of four neutrons remains bound at the point $g_{\rm exp}$ with this potential. For three neutrons, the considered triplet potential acts as a strong repulsion. This may be caused by a somewhat different action of the Pauli principle in three- and four-neutron systems. Using the presented spin-dependent potential leads to very high values of the critical coupling constants: $g_{\rm cr}(3_{L=1}) = 4.27 g_{\rm cr}(2)$, and $g_{\rm cr}(3_{L=0}) = 8.88 g_{\rm cr}(2)$. This is much worse than all the spinless results presented above. Therefore, the bound states in the three-neutron system cannot exist if two neutrons remains unbound even for potentials (18 and 19). Our attempts to modify the triplet potential to obtain the trineutron binding turned out unsuccessful.

5. Shell Oscillatory Estimates for Multineutron Systems

The high-precision calculations based on many-particle equations like (6) meet considerable difficulties, when one tries to analyze the systems of more than four particles. Realizable now are only the rough estimates of multineutron systems with higher number of neutrons. To investigate the binding conditions of different multineutron systems from one point of view, we consider the oscillatory approximation for the trial wavefunctions of simple systems of 2, 3, and 4 neutrons and closed-shell systems of 8, 20, and 40 neutrons. The energy functionals for such N-neutron systems can be calculated relatively simply in standard way. These functionals depend only on the neutron mass M and the variational parameter a as follows:

$$N=2,(L=0): E = \min_a \left\{ \frac{1}{Ma^2}\frac{3}{2} + \langle 0|V_s^+(ra)|0\rangle \right\},$$

$$N=2,(L=1): E = \min_a \left\{ \frac{1}{Ma^2}\frac{5}{2} + \right.$$

$$\left. + \frac{2}{3}\langle 0|r^2 V_t^-(ra)|0\rangle \right\},$$

$$N=3,(L=0): E = \min_a \left\{ \frac{5}{Ma^2} + \frac{1}{3}\langle 0|\Big(r^4 - 3r^2 + \right.$$

$$\left. + \frac{15}{4}\Big) V_s^+(ra)|0\rangle + \langle 0|r^2 V_t^-(ra)|0\rangle \right\},$$

$$N=3,(L=1): E = \min_a \left\{ \frac{4}{Ma^2} + \right.$$

$$\left. + \frac{3}{2}\langle 0|V_s^+(ra)|0\rangle + \langle 0|r^2 V_t^-(ra)|0\rangle \right\},$$

$$N=4,(L=0): E = \min_a \left\{ \frac{1}{Ma^2}\frac{13}{2} + \langle 0|\Big(\frac{1}{3}r^4 - \right.$$

$$\left. - r^2 + \frac{13}{4}\Big) V_s^+(ra)|0\rangle + 2\langle 0|r^2 V_t^-(ra)|0\rangle \right\},$$

$$N=8,(L=0): E = \min_a \left\{ \frac{1}{Ma^2}\frac{33}{2} + \langle 0|\Big(r^4 - \right.$$

$$\left. - r^2 + \frac{31}{4}\Big) V_s^+(ra)|0\rangle + 12\langle 0|r^2 V_t^-(ra)|0\rangle \right\},$$

$$N=20,(L=0): E = \min_a \left\{ \frac{1}{Ma^2}\frac{117}{2} + \frac{1}{64}\langle 0|(16r^8 - \right.$$

$$-96r^6 + 776r^4 - 680r^2 + 1945) V_s^+(ra)|0\rangle +$$

$$\left. + \frac{3}{2}\langle 0|r^2(4r^4 - 12r^2 + 55) V_t^-(ra)|0\rangle \right\},$$

$$N=40,(L=0): E = \min_a \left\{ \frac{1}{Ma^2}\frac{297}{2} + \frac{1}{256}\langle 0|(\frac{64}{9}r^{12} - \right.$$

$$-\frac{320}{3}r^{10} + \frac{3632}{3}r^8 - \frac{15392}{3}r^6 + 17884r^4 -$$

$$-13180r^2 + 21935) V_s^+(ra)|0\rangle + \frac{1}{512}\langle 0|r^2(512r^8 -$$

$$\left. -5120r^6 - 35072r^4 - 74496r^2 + 164640) V_t^-(ra)|0\rangle \right\}. \qquad (20)$$

Here, N is the number of particles, and L is the total angular momentum of a system. The kinetic energy term in these equations properly accounts for the center-of-mass motion. The matrix elements of even singlet and odd triplet potentials are defined with "oscillator" weight as

$$\langle 0|f(r)|0\rangle = \frac{1}{\pi^{3/2}} \int d\mathbf{r} \exp(-r^2) f(r). \qquad (21)$$

Of course, we must mention that the oscillatory approximation can describe well only strongly bound systems. But, for such loosely bound systems as multineutrons, the oscillatory approximation can be only a rough variational estimate by the order of magnitude. In the oscillatory approximation, we considered the

T a b l e 2. Critical coupling constants for different Fermi-systems with different numbers of particles in the oscillatory approximation with the spin-independent potential $V(r) = -g\exp(-r^2)$ (for the three-particle system, values for different angular momenta are presented)

Number of particles N	$g_{\rm cr}(N)$	$g_{\rm cr}(N)/g_{\rm cr}(2)$
2 ($L = 0$)	3.897	1.000
3 ($L = 0$)	10.408	2.671
3 ($L = 1$)	5.649	1.45
4	4.283	1.099
8	2.748	0.705
20	1.853	0.475
40	0.565	0.145

T a b l e 3. Critical coupling constants for different Fermi-systems with different numbers of particles in the oscillatory approximation with the spin-independent potential (18) (for the three-particle system, values for different angular momenta are presented)

Number of particles N	$g_{\rm cr}(N)$	$g_{\rm cr}(N)/g_{\rm cr}(2)$
2 ($L = 0$)	941.84	1.000
3 ($L = 0$)	5453.9	5.791
3 ($L = 1$)	712.05	0.756
4	832.15	0.884
8	274.39	0.291
20	203.22	0.216
40	26.32	0.0279

T a b l e 4. Critical coupling constants for different multineutron systems with the spin-dependent potential (18), (19) (for the three-particle system, values for different angular momenta are presented)

Number of particles N	$g_{\rm cr}(N)$	$g_{\rm cr}(N)/g_{\rm cr}(2)$
2	941.84	1.000
3 ($L = 0$)	–	–
3 ($L = 1$)	10190.28	10.82
4	—	—
8	24221.73	25.72
20	15170.02	16.11
40	45514.18	48.32

systems with potentials independent of spin: the attractive Gaussian potential, double-well potential (18), and the spin-dependent potential (18)–(19). The results of calculations are presented in Tables 2–4.

It is clear from Tables 2 and 3 that, for both spin-independent potentials, the critical coupling constants, at which a bound N-particle state appears below the two-particle threshold, decrease monotonically with increase in the number of particles starting from 3 ones. We note that both the considered potentials do not satisfy the saturation condition and must lead for large numbers of particles to a collapse. We see that the critical coupling constant is lower than the two-particle one (only with such a requirement, we can talk about a bound multineutron system) already for a system of 8 particles even in the simple Gaussian potential. The further decrease of the critical coupling constants for 20- and 40-particle systems indicates a trend of the tighter binding of heavy multineutron systems, what is a consequence of the simple non-saturating interaction. Nevertheless, as the oscillatory approximation gives the upper bound for the energy, we may expect that the octaneutron system ^8n will be stable in precise calculations. When we consider the spin-independent potential (18) in the oscillatory approximation, a system of particles with angular mommentum $L = 1$ is already bound with two unbound particles. This result is in agreement with high-precision three-particle calculations (Fig. 2). As for four fermions, the high-precision results [9] agree qualitatively with those in Table 3, because $g_{\rm cr}(4) < g_{\rm cr}(2)$. For heavier multineutrons, the tendency of stronger binding remains, since $g_{\rm cr}(N)$ decreases monotonically with rise of N.

The results of Tables 2 and 3 for the spinless case give a clear evidence that $g_{\rm cr}(3_{L=1}) < g_{\rm cr}(3_{L=0})$, and the energy level for the angular momentum $L = 1$ is always deeper than that for $L = 0$ for three-fermion system. Moreover, the critical constant for three particles with $L = 1$ is relatively close to the critical constant of four particles for all spin-independent interactions (we see the same situation in precise calculations).

The less simple situation is for realistic spin-dependent interaction potentials (Table 4). First, the oscillatory approximation cannot lead to the binding of any multineutron system, Second, the anomalous irregularity in the behavior of $g_{\rm cr}(N)$ is obtained for relatively large numbers of particles, $N \sim 40$. At last, the three-fermion system with $L = 0$ and the four-fermion system cannot be bound for arbitrarily great coupling

constants. Nevertheless, the precise calculations showed that a tetraneutron with this potential is bound [9]. This is the obvious drawback of the oscillatory approximation.

6. Conclusion

Our investigation shows that the binding conditions for a trineutron are much better in the state with angular momentum $L = 1$ than those with $L = 0$ for all the considered interaction potentials. The spin-independent potential (18), which binds a system of four fermions with zero total spin [9] with two unbound particles, binds also a system of three fermions in the doublet state with spin $S = 1/2$ and angular momentum $L = 1$. Of course, we have to pay a high price for the binding of the system, namely to violate the charge independence of nuclear forces. In the realistic case with spin-dependent potentials and with the triplet potential in the form (19), the binding of a trineutron disappears. The studies with other potentials showed that there is no possibility to build a potential of the same type as (18, 19) that can bind a trineutron and does not contadict the other requirements: first of all, it is the absence of the bound state of a dineutron. We demanded also a negative two-neutron scattering phase in the triplet state (effective repulsion) and the absence of resonances in the two-neutron system.

Thus, we showed that the binding conditions of a trineutron are unsatisfactory for a wide variety of interaction potentials in the states with $L = 0$ and $L = 1$ and mostly worse than those for a tetraneutron [9]. We can expect that the binding conditions will be better for heavier multineutron systems, first of all, an octaneutron ^8n. This tendence is mostly confirmed by rough oscillatory estimates.

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ВІДНОСНО УМОВ ЗВ'ЯЗУВАННЯ ТРЬОХ ФЕРМІОНІВ І НЕМОЖЛИВОСТІ ІСНУВАННЯ ТРИНЕЙТРОНА

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Р е з ю м е

Досліджено умови існування зв'язаного стану трьох ферміонів в стані з ненульовим орбітальним моментом $L = 1$ для широкого класу потенціалів взаємодії. Отримано, що для чисто притягувальних потенціалів взаємодії та стандартних потенціалів з притягуванням на більших відстанях і короткодійним відштовхуванням відсутнє зв'язування трьох ферміонів, якщо два з них не зв'язані. Тим більше відсутній зв'язок трьох нейтронів в цьому випадку, якщо параметри потенціалів узгоджені з низькоенергетичними даними двох нейтронів. Для потенціалів взаємодії з двома режимами притягування отримано можливість зв'язування трьох ферміонів з $L = 1$ у відсутності зв'язку між двома. В реалістичних умовах міжнейтронної взаємодії навіть з двома режимами притягування зв'язаний стан трьох нейтронів відсутній, на відміну від отриманого раніше зв'язку тетранейтрона.