ON THE NONLOCAL SYMMETRIES OF THE MAXWELL EQUATIONS AND THE CONSERVATION LAWS

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We have found two new unitary representations, relative to which the free Maxwell equations are invariant and which are determined by nonlocal generators of the universal covering Poincare group. It is shown that both representations, like a local one, describe a field with mass $m = 0$ and spin $s = 1$ (helicity $h = \mp 1$). We have proposed a Lagrangian in terms of the intensities $(\vec{E}, \vec{H}) = \vec{\mathcal{E}} \equiv \vec{E} - i\vec{H}$ of an electromagnetic field, which gives the physically adequate correspondence "symmetry generator – conservation law" using the usual method based on the Noether theorem. In this way, not appealing to potentials, we have obtained the standard series of 15 Poincare $(P_\mu, J_{\mu\nu})^{\rm loc}$ and conformal (D, K_μ) conservation laws as a result of the local conformal $C(1,3)$-symmetry of the free Maxwell equations and two sets $(P_\mu, J_{\mu\nu})^{\rm I,II}$ of basic dynamical variables — the consequences of the mentioned nonlocal symmetries, and $(P_\mu, J_{\mu\nu})^{\rm I} = (P_\mu, J_{\mu\nu})^{\rm loc}$. The quantities $P_\mu, J_{\mu\nu}$ are presented in terms of momentum-helicity amplitudes, and it is shown that the collection $(P_\mu, J_{\mu\nu})^{\rm I}$ requires the Bose-quantization of the field $\vec{\mathcal{E}}$, whereas the collection $(P_\mu, J_{\mu\nu})^{\rm II}$ requires the Fermi-quantization of this field. The last one is referred to a hypothetical massless particle with helicity $h = \mp 1$ obeyed the Fermi—Dirac statistics. Both types of quantization use the Fock space with a definite metric and satisfy the principle of microcausality. We propose to call the hypothetical particle under consideration as a Fermi-photon (F-photon).

1. Introduction

The present work is a sequel of the cycle of our works [1—8] aimed at the construction of quantum electrodynamics in terms of the intensities of an electromagnetic field.

We note that, as an electromagnetic field, we always mean the field of intensities in the real $(\vec{E}, \vec{H})$ or complex-valued $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$ forms (for the sake of convenience and in the work with irreducible representations, we deal with just the complex-valued form). Here, we do not involve the 4-vector-potential $A = (A^\mu)$ as an object of the electromagnetic field.

In [8], we have found a nonlocal representation of the universal covering $\mathcal{P}$ of the proper orthochronous Poincare group $\mathrm{P}_+^\uparrow$, relative to which not only the complete system of the Maxwell equations for a free electromagnetic field $(\vec{E}, \vec{H})$ is invariant, but its rot-subsystem as well. In [8, 9], the conservation laws associated with generators of the corresponding representation of this group were established. However, the generator $\widehat{p}_0$ in the established series of conservation laws is associated with a component of the Lipkin zilch-tensor [10, 11], rather than with the energy of the electromagnetic field which is set by a corresponding component of the tensor of energy-momentum of this field. Unsatisfactory is also the expression for the angular momentum of an electromagnetic field which was obtained in this series.

Earlier, we used different Lagrangians to determine the dynamical variables of the electromagnetic field [3—6] and found [7] the complete collection of first-order conservation laws (of the type of a Lipkin zilch-tensor [10, 11]) for this field without appealing to the Noether theorem. Here, we involve no different Lagrangians, but the different collections of symmetry operators. In this work, we use a single Lagrangian, for which the standard operators of the well-known local representation of the group $\mathcal{P}$ – the generators of 4-translations p_μ and 4-rotations $j_{\mu\nu}$ – correspond to just the standard (rather than zilch) conservation laws of energy, momentum, angular momentum, and boost angular momentum of the electromagnetic field by the Noether theorem. We have succeeded to obtain this result for the first time (the statement of the problem and its partial solution can be found in [3—6]). That is, we have obtained the adequate correspondence "symmetry operator → conservation law" on the basis of the Noether theorem. This allows us to associate the adequate conservation laws with the generators of the algebra of invariance also for new nonlocal symmetry operators introduced in the present work.

Here, we have found else two other (in addition to those in [8, 9]) nonlocal representations of the Poincare group. It is shown that just these representations are physically satisfactory: they correspond to the conservation laws which possess a clear physical interpretation. The Casimir operators of these two nonlocal $\mathcal{P}$-representations coincide with each other and, according to the Bargman—Wigner classification, are referred to a field with mass $m = 0$ and spin $s = 1$ (helicity $h = \mp 1$). For the first collection of nonlocal $\mathcal{P}$-generators, we have obtained, by using the Noether theorem, the same standard list of basic integral laws of conservation — the dynamical variables $(P_\mu, J_{\mu\nu})^{\rm I}$ as functionals of the field of intensities (i.e. the energy P_0, momentum $\vec{P}$, spatial angular momentum $\vec{J}$, and boost angular momentum $\vec{N}$), as for the local $\mathcal{P}$-representation. On the basis of the analysis in terms of the momentum-helicity amplitudes of the dynamical variables obtained, by the Noether theorem, for the other collection of nonlocal $\mathcal{P}$-generators, we obtained the collection $(P_\mu, J_{\mu\nu})^{\rm II}$, in which the energy $P_0^{\rm II}$ of a classical electromagnetic field is sign-indefinite, and the spatial $\vec{J}$ and boost $\vec{N}$ angular momenta coincide with the corresponding quantities for the first collection, i.e. they have standard form. This means that if the first collection $(P_\mu, J_{\mu\nu})^{\rm I}$ requires the Bose-quantization of the field of intensities, the second collection $(P_\mu, J_{\mu\nu})^{\rm II}$, in which the energy $P_0^{\rm II}$ is sign-indefinite, requires the Fermi-quantization of the field of intensities. We emphasize that, for both collections of generators, the spatial angular momentum $\vec{J}$ of the classical electromagnetic field coincides with the standard expression which was studied, in particular, in experiments [12].

As distinct from the standard local representation of the group $\mathcal{P}$, the nonlocal representations obtained by us are unitary.

2. Applied Notions, Notations, and Forms of Equations

We use the standard relativistic notations which are accepted, for example, in [13, 14]. In particular, we use a system of units, in which $\hbar = c = 1$. The Greek indices take values 0,1,2,3, the Latin ones are 1,2,3, and the metric tensor is chosen as $g^{\mu\nu} = g_{\mu\nu}$, $g \equiv (g^\mu_\nu) = \mathrm{diag}(+---)$.

By $\mathcal{P}$, we denote the universal covering of the proper orthochronous Poincare group $\mathrm{P}^\uparrow_+ = \mathrm{T}(4) \times \mathrm{L}^\uparrow_+$, and $\mathcal{L} = \mathrm{SL}(2,\mathrm{C})$ stands for the universal covering of the proper orthochronous Lorentz group $\mathrm{L}^\uparrow_+$.

We recall the well-known fact [15] that the system of the free Maxwell equations

$$\partial_0 \vec{E} = \mathrm{rot}\vec{\mathrm{H}}, \quad \partial_0 \vec{\mathrm{H}} = -\mathrm{rot}\vec{\mathrm{E}},$$

$$\mathrm{div}\vec{\mathrm{E}} = 0, \quad \mathrm{div}\,\vec{\mathrm{H}} = 0; \quad \partial_0 \equiv \frac{\partial}{\partial \mathrm{x}^0} \tag{1}$$

is invariant relative to a local representation of the group $\mathcal{P}$. However, the 6-component field $(\vec{E}, \vec{H})$ is transformed by the reducible $(0,1)\otimes(1,0)$ representation of the group $\mathcal{L} = \mathrm{SL}(2,\mathrm{C})$. We recall (see, e,g,, [1]), that the complex characteristic of the electromagnetic field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$ [16,17] is transformed by the irreducible $(0,1)$ representation of the group $\mathcal{L}$.

In these terms, the Maxwell equations are written as

$$\partial_0 \vec{\mathcal{E}} = i\mathrm{rot}\vec{\mathcal{E}}, \quad \mathrm{div}\vec{\mathcal{E}} = 0. \tag{2}$$

As seen, Eqs. (1) and (2) have a clearly noncovariant form. However, the covariant form of Eqs. (1) is well known, and Eq. (2) can be easily rewritten in a clearly covariant form. For this purpose, we define the complex-valued tensor

$$\mathcal{B} \equiv (\mathcal{B}^{\mu\nu}) : \mathcal{B}^{0j} = -\mathcal{B}^{j0} = \mathcal{E}^j, \quad B^{jk} = i\varepsilon^{jkl}\mathcal{E}^l \tag{3}$$

in terms of the object $\vec{\mathcal{E}} = (\mathcal{E}^j)$. This tensor is self-dual, namely

$$\varepsilon\mathcal{B}^{\mu\nu} \equiv \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\mathcal{B}_{\rho\sigma} = i\mathcal{B}^{\mu\nu} \leftrightarrow \varepsilon\vec{\mathcal{E}} \equiv \vec{H} + i\vec{E} = i\vec{\mathcal{E}}. \tag{4}$$

In terms of tensor (3), Eqs. (2) take the form

$$\partial_\nu \mathcal{B}^{\mu\nu} = 0; \quad \partial_\nu \equiv \frac{\partial}{\partial x^\nu}. \tag{5}$$

Equations (2), like (5), are invariant relative to a local representation of the group $\mathcal{P}$ generated by the irreducible (0,1)-representation of the group $\mathcal{L}$. This $\mathcal{P}$-representation is set by the generators

$$q_A : p_\mu = i\partial_\mu, \quad j_{\mu\nu} = m_{\mu\nu} + s_{\mu\nu}, \tag{6}$$

where

$$m_{\mu\nu} \equiv x_\nu p_\mu - x_\mu p_\nu, \tag{7}$$

the spin matrices $s_{\mu\nu} = -s_{\nu\mu}$ for the field $\vec{\mathcal{E}} = (\mathcal{E}^j)$ belong to the (0,1)-representation of the group $\mathcal{L}$ and have the form

$$s_{12} = \begin{vmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad s_{23} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{vmatrix},$$

$$s_{31} = \begin{vmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{vmatrix}, \quad s_{0j} = \frac{i}{2}\varepsilon^{jkl}s_{kl}, \tag{8}$$

In terms of tensor (3), they look as

$$s_{\mu\nu}\mathcal{B}_{\rho\sigma} \equiv (s_{\mu\nu}\mathcal{B})_{\rho\sigma} = i(g_{\mu\rho}\mathcal{B}_{\nu\sigma} + $$

$$+ g_{\rho\nu}\mathcal{B}_{\sigma\mu} + g_{\nu\sigma}\mathcal{B}_{\mu\rho} + g_{\sigma\mu}\mathcal{B}_{\rho\nu}). \tag{9}$$

As the parameters of the Poincare group, we chose real parameters, namely: 4-translations or shifts $a = (a^\mu)$ and the angles of rotations $\omega^{\mu\nu} = -\omega^{\nu\mu}$ in the planes $\mu\nu$. Therefore, the $\mathcal{P}$-generators associated with the indicated parameters are not the operators q_A (6), but the operators $-iq_A$. Generators (6) satisfy explicitly the covariant commutation relations of the group $\mathcal{P}$:

$$[p_\mu, p_\nu] = 0, \quad [p_\mu, j_{\rho\sigma}] = ig_{\mu\rho}p_\sigma - ig_{\mu\sigma}p_\rho,$$

$$[j_{\mu\nu}, j_{\rho\sigma}] = -i(g_{\mu\rho}j_{\nu\sigma} + g_{\rho\nu}j_{\sigma\mu} + g_{\nu\sigma}j_{\mu\rho} + g_{\sigma\mu}j_{\rho\nu}). \quad (10)$$

3. The Noetherian Way to the Determination of the Standard Conservation Laws for the Electromagnetic Field in Terms of Intensities

For the field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$, we will find the conservation laws which are consequences of the symmetry operators (6) and the application of the standard method of the Noether theorem. It is easy to see that the Lagrangian

$$L = \frac{i}{2}\{\bar{\vec{\mathcal{E}}}[\partial_0\vec{\mathcal{E}} - i\mathrm{rot}\vec{\mathcal{E}}] - \vec{\mathcal{E}}[\partial_0\bar{\vec{\mathcal{E}}} + i\mathrm{rot}\bar{\vec{\mathcal{E}}}]\},$$

$$\bar{\vec{\mathcal{E}}} = \vec{\mathcal{E}}^* \cdot (-\widehat{h}), \quad (11)$$

leads to the rot-subsystem of the Maxwell equations (2) and the conjugate equations equivalent to it. Here, $\widehat{h}$ is the operator of helicity,

$$\widehat{h} = \frac{\vec{s}\cdot\vec{p}}{\widehat{\omega}}, \quad \widehat{\omega} \equiv \sqrt{\vec{p}^{\,2}} = \sqrt{-\Delta}. \quad (12)$$

We note that, in order to obtain the rot-subsystem of the Maxwell equations on the basis of Lagrangian (11), it is sufficient to consider a complex conjugate quantity $\vec{\mathcal{E}} \rightarrow \bar{\vec{\mathcal{E}}} = \vec{\mathcal{E}}^*$ as the conjugate Lagrangian variable in (11). However, the Noetherian (i.e. by the Noether theorem) analysis of the dynamical variables of the field $\vec{\mathcal{E}}$, which was performed by us, convinces that only the conjugation $\vec{\mathcal{E}} \rightarrow \bar{\vec{\mathcal{E}}} = \vec{\mathcal{E}}^* \cdot (-\widehat{h})$ chosen in formula (11) ensures the adequate correspondence of the $\mathcal{P}$-generators (6) to the well-known conservation laws (of energy, momentum, spatial and boost angular momenta) for the electromagnetic field. The specific conjugation $\vec{\mathcal{E}} \rightarrow \bar{\vec{\mathcal{E}}} = \vec{\mathcal{E}}^* \cdot (-\widehat{h})$ in (11) is chosen on the basis of reasonings analogous to those which are inherent in the Dirac theory of a spinor field (we recall the known relation $\bar{\Psi} = \Psi^\dagger\gamma^0$).

In connection with the use of pseudo-differential operators of the type $\widehat{\omega} \equiv \sqrt{\vec{p}^{\,2}} = \sqrt{-\Delta}$ and their functions in (12) and especially further in nonlocal representations of the group $\mathcal{P}$, we give the explanation in order to guarantee the mathematical correctness of the proposed consideration. The mathematical correctness requires to indicate the set including the object of an equation which is Eq. (2) in this case. The further analysis will prove that the mathematical correctness of the consideration is ensured by the choice of the Schwartz space of generalized functions $\mathbf{S}^{3*} = (\mathbf{S}(\mathrm{M}(1,3)) \times \mathbf{C}^3)^*$, where $\mathbf{S}^3 = \mathbf{S}(\mathrm{M}(1,3)) \times \mathbf{C}^3$ is the space of test Schwartz functions of points $x \in \mathrm{M}(1,3)$ in the Minkowski space-time. (Here, the subscript * means the conjugation by the topology of the Schwartz space of generalized functions). We recall that the space $\mathbf{S}^3 = \mathbf{S}(\mathrm{M}(1,3)) \times \mathbf{C}^3$ is compact in the space $\mathbf{S}^{3*} = (\mathbf{S}(\mathrm{M}(1,3)) \times \mathbf{C}^3)^*$. Therefore, it is expedient to choose it as the joint domain of definition of all the below-considered operators, including the pseudo-differential operators (12). In this case, it turns out that the space $\mathbf{S}^3 = \mathbf{S}(\mathrm{M}(1,3)) \times \mathbf{C}^3$ is simultaneously the range of values, firstly, of all operators used here and those series of various products of these operators, whose numerical coefficients are determined by functions convergent in the space $\mathbf{S}^3 = \mathbf{S}(\mathrm{M}(1,3)) \times \mathbf{C}^3$. Therefore, we will construct the specific algebraic structures from the given operators and their action on $\vec{\mathcal{E}} \in \mathbf{S}^{3*}$ in the set $\mathbf{S}^3 \subset \mathbf{S}^{3*}$. To avoid the complication by excessive mathematical details, we will not use the functional form of generalized functions $\mathbf{S}^{3*}$, as it is in the standard axiomatic approaches (see, e.g., [18, 19]).

The set of solutions of the rot-equations (the set of extremals of Lagrangian (11)) includes the general solution $\vec{\mathcal{E}}^{\mathrm{tr}}(x)$ of the free equations (2) (a superposition of transverse waves), as well as the additional static set of solutions $\vec{\mathcal{E}}^{\mathrm{lon}}(\vec{x})$ (a superposition of longitudinal waves). These solutions in terms of momentum-helicity amplitudes look as

$$\vec{\mathcal{E}}^{\mathrm{tr}}(x) = \frac{1}{(2\pi)^{3/2}}\int d^3k(c^1(\vec{k})e^{-ikx} + c^{*2}(\vec{k})e^{ikx})\vec{e}_1(\vec{k}),$$

$$kx \equiv \omega t - \vec{k}\vec{x}, \quad (13)$$

$$\vec{\mathcal{E}}^{\mathrm{lon}}(\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3kc^3(\vec{k})e^{i\vec{k}\vec{x}}\vec{e}_3(\vec{k}). \quad (14)$$

Here, $c^{1,2}(\vec{k})$ are the momentum-helicity amplitudes of transverse electromagnetic waves, $c^3(\vec{k})$ is the amplitude of a static longitudinal wave, and the 3-component functions $\left\{\vec{e}_1(\vec{k}), \vec{e}_2(\vec{k}), \vec{e}_3(\vec{k})\right\}$ are the eigenvectors of the quantum-mechanical operator of helicity h:

$$h = \vec{s}\cdot\vec{k}/\omega, \quad h\vec{e}_j(\vec{k}) = \lambda\vec{e}_j(\vec{k}),$$

$$\lambda = \mp 1, 0, \quad j = 1, 2, 3. \tag{15}$$

The explicit form of unit vectors $\left\{\vec{e}_1(\vec{k}), \vec{e}_2(\vec{k}), \vec{e}_3(\vec{k})\right\}$ is as follows:

$$\vec{e}_1 = \frac{1}{\sqrt{2}}(\vec{R}_1 - i\vec{R}_2), \quad \vec{e}_2 = \vec{e}_1{}^* = \frac{1}{\sqrt{2}}(\vec{R}_1 + i\vec{R}_2),$$

$$\vec{e}_3 = \vec{R}_3 = \frac{\vec{k}}{\omega}, \tag{16}$$

where the unit vectors

$$\vec{R}_1 = \frac{1}{\sqrt{k^1k^1 + k^2k^2}} \begin{pmatrix} k^2 \\ -k^1 \\ 0 \end{pmatrix},$$

$$\vec{R}_2 = \frac{1}{\omega\sqrt{k^1k^1 + k^2k^2}} \begin{pmatrix} k^1k^3 \\ k^2k^3 \\ -k^1k^1 - k^2k^2 \end{pmatrix}, \vec{R}_3 \equiv \frac{\vec{k}}{\omega} \tag{17}$$

form the basis of a local frame. In this case, the vectors $\vec{R}_1$, $\vec{R}_2$ define the planes of plane-polarized electromagnetic waves normal to the momentum $\vec{k}$ (to the wave vector $\vec{k}$ of a plane-polarized electromagnetic wave).

We now give the explicit form of the Casimir operators for the local $\mathcal{P}$-generators of $\mathcal{L}$-irreducible (0,s) or (s,0) fields (in particular, for generators (6)):

$$p^2 \equiv p^\mu p_\mu = -\partial^\mu \partial_\mu,$$

$$W = w^\mu w_\mu = \vec{s}^2 p^2 = -s(s+1)\partial^\mu\partial_\mu, \tag{18}$$

where w^μ is the Pauli—Lubanski vector,

$$w^\mu \equiv -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} j_{\nu\rho} p_\sigma \to w^0 = \vec{j}\cdot\vec{p} = \vec{s}\cdot\vec{p},$$

$$\vec{j} \equiv (j_{23}, j_{31}, j_{12}). \tag{19}$$

The additional Casimir operator for a free field $\vec{\mathcal{E}}$ is

$$w_0 = \vec{s}\cdot\vec{p} \leftrightarrow \widehat{h} = \frac{\vec{s}\cdot\vec{p}}{\widehat{\omega}}. \tag{20}$$

As known, the field with zero mass satisfies, by definition, the equation

$$p^2\vec{\mathcal{E}} \equiv -\partial^\mu\partial_\mu\vec{\mathcal{E}} = 0, \tag{21}$$

and the imposition of the $\mathcal{P}$-invariant condition (21) additionally to the rot-subsystem of the Maxwell equations (2) is equivalent, as it is easy to see, to the imposition of the condition

$$\mathrm{div}\vec{\mathcal{E}} = 0 \tag{22}$$

which nullifies (14) and, thus, ensures the transversality of the free electromagnetic field. In other words, the imposition of condition (21) or (22) cuts the longitudinal solutions (14) from the set of solutions $\{\vec{\mathcal{E}}^{\mathrm{tr}}\} \cup \{\vec{\mathcal{E}}^{\mathrm{lon}}\}$ of the Maxwell rot-equations. Thus, a free electromagnetic field can be alternatively called the field which is described by the full system of the Maxwell equations (2) or the field which is described by Lagrangian (11) (i.e. by the rot-subsystem of Eqs. (2)) and the additional $\mathcal{P}$-invariant condition (21).

The formula of the Noether theorem, which is a consequence of the local $\mathcal{P}$-invariance of both the theory of a free electromagnetic field and Lagrangian (11), looks as

$$q_A \to Q_A = \frac{1}{2}\int d^3x \left(\bar{\vec{\mathcal{E}}}(x) q_A \vec{\mathcal{E}}(x)\right) + \text{Herm.con.}, \tag{23}$$

where $q_A = (p_\mu, j_{\mu\nu})$ is the collection of generators (6). By formula (23) for ten operators (6), we get the list of the conservation laws, namely the dynamical variables $(P_\mu, J_{\mu\nu})$ as functionals of the field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$. The physical content of these dynamical variables becomes clear if we express them, for example, in terms of the quantum-mechanical momentum-helicity amplitudes $c^1(\vec{k})$, $c^2(\vec{k})$, whose fixed form defines (by formula (13)) a fixed state of the classical electromagnetic field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$. The substitution (13) and (6) in (23) leads to the following result for $Q_A = (P_\mu, J_{\mu\nu})$:

$$q_A \to Q_A = \int d^3k \mathcal{C}^\dagger \tilde{q}_A \mathcal{C}, \quad \mathcal{C} \equiv \begin{pmatrix} c^1(\vec{k}) \\ c^2(\vec{k}) \end{pmatrix}. \tag{24}$$

Here, c^1, c^2 are the momentum-helicity amplitudes, and the operators $\tilde{q}_A$ (the densities of Q_A in the space $\{\mathcal{C}\}$) are the images of the generators $q_A = (p_\mu, j_{\mu\nu})$ (6) in the Hilbert space $\{\mathcal{C}\}$ of the collection of quantum-mechanical momentum-helicity amplitudes of a photon (i.e. in the space $\mathbf{H_1} = \{\mathcal{C}\}$ of one-photon quantum-mechanical states). Their explicit form is as follows:

$$\tilde{p}_0 = \omega, \quad \tilde{p}_l = k_l, \quad \tilde{j}_{kl} = \tilde{m}_{kl} + \tilde{s}_{kl}, \quad \tilde{j}_{0l} = \tilde{m}_{0l} + \tilde{s}_{0l}, \tag{25}$$

where

$$\tilde{m}_{jl} = \tilde{x}_j k_l - \tilde{x}_l k_j, \quad \tilde{m}_{0l} = -\frac{1}{2}\{\tilde{x}_l, \omega\}, \quad \tilde{x}_l \equiv -i\frac{\partial}{\partial k^l},$$

$$\tilde{s}_{jl} = \varepsilon^{jlm}\tilde{s}^m, \quad (\tilde{s}^m) \equiv \vec{\tilde{s}} = \frac{\omega}{k^1k^1 + k^2k^2}(-k^1, k^2, 0),$$

$$(-\tilde{s}_{0l}) = \frac{k^3}{k^1k^1 + k^2k^2}(k^2, k^1, 0). \tag{26}$$

Theorem 1. *The operators* $(\tilde{p}_\mu,\ \tilde{j}_{\mu\nu})$ *given by formulas (25) and (26) in the quantum-mechanical Hilbert space* $\{\mathcal{C}\}$ *satisfy the explicitly covariant commutation relations (10) for* $\mathcal{P}$*-generators of a local representation of the group* $\mathcal{P}$*. The operators of the dynamical variables* $Q_A = (P_\mu, J_{\mu\nu})$ *(23) of a quantized electromagnetic field* $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$ *which are given by (24) and (25), in which the amplitudes* $c^1(\vec{k})$ *and* $c^2(\vec{k})$ *satisfy the commutation relations*

$$\left[c^r(\vec{k}), c^{\dagger r'}(\vec{k}')\right] = \delta_{rr'}\delta(\vec{k}-\vec{k}'), r, r' = 1, 2, \qquad (27)$$

satisfy the commutation relations (10) in the Fock space of a quantized electromagnetic field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$.

Proof. The validity of the assertions of Theorem 1 can be verified by the direct calculation of the commutation relations (10) with the use of the Wick theorem firstly for the operators $(\tilde{p}_\mu, \tilde{j}_{\mu\nu})$, and then, by using the obtained commutators and (27), for the operators $Q_A = (P_\mu, J_{\mu\nu})$. In this case, it is clear that the integral dynamical variables $P_\mu, J_{\mu\nu}$, as the functionals of the quantized field $\vec{\mathcal{E}}$, and also their products (in particular, in commutation relations) are redefined as the corresponding normal products of the operators of creation $c^{\dagger r}(\vec{k})$ and annihilation $c^r(\vec{k})$, q.e.d.

The proved theorem means, in particular, that the Noether theorem for Lagrangian (11) (i.e. formula (23)) gives the adequate, physically satisfactory correspondence "symmetry operator $q_A \to$ conservation law Q_A". Indeed, the dynamical variables $P_\mu, J_{\mu\nu}$, which are the consequences of the Noether theorem and Lagrangian (11), satisfy the commutation relations (10) for the group $\mathcal{P}$, like the generators of this group. This becomes obvious after the quantization of the field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$ (by the Bose-quantization (27) of momentum-helicity amplitudes). Moreover, the obtained dynamical variables $P_\mu, J_{\mu\nu}$ have clear physical content of the well-known laws of conservation of the energy, momentum, spatial and boost angular momenta of the (classical or quantized) electromagnetic field.

For comparison, we present the list of the conservation laws for the electromagnetic field (the basic dynamical variables $P_\mu, J_{\mu\nu}$) which is a consequence of such properties of the energy-momentum tensor of a free electromagnetic field as the symmetry and the zero trace (see, e.g., [20]). This tensor in terms of the self-dual tensor (3) takes the form

$$T^{\mu\nu} \equiv \frac{1}{4}\left(\mathcal{B}^{*\mu\alpha}\,\mathcal{B}^{\nu}_{\alpha} + \mathcal{B}^{\mu\alpha}\,\mathcal{B}^{*\nu}_{\alpha}\right). \qquad (28)$$

The mentioned collection of dynamical variables is given by the formulas

$$P_\rho = \int d^3x T_{\rho 0},$$

$$J_{\rho\sigma} = \int d^3x(x_\rho T_{\sigma 0} - x_\sigma T_{\rho 0}) \equiv \int d^3x \mathbf{m}_{\rho\sigma}(x), \qquad (29)$$

where

$$T_{00} = \frac{1}{2}\vec{\mathcal{E}}^* \cdot \vec{\mathcal{E}} = \frac{1}{2}\left(\vec{E}^2 + \vec{H}^2\right) \equiv \mathbf{p}_0(x),$$

$$(T^{l0}) = \frac{1}{2}\vec{\mathcal{E}}^* \times \vec{\mathcal{E}} = \vec{E}\times\vec{H} \equiv \vec{\mathbf{p}}(x),$$

$$(\mathbf{m}_{23}, \mathbf{m}_{31}, \mathbf{m}_{12}) = \vec{x}\times\vec{\mathbf{p}}, \qquad \mathbf{m}^{0l} = x_0\vec{\mathbf{p}} - \vec{x}\mathbf{p}_0. \qquad (30)$$

Substituting formula (13) for $\vec{\mathcal{E}}$ in (29) and (30), we get (on the proper normalization of the amplitudes $c^1(\vec{k}), c^2(\vec{k})$) the expressions for the dynamical variables $P_\mu, J_{\mu\nu}$ of the electromagnetic field. They coincide with (24), (25), and (26) obtained on the basis of Lagrangian (11), the Noether theorem, and the local $\mathcal{P}$-invariance of the field $\vec{\mathcal{E}}$.

We recall that the conservation laws (29) and (30) were obtained in [21] as consequences of the Poincare-symmetry of the equations of the electromagnetic field in terms of 4-potentials $A = (A^\mu)$ and the Noether theorem with the use of a Lagrangian in these terms. In [4], we paid attention to the fact that, for the electromagnetic field in terms of intensities $(\vec{E}, \vec{H})$, the conservation laws [21] are not direct consequences of the symmetry of the Maxwell equations (1) or (2), the Noether theorem, and some Lagrangian in terms of $(\vec{E}, \vec{H})$ as variational variables. In order to associate these conservation laws with a field $(\vec{E}, \vec{H})$ by using the Noether theorem, it is necesary to use (i) the symmetries of the Maxwell equations (1) or (2) (rather than the symmetries of the d'Alembert equation for $A = (A^\mu)$), (ii) a Lagrangian, where the variational variables are the intensities (rather than the potentials).

Just such a program is executed in this section. Thus, the laws of conservation of the energy, momentum, spatial and boost angular momenta of the field are first obtained here as consequences of both the local Poincare-symmetry of the Maxwell equations and the Lagrangian approach in terms of intensities. For the field $(\vec{E}, \vec{H})$ on the basis of the Noether theorem and the Lagrangian in terms of $(\vec{E}, \vec{H})$, we obtain, for the first time, the physically adequate correspondence "the generators of 4-translations and 4-rotations" → "conservation laws of energy, momentum, and 4-angular momentum of the electromagnetic field". That is, we establish the standard connection between the homogeneity and isotropy of the space-time and the conservation laws just for the electromagnetic field in terms of intensities. We note that the electromagnetic field can be directly observed in experiments even on the classical level (it is almost the single observable field besides, possibly, the gravitational one).

We recall that the maximal algebra of invariance of the Maxwell equations (2) in the class of Lie operators is the 16-dimensional algebra $C(1,3) \oplus \widehat{\varepsilon}$, where $C(1,3)$ is the algebra of the 15-parametric group of conformal transformations, and $\widehat{\varepsilon}$ is the Heaviside—Larmor—Rainich duality transformation:

$$\widehat{\varepsilon} : \vec{E} \to \vec{H}, \vec{H} \to -\vec{E}. \tag{31}$$

The generators of the local $C(1,3)$-representation for the field $\vec{\mathcal{E}}$ include the collection of $\mathcal{P}$-generators (6) added by the generators of dilations and properly conformal transformations which are expressed [22, 23] through $\mathcal{P}$-generators (6):

$$d = x^\mu p_\mu + i, \tag{32}$$

$$k_\mu = 2x_\mu d - x^\nu x_\nu p_\mu + 2x^\nu s_{\mu\nu}, \tag{33}$$

where d is given in (32), and $s_{\mu\nu}$ in (8) and (9).

The conformal $C(1,3)$-generators (32), (33) generate, by formula (23), the laws of conservations

$$d \to D = \frac{1}{2}\int d^3x \left(\bar{\vec{\mathcal{E}}}(x) d\vec{\mathcal{E}}(x)\right) + \text{Herm.con.}, \tag{34}$$

$$k_\mu \to K_\mu = \frac{1}{2}\int d^3x \left(\bar{\vec{\mathcal{E}}}(x) k_\mu \vec{\mathcal{E}}(x)\right) + \text{Herm.con.} \tag{35}$$

Substituting relation (13) for $\vec{\mathcal{E}}(x) = \vec{\mathcal{E}}^{\rm tr}(x)$ in (34) and (35), we obtain the Poincare dynamical variables (24)—(26) and the complete list of 15 conformal conservation laws in terms of the momentum-helicity amplitudes calculated by the standard method of the Noether theorem for the field $\vec{\mathcal{E}}$.

In terms of the complex-valued field $\vec{\mathcal{E}}$, the generator of transformation (31) (associated with the charge as a real quantity) is the operator $(-i\widehat{\varepsilon})$, i.e.

$$-i\widehat{\varepsilon}\vec{\mathcal{E}} = 1 \cdot \vec{\mathcal{E}}. \tag{36}$$

The charge conservation law calculated by formula (23) has the form

$$\widehat{\varepsilon} \to Q_{\widehat{\varepsilon}} = \frac{1}{2}\int d^3x \left(\bar{\vec{\mathcal{E}}}(x) \widehat{\varepsilon} \vec{\mathcal{E}}(x)\right) + \text{Herm.con.} \equiv 0. \tag{37}$$

This formula demonstrates the fact that the charge of a complex-valued electromagnetic field $\vec{\mathcal{E}}$ is strictly equal to zero.

The 32-dimensional algebra $C(1,3) \oplus C(1,3) \oplus \widehat{\varepsilon}$, which was established in [3] and used in [4—6], is the maximal algebra of invariance of the free Maxwell equations in the class of matrix-differential first-order operators and does not lead, according to formula (23), to any conservation laws which would be different from those obtain above.

4. New Nonlocal Symmetries of the Free Maxwell Equations and Conservation Laws

It was shown in [8] that the Maxwell equations (2) are also invariant relative to a nonlocal representation of the group $\mathcal{P}$, whose generators look as

$$\widehat{p}_0 = \widehat{\omega}, \quad \widehat{p}_l = p_l = i\partial_l, \quad \widehat{j}_{kl} = j_{kl} = m_{kl} + s_{kl},$$

$$\widehat{j}_{0l} = -x_0 p_l \widehat{h} - \frac{1}{2}\left\{x_l, \widehat{\omega}\right\} + \widehat{s}_l, \tag{38}$$

where s_{kl} are the same as those in (8), m_{kl} coincide with (7) for $\mu\nu{=}kl$,

$$\widehat{s}_l = \frac{(\vec{s} \times \vec{p})^l}{\widehat{\omega}}, \quad \vec{s} \equiv (s_{23}, s_{31}, s_{12}), \tag{39}$$

and $\widehat{h}$ and $\widehat{\omega}$ are given in (12).

In [8, 9], the conservation laws, namely the dynamical variables $(P_\mu, J_{\mu\nu})$ generated by (39) by formula (23), where the ordinary complex conjugation was used instead of the special conjugation introduced here by us, were established. However, the correspondence $q_A \to Q_A$ (symmetry operator $\to$ conservation law) obtained in such a way turns out to be physically unsatisfactory even for the local $\mathcal{P}$-generators. In particular, the operators $P_\mu, J_{\mu\nu}$ obtained by such a method do not satisfy Theorem 1. The substitution of the nonlocal $\mathcal{P}$-generators (38) in (23) with special conjugation gives also a physically unsatisfactory correspondence $q_A \to Q_A$. Below, we will obtain two new collections of physically satisfactory nonlocal $\mathcal{P}$-generators.

Theorem 2. *Two following collections of nonlocal* operators,

$$p_0^{\rm I} = -\widehat{\omega}\widehat{h}, \quad \widehat{p}_l^{\rm I} = i\partial_l = p_l, \quad j_{kl}^{\rm I} = j_{kl},$$

$$j_{0l}^{\rm I} = x_0 p_l + \frac{1}{2}\left\{x_l, \widehat{\omega}\right\}\widehat{h} - \widehat{s}_l \widehat{h}, \tag{40}$$

$$p_0^{\rm II} = \widehat{\omega}, \quad \widehat{p}_l^{\rm II} = -\widehat{h} i\partial_l, \quad j_{kl}^{\rm II} = j_{kl}^{\rm I}, \quad j_{0l}^{\rm II} = j_{0l}^{\rm I}, \tag{41}$$

where $j_{kl}, \widehat{h}, \widehat{\omega}, \widehat{s}_l$ *are set, respectively, in (6), (12), and (39), satisfy the commutation relations (10) for* $\mathcal{P}$*-generators in the covariant form and are the transformations of invariance of the complete system of the free Maxwell equations (2) (and even of the rot-subsystem of the free Maxwell equations). The Casimir operators for generators (40) and (41) coincide and look as*

$$\left(p^{(\mathrm{I,II})}\right)^2 = \left(p_0^{(\mathrm{I,II})}\right)^2 - \left(\vec{p}^{(\mathrm{I,II})}\right)^2 \equiv 0,$$

$$W = w^\mu w_\mu \equiv 0, \quad w_0 = \vec{s}\cdot\vec{p} = \widehat{h}\widehat{\omega}. \qquad (42)$$

By formula (23) of the Noether theorem, the collections of generators (40) and (41) give two physically satisfactory, but fundamentally different collections $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$, $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$ *of the dynamical variables of the electromagnetic field. The collection* $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$ *coincides with the collection* $(P_\mu, J_{\mu\nu}) \in Q_A$ *(24), (25). As for the collection* $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$*, we have* $J^{\mathrm{II}}_{\mu\nu} = J^{\mathrm{I}}_{\mu\nu}$*, whereas the energy and momentum in the classical theory differ significantly from (24) and (25) and take the form*

$$P_0^{\mathrm{II}} = \int d^3k\omega\left(\left|c^1\right|^2 - \left|c^2\right|^2\right) \equiv \int d^3k\omega\mathcal{C}^\dagger\sigma^3\mathcal{C},$$

$$P_l^{\mathrm{II}} = \int d^3k k_l\left(\left|c^1\right|^2 - \left|c^2\right|^2\right) \equiv \int d^3k k_l\mathcal{C}^\dagger\sigma^3\mathcal{C}. \qquad (43)$$

As distinct from the collection $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$ *which satisfies Theorem 1, the collection of the operators* $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$ *satisfies the commutation relations (10) in the Fock space if the momentum-helicity amplitudes as operators in this space satisfy the anticommutation relations*

$$[c^r(\vec{k}), c^{\dagger r'}(\vec{k}')]_+ = \delta_{rr'}\delta(\vec{k}-\vec{k}'), \qquad (44)$$

where $[A, B]_\mp \equiv AB \mp BA$*, and* $r, r' = 1, 2$.

Proof. Theorem 2 is proved by the direct calculation of commutators (10) for each of the collections of generators (40) and (41) and by the calculation of the commutator of each generator from (40) and (41) with the operators of the Maxwell equations (2) and their rot-subsystem. The explicit form of the collections $(P_\mu, J_{\mu\nu})^{\mathrm{I,II}}$ of the dynamical variables of the electromagnetic field is calculated by formula (23) for the generators q_A, respectively, from the collections (40) and (41) with the substitution of the general solution $\vec{\mathcal{E}}(x) = \vec{\mathcal{E}}^{\mathrm{tr}}(x)$ (13) of the free Maxwell equations (2) in (23). The collection of the dynamical variables $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$ satisfies the $\mathcal{P}$-table (10), because it coincides with the local collection $(P_\mu, J_{\mu\nu}) \in Q_A$ (24), (25). As for the commutation relations for the dynamical variables $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$ in the Fock space, they are proved with the use of (44). In this case, the integral dynamical variables $(P_\mu, J_{\mu\nu})^{\mathrm{I,II}}$ and their products (in particular, in the commutation relations for each collection) are redefined as normal products of Bose- or, respectively, Fermi-operators $c^{\dagger r}(\vec{k}), c^r(\vec{k})$. In this case, we used the Wick theorem in the proof of the relevant equalities. q.e.d.

We note that, as distinct from the generators of the local representation (6), all the generators of collections (40) and (41) are presented in terms of the Hermitian operators $\vec{x}, \vec{p} = -i\nabla$, $\vec{s} = (s_{23}, s_{31}, s_{12})$ (8), $\widehat{s}_l$ (39) and are Hermitian too. Therefore, each of the representations of the group $\mathcal{P}$, which are set by two realizations (40) and (41), is unitary. For example, due to the Hermitian property of operators (40) and (41), the exponential series

$$U(a,\omega) = \exp(ia^\rho p_\rho^{\mathrm{I,II}} + \frac{i}{2}\omega^{\rho\sigma} j_{\rho\sigma}^{\mathrm{I,II}}) \qquad (45)$$

is an operator convergent in $\mathbf{S}^3 \subset \mathbf{S}^{3*}$, which can be extended by the known method (see, e.g., [19]) to a unitary operator in the whole $\mathbf{S}^{3*}$. Thus, the $\mathcal{P}$-representations, which are given by formulas in (45), are unitary groups of invariance of Eq. (2).

Based on the unitarity of the representations which are realized by generators (40) and (41), the explicit form of the Casimir operators (42) for these realizations, and the Bargman—Wigner classification, we may draw the following conclusion. Both $\mathcal{P}$-representations generated by (40) and (41) are referred to the field with mass $m = 0$ and helicity ∓ 1. Nevertheless, the $\mathcal{P}$-representations, which are realized by the collections of operators (40) and (41), are different in the sense that their consequences are different collections of dynamical variables $(P_\mu, J_{\mu\nu})^{\mathrm{I,II}}$ of the field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$ which require different procedures of quantization: the Bose-quantization for the first collection and the Fermi-quantization for the second one.

We note that the unitary $\mathcal{P}$-representations, which are set by the nonlocal generators (40) and (41), are induced, in fact, by $\mathcal{P}$-representations in the space $\{\vec{\mathcal{E}}\} = \mathbf{S}^{3*}$ (see, for comparusion, induced representations for a spinor field [14]). For $\mathcal{P}$-generators (6) of a local $\mathcal{P}$-representation, the induced $\mathcal{P}$-representation is set by the generators of collection (40). An analog of generators (6) of the local $\mathcal{P}$-representation for the second induced $\mathcal{P}$-representation (41) are quasilocal operators

$$\widehat{p}_0 = -\widehat{h}i\partial_0, \quad \widehat{p}_j = -\widehat{h}i\partial_j, \quad \widehat{j}_{kl} = j_{kl}, \quad \widehat{j}_{0l} = j_{0l}, (46)$$

where $\widehat{h}$ is the helicity operator, and j_{kl} and j_{0l} are the same as those in (6). The operators $\widehat{q}_A$ (46) satisfy the commutation relations (10) for $\mathcal{P}$-generators, set the transformation of invariance of the Maxwell equations (2), and give, by the Noether formula (23) the same collection $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$ of dynamical variables of the field $\vec{\mathcal{E}}$ as generators (41) of the second induced $\mathcal{P}$-representation.

We emphasize that both the standard Bose-quantization of the field $\vec{\mathcal{E}}$ which uses the collection $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$ of dynamical variables and the Fermi-quantization of the field $\vec{\mathcal{E}}$ which uses the collection $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$ lead to the same relation for the corresponding commutators (anticommutators) of Heisenberg quantized fields. The direct calculations show that, in terms of the covariant complex-valued field $\mathcal{B}^{\mu\nu}$, these commutators (both for a Bose-quantized field and the Fermi-quantized field $\widehat{\mathcal{B}}^{\mu\nu}$) have the form standard for a free electromagnetic field:

$$\left[\widehat{\mathcal{B}}_{\mu\nu}(x), \widehat{\mathcal{B}}_{\rho\sigma}(x')\right]_{\mp} = \left[\widehat{\mathcal{B}}^{\dagger}_{\mu\nu}(x), \widehat{\mathcal{B}}^{\dagger}_{\rho\sigma}(x')\right]_{\mp} = 0,$$

$$\left[\widehat{\mathcal{B}}_{\mu\nu}(x), \widehat{\mathcal{B}}^{\dagger}_{\rho\sigma}(x')\right]_{\mp} = 2i[g_{\mu\rho}\partial_\nu\partial_\sigma - g_{\mu\sigma}\partial_\nu\partial_\rho + g_{\nu\sigma}\partial_\mu\partial_\rho -$$

$$-g_{\nu\rho}\partial_\mu\partial_\sigma + i\varepsilon_{\mu\nu\alpha\beta}(\delta^\alpha_\rho\delta^\beta_\sigma - \delta^\alpha_\sigma\delta^\beta_\rho)]D_0(x - x'), \qquad (47)$$

where

$$D_0(x) \equiv \frac{i}{(2\pi)^3}\int \frac{d^3k}{2\omega}\left(e^{-ikx} - e^{ikx}\right) \qquad (48)$$

is the Pauli—Jordan commutator function for the field with zero mass.

5. Conclusions

In the present work, we have obtained two main results.

The first result consists in the determination of the conservation laws for an electromagnetic field in terms of intensities on the basis of the Lagrangian introduced here in these terms, the Noether theorem, and the generators of the local geometric symmetries of the Maxwell equations. We have first found such a Lagrangian, for which the Noetherian correspondences between the symmetry operators and the conservation laws are physically adequate. That is, by using the Noether theorem, we have first obtained the following correspondences: between the invariance relative to time translations and the conservation law for the energy P_0 of the field $(\vec{E}, \vec{H})$ (to be more exact, of the irreducible electromagnetic field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$), between the invariance relative to spatial translations and the conservation law for the momentum $\vec{P}$ of the field $\vec{\mathcal{E}}$, between the invariance relative to spatial rotations and the conservation law for the spatial angular momentum $\vec{J}$ of the field $\vec{\mathcal{E}}$, between the invariance relative to purely Lorentz space-time rotations and the conservation law for boost angular momentum $\vec{N}$ of the field $\vec{\mathcal{E}}$. Analogous adequate correspondences are obtained also for conformal symmetries. Thus, the problem solved in [21] for the field of potentials A^μ (the completely different initial object) is first solved for the field of intensities $\vec{\mathcal{E}}$ as an initial object of the electromagnetic field. The nontriviality of this problem was discussed in [3—6], where it was solved only partially.

The second result is the derivation of two dynamically nonequivalent collections $(p_\mu, j_{\mu\nu})^{\mathrm{I,II}}$ (40) and (41) of nonlocal $\mathcal{P}$-generators which generate two physically nonequivalent unitary representations (45) of the group $\mathcal{P}$ as a group of invariance of the Maxwell equation (2) for the classical irreducible electromagnetic field $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$. Moreover, the analysis by the Bargman—Wigner classification proves that each of these collections $(p_\mu, j_{\mu\nu})^{\mathrm{I,II}}$ is referred to the field with mass $m = 0$ and spin $s = 1$ (helicity $h = \mp 1$). For both collections $(p_\mu, j_{\mu\nu})^{\mathrm{I,II}}$ of nonlocal $\mathcal{P}$-generators given by (40) and (41), we have found, by using the Noether formula (23) (which is physically adequate and is verified for the local generators of the standard $\mathcal{P}$-symmetry), two series of basic conservation laws, namely the dynamical variables $(P_\mu, J_{\mu\nu})^{\mathrm{I,II}}$ of the field $\vec{\mathcal{E}}$. It is important that the dynamical variables $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$, which were obtained as the Noetherian consequences of the nonlocal generators $(p_\mu, j_{\mu\nu})^{\mathrm{I}}$ (40), coincide with the dynamical variables $(P_\mu, J_{\mu\nu})$ (24)—(26) obtained as the Noetherian consequences of the local generators $(p_\mu, j_{\mu\nu})$ (6) of the standard $\mathcal{P}$-symmetry. Each of the quantities of the series $(P_\mu, J_{\mu\nu})^{\mathrm{I,II}}$ is given in terms of the momentum-helicity amplitudes, which proves the physical nonequivalency of these two collections. As seen from (24), (25), and (43), the energy P_0^{I} of the classical field $\vec{\mathcal{E}}$ is sign-definite in the collection $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$. But, in the collection $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$, the energy P_0^{II} of this field is sign-indefinite (see formula (43)). Therefore (see Theorem 2), the collection $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$ requires the Fermi-quantization of the field $\vec{\mathcal{E}}$. Moreover, this quantization, like the Bose-quantization of the field $\vec{\mathcal{E}}$ which uses the collection $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$, is causal (see formulas (47)).

Thus, in the Fock space $\mathbf{H}^{\mathrm{F}}$ with the definite metric for a free quantum field $\vec{\mathcal{E}}$, the subspace

$\mathbf{H}^{\mathrm{SF}} \subset \mathbf{H}^{\mathrm{F}}$ of symmetric states describes the states of an arbitrary system of standard photons (as Bose-particles, B-photons), whereas the subspace $\mathbf{H}^{\mathrm{AsF}} \subset \mathbf{H}^{\mathrm{F}}$ of antisymmetric states describes particles with mass $m = 0$ and spin $s = 1$ (helicity $h = \mp 1$) which obey the Fermi—Dirac statistics. We propose to call these additional particles as Fermi-photons or F-photons, by clearly understanding that they are only the consequences of the additional theoretical model proposed here, i.e., they are hypothetical particles.

In the present work, we have comprehensively considered two possibilities of the quantization for just a free electromagnetic field. We are based on the fact that the theory of the free electromagnetic field has not only a purely heuristic, but also practical value: indeed, all the fields follow asymptotically (at the infinity in the Minkowski space $M(1,3)$) to the states of free fields. We have not analyzed the reason, for which such exotic particles, as F-photons, have not been observed till now. We only indicate that, as it will be shown in our next work, such Bose—Fermi dualism is characteristic of not only the electromagnetic, but also massless spinor field, in particular, a neutrino.

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ПРО НЕЛОКАЛЬНІ СИМЕТРІЇ РІВНЯНЬ МАКСВЕЛЛА ТА ЗАКОНИ ЗБЕРЕЖЕННЯ

І.Ю. Кривський, В.М. Симулик

Р е з ю м е

Знайдено два нові, задані нелокальними генераторами, унітарні зображення універсальної покривної групи Пуанкаре, відносно яких є інваріантними вільні рівняння Максвелла і показано, що обидва зображення, як і локальне, описують поле маси $m = 0$ і спіна $s = 1$ (спіральності $h = \mp 1$). Запропоновано лагранжіан у термінах $(\vec{E}, \vec{H}) = \vec{\mathcal{E}} \equiv \vec{E} - i\vec{H}$ напруженостей електромагнітного поля, який за звичайною методикою теореми Нетер дає фізично адекватне зіставлення “генератор симетрії – закон збереження”. Цим шляхом, не апелюючи до потенціалів, одержано стандартну серію 15 Пуанкаре $(P_\mu, J_{\mu\nu})^{\mathrm{loc}}$- та конформних (D, K_μ)-законів збереження як наслідків локальної конформної $C(1{,}3)$-симетрії вільних рівнянь Максвелла, а також два набори $(P_\mu, J_{\mu\nu})^{\mathrm{I,II}}$ основних динамічних змінних — наслідків згаданих нелокальних симетрій, причому $(P_\mu, J_{\mu\nu})^I = (P_\mu, J_{\mu\nu})^{\mathrm{loc}}$. Величини $P_\mu, J_{\mu\nu}$ виражено в термінах імпульсно-спіральних амплітуд і показано, що набір $(P_\mu, J_{\mu\nu})^{\mathrm{I}}$ вимагає бозе-квантування поля $\vec{\mathcal{E}}$, а набір $(P_\mu, J_{\mu\nu})^{\mathrm{II}}$ вимагає фермі-квантування цього поля, тобто відноситься до гіпотетичної безмасової частинки зі спіральністю $h = \mp 1$, яка підлягає статистиці Фермі—Дірака. Обидва типи квантування використовують простір Фока з дефінітною метрикою і задовольняють принцип мікропричинності. Введену тут в розгляд гіпотетичну частинку запропоновано назвати фермі-фотоном (F-фотоном).