DOUBLED FIELD APPROACH TO YANG—MILLS THEORY REQUIRES NON-LOCALITY

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Doubling the Yang—Mills (YM) field with its dual partner, we apply the pattern which has been found to construct a "duality-symmetric" gravity with matter to the "duality-symmetric" YM theory in five space-time dimensions. Constructing the action, we conclude that dualizing a non-Abelian theory requires non-locality. We analyze the symmetries of the theory and equations of motion. The extension to the supersymmetric model is also demonstrated.

1. Introduction

Some of the fields of Superstring/M-theory spectrum are of special class which are called chiral p-forms, or chiral bosons. These fields play an important rôle in establishing various dualities between different sectors of M-theory, but dealing with them beyond the mass-shell, i.e. at the level of the effective Lagrangians, is not a simple task. There are several methods (see [1] for a review and for the comprehensive list of references) which were proposed to describe theories with self-dual or duality-symmetric fields. All of them could be split into the following major sets. The first one [2—6] contains the approaches that are duality-invariant but are not manifestly Lorentz-invariant. Introducing the auxiliary fields is not required, but the coupling to other fields, especially to gravity, may cause problems with establishing the consistency of such a coupling. The second set [7—12] is dealt with the auxiliary fields, whose inclusion restores the Lorentz covariance. The number of these auxiliary fields may vary from one to infinity.

Among the approaches with auxiliary fields, the formalism proposed by Pasti, Sorokin, and Tonin (PST) [12] takes a special place. It is manifestly Lorentz-covariant and is minimal in a sense of having the only an auxiliary field entering the action in a non-polynomial way. The successful application of the PST approach to the construction of different field theories of chiral p-forms, super-p-branes with worldvolume chiral fields, and of different subsectors of supergravities has demonstrated the advantages of this approach and its compatibility with supersymmetry (cf. [1] and references therein). However, a gap for applying the PST formalism is a YM theory.

It is worth noting that the problem of dualizing a non-Abelian gauge theory has been intensively studied in the literature. Getting rid of self-interactions, it is straightforward to apply the machinery of dualization to the case. But the attempts to go beyond the free theory have faced with the troubles. The latter can be summarized in the "no-go" theorems [3, 13] which forbid a trivial generalization of the well-known electric-magnetic duality of the Maxwell theory. The point is that the Poincaré lemma is not directly generalized when dealing with a YM covariant derivative, which prevents, in its turn, the straightforward application of the PST formalism to formulate a duality-symmetric YM theory [14, 15].

Searching for a dualization of a non-Abelian theory becomes now important from the point of view of pushing forward the doubled field approach of [16, 17] to non-maximal supergravities. On the supergravity side, much has been done in the dualization program for maximal supergravities in diverse dimensions [16, 17] that can be obtained from $D = 11$ or $D = 10$ IIA/B supergravities by the toroidal dimensional reduction. There, by doubling the fields of the gauge and scalar sectors of supergravities, it has been demonstrated that the original equations of motion of a theory admit a representation in terms of the Bianchi identities for dual fields. Moreover, the dynamical content of a theory is encoded into the so-called twisted self-duality condition relating the original and the dual field strengths. The way to go beyond the mass-shell by lifting the approach of [17] onto the level of the proper action was proposed for $D = 11$ and $D = 10$ type IIA supergravities in [18, 19]. However, the doubled field approach cannot be directly applied to non-maximal supergravities in $D = 10$ and to low-dimension gauged supergravities, where non-Abelian fields become a part of the supergravity multiplet.

Therefore, to realize the aforementioned program, we are forced to figure out a way to deal with non-Abelian fields. One of the ways is to find a generalization of the Hodge star notion to a non-Abelian case. Such a generalization has been proposed in [20] and requires essentially a non-local consideration since it is based on a loop space formulation of the gauge theory. We are aimed to reach the same conclusion on the non-local character of the dualization of a non-Abelian theory [21, 22] being on the ground of the standard approach to the YM theory.

Another point of interest in searching for the YM dualization is the apparent fact that the YM theory possesses the same feature of having self-interactions as that of General Relativity. Therefore, studying the properties of the doubled field approach to a (super)YM model (SYM model) would be helpful in pushing forward the same approach to the (super)gravity case, especially if fermions will be taken into account. We are mostly interested in the dualization of $D = 11$ supergravity in a spirit of the recent study of a hidden symmetry group of the M-theory [27—43]. However, we could simplify the task with noting a tight relation between five-dimensional simple supergravity and $D = 11$ supergravity [23]. Hence, we would figure out (some of the) features of the eleven-dimensional theory considering just the five-dimensional model. That is why we will study a SYM theory in the $D = 5$ modeling of a situation in supergravities.

As a preliminary step in recovering the doubled field action for the YM theory, we have to find, without an appeal to a method of constructing such an action, a convenient representation of the YM equation of motion in a way that allows us to present the latter as the Bianchi identity for a dual field. It turns out to be convenient to write down the YM equation of motion in a form which is very similar to the dynamics of the Maxwell theory with an electric-type source. Such a representation suggests a way of extracting the dual field after that we can apply the machinery of the PST approach to construct the action, from which the duality relations between the YM field and its dual partner will follow as equations of motion. This task is accomplished in the next section to realize that non-locality has to be involved into the game. There we also discuss the symmetry properties of the model and derive the equation of motion for the dual field. It is worth to point out that the dual field dynamics is described, nevertheless, by a local equation. In Section 3, we discuss, in brief, the supersymmetric extension of the model, and our conclusions are summarized in the last section.

2. Doubled Fields Action of the $D = 5$ YM Theory and its Symmetries

We get started with the following action for the YM theory in $D = 5$

$$S_{\rm YM} = -\frac{1}{2}{\rm Tr} \int\limits_{\mathcal{M}^5} F^{(2)} * F^{(2)}, \qquad (1)$$

where $F^{(2)} = dA^{(1)} - \frac{1}{2}ig[A^{(1)}, A^{(1)}]$ is the field strength in the adjoint representation of a semisimple non-Abelian group, g is a coupling constant, and the wedge product between forms has to be assumed. The action S_{YM} and the gauge fields equation of motion

$$D * F^{(2)} = 0 \qquad (2)$$

are invariant under the local non-Abelian gauge transformations

$$\delta A^{(1)} = \frac{1}{g} D\alpha^{(0)} \equiv \frac{1}{g} d\alpha^{(0)} + i[A^{(1)}, \alpha^{(0)}], \qquad (3)$$

where we have introduced the covariant derivative $D = d + ig[A^{(1)},]$, whose action on forms is defined by

$$D\Omega^{(n)} = d\Omega^{(n)} + (-)^n \, ig[A^{(1)}, \Omega^{(n)}]. \qquad (4)$$

To apply the Poincaré lemma, let us present the equation of motion (2) in a slightly different form by extracting the part with the usual non-covariant derivative and separating the free part from that describing the self-interaction of the YM field. Taking into account the definition of the YM field strength, we get

$$d(*dA^{(1)}) = *J^{(1)} \qquad (5)$$

with

$$*J^{(1)} = ig[A^{(1)}, *F^{(2)}] + d * \left(\frac{1}{2}ig[A^{(1)}, A^{(1)}]\right). \qquad (6)$$

Since $d^2 = 0$, in the trivial topology setting, one could notice

$$*J^{(1)} = d * G^{(2)}, \tag{7}$$

where $G^{(2)}$ is a function of the YM potentials $A^{(1)}$ and their derivatives and, as we will see in what follows, is the source of non-locality, since we can formally resolve (7) through the non-local expression

$$*G^{(2)} = d^{-1} * J^{(1)}. \tag{8}$$

Here we have introduced the operator inverse to d, whose action can be understood as follows. Let us introduce a "Green" function to the equation

$$dh(x) = \delta^5(x), \tag{9}$$

with the Dirac delta-function on the r.h.s. Then on an arbitrary form at a space-time point x

$$d^{-1}(x)\omega^{(p)}(x) = (-)^p \int d^5y\, h(x-y)\ \omega^{(p)}(y). \tag{10}$$

To make a sense, the latter expression should only deal with the "Green" functions that act on a causality-related space-time region. We should also emphasize that one should take care in treating d^{-1} (see [44] for details), taking firstly $dd^{-1} = d^{-1}d = \mathrm{id}$ in expressions. For instance, $ddd^{-1}\dots$ has to be equal to $d\dots$, rather than to zero.

The dual to the one-form $J^{(1)}$ is a conserved "current" which is the YM analog of the gravity Landau—Lifshits pseudotensor (see [24]), and Eq. (5) is an analog of the equation of motion of the Maxwell field with an electric-type current. Note that the "current" entering the r.h.s. of (5) is not gauge invariant. But, since the l.h.s. of the same equation is not gauge invariant too, the latter compensates the former leaving Eq. (5) to be invariant under the local gauge transformations (3).

Having representation (5), we can double the YM field with its "dual" partner (a reason why we should call the partner "dual" will become clear in a minute) and write down this equation of motion as the Bianchi identity for the YM "dual" field,

$$dB^{(2)} = *(dA^{(1)} - G^{(2)}), \tag{11}$$

or, equivalently,

$$\mathcal{F}^{(3)} = 0, \qquad \mathcal{F}^{(3)} = dB^{(2)} - *(dA^{(1)} - G^{(2)}). \tag{12}$$

Indeed, applying the operator d to (11) or (12) leads to the YM equation of motion (2)

$$d\mathcal{F}^{(3)} = D * F^{(2)} = 0. \tag{13}$$

Therefore, an equivalent way of a description of the YM theory is to find the action, from which eq. (12) will follow as an equation of motion.

However, the action we shall construct should be gauge invariant as well as the equations of motion which will follow from that action. Therefore, we get to inspect the gauge invariance more closer. To this end, we recall that the action of the local gauge transformations (3) on the YM field strength results in the rotation of the latter in a group space, i.e. $\delta F^{(2)} = -i[\alpha^{(0)}, F^{(2)}]$. To find the similar transformation law for the $\mathcal{F}^{(3)}$, it is convenient to present (5) as

$$d * F^{(2)} = *\tilde{J}^{(1)},$$

$$*\tilde{J}^{(1)} = ig[A^{(1)}, *F^{(2)}] = d * \tilde{G}^{(2)}. \tag{14}$$

Then, using the $F^{(2)}$ gauge transformation law, one can derive the transformation of $*\tilde{G}^{(2)}$ from (14) as

$$\delta{*\tilde{G}^{(2)}} = -i[\alpha^{(0)}, \mathcal{F}^{(3)} + *F^{(2)}] - d^{-1}\left(i[d\alpha^{(0)}, \mathcal{F}^{(3)}]\right) \tag{15}$$

which is a non-local gauge transformation in view of the non-local character of this quantity.

To require $\delta\mathcal{F}^{(3)} = -i[\alpha^{(0)}, \mathcal{F}^{(3)}]$, one has to assign the following non-local non-Abelian gauge transformation to the $B^{(2)}$ field:

$$\delta\left(dB^{(2)}\right) = d^{-1}\left(i[d\alpha^{(0)}, \mathcal{F}^{(3)}]\right). \tag{16}$$

Concerning the gauge invariance of equations of motion, we should emphasize that the standard YM equation of motion (2) is only on-shell invariant under the action of (3) since $\delta(D * F^{(2)}) = -i[\alpha^{(0)}, D * F^{(2)}]$. The same concerns the $d\mathcal{F}^{(3)} = 0$ since this expression is gauge-invariant only on the shell of the duality relation $\mathcal{F}^{(3)} = 0$.

Therefore, our attempt to stay on the ground of applying the usual Poincaré lemma to the non-Abelian case has faced with the necessity of dealing with non-locality due to the "source"-like terms which appear in the non-Abelian gauge field equation of motion after fitting the later to the application of the Poincaré lemma. It is easy to see that non-locality disappears in the zero gauge coupling constant limit $g \to 0$, when $G^{(2)} \to 0$, $\mathcal{F}^{(3)} \to dB - *dA$, $\delta A = d\tilde{\alpha}^{(0)}$, $\delta B^{(2)} = d\alpha^{(1)}$, and therefore we effectively deal with $\mathcal{N}$ copies of the Abelian duality-symmetric fields, where $\mathcal{N}$ is the dimension of the non-Abelian group. However, it does not contradict the "no-go" theorem of [15] since the extension of a system of $\mathcal{N}$ copies of free duality-symmetric Abelian fields to a non-Abelian system comes through introducing the non-local quantities. Another feature of the construction that has to be noticed consists in a non-equivalence of the original YM field and its "dual" partner since the non-Abelian extension of the latter is formed by the part containing the self-interaction of the former. Indeed, the equation of motion for the dual field is

$$d(*dB^{(2)}) = -dG^{(2)}, \tag{17}$$

that follows from the duality relation $*\mathcal{F}^{(3)} = 0$. However, one can essentially simplify this equation by taking the Hodge identity

$$d\triangle^{-1}\delta + \delta\triangle^{-1}d = 1 \tag{18}$$

into account, where δ is the co-derivative and $\triangle^{-1}$ is the inverse to the Laplacian operator $\triangle = d\delta + \delta d$. Using the Hodge identity, one can present $*\mathcal{F}^{(3)} = 0$ as

$$*dB^{(2)} = F^{(2)} - *\triangle^{-1}\delta * \tilde{J}^{(1)}, \tag{19}$$

and since the last term on the r.h.s. of the latter equation is a closed form, the equation of motion of $B^{(2)}$ is

$$d(*dB^{(2)}) = -\frac{1}{2} ig\ d\left([A^{(1)}, A^{(1)}]\right). \tag{20}$$

Therefore, there is not a symmetry similar to the symmetry under duality rotations in the Maxwell theory that is closely related to the "no-go" theorem of [3]. Collecting all these facts in mind we refer to $B^{(2)}$ field as to a "dual" field.

Let us now turn to the construction of the action, from which the duality relation $\mathcal{F}^{(3)}$ will follow as an equation of motion. Taking an analogy with the gravity case considered in [24] into account, it is quite natural to guess the term

$$S_{\rm PST} = \frac{1}{2} {\rm Tr} \int\limits_{\mathcal{M}^5} v\ \mathcal{F}^{(3)}\ i_v\mathcal{F}^{(2)}, \tag{21}$$

as the main candidate that has to be added to action (1). Here, the one-form v is constructed out of the PST scalar field $a(x)$ ensuring the covariance of the model,

$$v = \frac{da(x)}{\sqrt{-(\partial a)^2}}, \tag{22}$$

$\mathcal{F}^{(3)}$ has appeared in (12), and

$$\mathcal{F}^{(2)} = dA^{(1)} - *(dB^{(2)} + *G^{(2)}), \quad \mathcal{F}^{(3)} = - * \mathcal{F}^{(2)}. \tag{23}$$

It is clear from the previous discussion that the generalized field strengths (12) and (23) are the objects covariant under the gauge transformations, although the quantities entering them are not covariant, and action (21) is invariant under the gauge transformations (3), (15), and (16) which leave the PST scalar intact.

To prove the relevance of the proposed term, let us consider a general variation of (21). The standard manipulations (see [18, 44] for details) result in

$$\delta\mathcal{L}_{\rm PST} = {\rm Tr}\left(\delta B^{(2)} + \frac{\delta a}{\sqrt{-(\partial a)^2}} i_v\mathcal{F}^{(3)}\right)\ d(v\ i_v\mathcal{F}^{(2)}) -$$
$$-{\rm Tr}\left(\delta A^{(1)} + \frac{\delta a}{\sqrt{-(\partial a)^2}} i_v\mathcal{F}^{(2)}\right)\ d(v\ i_v\mathcal{F}^{(3)}) -$$
$$-{\rm Tr}\ \delta(*G^{(2)})\ v\ i_v\mathcal{F}^{(2)} - {\rm Tr}\ \delta A^{(1)}\ d\mathcal{F}^{(3)}, \tag{24}$$

where we have omitted the total derivative term.

The last term of (24) is precisely the term, whose contribution is cancelled against the variation of $S_{\rm YM}$. Therefore, the complete action $S = S_{\rm YM} + S_{\rm PST}$ is invariant under the non-Abelian gauge transformations and the following two sets of special symmetry [12]:

$$\delta a(x) = 0, \qquad \delta A^{(1)} = da\ \varphi^{(0)},$$
$$(d\delta B^{(2)} + \delta * G^{(2)}) = da\ d\varphi^{(1)} \Longrightarrow$$
$$\delta B^{(2)} = da\ \varphi^{(1)} - d^{-1}\delta(*G^{(2)}), \tag{25}$$

$$\delta a(x) = \Phi(x), \quad \delta A^{(1)} = -\frac{\delta a}{\sqrt{-(\partial a)^2}} i_v\mathcal{F}^{(2)},$$
$$\delta B^{(2)} = -\frac{\delta a}{\sqrt{-(\partial a)^2}} i_v\mathcal{F}^{(3)} - d^{-1}\delta(*G^{(2)}). \tag{26}$$

Let us now discuss how these special symmetries do the job. The equations of motion of $B^{(2)}$ and $A^{(1)}$ that follow from the action $S = S_{\rm YM} + S_{\rm PST}$ are

$$d(v\ i_v\mathcal{F}^{(2)}) = 0, \tag{27}$$

$$d(v\ i_v\mathcal{F}^{(3)}) + \frac{{\rm Tr}(v\ i_v\mathcal{F}^{(2)}\ \delta * G^{(2)})}{\delta A^{(1)}} = 0, \tag{28}$$

where we have used that $*G^{(2)}$ is a function of the YM potentials $A^{(1)}$.

The general solution to the equation of motion (27) is [12]

$$v\ i_v\mathcal{F}^{(2)} = da\ d\xi^{(0)}. \tag{29}$$

In view of symmetry (25) with $\varphi^{(0)} = \xi^{(0)}$, relation (29) yields

$$i_v\mathcal{F}^{(2)} = 0 \quad \rightsquigarrow \quad \mathcal{F}^{(2)} = 0. \tag{30}$$

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Taking the latter into account and using the same trick, one can obtain

$$i_v\mathcal{F}^{(3)} = 0 \quad \rightsquigarrow \quad \mathcal{F}^{(3)} = 0 \tag{31}$$

from (28). It becomes clear that the equation of motion of the PST scalar $a(x)$

$$\mathrm{Tr}\left(i_v\mathcal{F}^{(3)}\ d(v\ i_v\mathcal{F}^{(2)}) - i_v\mathcal{F}^{(2)}\ d(v\ i_v\mathcal{F}^{(3)})\right) = 0 \tag{32}$$

does not contain a new dynamical information and is satisfied identically as a consequence of the equations of motion (27) and (28). Indeed, Eq. (32) is the Noether identity which is a reflection of a local symmetry which is nothing but the symmetry under (26).

Therefore, we have proved that the action $S = S_{\mathrm{YM}} + S_{\mathrm{PST}}$ is the one we are looking for. The action possesses the special symmetries (25) and (26) which have to be used to derive the duality relations between the YM field and its dual partner from equations of motion and to establish the auxiliary nature of the PST scalar field. Owing to symmetry (26), the PST scalar field does not spoil the original content of the theory and is a purely auxiliary field.

3. Supersymmetric Extension of the Duality-symmetric Action

To extend this construction to the supersymmetric case, we recall that the supersymmetric counterpart of the D=5 YM theory is described by

$$S_{\mathrm{SYM}} = -\frac{1}{2}\mathrm{Tr}\int\limits_{\mathcal{M}^5} F^{(2)} * F^{(2)} + i\bar{\lambda}\Gamma^a D\lambda\Sigma_a + \ldots \tag{33}$$

where the four-form Σ_a is defined by

$$\Sigma_a = \frac{1}{4!}\epsilon_{abcde}E^bE^cE^dE^e \tag{34}$$

with the vielbeins E^a, and $D\lambda$ is the covariant derivative of the gaugino field. Note that we have kept only the terms essential for the consideration in what follows neglecting the scalars and auxiliary fields which cast the off-shell $N = 2$ YM supermultiplet in $D = 5$.

Action (33) is, in particular, invariant under the following global supersymmetry transformations:

$$\delta_\epsilon A^{(1)} = -\frac{i}{2}\bar{\epsilon}\Gamma^{(1)}\lambda, \qquad \delta_\epsilon\lambda = \frac{1}{2} * (*F^{(2)}\ \Gamma^{(2)})\epsilon. \tag{35}$$

Here, we have used the following notation for gamma-matrices:

$$\Gamma^{(n)} = \frac{1}{n!}E^{a_n}\ \ldots\ E^{a_1}\Gamma_{a_1\ldots a_n}. \tag{36}$$

To find the appropriate supersymmetry transformations for the doubled field version of the SYM theory, it is convenient to present the PST part of the action as

$$S_{\mathrm{PST}} = -\frac{1}{2}\ \mathrm{Tr}\int\limits_{\mathcal{M}^5} i_v\mathcal{F}^{(2)} * i_v\mathcal{F}^{(2)} \tag{37}$$

with

$$\mathcal{F}^{(2)} = F^{(2)} - *(dB^{(2)} + *\mathcal{G}^{(2)}), \tag{38}$$

where $\mathcal{G}^{(2)}$ is the extension of $\tilde{G}^{(2)}$ with a non-local term coming from the fermionic current.

Then it is easy to verify that the action $S = S_{\mathrm{SYM}} + S_{\mathrm{PST}}$ is invariant under the following global supersymmetry transformations:

$$\delta_\epsilon a = 0, \qquad \delta_\epsilon A^{(1)} = -\frac{i}{2}\bar{\epsilon}\Gamma^{(1)}\lambda,$$

$$\delta_\epsilon\lambda = \frac{1}{2} * \left(*[F^{(2)} + v\ i_v\mathcal{F}^{(2)}]\ \Gamma^{(2)}\right)\epsilon. \tag{39}$$

The supersymmetry transformation of the field dual to the YM one can be recovered from the requirement

$$\delta_\epsilon\left(*dB^{(2)} + \mathcal{G}^{(2)}\right) = 0. \tag{40}$$

Hence, the proposed extension of the PST technique to a non-Abelian case is compatible with supersymmetry but requires the non-local terms in the supersymmetry transformation of the dual field.

4. Conclusions

To summarize, we have presented the YM equation of motion in the form which is very likely to that of the Maxwell theory with an electric-type current. The YM "current" form so obtained encodes the self-interaction between the YM fields, does not possesses the local gauge invariance, and is a closed form. The latter allows one to present the "current" form as the curl of a "current potential" and, therefore, to rewrite the second-order YM equation of motion as the first-order Bianchi identity for the dual field. Since the form of the "current potential" is defined by a non-local expression, we have demonstrated that such a dualization of the YM field requires non-locality. But the latter does not spoil the general scheme of constructing the duality-symmetric theories in a spirit of the PST approach, though the trace of the non-locality can be observed in the gauge transformations of the doubled field YM action.

The same story happens in the gravity case [24, 44]. Indeed, after resolving the torsion free constraint in the first-order formulation of the Einstein-Hilbert action, one can present the gravity equation of motion in a form similar to Eq. (5). Therefore, within the standard approach, dualizing the gravity requires introducing the non-localities too. However, we have mentioned above that there is an alternative way of a non-Abelian generalization of the electric-magnetic duality based on the loop space formulation of the gauge theory in D=4 space-time dimensions [20]. An analog of such a formulation of the gravitation theory is nothing but the Ashtekar—Sen approach (see, e.g., [25, 26] for reviews). It would be interesting to figure out how the loop space approach could be reformulated to describe a duality-symmetric theory where the original and the dual potentials will appear on the equal footing.

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ФОРМАЛIЗМ ПОДВОЄНИХ ПОЛIВ В ТЕОРIЇ ЯНҐА—МIЛЛСА ПОТРЕБУЄ НЕЛОКАЛЬНОСТI

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Р е з ю м е

Подвоєння поля Янґа—Міллса дуальним партнером дозволяє застосувати схему побудови "дуально-симетричної" теорії Янґа—Міллса у п'яти просторово-часових вимірностях, що аналогічно схемі побудови "дуально-симетричної" гравітації з матерією. Будуючи дію цієї теорії, ми приходимо до висновку про необхідність введення нелокальності. Ми аналізуємо симетрії теорії та її рівняння руху. Демонструється узагальнення моделі на суперсиметричний випадок.