EXCITATION OF LF OSCILLATIONS AT THE INJECTION OF A RELATIVISTIC ELECTRON BEAM IN PLASMA

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UDC 533.992

The nonlinear process of excitation of radial low-frequency (LF) ion oscillations at the propagation of a relativistic electron beam (REB) in plasma with a given density profile is investigated. The nonlinear equation in the Lagrangian variables describing the radial oscillations of ions in the space charge field of the REB and in its own self-consistent electric field is obtained. The results of the numerical analysis of the indicated system of equations for a tubular REB and various transversal profiles of the plasma density are presented. Physical mechanisms of excitation and damping of LF ion oscillations are discussed.

1. Introduction

Processes of excitation of LF ion oscillations play an important role in the dynamics of high-current relativistic electron beams with long time duration (the order of several microseconds and more). The most known devices with highly expressed ion LF processes are plasma-filled microwave devices (e.g., a vircator with plasma anode [1], phasotron [2, 3]), as well as devices for collective acceleration of ions [4]. Earlier we have investigated the nonlinear dynamics of excitation of LF ion oscillations at the radial injection of an ion flow in the area where a high-current REB propagates [4]. In this study, the excitation of radial LF ion oscillations is investigated at the injection of an REB in plasma with a given initial profile of density.

2. Physical Model. Basic Equations

Let a tubular REB with inner radius r_1 and external radius r_2 propagate in an infinite cylindrical metal drift chamber of radius a. In the near-wall area $a > r > r_2$, a plasma layer is located. The system is inserted in an external magnetic field. Electrons of the beam are magnetized; the influence of the magnetic field on the ion movement is neglected. Let's suppose that the linear density of REB essentially exceeds the linear density of plasma particles. In this case, plasma electrons can be excluded from the consideration.

Under the forces of a REB space charge, ions will start to make radial oscillations. We will describe the electric field of the REB space charge in the approximation of given parameters of the beam (fixed density and velocity). As to the ion component, it will be described as completely self-consistent.

The radial electric field of the tubular REB space charge is described by the expression

$$E_{br} = -\frac{2I_b}{v_e a}\frac{1}{R}F(R), \qquad (1)$$

where I_b is the current of the electron beam, v_e is its velocity,

$$F(R) = \begin{cases} 1, & R \geq R_2, \\ \frac{R^2 - R_1^2}{R_2^2 - R_1^2}, & R_2 \geq R \geq R_1, \\ 0, & R \leq R_1, \end{cases}$$

$R = r/a$, and $R_{1,2} = r_{1,2}/a$. The field of the ions space charge will be described in terms of Lagrangian variables as follows. First, we will find out the electrical potential of the elementary infinitely thin cylindrical ion layer which propagates in the radial direction under the arbitrary pattern $r_L(r_0, t)$, where r_0 is the initial radial coordinate of the ion layer, $r_L(r_0, 0) = r_0$. The expression for the charge density of such a layer looks like

$$d\rho_i = en_i(r_0)\, r_0 dr_0 \frac{\delta(r - r_L)}{r_L},$$

where $n_i(r_0)$ is the initial distribution of the ion density along a radius, and e is the charge of an electron.

From the Poisson equation, we find the potential distribution for such a charge,

$$\Phi_G\{r, r_L\} = -4\pi e n_i(r_0)\, r_0 dr_0 L(r, r_L),$$

$$L(r, r_L) = \begin{cases} \ln\frac{r}{a}, & r \geq r_L, \\ \ln\frac{r_L}{a}, & r \leq r_L, \end{cases}$$

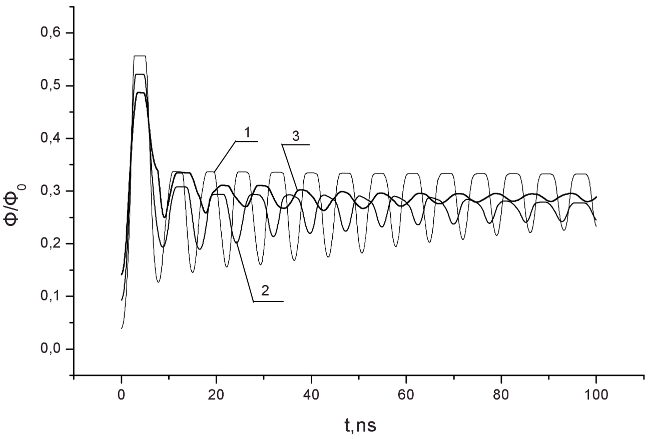

Fig. 1. Dependences of the electric potential on time at the external boundary of a REB ($n_{0i}/n_{0e} = 0.1$) *1* – $\Delta = 0.2$ cm, *2* – 0.5, *3* – 0.8

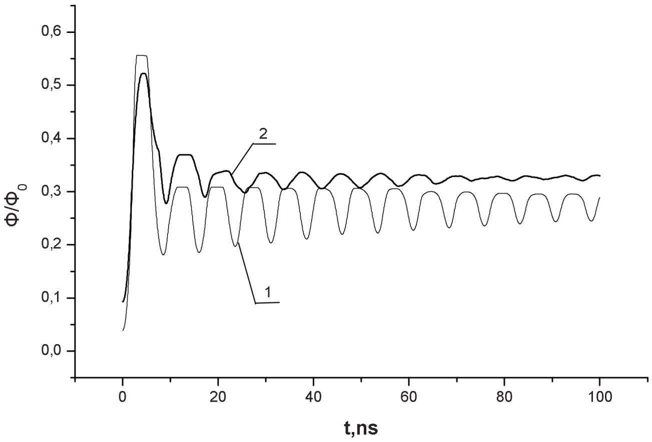

Fig. 2. Dependences of the electric potential on time at the external boundary of a REB ($n_{0i}/n_{0e} = 0.2$): *1* – $\Delta = 0.2$ cm, *2* – 0.5

and the distribution of the radial electric field,

$$E_G = 4\pi e n_i\left(r_0\right) r_0 dr_0 \frac{1}{r}\chi\left(r - r_L\right),$$

where $\chi(r - r_L)$ is the Heaviside function.

By integrating over the initial positions of elementary ion cylindrical layers, we find the potential and the electric field of the whole ion system as

$$\Phi = -\Phi_0 \int\limits_0^1 \Psi\left(R_0\right) R_0 dR_0 L\left(R, R_L\right), \qquad (2)$$

$$E_r = \frac{E_0}{R} \int\limits_0^1 \Psi\left(R_0\right) R_0 dR_0 \chi\left(R - R_L\right), \qquad (3)$$

where

$$\Phi_0 = 4\pi e a^2 n_{0i}, \quad E_0 = 4\pi e a n_{0i}, \quad R_0 = {}^{r_0}\!/_{a}, \quad R_L = {}^{r_L}\!/_{a},$$

$$n_i(r_0) = n_{0i}\Psi(R_0).$$

The function $\Psi\left(R_0\right)$ describes the initial distribution of the ion density.

By substituting the electric fields of both the REB (1) and the radial ion flow (3) into the equations of motion for ions, we obtain the closed self-consistent nonlinear equation

$$\frac{d^2 R_L}{d\tau^2} + \frac{1}{R_L} F\left(R_L\right) = \frac{2\alpha}{R_L} \int\limits_0^1 \Psi\left(R_0'\right) \chi\left(R_L - R_L'\right) R_0' dR_0', \qquad (4)$$

where $\alpha = \frac{1}{4}\frac{I_A}{I_b}\frac{v_e}{c}\frac{M}{m}\frac{\omega_{pi}^2 a^2}{c^2} = \frac{n_{0i}}{n_{0e}}\frac{a^2}{r_2^2 - r_1^2}$, n_{0e} is the initial REB density, $\omega_{pi}^2 = \frac{4\pi e^2 n_{0i}}{M}$ is the ion Langmuir frequency, m is the electron mass, M is the ion mass, $R_L' = R_L\left(R_0', \tau\right)$, $\tau = \omega_0 t$ is the dimensionless time, $\omega_0 = \frac{c}{a}\sqrt{\frac{2m}{M}\frac{I_b}{I_A}\frac{c}{v_e}}$ is the specific frequency of ion oscillations in the field of the REB space charge, and I_A=17 kA.

The integro-differential equation of ion motion is supplemented by the initial conditions

$$R_L\left(\tau = 0, R_0\right) = R_0, \frac{dR_L}{d\tau}|_{\tau=0} = 0. \qquad (5)$$

3. The Results of Numerical Analysis

The nonlinear equation of ion motion (4) has been solved by numerical methods for various configurations of a REB and the initial distributions of ion density.

In Fig. 1, the dependences of the electric potential of the ion flow (2) on time at the external boundary of a tubular REB $r = r_2$ for the ratio of densities of REB electrons and nitrogen ions $n_{0i}/n_{0e} = 0.1$ and various values of the thickness of the near-wall ion layer are presented. Other geometric parameters are taken as follows: the REB inner radius $r_1 = 1.1$ cm, its external radius $r_2 = 1.4$ cm, and the radius of a drift chamber $a = 2.5$cm. At the beginning of the process, there is a displacement of the ion layer as a whole to the axis and its acceleration. At that, the electric potential increases. Since the electric field inside the REB is absent, the

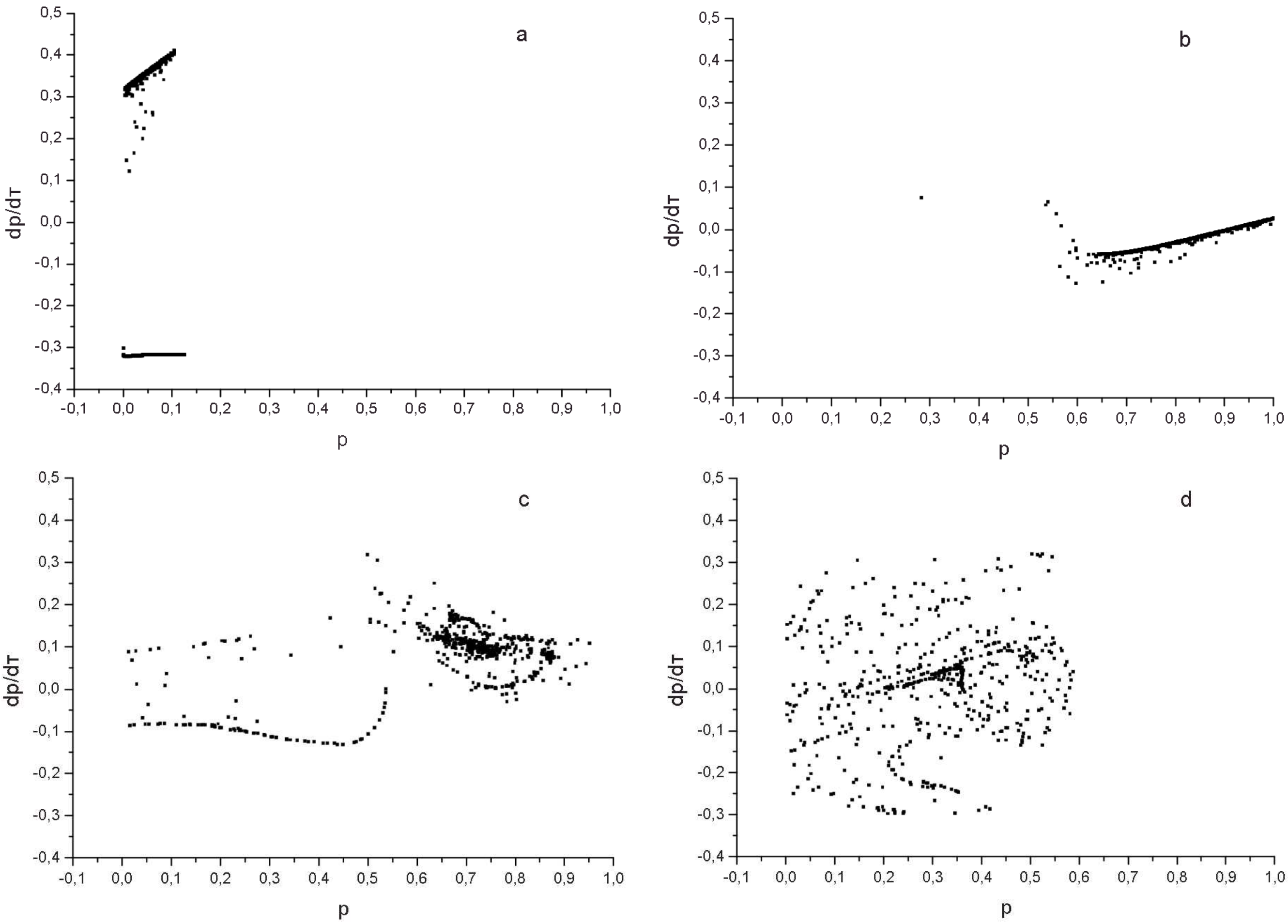

Fig. 3. Phase portraits of ions under $n_{0i}/n_{0e} = 0.1$: for $\Delta = 0.2$ cm at instants $a - t = 4$ ns, $b - t = 8$ ns, $c - t = 50$ ns; for $\Delta = 0.8$ cm at instant $d - t = 50$ ns

accelerated ion bunch inside the REB propagates with almost constant velocity and quickly, approximately during the time $\Delta t = 2$ ns, achieves the axis, and then goes out the REB area. During the time when the ion bunch is situated inside the REB, a potential remains constant. The motion of an ion bunch to the wall of a drift chamber is accompanied by a voltage decrease. The part of ions under the action of the Coulomb force hits the wall and vanishes from the system. Further, the process is repeated. The oscillatory movement of the ion layer is accompanied by an increase of its cross size, which is caused, on the one hand, by the action of ion space charge forces and, on the other hand, by the electric field gradient inside the layer and an anharmonicity of ion oscillations in the electric field of a tubular REB. Eventually, these effects result in the phase mixing of ions and an attenuation of potential oscillations. With increase in the initial thickness of the ion layer, the amplitude of oscillations and its attenuation time decrease. This is explained by acceleration of the process of phase mixing of ions during their radial oscillations. An increase in the ion density at a fixed thickness of the ion layer leads to the same results (Fig. 2).

The oscillatory movement of ions in the electric field of a REB is illustrated with phase portraits at various instants (Fig. 3) obtained for a thin ion layer $\Delta = 0.2$ cm and $n_{0i}/n_{0e} = 0.1$. At the instant $t = 4$ ns (Fig. 3,a), the ion layer reaches the center of a drift chamber and, at $t = 8$ ns (Fig. 3,b), reaches the chamber wall, where a part of ions is lost. Till this instant, the size of the bunch has strongly increased. However, the overwhelming part of ions is present outside the REB. Phase mixing results in slow losses of ions in the bunch.

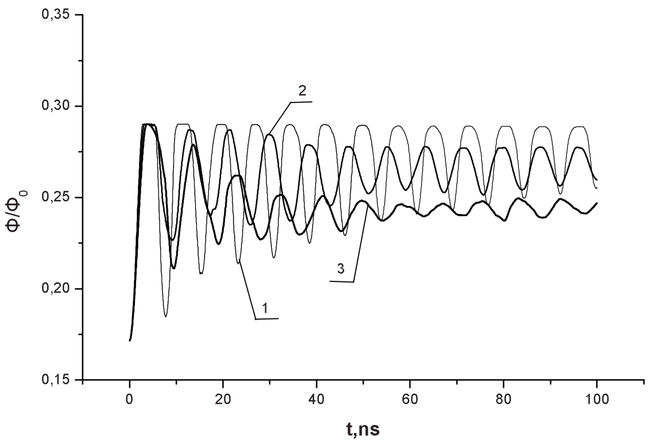

Fig. 4. Dependences of the electric potential on time at the external boundary of a REB ($\Delta = 2.5$ cm): *1* – $n_{0i}/n_{0e} = 0.2$, *2* – 0.4, *3* – 0.6

On the phase portrait at the instant $t = 50$ ns (Fig. 3,*c*), it is seen that the compact enough bunch of ions is outside of the REB in the near-wall area. This instant corresponds to a minimum of the potential. To the instant $t = 55$ ns, the ion bunch occurs inside the REB and the potential achieves its maximum. During the process of oscillations in a drift chamber, the ion bunch loses particles.

For a thick ion layer $\Delta = 0.8$ cm, the phase portrait at the instant $t = 50$ ns is shown in Fig. 3,*d*. Comparison of Fig. 3,*c* and Fig. 3,*d* obviously shows that an increase of the bunch thickness results in the acceleration of the phase mixing process.

Above, the radial oscillations of relatively thin ion layers in the space charge field of a REB and the self-consistent field of oscillating ions have been investigated. Let's consider now the case where ions fill completely a drift chamber. Such a situation is realized at the REB injection into plasma which fills completely a drift chamber. The plasma density is much lower than the REB density. For the taken geometric parameters, the plasma density should make less than 10 % of the REB density.

So, let's consider the tubular REB propagating in a homogeneous (at the initial instant) column of ions completely filling a drift chamber. In Fig. 4, we present the dependences of the potential on the external boundary of a tubular REB upon time. It is seen that the amplitude of oscillations of the potential decreases firstly and then remains practically constant. With increase in the ion density, its value decreases.

The phase portraits of ions show (Fig. 5) that, at the beginning of the process $t = 2.5$ ns (Fig. 5,*a*), the ions

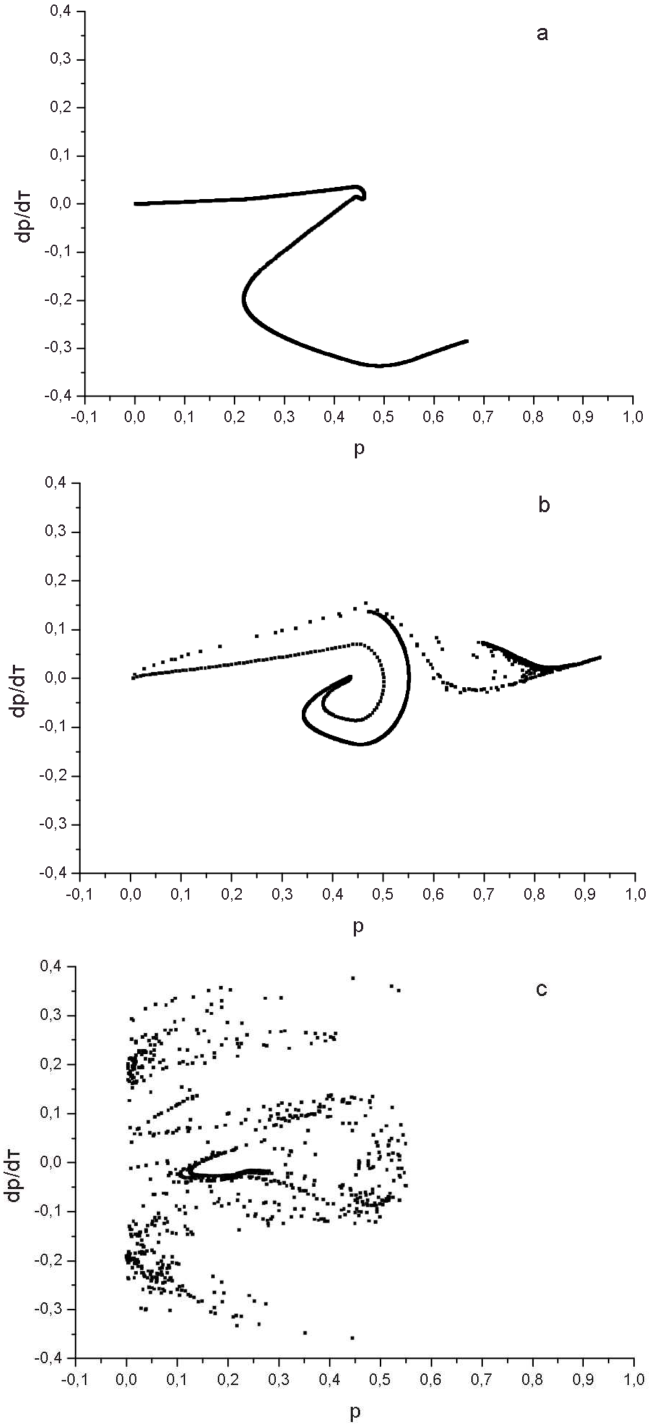

Fig. 5. Phase portraits of ions under $n_{0i}/n_{0e} = 0.02$, $\Delta = 2.5$ cm at instants: *a* – $t = 2.5$ ns, *b* – $t = 7.5$ ns, *c* – $t = 50$ ns

located inside the REB remain motionless, and ions which are located outside of the REB, start a radial movement and form the bunch which makes one turn on the phase plane and again reaches the near-wall area. At that, a new bunch of ions originates. Gyration of the bunch on the phase plane (Fig. 5,b) is accompanied by losses of some part of ions on the chamber wall, and the main part of ions is located outside of the REB. This instant $t = 7.5$ ns (Fig. 5,b) corresponds to the minimum of the potential. For large times $t = 50$ ns (Fig. 5,c), the local compact bunch is clearly seen in the area 0.75 cm $> r > 0.25$ cm, which is surrounded by a "cloud" of intermixed particles. Nevertheless, this formation oscillates in-phase that causes undamped oscillations of the potential. At the instant $t = 50$ ns, the overwhelming majority of ions is located inside the REB, and, in the area $r > 1.38$ cm, they are absent at all. A maximum on the curve of the potential corresponds namely to this instant. To the instant $t = 7.5$ ns corresponding to the minimal potential, the main part of ions are outside of a REB.

4. Summary

Thus, we have investigated the process of excitation of LF ion oscillations at the REB injection into plasma of the given initial density profile. The nonlinear equation in Lagrangian variables describing the radial movement of ions has been obtained.

Numerical calculations have shown that, in the case of a thin near-wall layer, the radial oscillations of ions lead to a slight damping of LF oscillations of the electric potential. Oscillations of the ion layer are accompanied by the increase of its thickness and losses of ions at the wall. These processes result in the slow phase mixing of ions and the damping of electric potential oscillations. With increase in the thickness of the near-wall ion layer, the amplitude of LF oscillations decreases, and their decrement grows. With increase in the ion density, the decrement of LF oscillations grows as well.

This work was partly supported by STCU Project No.1569.

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Received 11.05.07

ЗБУДЖЕННЯ НЧ-КОЛИВАНЬ ПРИ ІНЖЕКЦІЇ РЕЛЯТИВІСТСЬКОГО ЕЛЕКТРОННОГО ПУЧКА В ПЛАЗМУ

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Р е з ю м е

Досліджено нелінійний процес збудження поперечних іонних НЧ-коливань під час поширення релятивістського електронного пучка (РЕП) у плазмі з заданим профілем густини. Отримано нелінійне рівняння в лагранжевих змінних, яке описує радіальні коливання іонів у зовнішньому полі просторового заряду РЕП і у власному самоузгодженому електричному полі. Наведено результати чисельного аналізу вказаної системи рівнянь для трубчатого РЕП та різних поперечних профілів густини плазми. Обговорюються фізичні механізми збудження та загасання іонних НЧ-коливань.