VORTEX MOTION OF NUCLEONS AT ISOVECTOR DIPOLE EXCITATIONS OF NUCLEI

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Velocity fields for isovector dipole excitations in spherical nuclei have been considered in the framework of a semiclassical approach based on the Vlasov kinetic equation for a finite two-component Fermi system with moving surface. The velocity field was found to have a potential character in the energy range near the main maximum of a giant dipole resonance. However, at the energy of the high-energy maximum of the giant dipole resonance, which is caused by a neutron–proton asymmetry and dynamical surface effects, the velocity field reveals a vortex character in the near-surface region.

1. Introduction

In order to obtain more the information concerning the nature of collective excitations in nuclei, it is expedient to consider velocity fields. The latter describe the spatial distribution of the average velocity of nucleons at their collective motion. They can be determined in both quantum-mechanical [1–5] and semiclassical (macroscopic) approaches [5–8], which makes a direct comparison between them possible. In this case, it is important that the semiclassical approach should not used extra assumptions concerning the properties of the velocity field. For instance, in the liquid-droplet model [8], a nucleus is examined as incompressible vortex-free liquid, so that the velocity field has a potential character in this model. On the other hand, in the semiclassical approach, which is based on an explicit solution of the Vlasov kinetic equation for finite Fermi systems [9], the velocity field can be determined making use of the distribution function in the phase space, without making additional assumptions concerning the character of this field. In the framework of this approach, the velocity fields for isoscalar dipole resonances in spherical nuclei have been considered [7], and they were found to be similar to those obtained in the framework of the quantum-mechanical approach [4]. In particular, the low-energy isoscalar dipole resonance was demonstrated to have a vortex character.

In this work, the velocity fields for isovector dipole excitations were considered in the framework of the kinetic model [9]. In quantum-mechanical approaches of the random-phase-approximation (RPA) type, the velocity field for low-energy excitations was found to have an almost potential character in the range of the giant isovector dipole resonance and a vortex one for high-energy excitations. In work [10], a semiclassical approach [9] was used to study isovector dipole excitations in spherical nuclei, taking neutron-excess effects into account. It was found that the isovector dipole response function has two peaks in the energy range of giant isovector resonance in nuclei. It is of interest to study the velocity fields for resonances found in work [10], in order to elucidate the origin of those collective excitations and to compare them with the results of quantum-mechanical calculations. In Section 2, the velocity field at isovector dipole excitations is defined in the framework of the kinetic approach, and the relevant expression is deduced. The results of numerical calculations are presented in Section 3.

2. Velocity Field for Isovector Dipole Excitations

Consider a finite Fermi system consisting of neutrons and protons and confined by a moving spherical surface. At equilibrium, the average velocity of particles in the systems (the velocity field) is equal to zero. If the system is embedded into a weak external field, the velocity field will be different from zero. In the kinetic theory, the time Fourier-transform of the velocity field in the linear approximation is determined as follow:

$$\vec{u}(\vec{r},\omega) = \sum_{q=n,p} \tau_q \vec{u}_q(\vec{r},\omega), \tag{1}$$

$$\vec{u}_q(\vec{r},\omega) = \frac{1}{m\rho_0}\int d\vec{p}\,\vec{p}\,\delta\tilde{n}_q(\vec{r},\vec{p},\omega), \tag{2}$$

where the subscript q means neutrons ($q = n$) or protons ($q = p$); $\tau_n = 1$ and $\tau_p = -1$ in the case of isovector excitations; $\vec{r}$ and $\vec{p}$ are the radius-vector and the momentum of the particle, respectively; ρ_0 is the nuclear matter density at equilibrium; and $\delta\tilde{n}_q(\vec{r},\vec{p},\omega)$ is the variation of the particle distribution function in the phase space, caused by the action of an external field. Because isovector dipole excitations will be considered, the expression for the external field is taken in the form

$$V_q(\vec{r},t) = \beta\delta(t)a_q r Y_{10}(\theta), \qquad (3)$$

where $a_q = 2Z/A$ if $q = n$ and $a_q = -2N/A$ if $q = p$.

The velocity field will be considered in the XZ coordinate plane ($\vec{r} = (x, y = 0, z)$ or, in the spherical coordinates, $\vec{r} = (r, \theta, \varphi = 0)$), because such a representation is used in quantum-mechanical approaches of the RPA type [3]. In this case, the velocity field can be written down in the form

$$\vec{u}_q(r,\theta,\varphi=0,\omega) = u_z^q(r,\theta,\omega)\vec{e}_z + u_x^q(r,\theta,\omega)\vec{e}_x, \qquad (4)$$

where $u_x^q(r,\theta,\omega)$ and $u_z^q(r,\theta,\omega)$ are the projections of the velocity field vector onto the X and Z axes, respectively; and $\vec{e}_x$ and $\vec{e}_z$ are unit vectors directed along these axes.

Using the kinetic approach for the description of collective excitations in finite Fermi systems [9], the projections $u_x^q(r,\theta,\omega)$ and $u_z^q(r,\theta,\omega)$ of the velocity field for dipole excitations can be written down, after some transformations, in the form

$$u_z^q(r,\theta,\omega) = Y_{00}(\theta,0)u_{10}^q(r,\omega) - \sqrt{\frac{2}{5}}Y_{20}(\theta,0)u_{12}^q(r,\omega), \qquad (5)$$

$$u_x^q(r,\theta,\omega) = \sqrt{\frac{3}{5}}Y_{21}(\theta,0)u_{12}^q(r,\omega). \qquad (6)$$

The angular dependence of the velocity field is expressed in terms of spherical functions and coincides with the dependence obtained in quantum-mechanical approaches of the RPA type (see work [3]). The functions $u_{10}^q(r,\omega)$ and $u_{12}^q(r,\omega)$ in Eqs. (5) and (6) describe the radial dependence of the velocity field. The following expressions can be obtained for them:

$$u_{12}^q(r,\omega) = -i\sqrt{\frac{2}{3}}\pi\frac{1}{\rho_0}\frac{1}{r^2}\int d\varepsilon\int dll\times$$

$$\times\sum_{N=-1}^{1}\{-i[\delta\tilde{n}_{q,N}^{+}(r,\varepsilon,l,\omega) - \delta\tilde{n}_{q,N}^{-}(r,\varepsilon,l,\omega)]+$$

$$+\frac{N}{2}\frac{l}{p_r(r,\varepsilon,l)\,r}[\delta\tilde{n}_{q,N}^{+}(r,\varepsilon,l,\omega) + \delta\tilde{n}_{q,N}^{-}(r,\varepsilon,l,\omega)]\} \qquad (7)$$

and

$$u_{10}^q(r,\omega) = -i\sqrt{\frac{1}{3}}\pi\frac{1}{\rho_0}\frac{1}{r^2}\int d\varepsilon\int dll\times$$

$$\times\sum_{N=-1}^{1}\{i[\delta\tilde{n}_{q,N}^{+}(r,\varepsilon,l,\omega) - \delta\tilde{n}_{q,N}^{-}(r,\varepsilon,l,\omega)]+$$

$$+N\frac{l}{p_r(r,\varepsilon,l)\,r}[\delta\tilde{n}_{q,N}^{+}(r,\varepsilon,l,\omega) + \delta\tilde{n}_{q,N}^{-}(r,\varepsilon,l,\omega)\}, \qquad (8)$$

where $\delta\tilde{n}_{q,N}^{\pm}(r,\varepsilon,l,\omega)$ is the solution of the linearized Vlasov kinetic equation for a finite system with moving surface. While deriving Eqs. (5)–(8), a transition from the variables $(\vec{r},\vec{p})$ to new variables $(r,\varepsilon,l,\alpha,\beta,\gamma)$ [9] was made. Here, ε is the particle energy; l its angular moment; α, β, and γ are the Euler angles which determine the rotation to a coordinate system, whose z-axis is directed along the vector $\vec{l} = |\vec{r}\times\vec{p}|$ and y-axis along the vector $\vec{r}$.

Solving the linearized Vlasov equation with boundary conditions at the moving surface, we obtain the following expression for the particle distribution functions in the phase space [10]:

$$\delta\tilde{n}_{q,N}^{\pm}(r,\varepsilon,l,\omega) = \delta n_{q,N}^{0\,\pm}(r,\varepsilon,l,\omega)\times$$

$$\times\left(1 + \sum_{q'=n,p}\kappa_{qq'}\frac{\tilde{R}_{q'}^{v}(\omega)}{a_q a_{q'}}\right) + \delta\tilde{n}_{q,N}^{s\,\pm}(r,\varepsilon,l,\omega). \qquad (9)$$

Here, the function $\delta n_{q,N}^{0\pm}(r,\varepsilon,l,\omega)$ is the solution of the Vlasov equation for a system of noninteracting nucleons confined by a fixed surface [11],

$$\delta n_{q,N}^{0\,\pm}(r,\varepsilon,l,\omega) = -\beta\frac{\partial\bar{n}_0^q(\varepsilon)}{\partial\varepsilon}\times$$

$$\times\sum_{n=-\infty}^{\infty}\omega_{nN}(\varepsilon,l)e^{\pm i[\omega_{nN}(\varepsilon,l)\tau(r,\varepsilon,l)+N\gamma(r,\varepsilon,l)]}\times$$

$$\times\frac{Q_q(nN,\varepsilon l)}{\omega-\omega_{nN}(\varepsilon,l)+i\eta}, \qquad (10)$$

where $\bar{n}_0^q(\varepsilon)$ is the equilibrium distribution function in the Thomas–Fermi approximation [5, 6], $Q_q(nN,\varepsilon\ l) = (-1)^n\frac{a_q v_F^q}{R}\frac{1}{\omega_{nN}^2(\varepsilon,l)}$ is the classical limit for the radial matrix elements of the quantum-mechanical dipole operator, $\omega_{nN}(\varepsilon,l)=n\frac{2\pi}{T(\varepsilon,l)}+N\frac{\Gamma(\varepsilon,l)}{T(\varepsilon,l)}$ are single-particle frequencies, and $T(\varepsilon,l)$ and $\Gamma(\varepsilon,l)$ are the periods of the radial and "angular" particle motions.

The second term on the right-hand side of Eq. (9), $\delta\tilde{n}^{s\pm}_{q,N}(r,\varepsilon,l,\omega)$, is an extra term in the solution of the Vlasov equation, which is associated with the moving surface:

$$\delta\tilde{n}^{s\pm}_{q,N}(r,\varepsilon,l,\omega)=$$

$$=\frac{\partial\bar{n}^0_q(\varepsilon)}{\partial\varepsilon}\frac{e^{\pm i[\omega\tau(r,\varepsilon,l)+N\gamma(r,\varepsilon,l)]}}{\sin[\omega T(\varepsilon,l)+N\Gamma(\varepsilon,l)]}p(R,\varepsilon,l)\omega\delta R_q(\omega). \tag{11}$$

Here, $\delta R_q(\omega)$ are the variations of the equilibrium radius R of the system induced by the external field (3) [10], $2\tau(r,\varepsilon,l)|_{r=R}=T(\varepsilon,l)$, and $2\gamma(r,\varepsilon,l)|_{r=R}=\Gamma(\varepsilon,l)$.

The parameters $\kappa_{qq'}$ in Eq. (9) describe separable nucleon–nucleon interactions in the system bulk,

$$\kappa_{nn}=\kappa_{pp}=\frac{40\pi}{9A}\frac{\varepsilon_{\rm F}}{R^2}(F_0+F_0'),$$

$$\kappa_{np}=\kappa_{pn}=\frac{40\pi}{9A}\frac{\varepsilon_{\rm F}}{R^2}(F_0-F_0').$$

Here, F_0' and F_0 are the Landau parameters describing the isovector and isoscalar, respectively, nucleon-nucleon interactions in symmetric nuclear matter, and $\varepsilon_{\rm F}$ is the Fermi energy of nuclear matter.

The function $\tilde{R}^v_q(\omega)$ in expression (9) determines the variation of the mean field caused by a residual (in our model, separable) interaction in the system bulk [9]. In the case where an neutron-proton asymmetric system is subjected to dipole excitations, this function looks like [10]

$$\tilde{R}^v_q(\omega)=R_q(\omega)-\frac{1}{\beta}\times$$

$$\times\frac{R^2}{1-\kappa_{nn}\left(\frac{R^0_n(\omega)}{a_n^2}+\frac{R^0_p(\omega)}{a_p^2}\right)+(\kappa_{nn}^2-\kappa_{np}^2)\frac{R^0_n(\omega)}{a_n^2}\frac{R^0_p(\omega)}{a_p^2}}\times$$

$$\times\left[\left(1-\kappa_{nn}\frac{R^0_{q'}(\omega)}{a^2_{q'}}\right)[\chi^0_q(\omega)-\chi^0_q(0)]\delta R_q(\omega)+\right.$$

$$\left.+\kappa_{np}\frac{R^0_q(\omega)}{a_na_p}[\chi^0_{q'}(\omega)-\chi^0_{q'}(0)]\delta R_{q'}(\omega)\right]. \tag{12}$$

In expression (12), $R_q(\omega)$ is the function of the collective dipole response of the system confined by a fixed surface:

$$R_q(\omega)=$$

$$=\frac{R^0_q(\omega)\left(1-\kappa_{nn}\frac{R^0_{q'}(\omega)}{a^2_{q'}}+\kappa_{np}\frac{R^0_{q'}(\omega)}{a_na_p}\right)}{1-\kappa_{nn}\left(\frac{R^0_n(\omega)}{a_n^2}+\frac{R^0_p(\omega)}{a_p^2}\right)+(\kappa_{nn}^2-\kappa_{np}^2)\frac{R^0_n(\omega)}{a_n^2}\frac{R^0_p(\omega)}{a_p^2}},$$

where $R^0_q(\omega)$ is the single-particle response function of the system confined by a fixed surface [11],

$$R^0_q(\omega)=\frac{2}{h^3}\frac{8\pi^2}{3}\sum_{N=-1}^{1}\left|Y_{1N}\left(\frac{\pi}{2},\frac{\pi}{2}\right)\right|^2\int_0^\infty d\varepsilon\frac{\partial\bar{n}^0_q(\varepsilon)}{\partial\varepsilon}\times$$

$$\times\int_0^{\sqrt{2m\varepsilon}R}dll[-\omega_{nN}(\varepsilon,l)]T(\varepsilon,l)\frac{[Q_q(nN,\varepsilon\, l)]^2}{\omega-\omega_{nN}(\varepsilon,l)+i\eta}.$$

The function $\chi^0_q(\omega)$ in expression (12) describes dynamic surface effects and is determined as follows:

$$\chi_q(\omega)=-\frac{a_q}{R^2}\left(\frac{3A_q}{4\pi}-m\omega^2\frac{R^0_q(\omega)}{a_q^2}\right).$$

Thus, using the expressions given above, one can carry out numerical calculations of the velocity field in the XZ plane (Eqs. (5) and (6)) in the case of isovector dipole excitations.

3. Results of Calculations

The numerical calculations of the dipole velocity field (Eqs. (5) and (6)) were carried out in various approximations for an asymmetric system with the neutron number $N=126$ and the proton number $Z=82$. In the course of calculations, the following standard values of nuclear parameters were used: $r_0=1.12$ fm, $\varepsilon_{\rm F}=40$ MeV, $m=1.04$ MeV$\times(10^{-22}$ s$)^2$/fm^2. The values for the surface symmetry energy parameter Q and the Landau parameters F_0 and F_0' were taken from work [12]: $Q=35.4$ MeV, $F_0'=1.45$, and $F_0=-0.42$.

In Fig. 1, the strength function $S(E)$ describing the force distribution for isovector dipole excitations is depicted for various approximations. The strength function $S(E)$ is determined by the imaginary part of the response function $\tilde{R}(E)$ [10], where $E=\hbar\omega$. The

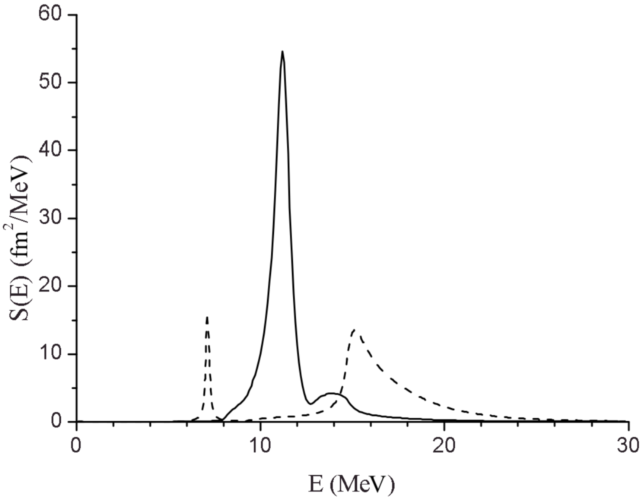

Fig. 1. Strength distributions in an asymmetric system with the neutron number $N = 126$ and the proton number $Z = 82$ in the case of isovector dipole excitations: the solid and dashed curves correspond, respectively, to the system with moving and fixed surface

dashed curve in Fig. 1 corresponds to the distribution of isovector dipole strength in the fixed-surface approximation ($\delta R_q(\omega) = 0$). The figure makes it evident that, in this case, the strength distribution has the resonance with a peak energy at 15 MeV. This peak corresponds to an isovector giant dipole resonance. In addition, the strength distribution is characterized by a resonance structure in the low-energy range (at about 7 MeV), which stems from the motion of the center of mass. The maximum strength position is determined by the isoscalar parameter F_0. The solid curve demonstrates the strength distribution for isovector dipole excitations in the moving-surface approximation, where nucleon-nucleon interaction is taken into account both in the bulk of the system and in its near-surface region. In such an approximation, the isovector dipole strength distribution is characterized by two maxima at energies of 11.8 and 13.9 MeV. The low-energy maximum describes the isovector giant dipole resonance in heavy nuclei. The high-energy maximum is generated in neutron-rich nuclei and is related to the dynamic surface effects being taken into account.

In Fig. 2, the results of numerical calculations of the dipole velocity field in the fixed-surface approximation and for the energy value which corresponds to the strength distribution maximum in this approximation (see Fig. 1, the dashed curve) are depicted. The figure demonstrates that particles move along trajectories that

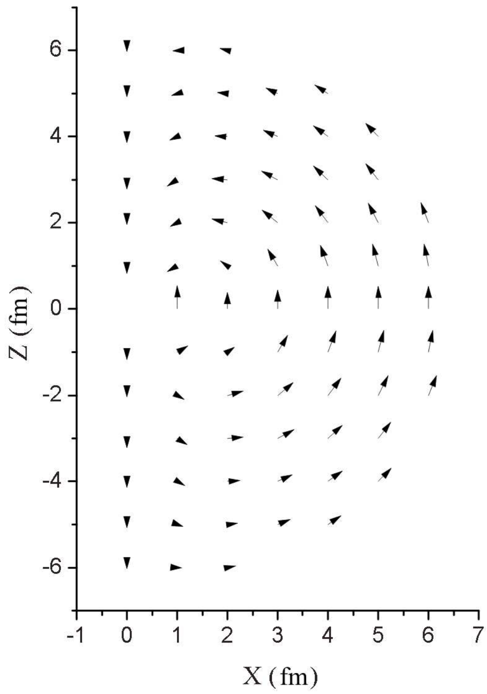

Fig. 2. Velocity field in the XZ coordinate plane of an asymmetric system with the neutron number $N = 126$ and the proton number $Z = 82$ calculated for the isovector giant dipole resonance in the fixed-surface approximation and at the energy $E_{\max} = 15$ MeV, which equals the energy of the resonance strength maximum (see Fig. 1)

are approximately parallel to the surface, i.e. the velocity field has a three-dimensional vortex character. Such a behavior of the velocity field agrees with the Steinwedel–Jensen model [1, 5].

Taking surface effects into account essentially changes the character of the dipole velocity field. It can be clearly seen from Fig. 3, which exhibits the dipole velocity fields calculated in the moving-surface approximation for the energies corresponding to low- and high-energy maxima (see Fig. 1, the solid curve). From Fig. 3,a, it is evident that the dipole velocity field at the energy of the low-energy maximum is the motion of particles along the z-axis (along the direction of action of the external force (3)) in the nucleus bulk. Small deviations from the motion along the z-axis are observed only in the near-surface region, owing to the boundary conditions of specular particle reflection at the moving surface. Such a behavior of the velocity field is in agreement with the results obtained in the Goldhaber–Teller model [1, 5].

However, in contrast to the Goldhaber–Teller model, the radial component of the velocity field $u^q_{12}(r, \omega)$ can

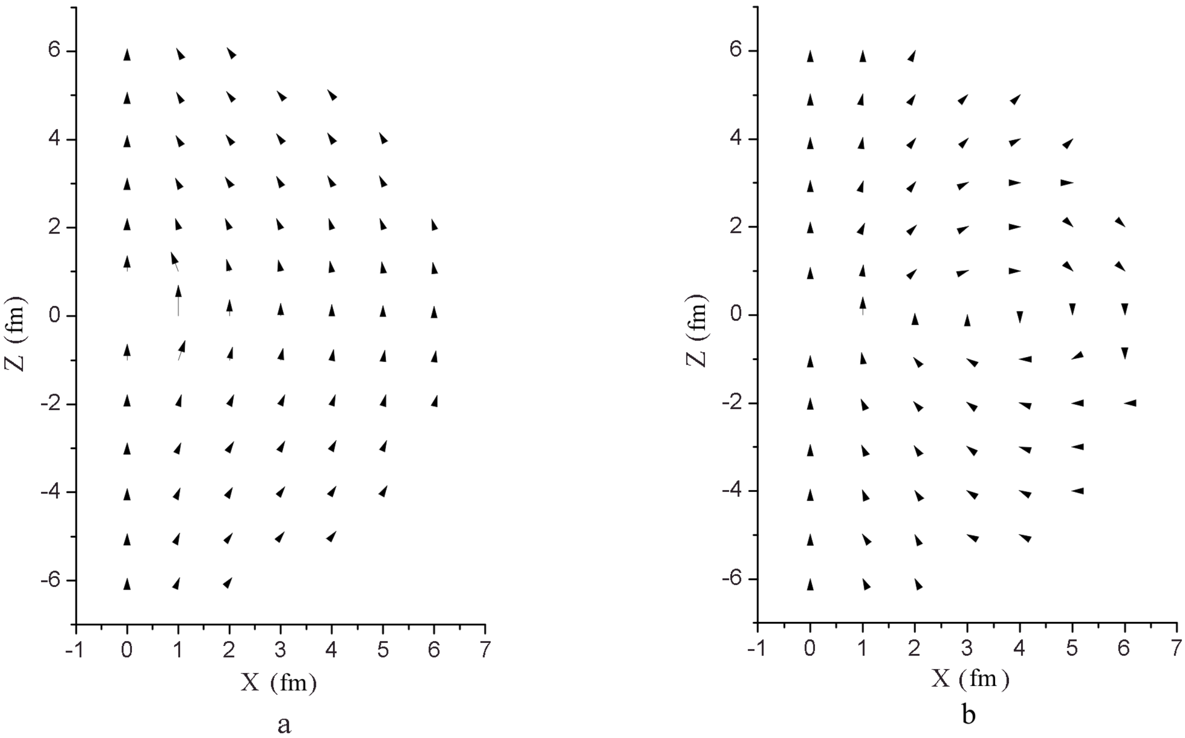

Fig. 3. Velocity fields in the XZ plane of an asymmetric system with the neutron number $N = 126$ and the proton number $Z = 82$ calculated for the isovector giant dipole resonance, in the moving-surface approximation, and at energies $E_{\max} = 11.8$ (a) and 13.9 MeV (b) which correspond to the energies of the low- and high-energy resonance strength maxima, respectively (see Fig. 1)

differ from zero in our model (see Eq. (7)). Really, at the energy of the high-energy maximum, the component $u^q_{12}(r,\omega)$ is the same order as the component $u^q_{10}(r,\omega)$ in the near-surface region. The velocity field at this energy has a vortex character in this region (see Fig. 3,b). The isovector dipole velocity fields obtained in this work agree with the results of corresponding quantum-mechanical calculations carried out in the RPA framework [1, 3].

4. Conclusions

The velocity fields for isovector dipole excitations in spherical nuclei have been studied in a semiclassical approach, which is based on the Vlasov kinetic equation for a finite two-component system with moving surface. The expression for the velocity field in the XZ coordinate plane, presented in terms of the particle distribution function in the phase space, has been considered. Taking advantage of a solution of a linearized Vlasov equation for a finite system with moving surface found in work [10], an analytical expression for the velocity field has been derived in the case of collective isovector dipole excitations. Numerical calculations have been carried out for the velocity field of a giant isovector dipole resonance, which proved to have two maxima in our approach. The velocity field was found to manifest an approximately potential character, as in the Goldhaber–Teller model, at the low-energy resonance energy. But, at the energy of the high-energy resonance, which emerges in neutron–proton asymmetric systems with a moving surface, the velocity field for isovector dipole excitations reveals a vortex character.

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ВИХРОВИЙ РУХ НУКЛОНІВ ПРИ ІЗОВЕКТОРНИХ ДИПОЛЬНИХ ЗБУДЖЕННЯХ ЯДЕР

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Р е з ю м е

Поля швидкостей для ізовекторних дипольних збуджень в сферичних ядрах розглянуто в рамках напівкласичного підходу, що спирається на кінетичне рівняння Власова для скінченних двокомпонентних фермі-систем з рухомою поверхнею. Знайдено, що в діапазоні енергій основного максимуму гігантського дипольного резонансу поле швидкостей має потенціальний характер. Проте поле швидкостей проявляє вихровий характер в поверхневій області для енергії високоенергетичного максимуму гігантського дипольного резонансу, який зумовлений нейтрон-протонною асиметрією та динамічними ефектами поверхні.