CROSS-SECTIONS OF ELECTRIC AND MAGNETIC LIGHT ABSORPTION BY SPHERICAL METALLIC NANO-PARTICLES. THE EXACT KINETIC SOLUTION

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The electric and magnetic cross-sections of light absorption by spherical metal particles, whose nanometer dimensions can be smaller than the electron free path, have been calculated. The approach is based on solving the kinetic equation with diffuse boundary conditions describing the reflectance of an electron from the interior side of the particle's walls. Analytical expressions, which allow the value of the cross-section to be determined in terms of the particle's radius and the frequency of incident electromagnetic radiation, have been obtained in the case, where the skin-layer is thicker than the characteristic size of the particle.

1. Introduction

The properties of small metallic particles (SMPs) substantially differ from those of a metal ingot. The reasons for that include not only the probable influence of quantum-mechanical finite-size effects — important by themselves [1, 2] — but may have the merely classical aspects as well. The classical Mie theory of interaction between electromagnetic (EM) radiation and spherical metal particles [3], which is based on the local equations of macroscopic electrodynamics, cannot be applied in the cases where the mean free path of electron l becomes either equal to or larger than the particle's size a. In this case, the collisions of electrons with the particle's surface essentially affect the response of the particle to the external field, so that the kinetic theory of conduction electrons in metal should be applied to calculate the character of interaction between light and the particle [4].

Usually, the particle's size is considered to be much smaller than the length of the EM wave. Under such conditions (see, e.g., work [5]), both the electric (electric absorption) and magnetic (magnetic absorption) components of the EM waves are known to contribute to the absorption by non-magnetic materials. A many papers on this topic have been published, and the principal aspects of the problem have been elucidated. The cases of electric and magnetic absorption by a spherical particle with $a \gg l$ have been studied especially well [6–10]. An analytical expression for magnetic absorption in the opposite case, where $a \leq l$, has been derived as well [11–15]. The results of those researches were summarized in monographies [4, 5, 16, 17]. The natural question arises: what is the use of one more paper on such a well studied subject? The answer is as follows.

First, in this work we derived analytical expressions for electric and magnetic absorption, which all the limiting cases obtained in others works are resulted from. Second, the resonance oscillations of the optical conductivity, which are caused by coming closer the frequency of the EM-wave and that of electron oscillations between particle's walls, were studied only in a few papers (see, e.g., works [12–14]). The electric oscillations were dealt with in work [12], while the magnetic ones in works [13, 14]. Nevertheless, explicit analytical expressions for oscillatory terms were obtained in none of them. The only analytical result has been derived, in the framework of the kinetic approach, for magnetic absorption, but the collision frequency was not taken into account at that [13, 14]. However, the explicit expressions for the electron scattering cross-section, which would take simultaneously into account the contributions of the electric and magnetic components of the EM wave, as well as the bulk and the surface electron scattering, were not written down. Nevertheless, they can be obtained for a spherical particle even if the collision frequency is made allowance for.

The purpose of this work was just to fill this blank, to derive exact analytical expressions for the cross-sections of electric and magnetic absorption by a spherical SMP in the framework of the kinetic method, and to carry out their comprehensive analysis.

2. Fundamentals

Let an EM wave with the components $\mathbf{E}$ and $\mathbf{H}$ of the electric and magnetic fields, respectively, fall onto an SMP with the radius R. Let λ, ω, and $\mathbf{k}$ be the length, frequency, and wave vector, respectively, of this EM wave, while $\mathbf{r}$ and t describe the spatial coordinate and time. We assume that $a \ll \lambda$. It allows us to consider the particle as such, which is embedded into the spatially uniform but oscillating in time $\mathbf{E}$ and $\mathbf{H}$ fields. The electric component of the wave induces a local electric field inside the SMP, which can be expressed in terms of the dielectric permittivity of the particle medium $\varepsilon(\omega)$: $E_{\text{in}} = 3E_0/[2+\varepsilon(\omega)]$ [18]. The magnetic wave component is responsible for the appearance of an eddy electric field $\mathbf{E}_{\text{ed}}$. If the skin-layer is thick ($\delta_H \gg R$), the field $\mathbf{H}_0$ can be regarded as uniform and constant one. Then, the Maxwell equations for the field $\mathbf{E}_{\text{ed}}$ look like

$$\text{rot}\,\mathbf{E}_{\text{ed}} = i\frac{\omega}{c}\mathbf{H}_0, \quad \text{div}\,\mathbf{E}_{\text{ed}} = 0 \tag{1}$$

and the boundary condition on the particle's surface is $\mathbf{E}_{\text{ed}}\mathbf{n}_S = 0$, where $\mathbf{n}_S$ is the normal to the surface of the sphere S. In this case, the contribution of eddy currents to absorption is maximal. Having found $\mathbf{E}_{\text{ed}}$ and knowing the current $\mathbf{j}_m(\mathbf{r})$ as well as $\mathbf{E}_{\text{in}}$ with the corresponding current $\mathbf{j}_e$, one can calculate the absorbed power by the formula

$$W = \frac{1}{2}\text{Re}\int_V d\mathbf{r}\,[\mathbf{j}_m(\mathbf{r})\,\mathbf{E}^*_{\text{ed}}(\mathbf{r}) + \mathbf{j}_e(\mathbf{r})\mathbf{E}_{\text{in}}(\mathbf{r})]. \tag{2}$$

We consider the case, where the characteristic dimensions of the particle can be smaller than the electron free path. Then, the wave energy is absorbed mainly owing to the collisions of conduction electrons with the interior surface of the particle. The corresponding calculations of the current must be carried out in the framework of the microscopic approach, according to which electrons follow the statistics of almost ideal degenerate Fermi-gas, and

$$\mathbf{j}(\mathbf{r}) = 2e\left(\frac{m}{2\pi\hbar}\right)^3 \iiint \mathbf{v}(\mathbf{r})\,f(\mathbf{r},\mathbf{v})\,d^3(v), \tag{3}$$

where $f(\mathbf{r},\mathbf{v})$ is the electron distribution function in the coordinate and velocity space, e the charge of the electron, and m its mass. The fields $\mathbf{E}_{\text{ed}}$ and $\mathbf{E}_{\text{in}}$ are responsible for the deviation of the electron distribution from the equilibrium Fermi one. Therefore, the total distribution function $f(\mathbf{r},\mathbf{v})$ is tried as the sum of an equilibrium part $f_0(\varepsilon)$, which depends on the electron's kinetic energy only, and a nonequilibrium addend $f_1(\mathbf{r},\mathbf{v})$, the linear approximation of which in the external field is determined from the Boltzmann kinetic equation

$$(\nu_V - i\omega)\,f_1(\mathbf{r},\mathbf{v}) + \mathbf{v}\frac{\partial f_1(\mathbf{r},\mathbf{v})}{\partial\mathbf{r}} +$$

$$+e\,(\mathbf{E}_{\text{in}} + \mathbf{E}_{\text{ed}})\mathbf{v}\frac{\partial f_0(\varepsilon)}{\partial\varepsilon} = 0, \tag{4}$$

where the collision integral is presented in the relaxation time approximation, and ν_V is the electron collision frequency in the particle's bulk. For the function $f_1(\mathbf{r},\mathbf{v})$ to be determined unambiguously, one has to specify the corresponding boundary conditions on the particle's surface. As such, we choose the conditions of diffuse electron reflection from the interior surface of the particle [15–17]: $f_1(\mathbf{r},\mathbf{v})|_S = 0$ and $v_n < 0$, where v_n is the normal component of the electron velocity with respect to the surface S. In order to solve the partial differential equation, the method of characteristics is used as a rule. In this case, the general expression for the solution $f_1(\mathbf{r},\mathbf{v})$ looks like [15]

$$f_1(\mathbf{r},\mathbf{v}) = -e\frac{\partial f_0}{\partial\varepsilon}\left[\mathbf{v}\mathbf{E}_{\text{in}} + \sum_{i,j=1}^{3}\alpha_{ij}v_i x_j\right]\times$$

$$\times\left(\frac{1-e^{-(\nu_V - i\omega)t}}{\nu_V - i\omega}\right), \tag{5}$$

where, in contrast to work [15], the item connected with the particle asymmetry is omitted, v_j is the j-th component of the velocity vector, $x_1 = x$, $x_2 = y$, and $x_3 = z$. The parameter t means the time of electron "motion" with the velocity $\mathbf{v}$ along the trajectory $\mathbf{r} = \mathbf{v}t + \mathbf{R}$ from the particle's surface to the point $\mathbf{r}$ and is determined by the formula

$$t = [\mathbf{r}\mathbf{v} + \sqrt{(\mathbf{r}\mathbf{v})^2 + (R^2 - r^2)v^2}]/v^2. \tag{6}$$

The radius-vector $\mathbf{R}$ specifies the coordinates of the surface, from which the electron starts to move (at $t = 0$). The augend in Eq. (5) is responsible for the absorption of the electric component of the EM wave, while the addend for the absorption of the magnetic one. The matrix members α_{ij} are the corresponding coefficients in the linear expansion of the eddy field and are determined from Eqs. (1) and the relevant boundary conditions. For a spherical particle, $\alpha_{ij} = \mp i \frac{\omega}{2c} H_k^{(0)}$. The subscripts $i \neq j \neq k$ vary as 1,2, and 3, which corresponds to the x-, y-, and z-projections, respectively. The matrix α is antisymmetric.

Expressions (5) and (6), together with the diffuse reflection conditions for electrons, completely define the solution of Eq. (4). The distribution function to be found allows, in its turn, current (3) and the average absorbed power (2) to be calculated.

3. Cross-Section of the EM-Wave Electric Absorption

The cross-section of absorption of the external electric field energy is defined as the ratio between the average dissipated power (2) and the average energy flux in the incident wave $c\mathbf{E}_0^2/(8\pi)$. Making use of expression (5) for the distribution function, as well as expressions (2) and (3), we find the following expression for the absorption cross-section:

$$S_E = \frac{e^2 m^3}{\pi^2 \hbar^3 c} \frac{1}{\mathbf{E}_0^2} \times$$

$$\times \mathrm{Re}\left[\frac{1}{\bar{\nu}} \iiint d\mathbf{r} \iiint d\mathbf{v} |\mathbf{v}\mathbf{E}_{\mathrm{in}}|^2 \delta(\varepsilon - \mu)\,(1 - e^{-\bar{\nu}\,t})\right], \tag{7}$$

where $\bar{\nu} = \nu_V - i\omega$, and the approximate equality $\partial f_0/\partial\varepsilon \approx -\delta(\varepsilon - \mu)$ was supposed. After integrating in Eq. (7) over all coordinates, we obtain

$$S_E(\omega, R) = 3\pi R^3 \frac{n\,e^2}{c v_{\mathrm{F}}^3} \frac{1}{\mathbf{E}_0^2} \times$$

$$\times \mathrm{Re}\left[\frac{1}{\bar{\nu}} \iiint d\mathbf{v}\; |\mathbf{v}\mathbf{E}_{\mathrm{in}}|^2 \psi(v)\; \delta(\varepsilon - \mu)\right], \tag{8}$$

where $v_{\mathrm{F}} = (2\mu/m)^{1/2}$ is the electron's velocity on the Fermi surface, μ is the Fermi energy, n is the electron concentration, and

$$\psi(v) = \frac{4}{3} - \frac{v}{R\bar{\nu}} + \frac{4v^3}{(R\bar{\nu})^3} - \frac{4v^2}{(2R\bar{\nu})^2}\left(1 + \frac{v}{2R\bar{\nu}}\right)\, e^{-\frac{2R}{v}\bar{\nu}}. \tag{9}$$

Now, let us introduce the collision frequency of the electron with the SMP's surface:

$$\nu_S = \frac{v_{\mathrm{F}}}{2R}. \tag{10}$$

For example, for a particle with $R = 150$ Å, provided that $\nu_V \approx 1 \times 10^{13}$ s^{-1} and $v_{\mathrm{F}} \approx 0.8 \times 10^8$ cm/s, the frequency $\nu_S \approx 2.67 \times 10^{13}$ s^{-1} is almost three times higher than the collision frequency in the particle's bulk. Analogous estimations show that the frequencies of electron collisions with the surface and in the bulk become comparable ($\nu_S \approx \nu_V$) only for particles with $R \approx 400$ Å.

Remaining integration in the velocity space, owing to the presence of the δ-function, does not bring about any difficulty. Therefore, we write down the final result

$$S_E(\omega, R) = 2S_0^E \frac{\omega_p^2}{\nu_V^2 + \omega^2}\, d(\omega) \times$$

$$\times \left\{ \frac{\nu_V}{3\nu_S} - \frac{1}{2}\frac{\nu_V^2 - \omega^2}{\nu_V^2 + \omega^2} + \nu_S^2 \frac{(\nu_V^2 - \omega^2)^2 - 4\nu_V^2\omega^2}{(\nu_V^2 + \omega^2)^3} - \right.$$

$$-\nu_S \frac{\cos\left(\frac{\omega}{\nu_S}\right)\, e^{-\frac{\nu_V}{\nu_S}}}{(\nu_V^2 + \omega^2)^2} \left[\nu_V(\nu_V^2 - 3\omega^2) + \right.$$

$$\left. + \nu_S \frac{(\nu_V^2 - \omega^2)^2 - 4\nu_V^2\omega^2}{\nu_V^2 + \omega^2}\right] + \nu_S \frac{\sin\left(\frac{\omega}{\nu_S}\right)\, e^{-\frac{\nu_V}{\nu_S}}}{(\nu_V^2 + \omega^2)^2} \times$$

$$\left. \times \left[\omega(3\nu_V^2 - \omega^2) + \nu_S \frac{4\nu_V\omega\;(\nu_V^2 - \omega^2)}{\nu_V^2 + \omega^2}\right] \right\}, \tag{11}$$

where the notations

$$S_0^E = \pi R^2 \left(\frac{v_{\mathrm{F}}}{c}\right), \tag{12}$$

$$d(\omega) = \frac{E_{\mathrm{in}}^2}{E_0^2} = \frac{9}{[\varepsilon'(\omega) + 2]^2 + [\varepsilon''(\omega)]^2}, \tag{13}$$

$$\varepsilon'(\omega) = 1 - \frac{\omega_p^2}{(\nu_V + 3\nu_S/2)^2 + \omega^2},$$

$$\varepsilon''(\omega) = \frac{\omega_p^2}{\omega} \frac{\nu_V + 3\nu_S/2}{(\nu_V + 3\nu_S/2)^2 + \omega^2}, \tag{14}$$

were introduced, $\varepsilon'(\omega)$ and $\varepsilon\prime\prime(\omega)$ are the real and imaginary parts, respectively, of the dielectric permittivity, and $\omega_p = \sqrt{4\pi n e^2/m}$ is the plasma oscillation frequency. The quantity ν_V in Eq. (14) was

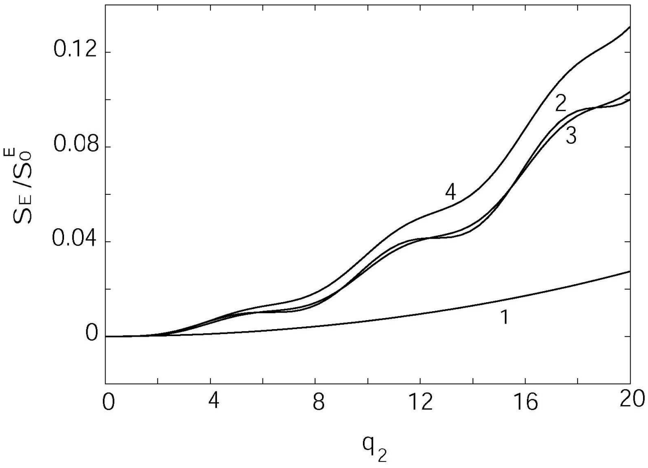

Fig. 1. Dependences of the cross-section ratio for electron absorption, stimulated by the electric component of the EM-wave, in a spherical SMP ($R = 150$ Å) on the frequency ratio $q_2 = \omega/\nu_S$. The contributions of merely bulk (at $\nu_V = 10^{13}$ s^{-1}) and merely surface (at $\nu_V = 0$) electron collisions are depicted by curves *1* and *2*, respectively. Curve *3* corresponds to the contribution of surface electron collisions with a correction for the influence of electron collisions in the bulk, and curve *4* to the contribution sum

formally substituted by $\nu_V + 3\nu_S/2$ [11] in order to ensure the correct limiting transition to the case of small ν_V.

Formula (11) is a general analytical expression for electron diffuse scattering in an electric field inside a spherical metal particle. It takes into account merely bulk electron collisions and collisions of electrons with the surface, as well as their interference. The part that is responsible for merely bulk collisions (the Drude classical case) can be singled out by taking $a \gg l$ or, in terms of the frequency, $\nu_V \gg \nu_S$ in Eq. (11). In so doing, only the first summand in the braces evidently remains:

$$S_E^{\rm cl}(\omega, R) = V\frac{\omega}{c}\varepsilon''(\omega)d(\omega), \tag{15}$$

where V is the particle's volume. Expression (15) corresponds to the well-known classical result for electron scattering cross-section in an electric field [18]. It can also be obtained immediately from formula (8), by putting $\psi(v) \approx 4/3$ and carrying out integration.

Provided that the frequency of bulk collisions $\nu_V \to 0$, the classical cross-section $S_E^{\rm cl} \to 0$.

The sum of all the terms, but the first, in expression (11) describes the contribution of diffuse scattering of electrons from the interior surface of the particle, taking into account the influence of electron collisions in the particle's bulk that are caused by the EM-wave electric field inside the particle. These summands dominate if the mean free path of the electron exceeds the particle's dimensions.

In the case, where — formally — the frequency of bulk collisions $\nu_V \to 0$, we obtain, from Eq. (11), the final dependence, which differs from the classical formula:

$$S_E^{\nu_V\to 0}(\omega, R) = S_0^E d(\omega)\frac{\omega_p^2}{\omega^2}\times$$

$$\times\left[1 + 2\,\left(\frac{\nu_S}{\omega}\right)^2\left(1 - \cos\left(\frac{\omega}{\nu_S}\right)\right) - 2\,\frac{\nu_S}{\omega}\sin\left(\frac{\omega}{\nu_S}\right)\right]. \tag{16}$$

This expression is attributed to merely surface collisions, without any influence of electron collisions in the bulk. In the limiting cases of high ($\omega \gg \nu_S$) and low ($\omega \ll \nu_S$) frequencies, Eq. (16), making allowance for Eq. (12), brings about

$$S_E^{\nu_V\to 0}(\omega, R)|_{\omega\gg\nu_S} \approx 6\pi V\frac{ne^2}{mc}\frac{\nu_S}{\omega^2}d(\omega),$$

$$S_E^{\nu_V\to 0}(\omega, R)|_{\omega\ll\nu_S} \approx \frac{3}{2}\pi V\frac{ne^2}{mc}\frac{1}{\nu_S}d(\omega). \tag{17}$$

Expressions (17) coincide with the result that was obtained earlier in work [15] for W_e, if one takes into account that $W_e = c\mathbf{E}_{\rm in}^2 S_E/(8\pi d(\omega))$.

The bulk scattering depends as R^3 on the particle's radius, whereas the surface one contains summands that depend on R or R^2, or are independent of R altogether.

To carry out calculations, the following values of the key parameters were chosen: $\nu_V = 1\times 10^{13}$ s^{-1}, $\omega_p = 5\times 10^{15}$ s^{-1}, $v_{\rm F} = 0.8\times 10^8$ cm/s, and $n = 10^{22}$ cm^{-3}. This set results, in particular, in $l = v_{\rm F}/\nu_V = 800$ Å.

In Fig. 1, the dependence of the relative cross-section of electron scattering in the electric field in a spherical SMP with $R = 150$ Å on the EM-wave frequency is depicted. [1] Individual contributions, connected with bulk (curve *1*) and surface (curve *2*) scattering, are shown, as well as the influence of bulk electron collisions on the electron scattering by the surface (curve *3*). The total cross-section (*1* + *3*) is illustrated by curve *4*.

From Fig. 1, one can see that the contribution of the electric field to the cross-section of electron

[1] The estimations show that, even at the collision frequency when the skin-layer thickness is minimal, the latter is almost eight times larger than the value selected for R; therefore, the condition $\delta_H \gg R$ is satisfied.

scattering appreciably grows as the frequency of the EM-wave increases. In so doing, electron scattering by the particle's surface plays the dominating role and is responsible for the frequency oscillations of the relative scattering cross-section, which, as we can assert, arise only if kinetic effects are made allowance for, being connected with the conditions of electron reflection from the particle's surface.

4. Cross-Section of the EM-Wave Magnetic Absorption

Taking expressions (2), (3), and the second term in the square brackets of Eq. (5) into account, we find the following expression for the cross-section of magnetic absorption:

$$S_M = \frac{8\pi e^2 m^3}{(2\pi\hbar)^3 c}\frac{1}{\mathbf{H}_0^2}\mathrm{Re}\left\{\frac{1}{\bar\nu}\iiint d\mathbf{r}\iiint d\mathbf{v}\,\delta(\varepsilon-\mu)\times\right.$$

$$\left.\times\sum_{i\,j\,l\,k}^{3}\alpha^*_{i\,j}\alpha_{l\,k}\upsilon_i x_j \upsilon_l x_k\,\left(1-e^{-\bar\nu\,t}\right)\right\}. \qquad (18)$$

The calculation scheme is similar to that used above for the case of electron scattering in the electric field. Making use of Eqs. (1)—(3) and integrating over coordinates, we obtain the expression for the cross-section of electron absorption in the circuital magnetic field, which, following the work [15], can be presented in the form

$$S_M = \frac{3}{2}\pi R^3\frac{ne^2}{c\upsilon_{\mathrm{F}}^3\mathbf{H}_0^2}\times$$

$$\times\mathrm{Re}\left[\frac{1}{\bar\nu}\iiint d\mathbf{v}\,\delta(\varepsilon-\mu)\psi_1(\upsilon)\sum_{i\,j}^{3}\,|\alpha_{i\,j}|^2R_j^2\upsilon_i^2\right], \qquad (19)$$

where

$$\psi_1(\upsilon)=\frac{8}{15}-\frac{\upsilon}{2R\bar\nu}+\frac{4\upsilon^3}{(2R\bar\nu)^3}-\frac{24\upsilon^5}{(2R\bar\nu)^5}+$$

$$+\frac{8\upsilon^3}{(2R\bar\nu)^3}\left(1+\frac{3\upsilon}{2R\bar\nu}+\frac{3\upsilon^2}{(2R\bar\nu)^2}\right)e^{-\frac{2R\bar\nu}{\upsilon}}. \qquad (20)$$

Formula (19) determines the magnetic absorption cross-section and includes both the terms that are responsible for merely bulk electron scattering (owing to electron collisions with phonons, lattice defects, and so on) and the terms associated with merely surface scattering of electrons. It also takes into account the interference between the electron collisions in the bulk and on the surface of the particle. In the case of a homogeneous spherical particle, the sum in Eq. (19) is equal to

$$\sum_{i,j=1}^{3}|\alpha_{ij}|^2\upsilon_i^2R_j^2 = 3\mathbf{H}_0^2\upsilon^2\left(\frac{\omega}{2c}\right)^2R^2. \qquad (21)$$

Further integration over the velocities, owing to the presence of the δ-function, can be carried out in the most general case. Although the expression obtained looks rather cumbersome, we write it down below, because it is the exact result obtained in the framework of the kinetic approach:

$$S_M(\omega,R) = S_0^M\frac{\omega^2}{\nu_V^2+\omega^2}\left\{\frac{8}{15}\frac{\nu_V}{\nu_S}-\right.$$

$$-\frac{\nu_V^2-\omega^2}{\nu_V^2+\omega^2}+4\,\nu_S^2\frac{(\nu_V^2-\omega^2)^2-4\,\nu_V^2\omega^2}{(\nu_V^2+\omega^2)^3}-$$

$$-24\,\nu_S^4\frac{\nu_V^2-\omega^2}{(\nu_V^2+\omega^2)^5}\left[(\nu_V^2-\omega^2)^2-12\,\nu_V^2\omega^2\right]+$$

$$+e^{-\frac{\nu_V}{\nu_S}}\frac{8\,\nu_S^2}{(\nu_V^2+\omega^2)^3}\left\{\cos(\omega/\nu_S)\left[(\nu_V^2-\omega^2)^2-\right.\right.$$

$$-4\,\nu_V^2\,\omega^2+3\,\nu_V\nu_S\frac{(\nu_V^2-\omega^2)^2-4\omega^2(2\,\nu_V^2-\omega^2)}{\nu_V^2+\omega^2}+$$

$$\left.+3\,\nu_S^2(\nu_V^2-\omega^2)\frac{(\nu_V^2-\omega^2)^2-12\,\nu_V^2\omega^2}{(\nu_V^2+\omega^2)^2}\right]-$$

$$-\sin(\omega/\nu_S)\left[4\,\nu_V\omega(\nu_V^2-\omega^2)+\right.$$

$$+3\,\omega\,\nu_S\frac{(\nu_V^2-\omega^2)^2+4\nu_V^2(\nu_V^2-2\omega^2)}{\nu_V^2+\omega^2}+$$

$$\left.\left.\left.+6\,\nu_V\omega\,\nu_S^2\frac{3\,(\nu_V^2-\omega^2)^2-4\,\nu_V^2\omega^2}{(\nu_V^2+\omega^2)^2}\right]\right\}\right\}, \qquad (22)$$

where

$S_0^M = (\pi R^2)^2ne^2\upsilon_{\mathrm{F}}/(2mc^3)$.

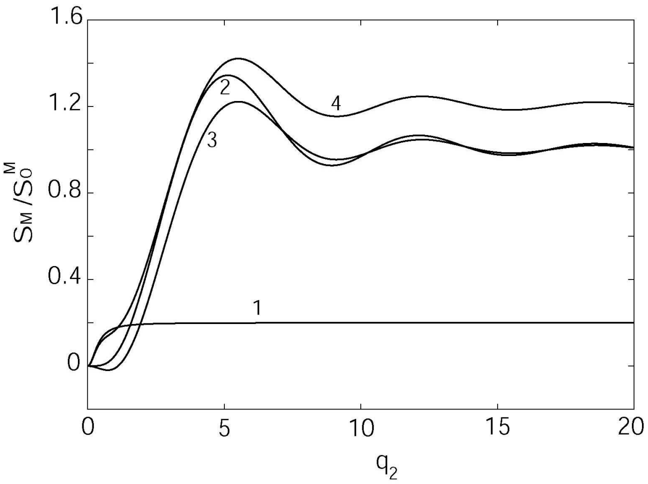

Fig. 2. The same as in Fig. 1, but for the magnetic component of the EM-wave

The specific mechanisms of electron scattering are taken into account in the frequency ratio ν_S/ν_V. For example, if $\nu_V \gg \nu_S$, electron scattering in the SMP's bulk plays the leading role in the absorption of wave energy, as it did in the previous case. On the contrary, if $\nu_V \ll \nu_S$, electron scattering by the particle's surface, influenced by bulk collisions, dominates. One can easily verify that, while considering the first case, the first summand in the braces in Eq. (22) prevails, and we obtain the classical result [18]

$$S_M^{\rm cl}(\omega, R) = \frac{2}{15}\pi R^5 \frac{\omega^3}{c^3}\varepsilon''(\omega). \tag{23}$$

The identical expression can also be obtained from Eq. (19), if one takes Eq. (20) into consideration and puts $\mathrm{Re}\{\psi_1(v)/\bar{\nu}\} \approx R/3v$ or $\psi_1 \approx 8/15$. The sum of all the rest summands in formula (22) describes the contribution of kinetic effects related to the influence of the circuital magnetic field on the diffuse reflection of the electron from the interior SMP's surface. They correspond to the case $\nu_V \ll \nu_S$, i.e. $l \gg a$, where surface electron scattering dominates.

Provided that the collision frequency $\nu_V \rightarrow 0$, Eq. (22) leads to

$$S_M^{\nu_V \rightarrow 0}(\omega, R) = S_0^M \left\{ 1 + 4\left(\frac{\nu_S}{\omega}\right)^2 + 24\left(\frac{\nu_S}{\omega}\right)^4 + 8\left(\frac{\nu_S}{\omega}\right)^2 \left[\left(1 - 3\left(\frac{\nu_S}{\omega}\right)^2\right) \times \cos\left(\frac{\omega}{\nu_S}\right) - 3\frac{\nu_S}{\omega}\sin\left(\frac{\omega}{\nu_S}\right)\right]\right\} \tag{24}$$

This result coincides with the solution found earlier in works [13, 14].

We can also obtain the low- (LF) and high-frequency (HF) limits of expression (24), which are defined by the relationship between the frequencies ω and ν_S. In the high- ($\omega \gg \nu_S$) and low-frequency ($\omega \ll \nu_S$) cases, the corresponding asymptotes are

$$S_M^{\rm HF}(R) \approx \frac{3}{4} V\,\pi R^2 \frac{ne^2}{mc^3}\,\nu_S \equiv S_0^M,$$

$$S_M^{\rm LF}(\omega, R) \approx \frac{V}{8}\pi R^2 \frac{ne^2}{mc^3}\frac{\omega^2}{\nu_S}, \tag{25}$$

which coincide with the results obtained by us earlier [19].

Figure 2 exhibits the frequency dependences of the relative absorption cross-sections for a merely magnetic component of the EM-wave, associated with the electron collisions both in the bulk (curve *1*) and with the particle's surface (curve *2*), as well as with their interference (curve *3*). The relevant calculations were carried out by formula (22), making use of the same parameter values, as for the electric field. The contribution of the magnetic field to the electron scattering cross-section reaches the maximal value $S_M/S_0^M \approx 1.42$ at the frequency $\omega \approx 6\nu_S$ and starts to oscillate, with some damping, about the value $S_M/S_0^M \approx 1.21$ with the period of about $7\nu_S$. The oscillations become indistinguishable at frequencies $\omega > 20\nu_S$. For particles with $R = 150$ Å, the quantity $S_0^M \approx 1.87 \times 10^{-16}$ cm^2. The diffuse scattering of electrons on the SMP's surface, similarly to the case with the electric field, starts to dominate over the bulk one as the frequency of the incident EM-wave grows (in the magnetic field case, at $\omega \gtrsim 1.65\nu_S$) and becomes more than five times larger already at $\omega > 4\nu_S$. Curve *2* in Fig. 2 corresponds to merely surface collisions. The behavior of curve *3* allows us to draw a conclusion that expression (24) describes well electron magnetic absorption (both in the bulk and on the surface) in the frequency range $\omega < 5\nu_S$.

The total cross-section of EM-wave scattering is the sum

$$S(R, \omega) = S_E(R, \omega) + S_M(R, \omega), \tag{26}$$

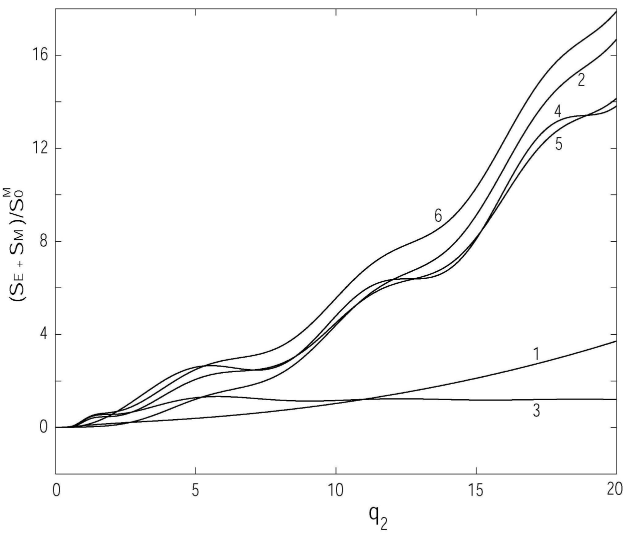

Fig. 3. Dependences of the ratio between electric and magnetic light absorption cross-sections in a spherical SMP ($R = 150$ Å) on the frequency ratio $q_2 = \omega/\nu_S$. The contributions of the particle's bulk (at $\nu_V = 10^{13}$ s^{-1}) and surface (at $\nu_V = 0$) are exhibited by curves *1* and *4*, respectively. Curves *2* and *3* correspond to the contributions of only electric or magnetic component of the EM-wave, respectively. Curve *5* corresponds to the surface electron collisions with a correction for the influence of electron collisions in the bulk, and curve *6* to the sum of the (volume + surface) contributions

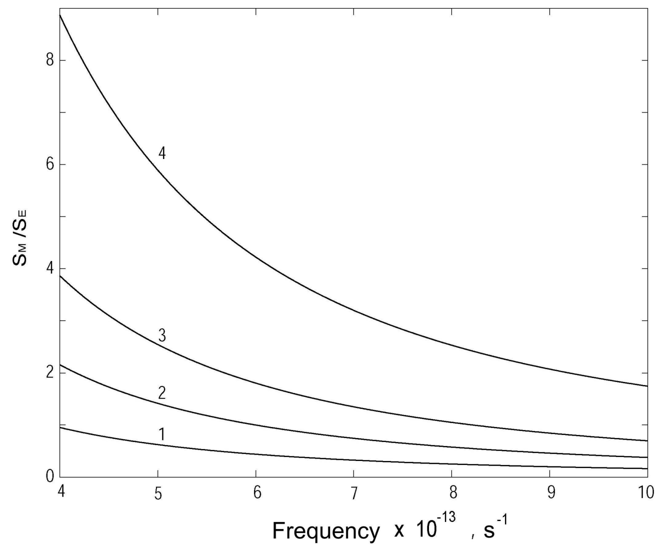

Fig. 4. Frequency dependences of the ratio between the cross-sections of magnetic and electric light absorption in SMPs with various radii R =50 (*1*), 75 (*2*), 100 (*3*), and 150 Å (*4*)

where $S_E(R, \omega)$ and $S_M(R, \omega)$ are defined by expressions (11) and (22), respectively.

In Fig. 3, the summed up frequency dependences of the electric and magnetic absorption cross-sections are plotted. As is seen from the figure, the (bulk + surface) contribution of the electric EM-wave component (curve *2*) to total (electric + magnetic) absorption starts to dominate over the (bulk + surface) contribution of the magnetic component (curve *3*) at frequencies $\omega \geq 5.5\nu_S$ and, afterwards, substantially grows as the EM-wave frequency increases. In so doing, the (electric + magnetic) contribution of surface electron scattering (curve *4*) remains dominating over the (electric + magnetic) contribution of bulk electron scattering (curve *1*). The total cross-section of (electric + magnetic and volume + surface) scattering (curve *6*) can also be described well by sum (26), where the summands are expressions (16) and (24) at frequencies $\omega < 5\nu_S$ (see curve *5*). However, at frequencies $\omega \geq 5\nu_S$, the sum of these expressions describes well only the contribution of the total (electric + magnetic) surface scattering of electrons.

At last, let us analyze the ratio between the magnetic and the electric electron scattering cross-section in the SMP. In Fig. 4, the frequency dependences of the quantity S_M/S_E are depicted for SMPs with various radii. For numerical calculations, the basic expressions (11) and (22) were used. Such particle's dimensions were selected, for which the dependences concerned would clearly illustrate the frequencies, at which the contribution of the magnetic component of the EM-wave to absorption becomes comparable with that of the electric component. These frequencies are of about 4×10^{13} s^{-1} for 50-Å particles, about 6×10^{13} s^{-1} for 100-Å particles, and about 8.1×10^{13} s^{-1} for 150-Å ones. For example, the contributions of the electric and magnetic components to absorption at a frequency of about 4×10^{13} s^{-1} are approximately identical for a particle with the radius of 50 Å. At the same time, for a 75-Å particle, the contribution of the magnetic component is twice, for a 100-Å particle, four times, and for a 150-Å particle, ten times as large as the contribution of the electric one. (The trend of the curves is not so smooth, as is shown in Fig. 4, for particles with larger radii and at higher frequencies, when the oscillatory terms in expressions (11) and (22) start to make appreciable contributions.)

The cross-section ratio can be roughly estimated in the case of merely surface scattering ($\nu_V = 0$), in the high- and low-frequency limits of EM-wave absorption.

With the help of relationships (17) and (25), we obtain

$$\frac{S_M}{S_E}\bigg|_{\rm HF} = \frac{1}{8\,d(\omega)}\left(\frac{\omega}{c}R\right)^2, \quad \frac{S_M}{S_E}\bigg|_{\rm LF} = \frac{1}{12\,d(\omega)}\left(\frac{\omega}{c}R\right)^2. \tag{27}$$

The factor $d(\omega)$ couples the internal and external electric fields (see Eq. (12)). Provided that the frequency is fixed, the ratio between the magnetic and the electric absorption cross-section grows as R^2.

5. Conclusions

Exact analytical expressions have been derived in the framework of the kinetic approach for the cross-section of light absorption, under the action of either an electric or a magnetic field, in a spherical SMP, whose radius is much smaller than the wavelength of the incident EM-wave and the skin-layer thickness. The contribution of the electric component has been demonstrated to become dominating as the EM-wave frequency grows. The contributions of electron collisions in the particle's bulk and with the particle's surface to the electron scattering inside the SMP have been singled out. The resonance oscillations of the optical conductivity, caused by approaching the EM-wave frequency and the frequency of electron oscillations between the particle's walls each other, have been studied, and the explicit analytical expressions for oscillatory terms have been obtained. The contributions of magnetic and electric absorption have been confronted at various frequencies and for various SMP's radii. The EM-wave frequencies have been found, at which the total (bulk + surface) magnetic and electric absorption become equal. The frequency intervals, where magnetic absorption is either higher or lower than electric one, have been indicated. An inflection point has been found in the dependence of the magnetic electron scattering cross-section on the EM-wave frequency, which is located at the frequency of electron oscillations between the particle's walls.

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ПЕРЕРІЗ ЕЛЕКТРИЧНОГО ТА МАГНІТНОГО ПОГЛИНАННЯ СВІТЛА СФЕРИЧНИМИ МЕТАЛЕВИМИ ЧАСТИНКАМИ НАНОМЕТРОВИХ РОЗМІРІВ. ТОЧНИЙ КІНЕТИЧНИЙ РОЗВ'ЯЗОК

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Р е з ю м е

Проведено обчислення електричного і магнітного перерізів поглинання світла металевими частинками сферичної форми нанометрових розмірів, які можуть бути меншими від довжини вільного пробігу електрона. Підхід базується на використанні кінетичного рівняння з дифузними умовами відбиття електрона від внутрішніх стінок частинки. Для випадку, коли товщина скін-шару є великою у порівнянні з характерними розмірами частинки, отримано точні аналітичні вирази, що дозволяють визначити величину перерізу в залежності від радіуса частинки та частоти падаючого електромагнітного (ЕМ) випромінювання.