MAGNETIC PROPERTIES OF QUANTUM RINGS IN THE PRESENCE OF SPIN-ORBIT INTERACTION

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Theoretical calculations of the influence of spin-orbit (SO) interaction on the magnetization and the magnetic susceptibility of small semiconductor quantum rings are presented. Those characteristics demonstrate quite an interesting behavior at low temperatures. The abrupt changes of the magnetization and the susceptibility at low magnetic fields are attributed to the crossing of the spin-split electron levels in the energy spectrum. The split of the levels is happened due to the SO interaction. Detailed calculations, where the parameters of an InSb semiconductor quantum ring were used, demonstrated the enhancement of the ring paramagnetism. There is also another possibility to control the effect by external electric fields.

1. Introduction

Modern progress in technology enabled the electron energy levels in semiconductor quantum dots and rings of various kinds to be studied in more detail. There appears also an opportunity to vary the number of electrons or to stabilize the chemical potential in rings and dots [1, 2]. During last decades, the orbital and spin magnetizations of such systems have been thoroughly studied by a plenty of scientists [3–12]. The interest in this effect can be explained by the fact that the magnetization supplies researchers with additional information concerning the many-particle dynamics of the rings embedded into an external magnetic field. In addition, studying the magnetic properties of quantum rings may provide us with means that are necessary to control the electron magnetization in nano-sized structures.

The electron's spin affects the magnetic properties of semiconductor quantum dots and rings [1, 13–15]. Among other spin-dependent interactions, the SO one plays an important role in the formation of the energy spectrum of nanostructures that include semiconductors of the III–V groups [16]. The spin-orbit interaction partially eliminates the spin degeneration of energy levels even in the absence of an external magnetic field. This circumstance substantially modifies the electronic properties of semiconductor nanostructures [17–22].

This work aimed at examining the influence of the SO interaction on the magnetic properties of quantum rings in low magnetic fields. We have calculated the magnetization and the magnetic susceptibility of quantum rings with a confining effective potential for electrons, taking the SO interaction into account. The magnetic field applied along the symmetry axis of a ring brings about an involved structure of the energy spectrum. The behavior of the energy levels and the thermodynamic properties have been considered theoretically in works [4, 6, 11, 12]. The well-known spin splitting has been calculated in work [22] in the framework of the parabolic-potential model for semiconductor quantum dots with the relevant parameters of InSb and InAs. The spin-orbit splitting in the zero magnetic field leads to the crossing of energy levels in weak external magnetic fields (similar to the Paschen–Back effect) and provokes unusual properties of quantum rings.

In order to demonstrate the influence of the SO interaction on the magnetization and the susceptibility

of quantum rings, we included the Rashba term [16, 19] into the SO interaction potential.

2. General Consideration of the Problem

The simple model potential that is considered in this work corresponds to the case of an isolated ring located in the XY plane, and its analytic form looks like [10, 32]

$$V_c(r) = \frac{a_1}{r^2} + a_2 r^2 - V_0, \tag{1}$$

where $V_0 = 2\sqrt{a_1 a_2}$. This model is rather flexible: both the radius and the width of the ring can be selected independently, by a proper choice of only two model parameters, a_1 and a_2. In particular, it can describe the properties of a quantum dot as well. This model, provided that no SO interaction is taken into account, enables the energy spectrum and the magnetization at the zero temperature, as well as the wave functions in a constant magnetic field applied normally to the ring, to be obtained analytically [10]. In the framework of this model, we can analyze the electron states of a perfect two-dimensional (2D) ring in detail.

Potential (1) has the following properties. There is a minimum $V(r_0) = 0$ at

$$r = r_0 = \left(\frac{a_1}{a_2}\right)^{\frac{1}{4}}, \tag{2}$$

where r_0 is defined as the average radius of the ring. In the vicinity of $r = r_0$, the potential has the simple parabolic form (Fig. 1)

$$V(r) \approx \frac{1}{2} m\omega_0^2 (r - r_0)^2,$$

where the quantity $\omega_0 = \sqrt{\frac{8a_2}{m}}$ characterizes the size of the potential well, and m is the effective electron mass.

The model can also be applied to the description of several other physical systems:

(i) a one-dimensional (1D) ring ($r_0 = \text{const}$ and $\omega_0 \to \infty$);

(ii) a 2D straight wire ($\omega_0 = \text{const}$ and $r_0 \to \infty$);

(iii) a quantum dot $a_1 = 0$;

(iv) an isolated antidot $a_2 = 0$.

Let the magnetic field be applied normally to the XY planes. Then, the Hamiltonian of an electron is

$$H = -\frac{\hbar^2}{2m(E)}\left[\frac{\partial}{r\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\right] -$$

$$-\frac{i}{2}\hbar\omega_c(E,B)\frac{\partial}{\partial\varphi} + \frac{1}{8}m(E)\omega_c^2(E,B)r^2 + V_c(r) +$$

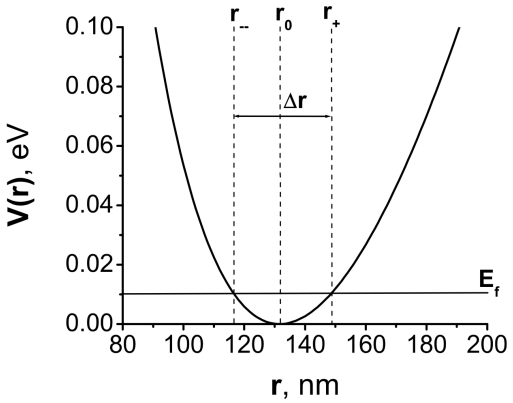

Fig. 1. Profile of the quantum ring potential

$$+V_{\mathrm{SO}}^R(r,\varphi) + \frac{1}{2}\sigma_z\mu_{\mathrm{B}}g(E)B, \tag{3}$$

where $\omega_c(E,B) = eB/m(E)$ is the electron cyclotron frequency. The electron–electron interaction is neglected. The effective electron mass is determined by the formula [19, 22, 24]

$$\frac{1}{m(E)} = \frac{1}{m(0)}\frac{E_g(E_g+\Delta)}{(3E_g+2\Delta)}\left[\frac{2}{E+E_g} + \frac{1}{E+E_g+\Delta}\right], \tag{4}$$

where E is the electron energy in the conduction band, $m(0)$ is the effective electron mass near the bottom of the conduction band, E_g and Δ are the energy gap width and the SO splitting of the valence band, respectively,

$$g(E) = 2\left[1 - \frac{m_0}{m(E)}\frac{\Delta}{3(E_g+E)+2\Delta}\right] \tag{5}$$

is the effective g-factor [25], $\mu_{\mathrm{B}} = e\hbar/2m_0$ is the Bohr magneton, and e and m_0 are the charge and the mass of a free electron, respectively.

The Rashba term in the SO interaction is determined by the formula [19, 21, 26, 27]

$$V_{\mathrm{SO}}^R(r,\varphi) = \sigma_z\alpha\frac{dV_c(r)}{dr}\left(k_\varphi + \frac{e}{2\hbar}Br\right), \tag{6}$$

where $k_\varphi = -i(1/r)\partial/\partial\varphi$, and α is the parameter of SO coupling introduced by Rashba.

Consider the Rashba potential

$$V_{\mathrm{SO}}^R(r,\varphi) = -2i\sigma_z\alpha a_2\frac{\partial}{\partial\varphi} + \sigma_z\alpha a_2 r^2\frac{eB}{\hbar} +$$

$$+2i\sigma_z\alpha a_1\frac{1}{r^4}\frac{\partial}{\partial\varphi} - \sigma_z\alpha a_1\frac{1}{r^2}\frac{eB}{\hbar} \tag{7}$$

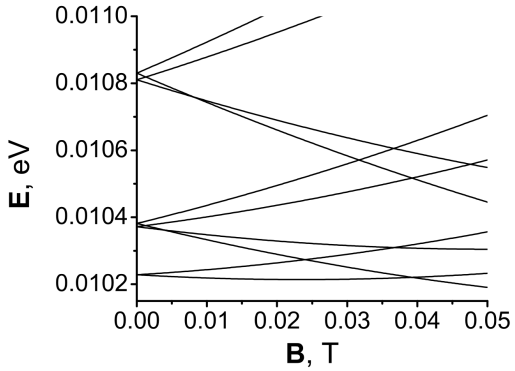

Fig. 2. Energy spectrum of an InSb ring, taking the SO interaction into account

in detail. If the third term on the r.h.s. of Eq. (7) is not taken into account, the energy eigenvalues and the characteristic wave functions of an electron look like

$$E_{n,l,\sigma} = \left(n + \frac{1}{2} + \frac{M}{2}\right)\hbar\omega - \frac{l}{2}\hbar\omega_C -$$

$$-V_0 + 2\sigma_z \alpha a_2 l + \frac{1}{2}\sigma_z \mu_{\mathrm{B}} g(E) B \tag{8}$$

and

$$\psi_{n,l,\sigma}(r,\varphi) = \frac{1}{\lambda}\left(\frac{\Gamma[n+M+1]}{2^{M+1} n! \left(\Gamma[M+1]\right)^2 \pi}\right)^{\frac{1}{2}} e^{-il\varphi} \times$$

$$\times e^{-\frac{1}{4}\left(\frac{r}{\lambda}\right)^2} \left(\frac{r}{\lambda}\right)^M {}_1F_1\left(-n, M+1, \frac{1}{2}\left(\frac{r}{\lambda}\right)^2\right), \tag{9}$$

respectively, where n is the principal quantum number, and l and σ_z are the projections of the orbital and spin quantum moments, respectively, of the electron onto the Z-axis,

$$\omega = \sqrt{\omega_C^2 + \frac{8a_2}{m(E)} + \frac{8}{m(E)}\sigma_z \alpha a_2 \frac{eB}{\hbar}}, \quad \lambda = \sqrt{\frac{\hbar}{m(E)\omega}},$$

and

$$M = \sqrt{l^2 + \frac{2a_1 m(E)}{\hbar^2} - \frac{2m(E)}{\hbar^2}\sigma_z \alpha a_1 \frac{eB}{\hbar}}. \tag{10}$$

The third term in Eq. (7) is taken into account in the framework of the small perturbation method. The first correction to the energy $\iint \psi^*_{n,l,\sigma}(r,\varphi)\, 2i\sigma_z \alpha a_1 \frac{1}{r^4}\frac{\partial}{\partial\varphi}\psi_{n,l,\sigma}(r,\varphi)\, r dr d\varphi$ is much smaller than the energy value in the zero-order approximation (Eq. (8)). We confine the calculations to the first order of smallness only.

Owing to the SO interaction, the electron energy levels with the orbital numbers $|l| > 0$ become split at $B = 0$ and intercrossed if the magnetic field exceeds a certain critical value (Fig. 2). The first cross-point of the lowest spin-split states satisfies the relationship [22]

$$\frac{\Phi}{\Phi_0} \approx \frac{\Delta \mathrm{E}}{\hbar\omega_0} \ll 1, \tag{11}$$

where Φ is the magnetic flux through the ring, ΔE is the energy of spin splitting at $B = 0$, and $\Phi_0 = \frac{hc}{e}$ is the quantum of the magnetic flux.

Below, we consider the influence of the SO interaction on the magnetic properties of a system of quantum rings. The temperature dependences of the average magnetization M and the magnetic susceptibility χ of such a system are coupled with the equilibrium value of the chemical potential by the equalities [28]

$$M = \sum_{n,l,s}\left(-\frac{\partial E_{n,l,s}}{\partial B}\right) f\left(E_{n,l,s} - \xi\right) \tag{12}$$

and

$$\chi = \frac{\partial M}{\partial B}, \tag{13}$$

where $f(E)$ is the Fermi distribution function, ξ is the chemical potential of the system which is determined by the equation

$$N = \sum_{n,l,s} f\left(E_{n,l,s} - \xi\right), \tag{14}$$

and N is the total number of electrons in the quantum ring.

3. Results of Calculations

For an InSb quantum ring, we selected the following values of the parameters: $m(0) = 0.014 m_0$ [29], $E_g = 0.24$ eV, $\Delta = 0.81$ eV [22, 24], and $\alpha = 500$ Å^2. For model calculations, the form of the ring is specified by the parameters $a_1 = 3$ meV(nm)$^2 10^6$ and $a_2 = 1000$ meV(nm)$^2 10^{-5}$. Then, the other parameters of the quantum ring are as follows: the internal radius $r_- = 117$ nm, the external radius $r_+ = 149$ nm, the width $\Delta r = 32$ nm, and the average radius $r_0 = 132$ nm.

The potential and the energy spectrum of the quantum ring are shown in Figs. 1 and 2, respectively. The numerical calculations were carried out for low magnetic fields $B = 0 \div 0.05$ T.

The magnetizations of the quantum ring with 1 to 6 electrons are shown in Fig. 3.

The step-like variation of the magnetization in the case with one or two electrons in the ring occurs owing to the crossing of energy levels with different $|l|$-values (Figs. 1 and 3,a). An analogous scenario is observed in the case with 5 or 6 electrons (Figs. 1 and 3,c). The spin-orbit interaction shifts the value of the magnetic field, at which the magnetization makes a jump.

An essentially different behavior of the magnetization versus the magnetic field is observed in the case of a ring with three or four electrons. Making allowance for the SO interaction results in magnetization jumps at low magnetic fields (Fig. 3,b). This phenomenon takes place owing to the crossing of the energy levels with $|l| = 1$.

The magnetization of a quantum dot with $N = 2$ or 6 electrons is zero at $B = 0$, which corresponds to the case of filled electron shells. The analysis of the energy spectrum of the quantum ring (Fig. 2) reveals that the energy levels E_{011} and E_{0-11} intercross if the magnetic field achieves the value $B = 1.3 \times 10^{-3}$ T. This circumstance stimulates an abrupt jump of the magnetization of the 4-electron quantum ring (Fig. 3,b). At the same time, owing to the SO interaction, the jump of the magnetization of the 3-electron ring takes place at a higher value of the magnetic field $B = 3.9 \times 10^{-3}$ T.

At nonzero but low enough temperatures ($k_{\mathrm{B}}T \ll \hbar\omega_0$, where k_{B} is the Boltzmann constant), the magnetization of a ring with $N = 1$, 2, 5, or 6 electrons satisfies the following rules: the rings with completely filled shells are diamagnetic, while the rings with partially filled shells manifest paramagnetic peaks (Fig. 4). The peak heights exponentially decrease ($\sim \exp(-k_{\mathrm{B}}T/\hbar\omega_0)$), and the magnetization tends to the value typical of the Landau diamagnetism at $k_{\mathrm{B}}T \approx \hbar\omega_0$ [28].

The peculiarities of the quantum ring magnetization process caused by the SO interaction manifest themselves brightly in the ring magnetic susceptibility. In Fig. 5, the dependences of the magnetic susceptibility on the magnetic field are shown for rings with 3 and 4 electrons. At zero temperature, the theoretical dependence of the differential magnetic susceptibility looks like the delta-function, which is associated with the magnetization jump that occurs owing just to the SO interaction.

At nonzero temperatures, the paramagnetic peaks also arise, but they have finite heights. The spin-orbit interaction for dots with 4 electrons results in the appreciable shift of the peak at $T \neq 0$. In case of the 3-electron dot, we observe three peaks. Only the medium

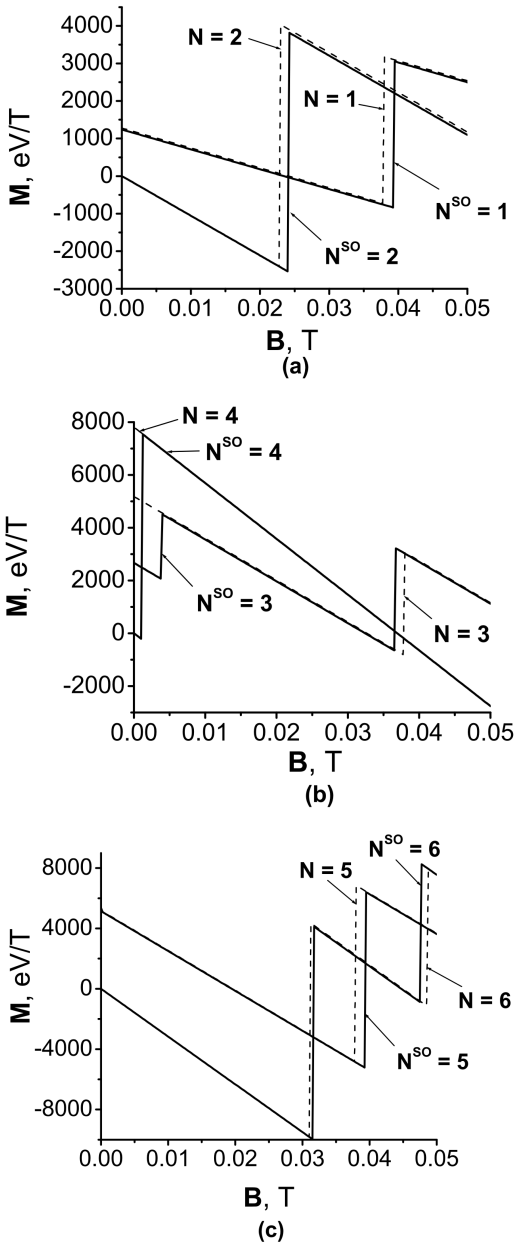

Fig. 3. Dependences of the quantum ring magnetization on the magnetic field at zero temperature for various numbers of electrons in the ring: (a) 1 and 2, (b) 3 and 4, and (c) 5 and 6. The superscript SO indicate that the SO interaction was taken into account

peak is coupled with the SO interaction. The positions of the peaks that are caused by the SO interaction can be controlled to some extent by applying an external electric field directed normally to the ring plane [16,19].

Consider the dependences of the magnetic field values B_1 and B_2, at which the energy levels with the

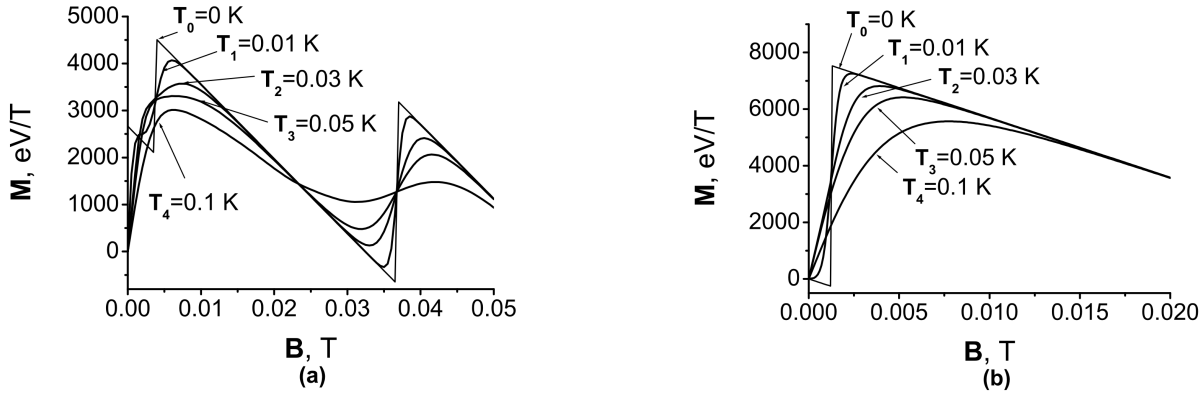

Fig. 4. Dependences of the 3- (a) and 4-electron (b) quantum ring magnetization on the magnetic field at various temperatures

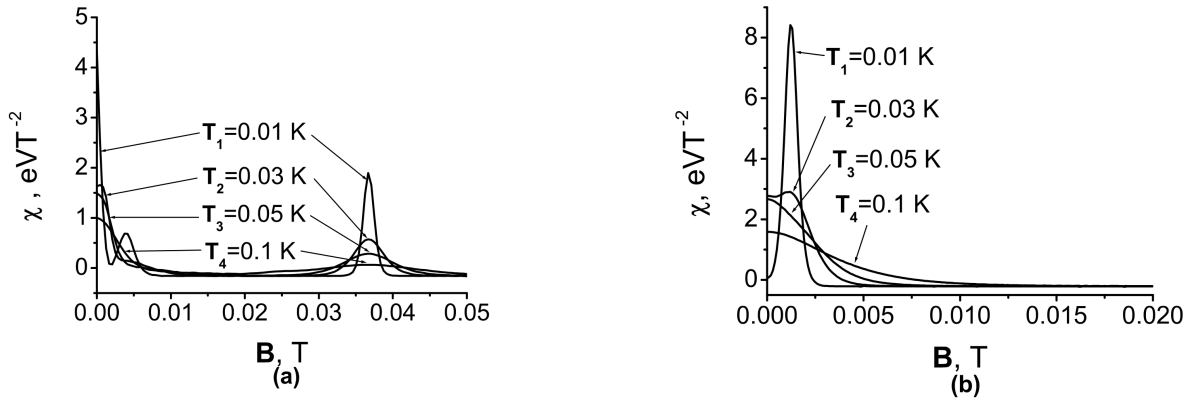

Fig. 5. Magnetic susceptibilities of the 3- (a) and 4-electron (b) quantum rings at various temperatures

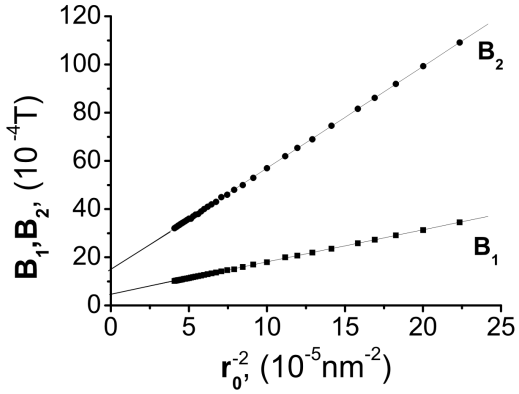

Fig. 6. Dependences of the magnetic field values, at which the corresponding levels intercross, on the reciprocal square of the average radius of a ring

quantum numbers ($n = 0$, $l = -1$, $\sigma_z = 1$) and ($n = 0$, $l = 1$, $\sigma_z = 1$) intersect for the first time and the energy levels with the quantum numbers ($n = 0$, $l = -1$, $\sigma_z = -1$) and ($n = 0$, $l = 1$, $\sigma_z = 1$) do it for the second time, respectively, on the average radius of the ring. The dependences of the magnetic fields B_1 and B_2 on the reciprocal square of the average radius of the quantum ring are linear (Fig. 6). Making use of these dependences, one can determine the magnetic fields, at which the levels considered intercross in the case of a quantum wire ($r_0 \to \infty$) with $a_2 = 0.01$ meV/nm^2 (which corresponds to a quantum wire of the parabolic form with the width $\Delta r = 32$ nm): $B_1 = 0.47 \times 10^{-3}$ T and $B_2 = 1.49 \times 10^{-3}$ T.

4. Conclusions

We have studied the consequences of taking the SO interaction into account, when considering the magnetization of quantum rings made up of InSb. The magnetization and the magnetic susceptibility of a ring with a few electrons have been calculated. The dependences of the influence of magnetic fields on the

quantities under investigation for infinite quantum wires have been considered. The features of the quantum rings made up of semiconductors of the III–V groups that were described above can be used in practical spintronics.

To summarize, we would like to emphasize that the interaction between electrons has been neglected in this article for the sake of simplicity. The electron-electron interaction alone can also induce the crossing of the energy levels [3–5, 30, 31]. In order to describe the multielectron system in full, it is necessary to consider it making allowance for both the SO and electron-electron interactions [2, 23]. In support of the idea stated above, the recent experiments [29] discovered that the electron-electron interaction in systems with strong localization can enhance the manifestation of the SO one.

The jumps of the magnetization and the peaks in the magnetic susceptibility, which are an obvious consequence of the electron energy level crossing in the system, stem from the SO interaction. The results described above have a transparent physical meaning, but should be verified experimentally.

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МАГНІТНІ ВЛАСТИВОСТІ КВАНТОВИХ КІЛЕЦЬ ПРИ ВРАХУВАННІ СПІН-ОРБІТАЛЬНОЇ ВЗАЄМОДІЇ

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Р е з ю м е

Наведено теоретичні розрахунки впливу ефекту спін-орбітальної (СО) взаємодії на намагніченість і магнітну сприйнятливість малих напівпровідникових квантових кілець. Ці характеристики демонструють досить цікаву поведінку при низьких температурах. Стрибкоподібна зміна намагніченості і сприйнятливості в малих магнітних полях є наслідком перетину спін-розщеплених електронних рівнів в енергетичному спектрі. Розщеплення енергетичних рівнів відбувається завдяки СО-взаємодії. Детальні чисельні розрахунки з використанням параметрів InSb напівпровідникових квантових кілець демонструють збільшення парамагнетизму в кільцях. Існує також додаткова можливість керувати ефектом за допомогою зовнішнього електричного поля.