DIRTY JOSEPHSON JUNCTIONS WITH INCOMPLETE BARRIER TRANSPARENCY

V.E. SAKHNYUK, A.V. SVIDZYNSKYJ

Lesya Ukrainka Volynskyi State University
(13, Volya Ave., Lutsk 43000, Ukraine; e-mail: sve@lab.univer.lutsk.ua)

UDC 538.945

The spatial behavior of the order parameter near the interface in a superconductor—insulator—superconductor (SIS) tunnel junction, provided that the impurity concentration in superconductors is arbitrary, has been investigated theoretically. The temperature is assumed to be close to the critical one. The transmission coefficient D of the barrier for electrons may vary in a wide range of values. The boundary conditions for the order parameter and the expression for the equilibrium current density have been obtained.

1. Introduction

In 1962, B. Josephson [1] made his famous prediction about the possibility for a stationary supercurrent to run through a contact between two superconductors separated by an insulating film, even in the absence of voltage drop across the contact. This discovery initiated the researches of spatially inhomogeneous compositions formed by two weakly coupled superconductors, with the whole body of investigations in this area being named "weak superconductivity". The substantial progress in studying the current states in superconducting junctions was achieved owing to the use of the model with a piecewise continuous profile of the order parameter $\Delta(\mathbf{r})$. A number of the results obtained on its basis are exposed in monography [2].

However, such a simplified model turns out invalid at temperatures close to the critical one, T_c. In this case, the order parameter varies over the lengths of the order of $\xi_0(1 - T/T_c)^{-1/2}$ and obeys the Ginzburg—Landau equation. Near the interface, at the distances not farther than ξ_0, the dependence $\Delta(\mathbf{r})$ varies quickly and satisfies the linear integral equation. This region is rather narrow, but it has to be studied to obtain the correct boundary condition for the Ginzburg—Landau equation. Such a self-consistent approach turned out fruitful for temperatures near the critical one (see [2, 3]). For example, the self-consistent problem concerning the impurity-free SIS junction at a temperature of about T_c has been solved in work [4]. Of course, the result obtained there must be correct for the SIS junctions as well, considered as a partial case.

In this work, we study the influence of the barrier transparency on the shape of the phase dependence of the current through the SIS junction. While solving this problem, it is important to elucidate the spatial behavior of the order parameter. This behavior is essentially governed by two factors: 1) availability of the weak coupling and 2) pair-breaking action of the current. The latter is especially important if the transparency of the barrier is not low. A similar problem, but in the limit $l \ll \xi_0$, was considered in work [5] on the basis of the Uzadel equations. The same problem was studied by other authors in the framework of model approaches [6, 7]. In our work, in contrast to works [5, 6, 7], the calculations were carried out for arbitrary concentrations of impurities. Our starting point was the system of Gor'kov's equations for Matsubara Green's functions, i.e. all the results obtained have microscopic origin. We attempted to advance analytical calculations as far as possible, not resorting to numerical calculations.

2. Behavior of the Order Parameter Near the IS Interface

Consider the behavior of the order parameter near the interface between two superconductors in a tunnel SIS junction. The half-spaces $z > 0$ and $z < 0$ are filled with superconductors with non-magnetic impurities, and, in the plane $z = 0$, there is an insulating film. At temperatures close to the critical one and in the spatial region near the interface, the order parameter is described by the system of linear integral equations [2], the dimensionless forms of which, taking into account the geometry of our problem, look like

$$\Delta(\zeta)=\frac{\rho}{2}\sum_n\int\limits_0^1\frac{dx}{x}\int\limits_{-\infty}^{\infty}d\zeta'\Delta_n(\zeta')\Big\{\exp\Big(-\frac{|2n'+1|}{x}|\zeta-$$

$$-\zeta'|\Big) + \mathrm{sign}\zeta\zeta' R(x)\exp\Big(-\frac{|2n'+1|}{x}(|\zeta|+|\zeta'|)\Big)\Big\}, \tag{1}$$

$$\Delta_n(\zeta) = \Delta(\zeta) +$$

$$+\frac{1}{2\lambda}\int_0^1 \frac{dx}{x}\int_{-\infty}^{\infty} d\zeta'\Delta_n(\zeta')\Big\{\exp\Big(-\frac{|2n'+1|}{x}|\zeta-\zeta'|\Big)+$$

$$+\mathrm{sign}\zeta\zeta' R(x)\exp\Big(-\frac{|2n'+1|}{x}(|\zeta|+|\zeta'|)\Big)\Big\}. \tag{2}$$

Here, $\zeta = z/\xi_0$ is the dimensionless variable, $\rho = |g|N(0)$ is the dimensionless coupling constant, $N(0)$ is the electron density of states on the surface of the Fermi sphere, $|2n'+1| = |2n+1| + 1/\lambda$, $\lambda = l/\xi_0$ is the dimensionless mean free path of electrons, and ξ_0 is the coherence length. The value of λ determines the degree of superconductor contamination: at $\lambda \ll 1$, the superconductor is extremely dirty, and, at $\lambda \gg 1$, it is pure enough.

In the limit case $\zeta \to \pm\infty$, the system of equations (1) and (2) has an asymptotically exact solution in the form of linear functions

$$\begin{aligned}&\Delta(\zeta) \overset{\mathrm{as}}{=} \Delta'_+\zeta + \Delta_+,\\ &\Delta_n(\zeta) \overset{\mathrm{as}}{=} \left|\frac{2n'+1}{2n+1}\right|(\Delta'_+\zeta+\Delta+),\end{aligned} \qquad \zeta\to+\infty, \tag{3}$$

$$\begin{aligned}&\Delta(\zeta) \overset{\mathrm{as}}{=} \Delta'_-\zeta + \Delta_-,\\ &\Delta_n(\zeta) \overset{\mathrm{as}}{=} \left|\frac{2n'+1}{2n+1}\right|(\Delta'_-\zeta+\Delta-),\end{aligned} \qquad \zeta\to-\infty. \tag{4}$$

Let us rewrite Eqs. (1) and (2) by introducing the symmetric and antisymmetric parts of both the order parameter $\Delta(\zeta)$ and the function $\Delta_n(\zeta)$ and by transforming the integrals in such a way that the integration is carried out over the semiaxis $\zeta > 0$. As a result, we obtain two systems

$$\begin{cases}\Delta_s(\zeta)=\int_0^\infty d\zeta'\Delta_{n,s}(\zeta')\{K(\zeta-\zeta')+K(\zeta+\zeta')\},\\ \Delta_{n,s}(\zeta)=\Delta_s(\zeta)+\\ \quad+\int_0^\infty d\zeta'\Delta_{n,s}(\zeta')\{\tilde K(\zeta-\zeta')+\tilde K(\zeta+\zeta')\},\end{cases} \tag{5}$$

and

$$\begin{cases}\Delta_a(\zeta)=\int_0^\infty d\zeta'\Delta_{n,a}(\zeta')\{K(\zeta-\zeta')+K_D(\zeta+\zeta')\},\\ \Delta_{n,a}(\zeta) = \Delta_a(\zeta)+\\ \quad+\int_0^\infty d\zeta'\Delta_{n,a}(\zeta')\{\tilde K(\zeta-\zeta')+\tilde K_D(\zeta+\zeta')\},\end{cases} \tag{6}$$

where

$$K(\zeta) = \frac{\rho}{2}\sum_n\int_0^1\frac{dx}{x}\exp\left\{-\frac{|2n'+1|}{x}|\zeta|\right\},$$

$$K_D(\zeta) = \frac{\rho}{2}\sum_n\int_0^1\frac{dx}{x}\tau(x)\exp\left\{-\frac{|2n'+1|}{x}|\zeta|\right\},$$

$$\tilde K(\zeta) = \frac{1}{2\lambda}\int_0^1\frac{dx}{x}\exp\left\{-\frac{|2n'+1|}{x}|\zeta|\right\},$$

$$\tilde K_D(\zeta) = \frac{1}{2\lambda}\int_0^1\frac{dx}{x}\tau(x)\exp\left\{-\frac{|2n'+1|}{x}|\zeta|\right\},$$

$$\tau(x) = 1-2D(x).$$

The exact solutions of system (5) are the constants

$$\Delta_s = A, \quad \Delta_{n,s} = \left|\frac{2n'+1}{2n+1}\right|A. \tag{7}$$

System (6) has no exact analytical solution, but it may serve as the basis for obtaining the results that are necessary for finding the limit value of the order parameter. The latter will be included into the expression for the density of the current that can run through the junction.

If ζ tends to infinity, we can obtain an asymptotically exact solution of system (6):

$$\Delta_a(\zeta) = C(\zeta+q_\infty), \ \Delta_{n,a}(\zeta) = \left|\frac{2n'+1}{2n+1}\right|C(\zeta+q_\infty). \tag{8}$$

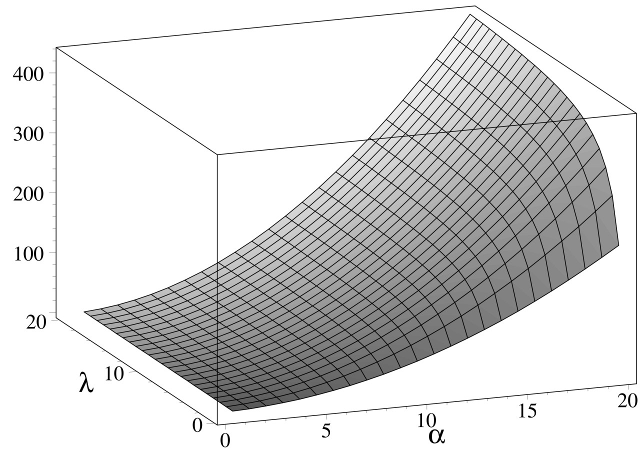

Fig. 1. Dependence of the q_∞-value on the transparency of the insulator α and the dimensionless mean free path of electrons $\lambda = l/\xi_0$

Substitute the following expressions for $\Delta_a(\zeta)$ and $\Delta_{n,a}(\zeta)$ (their asymptotes are indicated) —

$$\Delta_a(\zeta) = \zeta + q_\infty + \psi_a(\zeta), \qquad \lim_{\zeta\to\infty} \psi_a(\zeta) = 0,$$

$$\Delta_{n,a}(\zeta) = \left|\frac{2n'+1}{2n+1}\right| (\zeta+q_\infty)+\psi_{n,a}(\zeta), \; \lim_{\zeta\to\infty} \psi_{n,a}(\zeta) = 0$$

— into system (6). The global constant was put to unity, because it is not fixed by the system in any case. As a result, we obtain

$$\psi_a(\zeta) - \int\limits_0^\infty \{K(\zeta-\zeta') + K_D(\zeta+\zeta')\}\, \psi_{n,a}(\zeta')d\zeta' =$$

$$= \int\limits_0^\infty \{K(\zeta+\zeta') + K_D(\zeta+\zeta')\} \left|\frac{2n'+1}{2n+1}\right| \zeta' d\zeta' -$$

$$-q_\infty \int\limits_0^\infty \{K(\zeta+\zeta') - K_D(\zeta+\zeta')\} \left|\frac{2n'+1}{2n+1}\right| d\zeta', \qquad (9)$$

$$\psi_{n,a}(\zeta) - \psi_a(\zeta) - \int\limits_0^\infty \left\{\tilde{K}(\zeta-\zeta')+\tilde{K_D}(\zeta+\zeta')\right\} \psi_{n,a}(\zeta')d\zeta' =$$

$$= \int\limits_0^\infty \left\{\tilde{K}(\zeta+\zeta') + \tilde{K_D}(\zeta+\zeta')\right\} \left|\frac{2n'+1}{2n+1}\right| \zeta' d\zeta' -$$

$$-q_\infty \int\limits_0^\infty \left\{\tilde{K}(\zeta+\zeta') - \tilde{K_D}(\zeta+\zeta')\right\} \left|\frac{2n'+1}{2n+1}\right| d\zeta'. \qquad (10)$$

To find the constant q_∞, we use the method of quasi-orthogonality to *asymptotics* [8], which brings us to the following final result:

$$q_\infty = \frac{3\chi_1(\lambda)}{\chi(\lambda)} \int\limits_0^1 x^3 R(x)dx + \frac{12\chi(\lambda)}{\pi^2} \frac{\left(\int\limits_0^1 x^2 R(x)dx\right)^2}{\int\limits_0^1 xD(x)dx}, \qquad (11)$$

where

$$\chi(\lambda) = \sum_{n=-\infty}^{+\infty} \frac{1}{|2n'+1|(2n+1)^2},$$

$$\chi_1(\lambda) = \sum_{n=-\infty}^{+\infty} \frac{1}{(2n'+1)^2(2n+1)^2},$$

$$D(x) = \frac{p_0^2 x^2}{p_0^2 x^2 + K^2}, \quad R(x) = \frac{K^2}{p_0^2 x^2 + K^2}.$$

Expression (11) is one of the main results of this work. It will be used to obtain the boundary condition and to derive the formula for the current density. From Fig. 1, where the dependence of q_∞ on the dimensionless mean free path of electrons λ and the relative height of the barrier $\alpha = K/p_0$ is shown, one can see that, at large λ's, the values of q_∞ are determined by the barrier height only. This statement proves to be true by analytical calculations. Really, in the case of pure enough junctions ($l \gg \xi_0$ or $\lambda \gg 1$), we have $\chi_1(\lambda) \approx \frac{\pi^4}{48}$ and $\chi(\lambda) \approx \frac{7\zeta(3)}{4}$. Hence, the expression for q_∞ looks like

$$q_\infty = \frac{\pi^4}{28\zeta(3)} \int\limits_0^1 x^3 R(x)dx + \frac{21\zeta(3)}{\pi^2} \frac{\left(\int\limits_0^1 x^2 R(x)dx\right)^2}{\int\limits_0^1 xD(x)dx}. \qquad (12)$$

Since the coefficients in the asymptotes are interrelated by the equalities

$$\begin{aligned}&\Delta'_+ - \Delta'_- = 0, \quad \Delta_+ + \Delta_- = A,\\ &\Delta'_+ + \Delta'_- = C, \quad \Delta_+ + \Delta_- = Cq_\infty,\end{aligned} \tag{13}$$

the following relations are obtained:

$$\begin{aligned}&\Delta'_+ = \Delta'_-,\\ &\Delta_+ = \Delta_- + 2q_\infty \Delta'_-.\end{aligned} \tag{14}$$

They connect the coefficients in the right- and left-hand asymptotes and constitute the boundary conditions for the order parameter.

3. Boundary Condition for the Ginzburg—Landau Equation. Calculation of the Current Through the Junction

In the Ginzburg—Landau theory, the order parameter is known (see, e.g., work [2]) to satisfy the equation

$$\frac{\xi^2(T)}{\xi_0^2}\frac{d^2\Delta(\zeta)}{d\zeta^2} - \frac{1}{\Delta_\infty^2}|\Delta(\zeta)|^2\Delta(\zeta) + \Delta(\zeta) = 0. \tag{15}$$

The solution of this equation, provided a current through the junction, is tried in the form

$$\Delta(\zeta) = \exp\{\pm i\varphi/2\}\,\Delta_\infty f(\zeta)\exp\{2im\chi(\zeta)\}, \tag{16}$$

where $\chi(\zeta)$ is the continuous ($\chi(-0) = \chi(+0) = 0$) component of the order parameter phase, which determines the superconducting speed $\frac{d\chi(\zeta)}{d\zeta} = \xi_0 v_s(\zeta)$, and φ is the phase jump across the junction.

Let us substitute Eq. (16) into Eq. (15) and separate the imaginary and real parts. As a result, we obtain the equations

$$\frac{\xi^2(T)}{\xi_0^2}f''(\zeta) - \xi^2(T)4m^2v_s^2(\zeta)f(\zeta) + f(\zeta) - f^3(\zeta) = 0, \tag{17}$$

$$v_s(\zeta)f^2(\zeta) = \text{const}. \tag{18}$$

Let us also rewrite relations (14) in terms of the function $f(\zeta)$:

$$\begin{cases} f'_+\cos\varphi - 2mv_s(0)\xi_0 f_+\sin\varphi = f'_-,\\ f'_+\sin\varphi + 2mv_s(0)\xi_0 f_+\cos\varphi = 2mv_s(0)\xi_0 f_-,\\ f_+\cos\varphi = f_- + 2q_\infty f'_-,\\ f_+\sin\varphi = 4mv_s(0)\xi_0 q_\infty f_-.\end{cases} \tag{19}$$

This system has a nontrivial solution, provided that

$$4mv_s(0)\xi_0 q_\infty = \sin\varphi. \tag{20}$$

Taking this condition into account, Eqs. (19) yield

$$f_+ = f_-, \quad f'_+ = -f'_-, \quad \frac{f'_+}{f_+} = \frac{1}{2q_\infty}(1-\cos\varphi). \tag{21}$$

To find the equation for f_+ which is, in essence, the boundary condition for the Ginzburg—Landau equation, the third equality in (21) should be combined with another relation between f_+ and f'_+. The latter is derived as the first integral of Eq. (17), namely,

$$\frac{\xi(T)}{\xi_0}f'_+ - \frac{f_\infty^2 - f_+^2}{f_+}\sqrt{f_\infty^2 - 1 + \frac{1}{2}f_+^2} = 0. \tag{22}$$

Here, f_∞ is the value of the function $f(\zeta)$ at infinity.

For f_+, making use of Eqs. (21) and (22), the following equation is obtained:

$$f_+^6 - 2\left(1 + \frac{\xi^2(T)}{\xi_0^2 q_\infty^2}\sin^4\frac{\varphi}{2}\right)f_+^4 + f_\infty^2(4 - 3f_\infty^2)f_+^2 + $$

$$+2f_\infty^4(f_\infty^2 - 1) = 0. \tag{23}$$

Its solution, which agrees with the expression for f_+ at $f_\infty = 1$, looks like

$$f_+^2 = \pm\frac{2}{3}\sqrt{a^2 - 3\,b}\ \times$$

$$\times\sin\left(\frac{\pi}{6} - \frac{1}{3}\arctan\sqrt{\frac{4(a^2-3b)^3}{(9\,ba - 27\,c - 2\,a^3)^2} - 1}\right) - \frac{a}{3}, \tag{24}$$

where

$$a = -2\left(1 + \frac{\xi^2(T)}{\xi_0^2 q_\infty^2}\sin^4\frac{\varphi}{2}\right),$$

$$b = f_\infty^2(4 - 3f_\infty^2),$$

$$c = 2f_\infty^4(f_\infty^2 - 1). \tag{25}$$

The sign "+" is actual in Eq. (24) if $27c - 9ab + 2a^3 \geq 0$; otherwise, the sign "−" is valid.

Now, we are able to proceed to the derivaton of the expression for the density of the current which can run

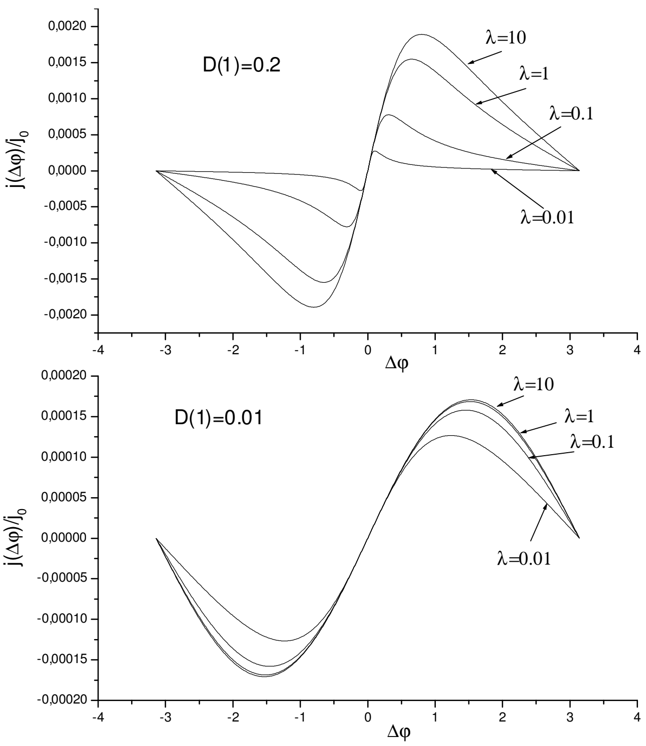

Fig. 2. Dependences of the current density on the phase difference between the junction's electrodes for various mean free paths of electrons. $T = 0.98T_c$ and $j_0 = \frac{env_0}{2p_0\xi_0}$

through the junction. The general formula of the Ginzburg—Landau theory for the current density in dirty superconductors [2],

$$j(\zeta) = i\frac{7\zeta(3)}{16\pi^2}\frac{env_0}{p_0\xi_0 T_c^2}\left(\Delta\frac{d\overset{*}{\Delta}}{d\zeta} - \overset{*}{\Delta}\frac{d\Delta}{d\zeta}\right)\chi(\xi_0/l), \qquad (26)$$

is taken as the initial one. Here,

$$\chi(\xi_0/l) = \frac{8}{7\zeta(3)}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2(2n+1+\xi_0/l)}.$$

We calculate the current in the region $z \gg \xi_0$, where the asymptote of the order parameter is linear. We substitute Eq. (3) into Eq. (26) and use relation (14) and expression (16) for the order parameter. Then, expression (26) for the current reads

$$j = \frac{7\zeta(3)}{16\pi^2}\frac{env_0\Delta_\infty^2}{p_0\xi_0 T_c^2}\frac{f_+ f_-}{q_\infty}\chi(\xi_0/l)\sin\varphi. \qquad (27)$$

Here, f_+ and f_- are the values of the function $f(\zeta)$ at $\zeta = +0$ and $\zeta = -0$, respectively. As follows from Eq. (21), they are identical and given by expression (24).

The final formula for the current, which can run through an SIS junction, provided an arbitrary concentration of impurities in the superconductor electrodes and an incomplete transparency of the insulating film, is

$$j = \frac{1}{3}\frac{env_0}{p_0\xi_0}\left(1-\frac{T}{T_c}\right)\frac{\chi(\xi_0/l)}{q_\infty}\left\{\mp\sqrt{a^2-3f_\infty^2(4-3f_\infty^2)}\,\times\right.$$

$$\left.\times\sin\left(\frac{\pi}{6}-\frac{1}{3}\operatorname{arctg}\mathcal{K}\right)-\frac{a}{2}\right\}\sin\varphi, \qquad (28)$$

$$\mathcal{K} = \sqrt{\frac{4(a^2-3f_\infty^2(4-3f_\infty^2))^3}{(9f_\infty^2(4-3f_\infty^2)a - 54f_\infty^4(f_\infty^2-1) - 2a^3)^2} - 1}. \qquad (29)$$

Thus, we obtained the expression for the coefficient q_∞ (11), which is valid for an arbitrary transparency of the insulator. It allowed us to determine the amplitude of the order parameter at the interface and, therefore, to study its spatial behavior. Based on these results, we found the current density (28) in the SIS tunnel junction for an arbitrary concentration of impurities and in the wide range of the electron transmission factor. From Fig. 2, it is evident that the current reveals a complicated dependence on the phase difference as the transparency factor grows. In the limit case $l \ll \xi_0$, our result agrees with the form of the current-versus-phase dependence obtained in work [5], where just this limit case was studied. The agreement with the general conclusions drawn by other authors in works [6, 7] is also achieved. That is, if the transparency factor grows, the maximal value of the current is attained at $0 < \varphi_{\max} < \pi/2$; the closer $\varphi_{\max}$ is to zero, the higher is the transparency.

In the opposite case $D \ll 1$, this dependence comes closer to a sinusoidal one. The origin of the complication of the current-versus-phase dependence is the especially nonlinear inverse effect of the current on the spatial variation of the order parameter, and this coupling manifests itself in the current. It is obvious that such an influence would be observed with a greater amplitude if the transparency were higher. The obtained result, besides its fundamental contribution to the study of Josephson junctions, may have practical implications. For example, the dependence $j(\varphi)$ may serve as a basis for the construction of the energy-versus-phase dependence for Josephson junctions, which is important while designing superconductor-based quantum bits [9] (see also review [10]).

1. *Josephson B.D.* // Phys Lett. — 1962. — **1**. — P. 251.
2. *Svidzinsky A.V.* Microscopic Theory of Superconductivity. In 2 vols. — Lutsk: Vezha, 2003. — Vol. 2 (in Ukrainian).
3. *Galaiko V.P., Svidzinsky A.V., Slyusarev V.A.* // Zh. Eksp. Teor. Fiz. — 1969. — **56**. — P. 835—840.
4. *Svidzinsky A.V., Golubev L.V.* // Teor. Mat. Fiz. — 1984. — **59**. — P. 129—138.
5. *Kupriyanov M.Yu.* // Pis'ma Zh. Eksp. Teor. Fiz. — 1992. — **56**, N 8. — P. 414—418.
6. *Sols F., Ferrer J.* // Phys. Rev. B — 1994. — **49**. — P. 15913.
7. *Freericks J.K., Nikolic B.K., Miller P.* // Int. J. Mod. Phys. B. — 2002. — **16**, N 4. — P. 531—561.
8. *Svidzinsky A.V., Sakhnyuk V.E.* // Condens. Matter Phys. — 2000. — **3**, N 3. — P. 683—696.
9. *Il'ichev E. et al.* // Appl. Phys. Lett. — 2002. — **80**. — P. 4184.
10. *Golubov A.A., Kupriyanov M.Yu., Il'ichev E.* // Rev. Mod. Phys. — 2004. — **76**. — P. 411—470.

Received 01.12.05
Translated from Ukrainian by O.I. Voitenko

ДЖОЗЕФСОНІВСЬКІ КОНТАКТИ ПРИ НЕПОВНІЙ ПРОЗОРОСТІ БАР'ЄРА ЗА НАЯВНОСТІ ДОМІШОК

В.Є. Сахнюк, А.В. Свідзинський

Р е з ю м е

Теоретично досліджено просторову поведінку параметра порядку поблизу границі в тунельному контакті за наявності домішок довільної концентрації. Температура вважається близькою до критичної. Коефіцієнт проходження D електронів крізь діелектрик може змінюватися в широкому інтервалі значень. Одержано граничні умови для параметра порядку, а також вираз для густини рівноважного струму.