DISPERSION PROPERTIES OF THE MAGNETOACTIVE DUSTY PLASMA WITH FERROMAGNETIC GRAINS

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The paper is devoted to the analysis of the dispersion properties of a dusty plasma with ferromagnetic grains in strong constant external magnetic fields. The dispersion of the magnetic permeability of this system is related to small vibrations of the magnetic dipole moments of grains around the force lines of the magnetic field. The numeric evaluations for the realistic parameters of this dusty plasma show that, in a very narrow band of frequencies in the HF range, the dielectric permittivity and magnetic permeability of the system may be negative simultaneously. This allows us to consider such a plasma as the left-handed medium in this frequency band.

The magnetic permeability of the infinite conventional electron-ion plasma and the dusty plasma with nonmagnetic grains is usually set to be equal to unity [1]. This means that the magnetic induction vector B is equal to the strength vector H, and the dispersion of the magnetic permeability is ignored [2].

The situation completely changes in the dusty plasma with ferromagnetic grains in a strong external magnetic field when the grains possess comparatively large individual magnetic moments. Some properties of such a plasma have been studied under the laboratory conditions. In particular, it was reported about the formation of linear chains of the magnetized grains and the rotational motion of a plasma column in the discharged plasma with ferromagnetic grains [3] in the external magnetic field.

In this paper, we consider the magnetic permeability tensor of such a plasma that is associated with the mechanical motion of the magnetic moments of the dusty grains embedded in the magnetoactive electron-ion plasma.

The grains are modelled as identical spheres of radius a with the built-in magnetic dipole moment d_m. Let us consider the dispersion of the permeability of the system of "cold magnetic dipoles" with a dipole moment d_m embedded in the electron-ion plasma in the strong magnetic field H_0 satisfying the condition

$$\frac{d_m H_0}{T} \gg 1. \tag{1}$$

Here, T is a temperature of the grain system. We also assume that

$$H_0 \gg 4\pi N_g d_m, \tag{2}$$

where $N_g \sim r^{-3}$ is the grain density number (r is a mean distance between grains). Inequality (2) means that the static magnetization of the dusty plasma, being associated with the orientation of individual dipoles along magnetic force lines, is small in comparison with the field H_0.

Now we find the response of this system of dipoles to a varying comparatively weak magnetic field

$$\vec{H}(\vec{r},t) = \vec{H} \exp\,[i\vec{k}\vec{r} - i\omega t]. \tag{3}$$

The dipoles slightly change their orientation with respect to the H_0 direction that is chosen parallel to the z-axis. Here, we use the conventional notation: $\vec{k}$

and ω are the wave vector and the frequency of a wave, respectively.

In the presence of the constant magnetic field H_0 and the varying magnetic field (3), the Lagrange function of a dipole can be presented in the form

$$L = \frac{m_g}{2}v^2 + \frac{J}{2}(\dot{\theta}^2 + \dot{\varphi}^2) +$$

$$+ d_m H_0 \cos\theta + \vec{d}_m \cdot \vec{H}(\vec{r}, t), \tag{4}$$

where $\vec{v}$ is a velocity of the translation motion of the mass center of the magnetic dipole with mass m_g and inertia moment J, ϕ and θ are the azimuthal and polar angles of the dipole orientation, respectively.

The Lagrange function (4) takes into account only the interaction between the dipole and the magnetic field. Inequality (2) allows us to neglect the magnetic dipole-dipole interaction between grains. The typical time of the grain translation motion by gravity is much larger than the period of vibrations of the dipole. In view of the spherical symmetry of grains, the interparticle Coulomb and gravitational forces do not affect the rotational degrees of freedom. The damping action of the plasma environment on the grain rotation will be taken into account later on.

In the long-wave approximation, when the wavelength of the varying field λ is large in comparison with the typical size of a magnetic dipole a, the force acting on the grain center of mass is of the order of $a/\lambda \ll 1$. This allows us to consider the grain center of mass as fixed and to ignore the coordinate dependence of the varying magnetic field (3).

The equations of motion of the rotational degrees of freedom that follow from (4) can be written as

$$\ddot{\varphi} = \frac{d_m}{J}\left\{\sin\theta[-H_x \sin\varphi + H_y \cos\varphi]\right\}\exp(-i\omega t),$$

$$\ddot{\theta} + \frac{d_m H_0}{J}\sin\theta = \frac{d_m}{J}\left[\cos\theta(H_x\cos\varphi + H_y \sin\varphi)\right.$$

$$\left. - H_z \sin\theta\right]\exp(-i\omega t). \tag{5}$$

The system of nonlinear equations (5) describe variations of the orientation of a grain magnetic moment with time.

Let the amplitudes of the varying magnetic field (3) be of the first order of smallness with respect to the amplitude of a constant field ($H_x, H_y, H_z \ll H_0$). Keeping this in mind and taking into account that, for "cold dipoles", $\theta \ll 1$, we solve system (5) by the method of step-by-step approximations. In the zero approximation, system (5) takes the form

$$\ddot{\varphi}_0 = 0, \quad \ddot{\theta}_0 + \omega_0^2\theta_0 = 0, \tag{6}$$

where

$$\omega_0 = \sqrt{d_m H_0/J} \tag{6a}$$

is the frequency of small vibrations of the grain magnetic moment orientation with respect to the z-axis. From the first equation of system (6), we see that the azimuthal angle of a magnetic dipole varies with time according to the law

$$\varphi_0 = \Omega t + \alpha, \tag{7}$$

where Ω and α are the integration constants. Obviously, the angular frequency Ω coincides with the thermal rotational frequency $\omega_T = \sqrt{T/J}$. Comparing Ω with the typical frequency of variation of $\theta(t)$, ω_0, we get $\omega_0/\omega_T = \sqrt{d_m H_0/T} \gg 1$ in accordance with the model of "cold" magnetic dipoles. This means that, in our model, the angular velocity of the azimuthal rotation of a dipole around the z-axis $\dot{\varphi}(t)$ is much slower than the frequency of oscillations or "trembling" of its polar angle $\dot{\theta}(t)$. In another words, while solving the first-order equation for $\theta_1(t)$, we can set φ_0 to be a constant.

Our model equations of motion (5) do not take into account the dissipative forces acting on grains. Plasma ions collide with the grain surface and damp its rotational motion. Further, we assume that the friction force is comparatively small. We include the braking moment in the first-order equation for $\theta_1(t)$ through the phenomenological relaxation time and write it down in the form

$$\ddot{\theta}_1 + {\omega_0}^2\theta_1 = \frac{d_m}{J}[H_x\cos\varphi + H_y\sin\varphi]\exp(-i\omega t) - \dot{\theta}_1/\tau. \tag{8}$$

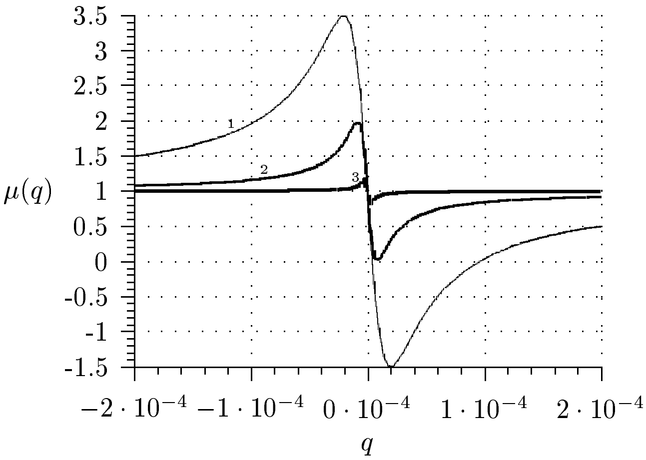
The evaluation of τ may be done with the help of the expression of the friction force moment acting on a spherical particle in the viscous liquid [6],

$$\tau = J/[8\pi\eta a^3], \tag{9}$$

where the viscosity η is given by the known expression [7]

$$\eta = T_i^{5/2} m_i^{1/2}/[e^4 L]. \tag{10}$$

Here, T_i is the temperature of ions, m_i is the ion mass, e is the electron charge, and L is the Coulomb logarithm.



The permeability μ as a function of $q = \omega^2/\omega_0^2 - 1$ for $\Omega_m = 5 \times 10^2$Hz, $\tau = 1$s and different ω_0. Curve *1*: $\omega_0 = 5 \times 10^4$Hz; Curve *2*: $\omega_0 = 1.25 \times 10^5$Hz; Curve *3*: $\omega_0 = 6.25 \times 10^5$Hz

The partial solution of Eq. (8) describes the driven oscillations of the magnetic dipole direction and takes the form

$$\theta_1(t) = \frac{d_m}{J} \frac{[H_x \cos\varphi + H_y \sin\varphi]}{\omega_0{}^2 - \omega^2 - i\omega/\tau} \exp(-i\omega t). \tag{11}$$

The substitution of (11) in the first equation of system (5) shows that the variation of the azimuthal angle with time is of the second order of smallness in the ratio of the amplitudes of the varying and constant magnetic fields.

Now it is possible to find the high frequency magnetization of a unit volume of the system of magnetic grains by comparing the magnetization

$$M_i = N_g d_m \tag{12}$$

with the phenomenological expression

$$M_i = \chi_{ij} H_j. \tag{12a}$$

In these formulas, χ_{ij} is the tensor of magnetic susceptibility, and the indices $i, j = x, y, z$. The components of the magnetic dipole moment induced by a varying magnetic field in our model are

$$d_{mx} = d_m \theta_1 \cos\varphi_0, \quad d_{my} = d_m \theta_1 \sin\varphi_0, \quad d_{mz} \cong d_m, \tag{13}$$

Substituting (11) in (13), we get the tensor of magnetic permeability $\mu(\omega) = \delta_{ij} + 4\pi\chi_{ij}$. Its real nonzero components take the form

$$\mu_{xx} = \mu_{yy} \equiv \mu(\omega) = 1 + \frac{\Omega_m^2 \left(\omega_0^2 - \omega^2\right)}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2/\tau^2}, \quad \mu_z = 1; \tag{14}$$

$$\Omega_m = \sqrt{\frac{2\pi N_g d_m^2}{J}}. \tag{14a}$$

We note here that the deduction of the permeability tensor (14) is analogous to the deduction of the permittivity tensor of a cold magnetoactive plasma. From this point of view, the frequency Ω_m may be treated as the collective mode of magnetic dipoles that is similar to the plasma frequency . In fact, substituting $d_m \sim e_m a$ and $J \sim m_g a^2$ in (14a), we obtain $\Omega_m^2 \sim 2\pi e_m^2 N_g/m_g$ that coincides with the Langmuir frequency to within a factor of 2 (e_m is the magnetic charge). This simple physical reasoning holds true in the long-wave approximation $\lambda^3 N_g \gg 1$. In fact, this inequality specifies the minimal length of an electromagnetic wave. As usual, while analyzing the dispersion properties of a dusty plasma, its magnetic permeability is set to be unity. The situation changes in a magnetoactive dusty plasma with ferromagnetic grains. The account of the permeability tensor (14) along with the tensor of dielectric permittivity of charged components in the Maxwell equations leads to a new dispersion equation. It splits into two equations. One of them reads

$$\mu(\omega) = 0. \tag{15}$$

Here, $\mu(\omega)$ is given by (14).

Another equation describes the electromagnetic waves in the dusty plasma with the participation of magnetic dipoles. In the case of ferromagnetic grains with anomalous magnetic moments, their contribution to the dispersion properties can be important. As in the conventional magnetoactive plasma, it is the biquadratic equation with respect to the refraction index kc/ω but with coefficients depending on $\mu(\omega)$. If the grains have no magnetic moments, i.e. $\mu(\omega) \to 0$, this equation coincides with the corresponding result for the conventional magnetoactive electron-ion plasma [7].

In the general case, the frequency dependence of the function $\mu(\omega)$ (14) is a Drude-type curve with two neighboring extreme points from the left and right sides of a point ω_0.

Figure 1 presents the dependences of permeability (14) on $q = \omega^2/\omega_0^2 - 1$ for the grains $a = 5$ μm, the same $\Omega_m\tau$, and three different values of $\omega_0\tau$; 1) Eq. (15) has no real roots; 2) Eq. (15) has one real root; 3) Eq. (15) has two real roots.

Equation (15) has real roots provided that

$$\Omega_m^2 \tau \geq 2\omega_0. \quad (16)$$

Here, we took into account that, for the small friction force, $\omega_0 \tau >> 1$. The equality and inequality signs indicate that (15) has one real or two real roots, respectively.

Under real conditions, inequality (2) leads to $\Omega_m << \omega_0$. Obviously, this condition is always true in the gaseous media with ferromagnetic impurities.

The solutions of the dispersion equation (15) describe the weakly damping vibrations of magnetic dipoles in the plane perpendicular to the constant magnetic field H_0. We may call them the magnetization vibrations. In this case, the varying magnetic field does not produce any electric field. The frequencies of these eigenvibrations are very close to ω_0.

The solutions of the biquadratic equation (for ω) (15) give the evaluation of the frequency band $\Delta\omega$, where $\mu(\omega) < 0$, as

$$\Delta\omega = \frac{1}{2}\frac{\Omega_m^2}{\omega_0}. \quad (17)$$

It is satisfied in a rather narrow frequency range with the specially tuned parameters of grains (d_m, N_g), the external magnetic field H_0, and the relaxation time τ.

The typical frequencies of this system ω_0 and Ω_m depend on the grain magnetic dipole moment d_m. For the grains of a typical size $a_0 \approx 10^{-6}$ cm, the one-domain approximation is true. The magnetic moment of such a grain $d_m \approx 10^{-16}$ erg/Gs. For larger ferromagnetic grains ($a > a_0$) in strong magnetic fields, their dipole moment may be evaluated as $d_m \approx 10^{-16} \times a^3/a_0{}^3$. The external magnetic field of the order of $H_0 \geq 10^3$ Gs "freezes" the orientation of our magnetic dipoles. The inertia moment of a spherical grain is $J = \frac{2}{5} m_g a^2$, and $m_g = \frac{4\pi}{3}\varrho a^3$ (ϱ is the density of a grain of radius a). In evaluations, we used the following parameters: $a \approx 5 - 10\mu$, $N_g \geq 10^6$ cm^{-3}, $H_0 \simeq 10^3 - 10^4$ Gs, when the typical frequency $\Omega_m \approx (5-7) \times 10^2$ and $\omega_0 \approx (2.5-5)\times 10^4$ (the frequency is measured in rad/s). The value of the relaxation time in (16) for these size of grains and the ion temperature in the argon plasma of 10^3 K is $\tau \approx (1-5)$ s according to (9), (10).

Let a linearly polarized monochromatic electromagnetic wave of the frequency in the range $\omega_0 + \Delta\omega$ ($\Delta\omega$ is given by (16)) with a wave vector k propagate along the y-axis in the media transversally to the constant magnetic field H_0. Let the varying electric component of this wave E_z be directed along the z-axis, and let the varying magnetic component be along the x-axis. The dispersion equation of this wave is

$$\omega = \frac{kc}{\sqrt{\varepsilon\mu}}. \quad (18)$$

In this equation, ε is the longitudinal component of the dielectric permittivity tensor of the ion component. It does not depend on the magnetic field H_0 and can be written as

$$\varepsilon \approx 1 - \omega_p^2/\ \omega^2, \quad (19)$$

where $\omega_p^2 = 4\pi e^2 N_i/\ m_i$ is the Langmuir frequency of ions, N_i is the ion density number, m_i is the ion mass. The above-presented numerical values of ω_0 correspond to the typical frequency of the ion motion in the plasma.

Now we consider the frequency range (17) in the vicinity of ω_0, where the transversal components of the permeability tensor (14) are negative. The electromagnetic wave in this frequency domain may propagate in the plasma under consideration, provided that the corresponding components of the dielectric permittivity tensor are negative as well. This could be done by the appropriate choice of the density number of ions. For example, choosing $N_i = 2 \times 10^8$ cm^{-3}, we obtain $\varepsilon \approx -400$. This magnitude of N_i (at the ion temperature $T_i = 10^3$K) provides also a large enough number of collisions between the argon ions and the grain to use the expression for the plasma viscosity (10).

By tuning ω slightly larger than ω_0, we can obtain $\mu \approx -1$. According to (18), the phase velocity of an electromagnetic wave passing through the dusty plasma under consideration (with $\varepsilon \approx -400$) lowered by 20 times.

Our numerical evaluations show that, by the appropriate tuning of the magnetized dusty plasma parameters, it is possible to obtain $\varepsilon < 0$ and $\mu < 0$ simultaneously. This allows us to claim that the above-described system can be considered as a possible candidate of the "left-handed media" with controlled parameters in the radio-frequency range.

In conclusion, we would like to note that, in the framework of our model, the magnetoactive dusty plasma with ferromagnetic grains in the comparatively narrow frequency range (17) from the right side of ω_0 has $\varepsilon < 0$ and $\mu < 0$ simultaneously. The frequency ω_0 lies within the HF frequency range. It is connected with the need to meet inequality (16). The situation is complicated by the fact that N_g cannot be made much more than 10^6 in the dusty plasma, and the inequality $\Omega_m \ll \omega_0$ is always true in strong magnetic fields.

A considerable increase of the frequency ω_0 and $\Delta\omega$ could be apparently obtained in solid composite materials with ferromagnetic grains. In such systems, the impurity density number N_g can be enlarged by many orders. This will result in a significant increase of Ω_m, which can shift, in turn, the domain, where $\mu < 0$, to the SHF range.

According to the current status of the research, all the known left-handed media were manufactured in the laboratory [8]. To our mind, such systems in the nature can occur in the atmosphere of the so-called brown dwarfs, where the electron-ion plasma consists of iron nanoparticles and there are very strong magnetic fields.

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ДИСПЕРСІЙНІ ВЛАСТИВОСТІ МАГНІТОАКТИВНОЇ ЗАПОРОШЕНОЇ ПЛАЗМИ З ФЕРОМАГНІТНИМИ ГРАНУЛАМИ

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Р е з ю м е

Робота присвячена аналізу дисперсійних властивостей запорошеної плазми з феромагнітними гранулами в сильному зовнішньому магнітному полі. Дисперсія магнітної проникності цієї системи пов'язана з малими коливаннями магнітних дипольних моментів гранул навколо напрямку силових ліній магнітного поля. Числові оцінки типових параметрів такої запорошеної плазми показують, що у вузькому діапазоні частот у ВЧ-області діелектрична проникність та магнітна сприйнятливість системи можуть бути негативними одночасно. Це дозволяє розглядати таку плазму як лівостороннє середовище у відповідній смузі частот.