SYSTEM OF TWO CHARGED AND ONE NEUTRAL PARTICLES STRONGLY INTERACTING WITH ONE ANOTHER IN THE CONTINUOUS SPECTRUM

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The Faddeev-type equations for a system of three strongly interacting particles, two of which are charged and the remaining neutral particle is bound to only one of them, have been transformed into a form suitable for the further investigation and calculations. In the modified equations, the principal parts associated with the Coulomb interaction can be separated, and the most complicated summands of the complete three-particle wave function can be written in terms of fast convergent series of the products of spherical functions. In addition, the integral equations for the radial wave functions of two dimensional variables have been derived in the basic approximation.

1. Introduction

In order to describe a nuclear system in the range of its continuous spectrum theoretically, provided that the kinetic energies of its parts which are unbound but interacting with one another are low, the long-range Coulomb interaction (CI) has to be considered in addition to the nuclear one. The elementary example of such a system is the system composed of three strongly interacting particles, two of which are charged and the remaining neutral particle is bound with only one of them (e.g., pd and πd systems). It is this case that will be considered in this work.

The known original Faddeev integral equations of motion [1] for three interacting particles, which are not of the Fredholm type if the CI is taken into account, have been modified in a number of works [2—7]. In particular, from the Faddeev-type equations which contain the CI, the main Coulomb singularity was excluded; but the arising, as the remainder, effective (Coulomb-nuclear) potential does not become as short-range as the potential of purely nuclear interaction, although it falls down depending on the distance appreciably more quickly than the Coulomb potential does.

In this work, the most complicated remainder, which arises after the main Coulomb contribution having been excluded from the complete wave function, is expanded in a fast convergent series of the products of spherical functions, and the equations for the sought radial functions of only two (of six) relative variables, which have dimensions of length, are constructed. In addition to the development of the general theory, the basic approximation for two zero-valued relative orbital moments $\vec{\ell}_1$ and $\vec{\ell}_2$ is considered in detail. We note that, in order to obtain a more accurate result, more summands in the expansion series should be taken into account, as compared with the solutions of problems in works [8, 9], where the CI was not considered. Therefore, the result obtained in the framework of the approximation $\ell_1 = \ell_2 = 0$ and taking the CI into account will be only an estimation at a qualitative level. From what follows, one can see that the result obtained can be numerically made more accurate, but the problem becomes more cumbersome at that.

The reformulated system of equations, which was obtained in works [5, 6] with regard for the CI, contains effective (nuclear-Coulomb) potentials with a finite radius of interaction. Therefore, we proceed from the results of those works and use, in the main, their notations.

In accordance with the results of work [6], the complete wave function Ψ of a three-particle system (taking no spins into account), provided that charged particle 1 is scattered by the system of two other bound particles – charged particle 2 and neutral particle 3, can be written down in the form

$$\Psi = (1 + G_0 T_{12}^{\mathrm{C}})(\Psi^{(12)} + \Psi^{(23)} + \Psi^{(31)}) \,, \quad (1)$$

where three components $\Psi^{(ij)}$ (hereafter, $i,j = 1,2,3$ and $i \neq j$) satisfy the system of coupled equations

$$\Psi^{(23)} = \Phi^{(23)} + G_0\tilde{T}^N_{23}(1 + G_0T^{\rm C}_{12})(\Psi^{(12)} + \Psi^{(31)})\,, \quad (2)$$

$$\Psi^{(31)} = G_0\tilde{T}^N_{31}(1 + G_0T^{\rm C}_{12})(\Psi^{(12)} + \Psi^{(23)})\,, \quad (3)$$

$$\Psi^{(12)} = G_0\tilde{T}^N_{12}(1 + G_0T^{\rm C}_{12})(\Psi^{(23)} + \Psi^{(31)})\,, \quad (4)$$

and the "free" term $\Phi^{(23)}$ in Eq. (2) is defined by the independent integral equation for the same three-particle system but with the "cut-off" interaction potential between particles, which is the sum of the nuclear potential $V^N_{23} \equiv V_{23}$ between particles 2 and 3 and the Coulomb potential $V^{\rm C}_{12}$ between particles 1 and 2:

$$\Phi^{(23)} = \Phi_{23} + G_0T^N_{23}G_0T^{\rm C}_{12}\Phi^{(23)}\,. \quad (5)$$

Here, Φ_{23} is the "initial" wave function which is the product

$$\Phi_{23} = \phi(1,23)\psi_{b_{23}}(23) \quad (6)$$

of the wave function $\phi(1,23)$ of the relative motion of particle 1 with respect to the system of particles (23) (a plane wave) and the internal wave function $\psi_{b_{23}}(23)$ of the system of bound particles 2 and 3 (b_{23} being the corresponding binding energy of the two-particle system).

Recall the definitions of operators that enter into Eqs. (1)—(5): $G_0 \equiv G_0(Z) = (Z - H_0)^{-1}$ is the free three-particle Green's function (operator); $Z = E + i0$; E is the total energy of the three-particle system; H_0 the operator of the kinetic energy of three particles [10, 11]; $T^{\rm C}_{12} \equiv T^{\rm C}_{12}(Z)$ is the two-particle Coulomb transition operator which obeys the operational equation $T^{\rm C}_{12} = V^{\rm C}_{12}(1+G_0T^{\rm C}_{12})$; $\tilde{T}^N_{ij} \equiv \tilde{T}^N_{ij}(Z)$ are the two-particle nuclear-Coulomb transition operators which satisfy the equation $\tilde{T}^N_{ij} = V_{ij}(1 + G^{\rm C}_{12}\tilde{T}^N_{ij})$; and $G^{\rm C}_{12} \equiv G^{\rm C}_{12}(Z) = G_0(1+T^{\rm C}_{12}G_0) \equiv (1+G_0T^{\rm C}_{12})G_0$ is the Coulomb Green's function (operator) of the system of three particles, two of which – particles 1 and 2 – are charged [6]. For further transformations and carrying out the general research, the two-particle transition operators $T^N_{ij} \equiv T^N_{ij}(Z)$, which are simpler than $\tilde{T}^N_{ij}$ ones, will be needed. The operators T^N_{ij} satisfy the equation $T^N_{ij} = V_{ij}(1+G_0T^N_{ij})$, being connected only with the potentials of nuclear interaction V_{ij}. Making use of the operators T^N_{ij}, the equation for $\tilde{T}^N_{ij}$ can be written down in another form: $\tilde{T}^N_{ij} = T^N_{ij}\left[1 + (G^{\rm C}_{12} - G_0)\tilde{T}^N_{ij}\right]$. Provided $V^{\rm C}_{12} \to 0$, it is evident that $T^{\rm C}_{12} \to 0$, $G^{\rm C}_{12} \to G_0$, $\tilde{T}^N_{ij} \to T^N_{ij}$, and $\Phi^{(23)} \to \Phi_{23}$.

2. Reformulation of the Equations of Motion for a System of Three Particles, two of which Being Charged

The system of Eqs. (1)—(4) for the complete wave function Ψ of a three-particle system with two charged particles is rather complicated both for the general study and numerical calculations. (The same is also true for Eq. (5), the solution of which, constituting a separate problem, is needed for finding the function Ψ.) Therefore, we transform these equations to some extent in order to give them a more convenient form, by either simplifying the complicated operators or getting rid of some of them. We will also propose a method for the numerical solution of the equations obtained.

For example, substituting the sum $V_{31} + V_{31}G^{\rm C}_{12}\tilde{T}^N_{31}$ for the operator $\tilde{T}^N_{31}$ in Eq. (3) (see Introduction) and making use of the definition $G^{\rm C}_{12} = (1 + G_0T^{\rm C}_{12})G_0$, we obtain

$$\Psi^{(31)} = G_0V_{31}(1 + G_0T^{\rm C}_{12})(\Psi^{(12)} + \Psi^{(23)}) +$$
$$+G_0V_{31}(1 + G_0T^{\rm C}_{12})\left[G_0\tilde{T}^N_{31}(1 + G_0T^{\rm C}_{12})(\Psi^{(12)} + \Psi^{(23)})\right]. \quad (7)$$

The expression enclosed in brackets in the term on the right-hand side of equality (7) is the $\Psi^{(31)}$ component according to definition (3). Therefore, taking Eq. (1) into account, we write down, instead of Eq. (7), the simpler relation

$$\Psi^{(31)} = G_0V_{31}\Psi\,, \quad (8)$$

where the right-hand side already contains the required complete wave function Ψ. In a similar way, we can get rid of the complicated operators $\tilde{T}^N_{12}$ in Eq. (4) and $\tilde{T}^N_{23}$in Eq. (2):

$$\Psi^{(12)} = G_0V_{12}\Psi\,, \quad (9)$$

$$\Psi^{(23)} - \Phi^{(23)} = G_0V_{23}\left[\Psi - (1 + G_0T^{\rm C}_{12})\Phi^{(23)}\right]. \quad (10)$$

Let us operate now on the left- and right-hand sides of Eqs. (8)–(10) with the Coulomb operator $(1+G_0T^{\rm C}_{12})$ and sum up the equations obtained, taking Eq. (1) and the definition of $G^{\rm C}_{12}$ into account. As a result, we obtain a single equation for the complete wave function Ψ:

$$\Psi - (1 + G_0T^{\rm C}_{12})\Phi^{(23)} = G^{\rm C}_{12}V\left[\Psi - (1 + G_0T^{\rm C}_{12})\Phi^{(23)}\right] +$$
$$+G^{\rm C}_{12}(V_{31} + V_{12})(1 + G_0T^{\rm C}_{12})\Phi^{(23)}\,, \quad (11)$$

where $V = V_{12} + V_{23} + V_{31}$ is the total nuclear potential of the three-particle system.

The function $\Phi^{(23)}$ that enters into Eq. (11), as well as the function Ψ, is unknown. Below, we show how it can be found by analyzing Eq. (5). Meanwhile, we consider $\Phi^{(23)}$ given. In this case, Eq. (11) determines only one unknown function Ψ.

It is convenient to introduce the function

$$\chi^{(23)} = (1 + G_0 T_{12}^{\mathrm{C}})\Phi^{(23)} \quad (12)$$

instead of $\Phi^{(23)}$ into Eq. (11). This new function has a simple physical meaning. If we rewrite Eq. (11) in the form

$$\Psi - \chi^{(23)} = G_{12}^{\mathrm{C}} V(\Psi - \chi^{(23)}) + G_{12}^{\mathrm{C}}(V_{31} + V_{12})\chi^{(23)} \,, \quad (13)$$

it becomes evident that when the nuclear interaction between particle 1 and two other particles is "switched off' (i.e., when $V_{12} = V_{31} = 0$), the equation obtained

$$\Psi - \chi^{(23)} = G_{12}^{\mathrm{C}} V_{23}(\Psi - \chi^{(23)}) \quad (14)$$

possesses the solution $\Psi = \chi^{(23)}$. It can also be seen from the initial Eqs. (1)–(4), because, in this case, $\Psi^{(31)} = \Psi^{(12)} = 0$, $\Psi^{(23)} = \Phi^{(23)}$, and $\Psi = (1 + G_0 T_{12}^{\mathrm{C}})\Psi^{(23)} = \chi^{(23)}$.

In the same way, let us transform Eq. (5) for $\Phi^{(23)}$ into the equation for the function $\chi^{(23)}$, simultaneously getting rid of the complicated operator T_{23}^{N}. We emphasize that Eq. (5), unlike Eq. (11), does not depend on the nuclear potentials V_{12} and V_{31}, because the known function Φ_{23} and every operator that enters into Eq. (5), as well as the operator G_{12}^{C}, do not depend on them. Therefore, the functions $\Phi^{(23)}$ and $\chi^{(23)}$ also do not depend on the potentials V_{12} and V_{31} and are not changed if the latter are zeroed.

Making use of the equation for the operator T_{23}^{N}, we substitute the sum $V_{23} + V_{23} G_0 T_{23}^{N}$ for T_{23}^{N} in Eq. (5) (see Introduction) –

$$\Phi^{(23)} = \Phi_{23} + G_0 V_{23} G_0 T_{12}^{\mathrm{C}} \Phi^{(23)} +$$

$$+ G_0 V_{23}\left[G_0 T_{23}^{N} G_0 T_{12}^{\mathrm{C}} \Phi^{(23)}\right] \quad (15)$$

— and note that the expression in brackets on the right-hand side of Eq. (15) is equal, in accordance with Eq. (5), to the difference $\Phi^{(23)} - \Phi_{23}$. Acting on the equation obtained

$$\Phi^{(23)} = (1 - G_0 V_{23})\Phi_{23} + G_0 V_{23}(1 + G_0 T_{12}^{\mathrm{C}})\Phi^{(23)} \quad (16)$$

with the operator $1 + G_0 T_{12}^{\mathrm{C}}$ from the left, applying Eq. (12), and introducing the operator G_{12}^{C}, as well as the notation χ^{C} for the function which it is considered known (the analog of Φ_{23}),

$$\chi^{\mathrm{C}} = (1 + G_0 T_{12}^{\mathrm{C}})(1 - G_0 V_{23})\Phi_{23} \,, \quad (17)$$

we obtain the equation for the function $\chi^{(23)}$:

$$\chi^{(23)} = \chi^{\mathrm{C}} + G_{12}^{\mathrm{C}} V_{23} \chi^{(23)} \,. \quad (18)$$

In this equation, every quantity, but the function $\chi^{(23)}$, is considered known. It is clear that Eq. (18) replaces Eq. (5), because they are equivalent.

Below, we proceed from the transformed Eqs. (13) and (18), but the initial Eqs. (1)—(5) will be of use as well in order to substantiate some of our statements.

3. Expansion in Fast Convergent Series

Despite that the initial Eqs. (1)—(4) include both the short-range nuclear potentials V_{ij} and the long-range Coulomb one V_{12}^{C}, some parts of the complete wave function, namely, $\Psi^{(23)} - \Phi^{(23)}$, $\Psi^{(31)}$, $\Psi^{(12)}$, and the difference $\Psi - (1 + G_0 T_{12}^{\mathrm{C}})\Phi^{(23)} = \Psi - \chi^{(23)}$, can be expanded in fast convergent series of K-harmonics. However, it cannot be done for the difference $\Phi^{(23)} - \Phi_{23}$, owing to the presence of the potential V_{12}^{C} in Eq. (5). Therefore, in this section, we consider, for the moment, the expansion of the difference $\Psi - \chi^{(23)}$ in a fast convergent series.

Although there is the opportunity to expand $\Psi - \chi^{(23)}$ in a series of K-harmonics, similarly to what was done in works [8, 9], where the CI was absent, it would be an extremely inconvenient operation in our case because of the presence of the CI potential V_{12}^{C} in the total Hamiltonian. Therefore, in contrast to works [8, 9], it is reasonable to select six following variables as the relative ones [11]: two dimensional variables $r = |\vec{r}|$ and $\rho = |\vec{\rho}|$ and two pairs of corresponding angles $\Omega_1 \equiv (\theta_1, \phi_1)$ and $\Omega_2 \equiv (\theta_2, \phi_2)$. This set of angles defines the orientations of two three-dimensional vectors $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{\rho} = \vec{r}_3 - (M_1 \vec{r}_1 + M_2 \vec{r}_2)/(M_1 + M_2)$, where $\vec{r}_j$ and M_j are the radius-vector and the mass of the j-th particle, respectively. The efficiency of such a choice of the variables is explained by the fact that, in this case, the Coulomb potential depends only on the variable $V_{12}^{\mathrm{C}} \equiv V_{12}^{\mathrm{C}}(r)$. Now, we expand the difference $\Psi - \chi^{(23)}$ in a series of the products of spherical functions which looks like

$$\Psi - \chi^{(23)} = \sum_{l_1 m_1} \sum_{l_2 m_2} b_{l_1 m_1,\, l_2 m_2}(r, \rho) Y_{l_1 m_1}(\Omega_1) Y_{l_2 m_2}(\Omega_2). \quad (19)$$

Series (19), as well as the series of K-harmonics, is fast convergent, because, provided $r \to \infty$ and $\rho \to \infty$, the difference $\Psi - \chi^{(23)}$ and all radial functions $b_{l_1m_1,\,l_2m_2}(r,\rho)$ quickly tend to zero.

Let us substitute expansion (19) in the left- and right-hand sides of Eq. (13) and multiply the result by $Y^*_{l_1'm_1'}(\Omega_1)Y^*_{l_2'm_2'}(\Omega_2)$. After having integrated the obtained relation over the angular variables Ω_1 and Ω_2, we obtain the infinite system of coupled equations for the determination of the radial functions $b_{l_1m_1,\,l_2m_2}(r,\rho)$. In this work, we present only the result for $l_1' = m_1' = 0$ and $l_2' = m_2' = 0$ which will be referred to as $b(r,\rho) = b_{00,00}(r,\rho)$ below:

$$b(r,\rho) = \frac{1}{4\pi}\int d\Omega_1 \int d\Omega_2\, G_{12}^{\mathrm{C}}\, V\times$$

$$\times\sum_{l_1m_1}\sum_{l_2m_2} b_{l_1m_1,\,l_2m_2}(r,\rho)Y_{l_1m_1}(\Omega_1)Y_{l_2m_2}(\Omega_2)+$$

$$+\frac{1}{4\pi}\int d\Omega_1 \int d\Omega_2\, G_{12}^{\mathrm{C}}(V_{31}+V_{12})\chi^{(23)}\,. \tag{20}$$

The unknown function $\chi^{(23)}$ was considered given in Eq. (20). To find it, we introduce the asymptotic function χ_{23} which can be obtained from $\chi^{(23)}$ by letting $r \equiv |\vec{r}_1 - \vec{r}_2| \to \infty$. This function is equal to a product of the quickly decaying (at $|\vec{r}_2 - \vec{r}_3| \to \infty$) known wave function $\psi_{b_{23}}(23)$ describing the bound state of particles 2 and 3 and the function $\phi^{\mathrm{C}}(1,23)$ corresponding to the relative motion of the bound two-particle system (23) and the charged particle 1. The function $\phi^{\mathrm{C}}(1,23)$, according to work [12], is defined as

$$\phi^{\mathrm{C}}(1,23) = \phi(1,23)\,\exp\left\{i\frac{Z_1Z_2e^2}{\hbar v}\,\ln(k_{12}r - \vec{k}_{12}\vec{r})\right\},$$

$$k_{12} = |\vec{k}_{12}|\,, \tag{21}$$

where Z_1e and Z_2e are the charges of particles 1 and 2; v and $\vec{k}_{12}$ are their relative velocity and wave vector, respectively, at infinity; and e is the proton charge. Function (21) is transformed into the plane wave $\phi(1,23)$ if $V_{12}^{\mathrm{C}} \to 0$. Thus, the function $\chi_{23} = \phi^{\mathrm{C}}(1,23)\psi_{b_{23}}(23)$ is considered known, as well as $\Phi_{23} = \phi(1,23)\psi_{b_{23}}(23)$.

While the difference $\Phi^{(23)} - \Phi_{23}$ cannot be expanded in a fast convergent series of a complete set of angular functions, this can be done for the difference $\chi^{(23)} - \chi_{23}$ — in the same manner as for $\Psi - \chi^{(23)}$:

$$\chi^{(23)} - \chi_{23} =$$

$$= \sum_{l_1m_1}\sum_{l_2m_2}\chi_{l_1m_1,\,l_2m_2}(r,\rho)Y_{l_1m_1}(\Omega_1)Y_{l_2m_2}(\Omega_2)\,. \tag{22}$$

In order to obtain the equations for the radial functions $\chi_{l_1m_1,\,l_2m_2}(r,\rho)$, one may repeat the procedure of constructing the equations for $b_{l_1m_1,\,l_2m_2}(r,\rho)$ in Eq. (19). In so doing, the analog of Eq. (13) for the function $\chi^{(23)}$ is used:

$$\chi^{(23)} - \chi_{23} = G_{12}^{\mathrm{C}}V_{23}(\chi^{(23)} - \chi_{23})+$$

$$+G_{12}^{\mathrm{C}}V_{23}\chi_{23} + \chi^{\mathrm{C}} - \chi_{23}\,, \tag{23}$$

which stems directly from Eq. (18).

4. Equations for the Radial Functions in the Case $\ell_1 = \ell_2 = 0$

Without losing generality, consider, as an example, the simplest equation for the radial functions (20) which is included into the infinite chain of coupled integral equations.

First of all, consider a typical quadruple integral over the angular variables, which is available in the examined equations,

$$J(r,\rho) = \int d\Omega_1 \int d\Omega_2\, G_{12}^{\mathrm{C}}\, f(r,\rho,\Omega_1,\Omega_2)\,, \tag{24}$$

where $f(r,\rho,\Omega_1,\Omega_2)$ is an arbitrary function (it will be specified below);

$$G_{12}^{\mathrm{C}} \equiv (E + i0 - H_0 - V_{12}^{\mathrm{C}})^{-1} =$$

$$= \Big(E + i0 + \frac{\hbar^2}{2\mu_{12}}\,\Delta_r - V_{12}^{\mathrm{C}} + \frac{\hbar^2}{2\mu_{3(12)}}\,\Delta_\rho\Big)^{-1} =$$

$$= \Big(E{+}i0{-}T_r{-}V_{12}^{\mathrm{C}}(r){-}\frac{\hbar^2\vec{\ell}_r^{\,2}}{2\mu_{12}\,r^2}{-}T_\rho{-}\frac{\hbar^2\vec{\ell}_\rho^{\,2}}{2\mu_{3(12)}\rho^2}\Big)^{-1}, \tag{25}$$

is the Coulomb Green's function which was mentioned in Introduction and which enters into Eq. (20);

$$T_r = -\frac{\hbar^2}{2\mu_{12}}\,\frac{1}{r^2}\,\frac{\partial}{\partial r}\Big(r^2\,\frac{\partial}{\partial r}\Big),$$

$$T_\rho = -\frac{\hbar^2}{2\mu_{3(12)}}\,\frac{1}{\rho^2}\,\frac{\partial}{\partial \rho}\Big(\rho^2\,\frac{\partial}{\partial \rho}\Big), \tag{26}$$

$\mu_{12} = M_1M_2/M_{12}$ and $\mu_{3(12)} = M_3M_{12}/(M_3 + M_{12})$ are the reduced masses; $M_{12} = M_1 + M_2$; and $M_{1,2,3}$ are the masses of the particles. Now, we expand $f(r,\rho,\Omega_1,\Omega_2)$ in a series of spherical harmonics,

$$f(r,\rho,\Omega_1,\Omega_2) =$$

$$= \sum_{l_1 m_1} \sum_{l_2 m_2} f_{l_1 m_1, l_2 m_2}(r,\rho) Y_{l_1 m_1}(\Omega_1) Y_{l_2 m_2}(\Omega_2), \qquad (27)$$

and substitute relations (25)—(27) into Eq. (24):

$$J(r,\rho) = 4\pi G_{12}^{\mathrm{C}}(r,\rho) f_{00,00}(r,\rho)\,, \qquad (28)$$

where

$$G_{12}^{\mathrm{C}}(r,\rho) = \left(E + i0 - T_r - V_{12}^{\mathrm{C}}(r) - T_\rho\right)^{-1} \qquad (29)$$

is the "cut-off" Coulomb Green's function; and the function $f_{00,00}(r,\rho)$, according to Eq. (27), is defined by the expression

$$f_{00,00}(r,\rho) = \frac{1}{4\pi} \int d\Omega_1 \int d\Omega_2\, f(r,\rho,\Omega_1,\Omega_2)\,. \qquad (30)$$

In the infinite fast convergent series in Eq. (20), the main contribution is made by the term with $l_1 = m_1 = 0$ and $l_1 = m_1 = 0$; therefore, let us consider this simplest case. From Eq. (20), making use of relations (28)—(30), we obtain the linear nonhomogeneous integral equation for $b(r,\rho)$:

$$b(r,\rho) = \frac{1}{(4\pi)^2} G_{12}^{\mathrm{C}}(r,\rho) b(r,\rho) \int d\Omega_1 \int d\Omega_2\, V +$$

$$+ \frac{1}{4\pi} G_{12}^{\mathrm{C}}(r,\rho) \int d\Omega_1 \int d\Omega_2\, (V_{31} + V_{12}) \chi^{(23)}. \qquad (31)$$

In the same way, using Eqs. (22) and (23), we obtain the integral equation for the radial function $\chi(r,\rho) \equiv \chi_{00,00}(r,\rho)$:

$$\chi(r,\rho) = \frac{1}{(4\pi)^2} G_{12}^{\mathrm{C}}(r,\rho) \chi(r,\rho) \int d\Omega_1 \int d\Omega_2\, V_{23} +$$

$$+ \frac{1}{4\pi} G_{12}^{\mathrm{C}}(r,\rho) \int d\Omega_1 \int d\Omega_2 \times$$

$$\times [V_{23}\chi_{23} + (G_{12}^{\mathrm{C}})^{-1} (\chi^{\mathrm{C}} - \chi_{23})]. \qquad (32)$$

Since Eq. (32) contains only one unknown function $\chi(r,\rho)$, it is reasonable to solve firstly just this equation which can be rewritten in the compact form as

$$\chi(r,\rho) = G_{12}^{\mathrm{C}}(r,\rho) \chi(r,\rho) \overline{V}_{23} + C(r,\rho)\,, \qquad (33)$$

where $\overline{V}_{23} \equiv \overline{V}_{23}(r,\rho) = (4\pi)^{-2} \int d\Omega_1 \int d\Omega_2\, V_{23}(|\vec{r}_2 - \vec{r}_3|)$ and $C(r,\rho)$ (the term on the right-hand side of expression (32)) are known functions.

Let us introduce the eigenfunctions of the operators $T_r + V_{12}^{\mathrm{C}}(r)$ and T_ρ [12]

$$\phi_{k_{12}}(r) = \sqrt{\frac{2}{\pi}}\, k_{12} \exp\left[i k_{12} r - \frac{\pi \alpha_{12}}{2} \right] \times$$

$$\times \left(\frac{\pi \alpha_{12}}{sh(\pi \alpha_{12})} \right)^{1/2} F(1 + i\alpha_{12}, 2, -2ik_{12} r)\,,$$

$$\psi_{k_{3(12)}}(\rho) = \sqrt{\frac{2}{\pi}} \frac{\sin(k_{3(12)}\rho)}{\rho}, \quad \alpha_{12} = \frac{Z_1 Z_2\, e^2 \mu_{12}}{\hbar^2 k_{12}}\,, \qquad (34)$$

where $k_{3(12)}$ is the wave number of the relative motion of particle 3 and system (12). Taking advantage of the orthonormalization and completeness of functions (34), the integral equation (33) for $\chi(r,\rho)$ can be rewritten, according to Eqs. (28)—(30), in the form

$$\chi(r,\rho) - C(r,\rho) = \int_0^\infty dr' r'^2 \int_0^\infty d\rho' \rho'^2 \chi(r',\rho') \overline{V}_{23}(r',\rho') \times$$

$$\times \int_0^\infty dq\, \phi_q^*(r')\, \phi_q(r) \int_0^\infty d\kappa\, \frac{\psi_\kappa^*(\rho') \psi_\kappa(\rho)}{Z - \left(\frac{\hbar^2 q^2}{2\mu_{12}} + \frac{\hbar^2 \kappa^2}{2\mu_{3(12)}} \right)}. \qquad (35)$$

The integral over κ in Eq. (35) can be expressed in the form of an integral with infinite limits and, applying the residue theorem ($Z = E + i0$) and the Heaviside function $\Theta(x)$, can be calculated explicitly:

$$\frac{2}{\pi} \frac{1}{\rho\rho'} \frac{1}{2} \int_{-\infty}^\infty d\kappa\, \frac{\sin(\kappa\rho')\, \sin(\kappa\rho)}{E + i0 - \frac{\hbar^2}{2} \left(\frac{q^2}{\mu_{12}} + \frac{\kappa^2}{\mu_{3(12)}} \right)} =$$

$$= \frac{2}{\pi} \frac{1}{\rho\rho'} \left[P_>(q) \Theta(b_q) + P_<(q) \Theta(-b_q) \right]\,,$$

$$P_>(q) = -\frac{\mu_{3(12)}}{\hbar^2} \frac{\pi}{\sqrt{b_q}} [\exp(i\sqrt{b_q}\rho)\, \sin(\sqrt{b_q}\,\rho') \times$$

$$\times \Theta(\rho - \rho') + \exp(i\sqrt{b_q}\rho')\, \sin(\sqrt{b_q}\,\rho)\, \Theta(\rho' - \rho)]\,,$$

$$P_<(q) = -\frac{\mu_{3(12)}}{\hbar^2} \frac{\pi}{\sqrt{-b_q}} [\exp(-\sqrt{-b_q}\rho)\, \mathrm{sh}(\sqrt{-b_q}\,\rho') \times$$

$$\times \Theta(\rho - \rho') + \exp(-\sqrt{-b_q}\rho')\, \mathrm{sh}(\sqrt{-b_q}\,\rho)\, \Theta(\rho' - \rho)]\,,$$

$$b_q = \frac{2\mu_{3(12)}}{\hbar^2} \left(E - \frac{\hbar^2 q^2}{2\mu_{12}} \right)\,. \qquad (36)$$

On finding the function $\chi(r,\rho)$ and substituting it into Eq. (31), the integral equation (31) for the function $b(r,\rho)$ can be transformed in a similar manner as well.

5. Conclusions

1. The system of coupled integral equations obtained in the earlier works for separate parts of the complete wave function Ψ which describes the systems of three strongly interacting particles, two of which are charged and only one charged particle is bound with the neutral one, has been reduced to a single integral equation for Ψ.

2. The most complicated fragments of the complete wave function Ψ, which depend simultaneously on the nuclear and the Coulomb interaction, have been expressed as fast convergent series of the products of spherical functions. The infinite system of coupled integral equations for the radial functions, which depend on two dimensional variables, has been constructed; and the method of the numerical solution of the truncated system has been demonstrated.

3. The basic approximation with the zero relative moments has been considered as an example. For this case, the system of equations was reduced to two coupled integral equations for two radial functions. The equations of motion have been written down in the form which is convenient for the numerical solution.

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СИСТЕМА ТРЬОХ СИЛЬНОВЗАЄМОДІЮЧИХ ЧАСТИНОК, ДВІ З ЯКИХ ЗАРЯДЖЕНІ, У НЕПЕРЕРВНОМУ СПЕКТРІ

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Р е з ю м е

Маючи на меті зручність подальших досліджень і розрахунків, виконано перетворення рівнянь руху типу Фаддеєва для системи трьох сильновзаємодіючих частинок, з яких дві заряджені і тільки одна заряджена частинка зв'язана з нейтральною. Після відокремлення головних частин з кулоновою взаємодією (КВ), переформульовані рівняння дають можливість представити найбільш складні доданки повної хвильової функції трьох частинок у вигляді швидкозбіжних рядів за добутками сферичних функцій. Одержано також інтегральні рівняння в основному наближенні для радіальних хвильових функцій двох розмірних змінних.