A SPATIAL RGM BASIS FOR SCATTERING ^{4}He$(\alpha,\alpha)^4$He

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Within the resonating group method (RGM), the spatial basis taking into account the structure evolution of colliding nuclei is used in the calculation of the parameters of single-channel $\alpha-\alpha$ scattering. A simple two-level step-like approximation for the radial dependence of the oscillator width of the shell model potential of ^{4}He together with a modified Volkov nucleon-nucleon potential enables to describe all scattering phase shifts in the energy region $0 < E$(c.m.) $\leq$ 40 MeV if, at the internuclear distance less than 3.3 fm, the oscillator width decreases by ≈40% comparatively with its phenomenological value at large distances.

1. Introduction

During more than three last decades, the ^{4}He+^{4}He system is an important object of calculations aimed at the application of the concept of RGM [1] to reactions with $A \geq 4$ nuclei. The simple shell structure of a nucleus ^{4}He and the zero spin of channel fragments, together with the available systematic experimental data on scattering phase shifts, cause the appearance of numerous publications regarding this process in the framework of different calculation versions of RGM [2—4]. The obtained almost at once description of basic features of the energy dependence for scattering phase shifts at E(c.m.) $\leq$ 50 MeV has attenuated, probably, a little the interest to this process, though the reproduction of experimental data was incomplete even in the single-channel area E(c.m.) $\leq$ 17.3 MeV, and some physical aspects of performed calculations did not look quite clear. Basic works of that and the further periods built the reaction space with antisymmetrized products of the usual shell-model wave functions of ^{4}He nucleons. The attempts to improve the results by means of additives of diverse mathematical forms with varied amplitudes to the standard RGM wave function described for first time in [5] had no noticeable success. The inclusion of various hypothetical excited states and pseudostates ^{4}He in the reaction space, following [6,7], also insufficiently improves the calculated phase shift values. The same concerns to the search of a more successful form of nucleon-nucleon potential [8]. For the heavier system, ^{4}He + ^{6}Li, the RGM leads to significant discrepancies with the experimental data on elastic scattering [9, 10]. The elastic-scattering cross sections calculated in both works are essentially overestimated, approximately by a factor of 10, and the inclusion of other most probable reaction channels (α+^{6}Li*, d+^{8}Be, p+^{9}Be, and n+^{9}B) does not change the result [11]. The similarity of the results calculated with quite different forms of the spatial basis and nucleon-nucleon potential shows apparently the inadequacy of the basis built with usual shell-model or cluster wave functions for colliding ^{4}He nuclei.

In this connection, the present work proposes a modified version of the RGM spatial basis which deals with a structural interdependence of channel fragments. All theoretical and calculational aspects of the developed approach are verifed by means of their application to the elastic scattering ^{4}He$(\alpha,\alpha)^4$He. The presented concept of the basis supposes that the initial stage of structural changes for colliding ^{4}He nuclei consists in a variance of the nuclear shell-model potential which can be reflected by the corresponding variance of the oscillator parameter of the basis wave functions. It is evident that the scale of structural changes in the channel fragments is defined by the internuclear distance. Thus, the shell-model nuclear potential of ^{4}He becomes dependt on the distance between the channel fragments. Finally, the parameter $b = \sqrt{\frac{\hbar}{m\omega}}$ (oscillator width) for this potential depends on the internuclear distance r, i.e. we introduce the dependence $b(r)$. A specific form of $b(r)$ in the framework of the general RGM concept must be found from the initial Schrödinger equation for the multinucleon system using the same procedure as for the usual RGM channel wave function. The resulting system of integro-differential RGM dynamic equations for two mentioned functions is, however, essentialy more complicated than the usual RGM dynamic equation. In this situation, it seems resonable to preliminarily estimate the possible effect of the introduced factor by using simpler means. To this end, the present work uses a simple parametrized form of $b(r)$. Optimum values of the parameters are obtained from the fitting of the experimental data on elastic-scattering phase shifts in the energy region $0 < E$(c.m.) $\leq$ 40 MeV.

The necessary formalism of RGM is only slightly more complicated than the usual one. The corresponding dynamic equations are obtained in Section 1, as well as the parametrization of $b(r)$ used in calculations. The construction of the adequate nucleon-nucleon interaction for the ^{4}He + ^{4}He system is presented in Section 2. Section 3 describes the most important computational aspects together with the presentation of the main results of calculations.

2. Equation of Motion

When the cluster shell-model potential depends on the distance between the channel fragments, the RGM wave function for the single-channel scattering has the form

$$\Psi = \mathcal{A}_e \left[F(\vec{r})\, \Phi(\xi_1, r)\, \Phi(\xi_2, r) \right], \tag{1}$$

where $\vec{r}$ is the distance between the centers of mass of the first and second channel fragments, r is the absolute value of $\vec{r}$, Φ is the wave function of ^{4}He with arguments ξ_1 and ξ_2, being the collections of the cluster coordinates for the corresponding fragments, and the dependence on r follows from the dependence $b(r)$. This function is given by a four-nucleon Slater determinant built from nucleon wave functions, and each of the functions is the product of spatial, spin, and isospin nucleon functions. In all calculations, the $1s$-function of a harmonic oscillator is taken as the spatial factor of a nucleon wave function. Therefore, Φ is antisymmetric with respect to the permutations of nucleons inside the corresponding nucleus, $F(\vec{r})$ is the wave function which describes the relative movement of channel fragments, and the operator $\mathcal{A}_e$ antisymmetrizes the wave function of the system with respect to nucleon permutations between channel fragments.

The Hamiltonian (in the center-of-mass system) includes two terms: the operator of kinetic energy

$$K = -\frac{\hbar^2}{2m} \left[\sum_{i,j=1}^{3} \left(\delta_{ij} - \frac{1}{A_1} \right) \nabla_i \nabla_j + \right.$$

$$\left. + \sum_{i,j=5}^{7} \left(\delta_{ij} - \frac{1}{A_2} \right) \nabla_i \nabla_j + \frac{1}{\tilde{A}} \Delta \right] \tag{2}$$

and the nucleon interaction operator

$$I = \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{A} \left[v(r_{ij}) + \frac{e^2}{r_{ij}} \left(\frac{1-\tau_{iz}}{2} \right) \left(\frac{1-\tau_{jz}}{2} \right) \right] \tag{3}$$

where $\tilde{A}$ is the reduced mass of the system, $\tilde{A} = \frac{A_1 A_2}{A_1 + A_2}$, and m is the nucleon mass. The kinetic energy (2) is formed by the movement of nucleons relatively to the center of masses of the corresponding cluster and by the relative movement of clusters. The nucleon interaction (3) is the sum, which is carried out over all possible nucleon pairs, of the nuclear potential $v(r_{ij})$ and the Coulomb potential which is expressed through z-components of the isospin Pauli matrices, and r_{ij} is the distance between the nucleons i and j. It should be noted that, in the case under study, the last term in operator (2) acts also on the cluster wave functions Φ. It is expedient to introduce an operator $K^{(c)}$ representing the kinetic energy in the system of cluster coordinates corresponding to the nucleon distribution produced by the certain term of the operator $\mathcal{A}_e$ in (1). The operator $K^{(c)}$ is defined according to (2), where the first and second sums run over nucleons which belong to the corresponding cluster in the obtained distribution. For the same coordinate system, we introduce the operator of intercluster interaction $I^{(c)}$ defined accordingly to (3), where the nucleons with indices i and j belong, respectively, to the first and second clusters.

Multiplying the Schrödinger equation with the Hamiltonian $H = K + I$ from the left by the complex conjugate wave function (1), performing the integration over the nucleon spatial variables, and using the variational procedure described, in particular, in [12], we obtain a dynamic equation for $F(\vec{r})$. For the channel with orbital moment l, this equation can be presented in spherical coordinates, after the usual substitution $g_l(r) = r\, F_l(r)$, as

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2} E - V(r) \right] g_l(r) +$$

$$+ \int dr'\, N_l(r,r') \left[\frac{d^2}{dr'^2} - \frac{l(l+1)}{r'^2} + \frac{2\mu}{\hbar^2} E \right] g_l(r') -$$

$$- \int dr'\, U_l(r,r')\, g_l(r') +$$

$$+ \left\{ 2\, P(r) \left[a'(r) \frac{d}{dr} + a''(r) \right] + Q(r) (a'(r))^2 \right\} g_l(r) +$$

$$+ 2 \int dr'\, R_l(r,r') \left[a'(r') \frac{d}{dr'} + a''(r') \right] g_l(r') +$$

$$+\int dr' S_l(r,r')\,(a'(r'))^2 g_l(r')+$$

$$+\int dr'\,T_l^{(0)}(r,r')\,g_l(r')=0, \qquad (4)$$

where E is the relative kinetic energy of channel fragments, and μ is the reduced channel mass. The potential $V(r)$ is formed by the local components of the interaction matrix elements:

$$V(r)=\frac{2\mu}{\hbar^2}\langle\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\times$$

$$\times\mid I^{(c)}\mid \mathcal{A}_e\,\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\rangle_{\rm loc}. \qquad (5)$$

The nonlocal components of matrix elements form the integral kernels

$$U_l(r,r')=\frac{2\mu}{\hbar^2}\,rr'[\langle\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\times$$

$$\times\mid I^{(c)}\mid \mathcal{A}_e\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\rangle_{\rm nloc}]_l. \qquad (6)$$

The square brackets with a subscript denote the projection on a state with orbital moment l. The overlap kernels are given by nonlocal components of the corresponding matrix elements:

$$N_l(r,r')=\,rr'[\langle\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\times$$

$$\times\mid \mathcal{A}_e\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\rangle_{\rm nloc}]_l. \qquad (7)$$

The terms of the operator $K^{(c)}$ describing the internal cluster movement together with similar terms of I, which does not enter the operator $I^{(c)}$, give the total internal energy of channel fragments by acting on the wave function (1), which results in the appearance of the kinetic energy E in the equation (see for example [12, p.138]). The fourth, fifth, and sixth lines of the equation result from the spatial dependence of the shell-model potential form and depend on the derivatives of the parameter $a=b^{-2}=\frac{m\omega}{\hbar}$ of this potential. These derivatives are denoted with primes, in a different way than the derivatives of g_l, in order to emphasize that Eq. (4) does not define the function $a(r)$. The fourth, fifth, and sixth lines are produced by the action of the operator $K^{(c)}$.

The local functions $P(r)$ and $Q(r)$ and the integral kernels $R_l(r,r')$ and $S_l(r,r')$ are produced by the local and nonlocal components of matrix elements for the 1-st and 2-nd partial derivatives with respect to a:

$$P(r)=\,\langle\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\times$$

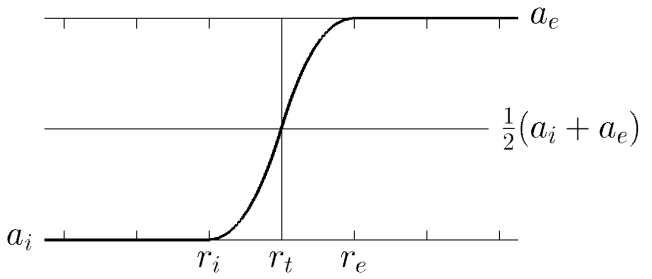

Fig. 1. Accepted dependence of the oscillator width of the shell-model potential of ^{4}He on the distance between ^{4}He nuclei. The notation is explained in the text

$$\times\left|\frac{\partial}{\partial a}\right|\mathcal{A}_e\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\rangle_{\rm loc}, \qquad (8)$$

$$Q(r)=\,\langle\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\left|\frac{\partial^2}{\partial a^2}\right|\times$$

$$\times\mathcal{A}_e\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\rangle_{\rm loc}, \qquad (9)$$

$$R_l(r,r')=\,r\,r'[\langle\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\left|\frac{\partial}{\partial a}\right|\times$$

$$\times\mathcal{A}_e\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\rangle_{\rm nloc}]_l, \qquad (10)$$

$$S_l(r,r')=\,r\,r'[\langle\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\mid\frac{\partial^2}{\partial a^2}\mid\times$$

$$\times\mathcal{A}_e\{\Phi(\xi_1,r)\,\Phi(\xi_2,r)\}\rangle_{\rm nloc}]_l. \qquad (11)$$

The last term in (4) is introduced in order to compensate the use of the operator $K^{(c)}$ instead of the kinetic operator K in the preceding terms of the equation. This term presents the result of the action of the operator $K-K^{(c)}$ and contains components of the integral kernels of the kinetic energy which are the most difficult for computations but sometimes can be omitted. This question is considered in Section 3. For the direct kinetic energy in (4), the operators K and $K^{(c)}$ give the same result. Expressions (5), (6), and (7) are the same as in the case with invariable a, but their final mathematical forms are more complicated due to the dependence $a(r)$.

To solve Eq. (4), one needs the explicit form of $a(r)$. At large r, it is obvious that $a(r)$ is constant, whose value must provide correct values of the binding energy and the radius of a ^{4}He nucleus. As for the behaviour of $a(r)$ at smaller r, one may expect only an enough smooth dependence, which is necessary for the continuity of $g_l(r)$ and the existence of its first two derivatives. These conditions will be undoubtely satisfied for any function with continuous first two derivatives. The present work uses a simple smooth step-like parametric form with the required properties (Fig. 1). The asymptotic value in the region $r \geq r_e$ in Fig. 1, further noted as a_e, is joined with a value (denoted as a_i) of the curve

$$\sum_{k=1}^{3} c_k \, (r - r_t)^{2k-1},$$

in the internal region $r \leq r_i$. This curve is symmetric relative to the point with the coordinates $r_t = \frac{1}{2}(r_i + r_e)$, $a_t = \frac{1}{2}(a_i + a_e)$. All coefficients c_k are determined from the continuity of the obtained function $a(r)$ and its first two derivatives at the points r_i and r_e.

3. Effective Nucleon-nucleon Interaction

Existing expressions for the nucleon-nucleon potential (NNP) enable one, as a rule, to describe more or less properly a part of certain phenomena, and the attempts to extend the region of their application were not successful. The experience of the use of different forms of NNP in the problem under study testifies to the same. One may expect only that the present study may be restricted to central NNP components. The accepted form of $a(r)$ requires the correct reproduction of the size and internal energy of an ordinary ^{4}He nucleus. From this point of view, the most preferable seems the Volkov potential [13]. Constructed just for the calculation of the internal energy of ^{4}He, it was also applied to the scattering.

The central Volkov potential is defined as some effective quantity averaged over spin-isospin states for all nucleon-nucleon pairs in a ^{4}He nucleus. The $^4\mathrm{He} + {}^4\mathrm{He}$ system includes additionaly the pairs where both nucleons have the same spin-isospin states. In the nucleon-nucleon scattering, the spatial component of the wave function for such a nucleon pair must be odd. Odd nucleon-nucleon states do not present in a nucleus ^{4}He and in the $^4\mathrm{He} + {}^4\mathrm{He}$ system. This means that the interaction of nucleons with identical spin-isospin states must be removed from the sum in expression (3) for I and $I^{(c)}$. Eight sets of NNP parameters presented in Table 1 in [13] allow one to reproduce the binding energy of ^{4}He with differente accuracies, discrepancies being up to 0.8 MeV. With the first parameter set (V1), in particular, the binding energy value is underestimated by 0.29 MeV. Further, the calculation [13] did not take the Coulomb interaction into account. The performed calculation showed that the Coulomb interaction supplementarily diminishes the binding energy value, and the discrepancies for the majority of potentials of Table 1 in [13] surpass 1 MeV.

In order to improve the situation, the Volkov potential

$$v(r_{ij}) = \left[(1-m) + mP_{ij}^r\right] \sum_{n=1}^{2} V_n \, \exp\left(-\frac{r_{ij}^2}{a_n}\right) \qquad (12)$$

was multiplied by a corrective coefficient, whose value was fitted from the comparison of the calculated and phenomenological values of the ^{4}He energy. For that purpose, the ^{4}He energy was calculated, analogously to [13], as the minimum value of the corresponding expression

$$\langle \Phi(\xi) \mid K(^4\mathrm{He}) + I(^4\mathrm{He}) \mid \Phi(\xi) \rangle \qquad (13)$$

relative to the variation of the ^{4}He potential parameter. In this case, the operator K is given by the first term (2), and the operator I is defined by (3) with $A = 4$. The ^{4}He wave function has no variable r, because a single nucleus is considered. In (12), the operator P_{ij}^r exchanges the spatial coordinates of the nucleons i and j, and the parameter m defines the contribution of the exchange processes. The hyperbola $\dfrac{1}{r_{ij}}$ in the Coulomb part of the operator $I(^4He)$ at small r_{ij} values ($r_{ij} \leq r_q$) was approximated by a linear combination of Gaussians as

$$\sum_{k=1}^{M} q_k \, \exp\{-\beta_k \, r_{ij}^2\}. \qquad (14)$$

Values of the parameters q_k, β_k were chosen to attain the best possible approximation up to a certain minimal value $r = r_0$ starting from point $r = r_q$ where curve (14) is joined with the hyperbola. The smaller r_0, the more M. To evaluate the necessary value of M, expression (13) was minimized with different values of M, which showed that the increase of M above 6 does not practically change the energy value. In this case, r_q was fixed at 5 fm, i.e. we go out actually of a ^{4}He nucleus. That is why we took $M = 6$ in the following calculations, which corresponds to $r_0 = 0.3$ fm.

The suitability of the modified NNP for the description of the nucleon-nucleon scattering was tested by means of the corresponding calculations for the p + n system. The necessary equation of motion can be obtained from (4) after leaving the first three lines only in its left part. The phase shifts for the energy interval $0 < E \leq 40$ Mev were determined following the procedure described in Section 3 for the $\alpha + \alpha$ system. For all eight sets of NNP [13], the phase shifts $\delta_0(E)$ obtained with modified and original values of the parameters practically coincided, the same was true for the scattering length and the effective radius. The parameters of the modified V1 NNP used in all further calculations are: $V_1 = -84.67$ MeV, $V_2 = 147.17$ MeV, $a_1 = 2.56$ fm^2, $a_2 = 0.6724$ fm^2. The notation corresponds to formula (12). The calculated binding energy of ^{4}He is 28.293 MeV. The original V1 gives the value of 28.01 MeV [13] which must be additionaly decreased because of the Coulomb interaction. The oscillator width of the ^{4}He nuclear potential for both NNPs has the same value: $a = 0.533$ fm^{-2}.

4. Computations and Results

The calculation of the $\alpha - \alpha$ elastic scattering phase shifts consists in the following procedures carried out for different collections of the parameters a_i, r_t, and m in the energy interval $0 < E \leq 40$ MeV. The numerical solution of Eq. (4) has been found in the spatial area, where the nuclear interaction is not negligible for l equal to 0, 2, 4, 6. Near the upper border of the spatial interval, the logarithmic derivative of the solution is matched to its usual asymptotic form composed from Coulomb wave functions. This gave the phase shifts which were compared, after subtracting the Coulomb ones, with the experimental values. It should be noted that the phase shifts were defined with uncertainity $n\pi$. The comparison with the experiment allows us to determine the region of the optimal values of a_i, r_t, and m.

The starting stage of calculations included the determination of the explicit expressions for the coefficients of Eq. (4). For this purpose, it is necessary to perform the integration in the matrix elements present in (5)—(11). In the corresponding integrands, the operator $\mathcal{A}_e$ produces a linear combination of terms of the form $\exp[-\sum_{i,j} c_{ij}(\vec{r}_i\vec{r}_j) + d\sum_i(\vec{r}\vec{r}_i) + d'\sum_i(\vec{r}'\vec{r}_i)]$, where $\vec{r}_k$ are the nucleon cluster coordinates (see [12]) which are the integration variables. The number of variables is 6 for the local functions (5), (8), and (9) and is 5 for kernels (6), (7), (10), and (11); $c_{ij} \geq 0$. In the matrix elements (8)—(11), each exponential has a polynomial factor formed from the integration variables, the maximum degree of the polynomial being 2 for (8) and (10) and 4 for (9) and (11). All these integrals can be calculated analytically [1]. The result is a function of the same type as in the integrand, but a function of the variables r and r', i.e. $\exp[d_1 r^2 + d_2(\vec{r}\vec{r}') + d_3 r'^2]$ with supplementary polynomial factors for (8)—(11). Finally, the considered matrix elements are linear combinations of such exponentials. Moreover, for local ones, $d_2 = d_3 = 0$. The obtained expressions are used then for the computation of functions (5)—(11) for different values of r. The matrix elements of $K^{(c)}$ which are necessary for the "residual" kernels $T^{(0)}$ have a similar simple form. The procedure for the kernels of the kinetic operator K is more complicated. First, we transform K to the cluster coordinate system which corresponds to a certain term of $\mathcal{A}_e$, and then apply it to the term of the wave function of the system which is produced by the same $\mathcal{A}_e$ term. Further, the result was transformed to the initial cluster coordinate system, and the integration was carried out. The procedure was repeated for all terms of the antisymmetrizer, and the sum of the results gave the necessary matrix element. Since all transformations of the cluster coordinates are linear, the operator obtained on the first step differs from the initial one (2) only by the numerical coefficients of differential operators. This operator act on the system wave function like initial one. Therefore, the resulting kernels $T^{(0)}$ have the same mathematical structure as (7)—(11) and are expressed in terms of the algebraic functions of the same structure with respect to r, r'.

For the numerical solution, Eq. (4) was written on the grid of equidistant values of r within the interval $0 < r \leq r_b$, where $r_b \approx 8$ fm is the matching radius. The integral kernels were written according to the Simpson formula in the interval $0 < r' \leq r_b$ with the same radial step h. The Coulomb interaction at small r was approximated following (14) with $M = 6, r_q = 5$ fm. All integral kernels in the spatial region, where $r \geq r_b$ or $r' \geq r_b$, were practically equal to zero. For the differential operators, we used a simple difference formula based on the three-point approximation of the solution by a quadratic polynomial. As a result, we constructed a system of linear equations for values of the unknown function g_l at each point of the discretized r-space. Solving the system, we obtained values of the logarithmic derivative of g_l at the matching point r_b.

The necessary value of a radial step h was found empirically. At first, the calculation was carried out with

an undoubtedly rough step $h \approx 0.2$ fm, than h was decreased, and the result of calculations was compared with the preceding one. In the calculations, we took such value of h, whose decrease does not essentialy change the result. In these testing calculations, the ^{4}He shell-model potential was fixed, i.e. $a_i = a_e$. The rest of computational details is described in [10, 11]. The eligible results were obtained already for $h \leq 0.12$ fm in the calculations with the full equation (4) and without its last term. The phase shifts obtained in these two versions can essentialy differ for the energy values near the ^{8}Be resonances. In the rest of the energy region, the differences were smaller than 2°. It is possible, however, to obtain the practically complete coincidence of the approximated and exact solutions due to a small change of m. Thus, the maximal divergence between the approximated solution with $m = 0.557$ and the exact one with $m = 0.56$ is smaller than 1°. Such a correction of the starting value of m, approximately 0.003, was sufficient for the practically complete coincidence of both results in the interval $0.5 \leq m \leq 0.7$. Since m is not fixed, and its value lies usually in the mentioned interval, the both versions of calculations can be considered as equivalent. Physically, this means that the omitted part of the kinetic energy with $T^{(0)}$ may be compensated by a very weak change of the interaction.

In the calculations with $a_i \neq a_e$, it is taken that a change of the nuclear ^{4}He potential occurs within two radial steps near r_t (Fig. 1). For the used form of $a(r)$, its second derivative is equal to zero at all points of the discretized space. The first derivative differs from zero at the point r_t only. The local terms of the dynamic equation, $V(r), P(r)$, and $Q(r)$ have different forms in the next three spatial regions which correspond to three possible values of a: $r \leq r_i, r = r_t, r \geq r_e$. The mathematical forms of the integral kernels are different for nine domains of their variables corresponding to nine possible pair combinations of three mentioned spatial regions. For this type of calculations, we also compared firstly the results obtained with the exact and approximated forms of Eq. (4). It is found that, for $a_i \neq a_e$, the divergence between the results grows quickly with increase of the difference between a_i and a_e. Since it is necessary to perform the calculations within the very wide intervals of values of the parameters ($0.38 \leq a_i \leq 1.5$ fm^{-2}, $0.5 \leq r_t \leq 5$ fm, $0.5 \leq m \leq 0.7$), the attempts to compensate the discrepancies by a change of m had no success. That is why, we used only the full form of (4) in the calculations with $a_i \neq a_e$.

The proximity of the calculated phase shifts to experimental data was evaluated by means of the least square criterion. The acceptable results for phase shifts with $l = 0, 2, 4, 6$ have been obtained within the following intervals of the parameters: $1.00 \leq a_i \leq 1.7$ fm^{-2}, $1.8 \leq r_t \leq 3.6$ fm, $0.67 \leq m \leq 0.7$. The best description of the experiment was attained for the parameters lying near a curve within the above-mentioned limits. Along such a curve, there are two "local"optima of the description: near $a_i = 1.156$ fm^{-2}, $r_t = 2$ fm, $m = 0.68$ and near $a_i = 1.563$ fm^{-2}, $r_t = 3.25$ fm, $m = 0.695$. The second optimum is absolute one, and this result is presented in Fig. 2. The first optimum gave almost the same description of the experiment. As the experimental data, we used the typical results of a number of authors collected in [8].

The used approximation of $a(r)$ enables us to study the dependence of the results on the steepness of the transition section of this function. This can be realized by means of a variation of the radial step within the admissible interval $h \leq 0.12$ fm. For this purpose, we carried out the calculation with different values of h: $0.02 \leq h \leq 0.12$ fm. These values of h correspond to the transition region width for $a(r)$ from 0.04 fm up to 0.24 fm (see Fig. 1). This procedure led only to a little variance of the trajectory of optimal values of the parameters in the space (a_i, r_t, m) without essential change of $\delta_l(E)$.

As seen from Fig. 2, the results of calculations practically completely describe the experimental data in the single-channel energy region. This indicates, in particular, that the calculation also gives correct values of the energy and the width for three first resonances of ^{8}Be. This means that, alongside with the nucleon-nucleon interaction inside ^{4}He, the used expression for NNP describes also quite well the effective nucleon interaction inside ^{8}Be and in the ^{4}He + ^{4}He system.

The good description of experiments reveals evidently the productivity of the proposed concept of the basis even with a very simple form of $a(r)$ used in the present work. As seen from Fig. 2, the calculation reproduces the main features ("smooth component") of the experimental dependence $\delta_l(E)$ also outside the single-channel region at $E \geq 17.3$ MeV, where, in particular, the channel $p+^7$Li is open. The irregularities superposed on the smooth component originate, first of all, from the neglected reaction channels. At the same time, it is natural to expect of the increasing influence of the spatial region with small r at higher energies. The fact that the calculation reproduces the dominating component $\delta_l(E)$ confirms a magistral tendence of the evolution of the structure of colliding nuclei ^{4}He, which is reflected by the used expression for $a(r)$.

The presence of two almost equivalent regions of the optimum values of the parameters indicates, most likely, that the real dependence $a(r)$ differs from the simple two-level approximation accepted here. The obtained results are insufficient for a more detailed conclusion concerning the form of $a(r)$. However, the present work demonstrates the possibility of a further development for the proposed concept of the reaction space. This could be realized by means of a more complicated parametrization for $a(r)$ or, in a strict way, by solving the system of dynamic RGM equations for two unknown functions: $a(r)$ and $g_l(r)$. The system can be constructed on the ground of the initial microscopic Schrödinger equation by a variational procedure. Since the wave function (1) is nonlinear in $a(r)$, the equations of the system should be more complicated than those considered here. The results presented here for the nuclei ^{4}He which are the most resistant to external influences, indicate, apparently, that the developed approach to the construction of the RGM basis is a potential resource for the improvement of RGM results for reactions with heavier nuclei.

5. Conclusion

The developed concept of the RGM spatial basis with a simple spatial dependence for the oscillator width of the ^{4}He nuclear potential provides the results for $\alpha - \alpha$ scattering phase shifts which are unattainable with the methods earlier employed for this purpose. The calculations reproduce quite well the experiment also outside the single-channel energy region up to $E = 40$ MeV. A modification of the Volkov nucleon-nucleon potential enables us to obtain the exact value for the ^{4}He(g.s.) energy with a correct value of the nucleus radius. The position and the width of low lying ^{8}Be resonances also are obtained exactly. The results show the essential (≈40%) contraction of the ^{4}He nuclear (shell-model) potential, when the distance between the colliding nuclei becomes smaller than ≈3.3 fm. Further, a more detailed information about the evolution of the nuclear potential for the colliding nuclei can be obtained by solving the system of dynamic equations, where the parameter of the nuclear potential is considered as an unknown function dependent on the internuclear distance additionally to the usual RGM wave function of relative movement. It is reasonable to expect that, for the collision of heavier nuclei, the introduced type of the spatial dependence of the nuclear potentials of channel fragments should be pronounced more intensively. Probably, the considered approach to the spatial basis can contribute to the study of the reasons of the recently observed [9,10] impressive divergence between the theoretical results and the experimental data for the scattering of ^{4}He by nuclei with $A > 4$.

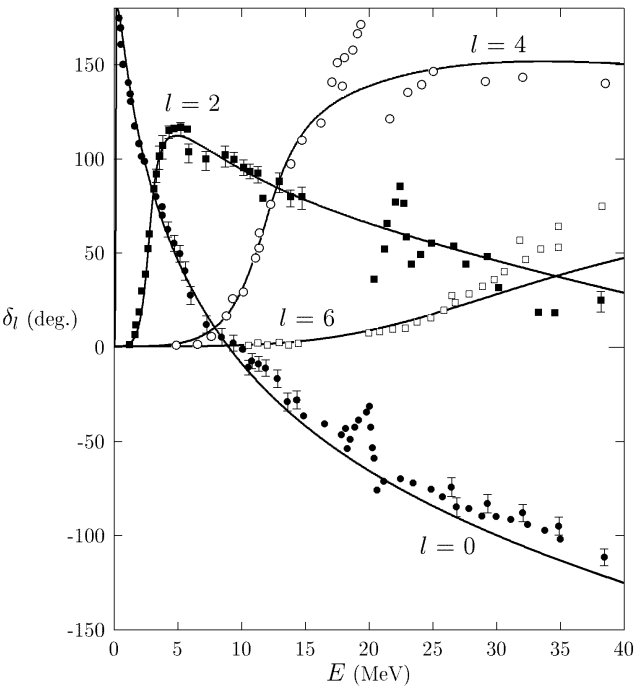

Fig. 2. Phase shifts for the scattering $^4\mathrm{He}\,(\alpha,\alpha)\,^4\mathrm{He}$ with orbital moments $l = 0, 2, 4, 6$. Lines with l values represent the results of calculations. Experimental data for $l = 0, 2, 4, 6$ are given by black points, black squares, open points, open squares, respectively

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ПРОСТОРОВИЙ БАЗИС МРГ ДЛЯ РОЗСІЯННЯ ^{4}He$(\alpha,\alpha)^4$He

Ю.Ю. Козир

Р е з ю м е

Просторовий базис МРГ, який враховує структурні зміни ядер в процесі їх зіткнень, застосовано для одноканального розрахунку $(\alpha-\alpha)$-розсіяння. В наближенні найпростішої дворівневої ступінчастої залежності осциляторного радіуса оболонкового потенціалу ^{4}He від між'ядерної відстані із застосуванням модифікованого нуклон-нуклонного потенціалу Волкова описано енергетичну залежність всіх фазових зсувів розсіяння в області $0 < E$(с.m.) ≤ 40 МеВ за умови, що осциляторний радіус при зближенні ядер на відстань, меншу за $\approx$3,3 фм, зменшується приблизно на 40% від його феноменологічного значення на дальній асимптотиці.