AVERAGE RESONANCE PARAMETERS OF ^{106,108}Cd ISOTOPES

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UDC 539.172

Making use of the method developed by the authors for the analysis of the experimental data concerning the differential cross section of neutron elastic scattering, the average resonance parameters S_0, S_1, R'_0, R'_1, $S_{1,1/2}$, and $S_{1,3/2}$ of the ^{106,108}Cd isotopes in the energy range $1-450$ keV have been determined. The values of the average resonance parameters, both obtained by the authors and available in the literature, have been analyzed for all cadmium isotopes.

1. Introduction

In our previous works, we have determined the strength functions S_0, S_1, $S_{1,1/2}$, and $S_{1,3/2}$ and the radii of potential scattering $R'_0$and R'_1 of neutrons with the orbital moments $l = 0$ and 1 by even isotopes of tin ($^{116-124}$Sn [1, 2]) and cadmium (^{110,112,116}Cd [3]). This work continues our researches, being devoted to the determination of the resonance parameters of the ^{106,108}Cd isotopes. The method of determination of resonance parameters is based on the possibility within the framework of the optical model to install the relationship of the coefficients of the expansion of the differential cross-section of elastic neutron scattering by even-even nuclei into a series of the Legendre polynomials with the average scattering matrix diagonal elements which are in turn expressed in terms of average resonance parameters. The complete sets of the average resonance parameters were determined by fitting the calculated values of σ_{el}, ω_1, and ω_2 to their experimental values. The parameters that were obtained, making use of this method, in works [1–3] describe well the experimental data for σ_{el}, ω_1, and ω_2 within the energy range $1 - 450$ keV.

Concerning ^{106,108}Cd nuclei, the literature data include both the complete sets of average resonance parameters and the values for separate parameters. The complete sets of the parameters concerned were determined only in work [4], while analyzing the experimental differential cross-section of elastic scattering of low-energy neutrons in the framework of the R-matrix theory. In work [5], where the individual parameters of separate resonances were analyzed, only the strength functions S_0 and S_1 have been determined; later, these functions were used as the basis for the system of recommended parameters [6–8]. The values of the parameter S_1 which were obtained in works [4, 5] are in general agreement, within the error limits, with one another; however, the magnitudes of the parameter S_0 have a discrepancy by almost a factor of four. This circumstance, together with other reasons, has stimulated our researches.

Below, the essence of the method is explained shortly, the results obtained are exposed and analyzed. Additionally, the dependences of the resonance parameters on the mass number A are analyzed for all Cd isotopes.

2. The Method of Determination of the Average Resonance Parameters

At energies up to about 450 keV, practically only neutrons with the orbital moments $l = 0$ and 1 are scattered by nuclei. In this case, the differential cross-section of neutron elastic scattering can be expanded in a series of the Legendre polynomials which looks like

$$\sigma_{el}(\mu) = \frac{\sigma_{el}}{4\pi}\{1 + \omega_1 P_1(\mu) + \omega_2 P_2(\mu)\}, \tag{1}$$

where $\mu = \cos\theta$, θ is the scattering angle, σ_{el} the integral cross-section of elastic scattering, P_l are the Legendre polynomials, ω_1 and ω_2 are the coefficients of the expansion. For even-even nuclei, provided that $\sigma_t \approx \sigma_{el}$, we obtained [1]

$$\omega_1 = \frac{6\pi\lambda\!\!^{-2}}{\sigma_{el}}(1 - \eta_{0\mathrm{Re}} - \eta_{1\mathrm{Re}} + \eta_{0\mathrm{Re}}\eta_{1\mathrm{Re}} + \eta_{0\mathrm{Im}}\eta_{1\mathrm{Im}}), \tag{2}$$

$$\omega_2 = \frac{2}{\sigma_{el}}(\sigma_{s1} + \pi\lambda\!\!^{-2} T_{1,3/2}), \tag{3}$$

where $\eta_l = \eta_{l\mathrm{Re}} + i\eta_{l\mathrm{Im}}$ is the l-th diagonal element of the averaged scattering matrix, σ_{s1} is the cross-section

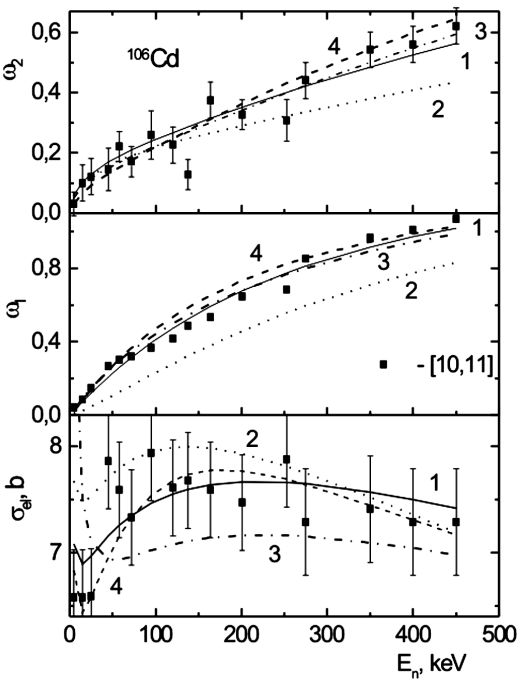

Fig. 1. Energy dependences of the quantities σ_{el}, ω_1, and ω_2 for a ^{106}Cd nucleus. Experimental data (■) were taken from works [10, 11]. The curves demonstrate the results of calculations with the resonant parameters obtained in this work (*1*), with the parameters of work [4] (*2*), with the recommended parameters [6] (*3*), and with the parameters obtained by the fitting procedure, where all the parameters were varied (*4*)

of potential scattering of neutrons with $l = 1$, and $T_{1,3/2}$ is the neutron penetrability factor at $l = 1$ and $j = 3/2$.

In the framework of the optical model, the cross-section σ_{el} of elastic scattering of neutrons with $l = 0$ and 1 is composed of the partial cross-sections of compound and potential scattering,

$$\sigma_{el} = \sigma_{c0} + \sigma_{c1} + \sigma_{s0} + \sigma_{s1}, \tag{4}$$

which are expressed through the matrix elements η_l. In the case of narrow resonances ($\Gamma \ll D$), these cross-sections have to coincide with the corresponding average cross-sections of the resonance theory [4, 9] which have the following forms:

$$\sigma_{c0} = \pi \lambda\!\!\!^{-2} T_0, \qquad \sigma_{c1} = 3\pi \lambda\!\!\!^{-2} T_1,$$

$$\sigma_{s0} = 4\pi \lambda\!\!\!^{-2} \sin^2 \delta_0 (1 - 0.5T_0),$$

$$\sigma_{s1} = 12\pi \lambda\!\!\!^{-2} \sin^2 \delta_1 (1 - 0.5T_1), \tag{5}$$

where $\delta_0 = \arcsin(\rho R_0^{\infty}) - \rho$, $\delta_1 = \arcsin(\rho v_1 R_1^{\infty}) - \rho + \operatorname{arctg}\rho$, $\rho = kR$, $v_1 = \frac{(kR)^2}{1+(kR)^2}$, $R = (1.23A^{1/3} + 0.8)$, $T_l = 1 - |\eta_l|^2 = 2\pi S_l v_l \sqrt{E}$, $R_l^{'} = R[1 - (2l+1)R_l^{\infty}]$, $\kappa = 1/\lambda\!\!\!^{-} = 0.21968\sqrt{E}A/(A+1)$.

If $\Gamma \ll D$, the matrix elements η_l can be presented in the form

$$\eta_{l\mathrm{Re}} = (1 - 0.5T_l)\cos 2\delta_l, \quad \eta_{l\mathrm{Im}} = (1 - 0.5T_l)\sin 2\delta_l. \tag{6}$$

Thus, the quantities σ_{el}, ω_1, and ω_2 in Eqs. (2)—(4) can be expressed in terms of the average resonance parameters making use of expressions (5) and (6). It allows the complete sets of the average resonance parameters S_0, S_1, R_0', R_1', $S_{1,1/2}$, and $S_{1,3/2}$ to be determined by fitting the calculated values of σ_{el}, ω_1, and ω_2 to their experimental ones. The parameter $S_{1,1/2}$ can be found by the relationship $S_1 = 1/3(S_{1,1/2} + 2S_{1,3/2})$. In the course of calculations, we used the corresponding computer code to fit experimental data by minimizing the discrepancy by the χ^2-criterion. All three quantities σ_{el}, ω_1, and ω_2 were fitted simultaneously; nevertheless, the χ^2-monitoring was possible for each separate quantity.

In order to determine the resonance parameters of ^{106,108}Cd nuclei, we used the experimental data on σ_{el}, ω_1, and ω_2 obtained by the authors of work [10] for the separated isotopes in the energy range $1 - 253$ keV. To enhance the reliability of final results, we also used the data obtained by the authors of work [11] for cadmium with natural isotope composition at energies of 275, 350, 400, and 450 keV. The motivation of the expediency of using those data, as well as the calculation procedure, was expounded more comprehensively in work [3], where the resonance parameters of ^{110,112,116}Cd nuclei were found.

3. Results of Calculations and the Analysis of Resonance Parameters

^{106}Cd. The experimental energy dependences of the σ_{el}, ω_1, and ω_2 values, taken from works [10,11] (the data of work [10] were averaged at the beginning of the energy interval), are depicted in Fig. 1 together with the results of our calculations.

The resonance parameters for this nucleus are known from the literature [4–8]. To investigate how they describe experimental values of σ_{el}, ω_1, and ω_2, the relevant calculations were carried out. Curves *2* in Fig. 1 correspond to the results of calculations making use of the parameters of work [4], namely, $S_0 = 0.27(10)$, $S_1 = 4.1(8)$, $R'_0 = 6.55(33)$, $R'_1 = 9.0(6)$, and $S_{1,3/2} = 3.57(33)$ (hereafter, the values of S_l and R'_l are presented in terms of 10^{-4} and fm units, respectively; the errors are indicated in parentheses). One can see from the figure that the results of calculations do not correspond to the experimental data at all (the quality of the description was estimated by the χ^2-criterion). Since the data of work [4] were confined to the energy interval below 253 keV, the values of the parameter R'_1 turned out to be underestimated, which manifested itself in the description of ω_2 at the end of the energy range. Moreover, the overestimation of the parameter R'_0 affected both the description of the cross-section σ_{el} in the first half of the energy interval and the description of ω_1.

The recommended parameters for the ^{106}Cd nucleus are $S_0 = 1.00(35)$ and $S_1 = 5.0(1.5)$ [6–8]. Using these values in the fitting procedure, we obtained the following values for the remaining parameters of the complete set: $R'_0 = 4.72$, $R'_1 = 14.87$, and $S_{1,3/2} = 2.36$. The corresponding results of calculations for σ_{el}, ω_1, and ω_2 are shown in Fig. 1 by curves *3*. It is evident from the figure that the cross-sections are not described satisfactorily: a too large value of the parameter S_0 (and, to some extent, of the parameter S_1) brought about a substantial underestimation of the parameter R'_0, which revealed itself in the description of the cross-sections.

In order to study the influence of the parameters on the description of the experimentally measured quantities σ_{el}, ω_1, and ω_2, the fitting procedure with variation of all the parameters was carried out. The resulting values are as follows: $S_0 = 0.51$, $S_1 = 7.10$, $R'_0 = 4.58$, $R'_1 = 14.25$, and $S_{1,3/2} = 2.83$. From Fig. 1, one can see that the experimental data are described rather satisfactorily (curves *4*); nevertheless, the values obtained for the parameters S_1 and R'_0 evidently go beyond the limits of their generally accepted dependences on A [6]. This may be a result of correlations between the parameters.

All three tested sets of the resonance parameters either do not describe experimental data or include such values that do not agree with the generally accepted dependences on A. To eliminate those shortcomings, we varied each parameter in the set, in turn, to minimize the χ^2 value while describing each of the experimental quantities σ_{el}, ω_1, and ω_2. In such a way, a new parameter set was determined: $S_0 = 0.25$, $S_1 = 3.8$, $R'_0 = 6.29$, $R'_1 = 12.18$, and $S_{1,3/2} = 3.40$. Figure 1 testifies that the calculations with this set of parameters describe well all experimental data (curves *1*). Moreover, the parameters agree with the available generally accepted dependences on A [6].

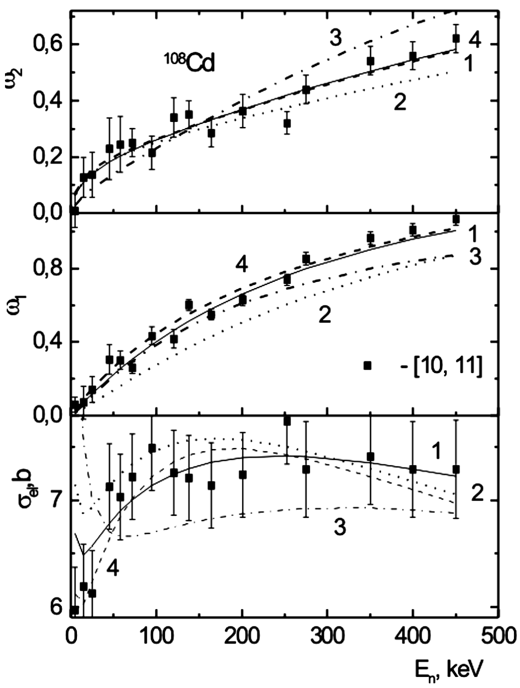

Fig. 2. The same as in Fig. 1 but for the ^{108}Cd nucleus

^{108}Cd. Figure 2 exposes the experimental data for σ_{el}, ω_1, and ω_2 taken from works [10, 11], as well as our results of calculations carried out similarly to the case of the ^{106}Cd nucleus. Curves *2* correspond to the results of calculations with the parameters of work [4]: $S_0 = 0.32(8)$, $S_1 = 4.4(7)$, $R'_0 = 5.99(33)$, $R'_1 = 10.25(80)$, and $S_{1,3/2} = 3.75(31)$. It is evident that the experimental data, analogously to those for the ^{106}Cd nucleus, are not described well by this set of resonance parameters.

The recommended parameters for this nucleus are $S_0 = 1.20(40)$ and $S_1 = 4.8(1.3)$ [6–8]. The fitting with the use of these values gave the following results for the remaining parameters of the set: $R'_0 = 4.25$, $R'_1 = 15.61$, and $S_{1,3/2} = 2.14$. The descriptions of the cross-sections at the beginning of the energy interval (curve *3*) evidence for a too large value of the parameter S_0. The cross-section σ_{c0} calculated, making use of this parameter, at an energy of 5 keV amounts to $\sigma_{c0} = 2\pi\lambda\!\!\!^{-2} T_0 \approx 6.8$ b [see formulae (4) and (5)], which exceeds the experimental value $\sigma_{el} \approx 6$ b. This resulted in a considerable inaccuracy of the values of other parameters, obtained by the fitting procedure. Especially, it is true for the parameter R'_0, the value of which does not agree with the dependence of this

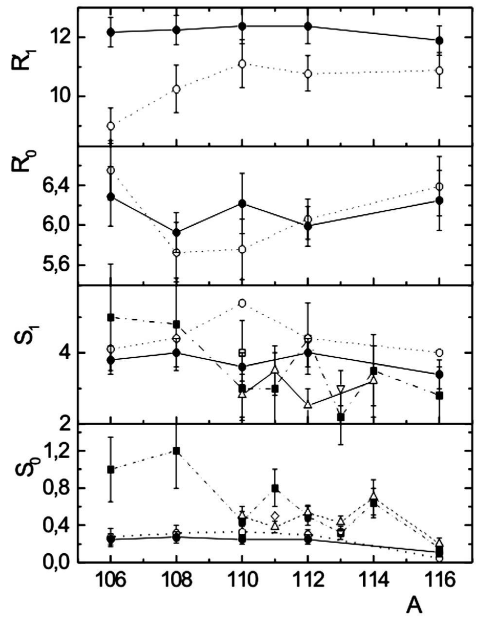

Fig. 3. Dependences of the strength functions S_0 and S_1 and the radii of potential scattering R_0' and R_1' on A for cadmium isotopes: • — this work, ○ — [4], ■ — [6], □ — [7], ◊ — [8], △ — [12], and ▽ — [13]

parameter on A. As a result, the description of experimental data turned out unsatisfactory.

Fitting, with all the parameters being varied, yielded the following results: $S_0 = 0.34$, $S_1 = 6.19$, $R_0' = 5.03$, $R_1' = 12.49$, and $S_{1,3/2} = 3.55$. The experimental data, as is seen from Fig. 2 (curves *4*), become described rather well; nevertheless, the value of the parameter R_0' is somewhat underestimated.

At last, using the way mentioned above, we determined a new set of the resonance parameters: $S_0 = 0.27$, $S_1 = 4.00$, $R_0' = 5.93$, $R_1' = 12.25$, and $S_{1,3/2} = 3.42$. It describes well the experimental data (curves *1*), moreover the minimal values of χ^2 among all the variants of calculations having been reached again.

From the aforesaid, the conclusion that the reliability of the parameters obtained is substantially governed by the accuracy of experimental data can be drawn. From Figs. 1 and 2, it is evident that the experimental data are characterized by rather considerable errors and their distributions by appreciable dispersions. In particular, the cross-sections of both isotopes are characterized by a rather sharp drop at the beginning of the energy interval (below of approximately 30 keV), given by first three experimental points. To check the influence of the latter on the values of the resonance parameters for the ^{108}Cd nucleus, additional calculations were carried out. These calculations showed that, in all variants of the calculations that had been fulfilled before, the exclusion of first three points had not resulted in basic variations of both the final values of the resonance parameters and the description of experimental data.

The values of the resonance parameters for ^{106,108}Cd nuclei that were obtained in this work are quoted in the table.

4. Dependences of the Resonance Parameters of Cadmium Isotopes on A

In addition to the analysis of the sets of the average resonance parameters for separate isotopes of cadmium, it is expedient to consider the dependences of those parameters on the mass number A within the whole chain of cadmium isotopes ($A = 106 \div 116$). Earlier, the dependences of the parameters S_0, S_1, and R_0' on A were studied in work [6] by analyzing the data available in the literature (such an analysis for R_1' was absent for the lack of relevant data). As a separate task of the cited work, the dependence of the parameter S_0 on A for cadmium, tin, and tellurium isotope chains was studied. The conclusion was drawn that the dependence $S_0(A)$ for these nuclei does not agree with the predictions of the optical model concerning both the behavior and the value of this parameter. After the data of work [4] had been published and we had obtained the data published in this work and work [3], a necessity to revise the dependences of the parameters S_0, S_1, R_0', and R_1' on A for cadmium isotopes arose. The strength functions S_0 and S_1, as well as the radii of neutron potential scattering R_0', and R_1', which were obtained by both us and the authors of works [4,6–8,12,13] for even cadmium isotopes, are depicted in Fig. 3.

S_0. From Fig. 3, one can see that there is a considerable inconsistency of the data obtained for S_0 by various authors.

The values of the parameter S_0 that were obtained by us and in work [4] are included into the complete sets of average resonance parameters which were determined by fitting σ_{el}, ω_1, and ω_2 to the experimental data averaged over the energy [10,11]. The analysis showed [3] that the dependences of these quantities on the energy reveal no noticeable isotope effects and fluctuate around their average values within the limits of relevant

Nucleus	$S_0\times10^4$	$S_1\times10^4$	R_0', fm	R_1', fm	$S_{1.1/2}\times10^4$	$S_{1.3/2}\times10^4$
^{106}Cd	0.25(4)	3.8(3)	6.29(21)	12.2(4)	4.6(6)	3.4(3)
^{108}Cd	0.25(4)	4.0(3)	5.93(22)	12.4(4)	5.2(6)	3.4(3)

available dispersions. In general, they agree with the data obtained by a lot of other authors for samples with the natural composition of isotopes [3]. Our results and the results of work [4] for the parameter S_0 do not reveal its significant dependence on A. The analysis of the calculation results that was carried out above allows a conclusion to be made that the values of the parameter S_0 recommended for the ^{106,108}Cd nuclei do not describe the experimental cross-sections at the beginning of the energy interval. Earlier, a similar conclusion has been made for ^{110,112}Cd isotopes [3].

In works [5, 12, 13], the values of the parameter S_0 were obtained by averaging the individual parameters of separated resonances. Therefore, they are not coordinated, in fact, with the average total cross-section and other resonance parameters. It is this reason, as well as the existing difficulties of the determination of resonance parameters (a small number of separated resonances, omission of weak resonances, unreliable identification of resonances), that can explain the chaotic spread of the S_0 parameter values for various A, which is evidently demonstrated in Fig. 3.

$\mathbf{S_1}$ and $\mathbf{R'_0}$. All the aforesaid holds true, to a large extent, for the parameter S_1 as well. Moreover, the methods that were used for the determination of resonance parameters in this work and in work [4] imply correlations between the parameters S_1 and R'_0 which are responsible for some ambiguity in their evaluating the latter [3]. This circumstance and various qualities, with which experimental data were described, led to appreciable divergences between the values of the parameters S_1 and R'_0 determined in this work and in work [4]. Nevertheless, for almost all cadmium isotopes and within the error limits, they are in agreement with one another and with the available recommended values [6]. Figure 3 demonstrates that these parameters, in accordance with the experimental data that were used in calculations, also reveal smooth, in general, dependences on A; but these dependences are not so expressive as that for the parameter S_0.

$\mathbf{R'_1}$. The values of this parameter and the parameter R'_0 for isotopes ^{106,108,110,112,116}Cd were determined for the first time in work [4]. Because of the restriction of the experimental data by energy which was mentioned above, they turned out regularly smaller than the results obtained by us in this work and in work [3] (see Fig. 3).

5. Conclusions

By analyzing the experimental differential cross-sections of neutron elastic scattering by the even cadmium isotopes ^{106,108}Cd, the complete sets of the average resonance parameters S_0, S_1, R'_0, R'_1, $S_{1,1/2}$, and $S_{1,3/2}$ have been determined. They describe well the experimental data for the quantities σ_{el}, ω_1, and ω_2; and their values, on the whole, agree with their available generally accepted dependences on A. It has been found that the values of the parameter S_0 obtained by us and the authors of work [4] for all Cd isotopes did not confirm the assertion about the sharp diminishing of this parameter with the growth of A [6]. It is caused by the fact that the recommended values for the parameter S_0 for the ^{106,108}Cd isotopes are obviously overestimated and do not correspond to the experimental data concerning σ_{el}, ω_1, and ω_2. The values of the parameter S_1 obtained by us agree with the recommended ones [4] within the error limits. The values of the parameter R'_1 that were determined in work [4] have been elaborated.

To summarize, the authors express their sincere gratitude to Prof. A.D. Fursat for his help in creating and improving the computer code, and to the Prof. A.V. Ignatyuk (PEI, Obninsk, Russia) for his participation in discussing the results of the work and his remarks.

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Received 30.09.05.
Translated from Ukrainian by O.I. Voitenko

СЕРЕДНІ РЕЗОНАНСНІ ПАРАМЕТРИ ІЗОТОПІВ ^{106,108}Cd

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Р е з ю м е

Середні резонансні параметри $S_0, S_1, R_0^{'}, R_1^{'}$, $S_{1,1/2}, S_{1,3/2}$ ізотопів кадмію ^{106,108}Cd визначено з аналізу експериментальних диференціальних перерізів пружного розсіяння нейтронів в області енергій 1 — 450 кеВ за допомогою розробленого авторами методу. Проведено аналіз отриманих у роботі та наявних у літературі резонансних параметрів усіх ізотопів кадмію.