NONFACTORIZED DISTORTED-WAVE IMPULSE APPROXIMATION IN THE THEORY OF QUASIFREE SCATTERING. ^{12}C(p, 2p)^{11}B REACTION AT AN ENERGY OF 156 MeV

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A new simple formalism of the nonfactorized distorted wave impulse approximation (DWIA) is proposed. This formalism that takes only the direct mechanism into account is used for the theoretical analysis of the ^{12}C(p,2p)^{11}B reaction at an energy of 156 MeV in the symmetric coplanar geometry. One-particle $1p_{3/2}$ and $1s_{1/2}$ bound states of protons in ^{12}C are generated by the Woods—Saxon shell potential. As the distorted waves, their three-dimensional analytic representations are used. Taking into account the distorting interactions in the input and output channels and employing a reliable description of proton bound states, we obtain a proper quantitative description of the angular correlation functions of the emitted protons. It is found that the dominant contribution to the reaction cross-section is due to a triplet state of the two colliding protons. At the same time, the descent way from the energy surface in the two-nucleon t-matrix is found to be of minor relevance.

Consider the following process. The initial state of the system $|i\rangle$ is comprised of a nucleus-target with the mass number A in the rest and an incident proton with the momentum $\mathbf{p}_0$. In the final state $|f\rangle$, we observe a residual nucleus $(A-1)$ and two emitted protons with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. The simplest description of this reaction is represented by a one-step mechanism that includes the direct interaction of the incident particle with a nucleus' nucleon, which is then extracted from the nucleus. This description is complicated to some extent by the presence of other nucleons in the nucleus. The distorting effects are caused by the interaction of the incident particle with the rest of nucleons of the nucleus-target and the interaction of the emitted particles with the residual nucleus via the corresponding optical potentials. The complications of the simple description may arise due to the exchange with the nucleus' nucleons, interaction between the reaction products in the final state, and as a result of the core excitation that generates the two-stage mechanism of the reaction. Uncertainties in the description of the reaction mechanism complicate significantly the interpretation of the structure of nuclear states.

The knocking-out of nucleons is interesting from the point of view of obtaining the information on the behavior of the nucleon-nucleon (NN) scattering amplitude beyond the energy surface and on the wave function of the intra-nucleus nucleon. Understanding the role of the effects beyond the energy surface allows us to overcome the uncertainties in the spectroscopic information about the investigated states and the momentum distributions of intra-nucleus nucleons. In principle, the information on the states beyond the energy surface can be obtained during investigations of the properties of bound nucleon systems (density, binding energy of the nuclear matter, low energy characteristics of three-nucleon nuclei, doublet width of the nd-scattering, etc.) which are very sensitive to the details of NN-forces. But this is not easy to be implemented. For this purpose, the nuclear reactions present more suitable means, because they allow us to clearly determine the kinematic characteristics in the input and output channels by adjusting, in this way, the deviation from the energy surface in the amplitude of the NN-collision. The empirical information obtained in one set of experiments can be used for the interpretation of other experiments.

The majority of early works devoted to (p,2p) reactions uses the expression for the two-nucleon t-

matrix at the energy surface which relates the square of the absolute value, $|t|^2$, to the corresponding experimental cross-section of the NN-scattering. This approximation works well at the energy surface for sufficiently high energies. But, at energies less than $\sim$ 200 MeV, any approximation at the energy surface is not satisfactory, and the situation becomes worse for deeper bound states. Therefore, in order to obtain the information on the effects beyond the energy surface from the experimental data at such energies, one should take into account the distortion of waves in the input and output channels. The distortion effects somewhat reduce the sensitivity of the cross-sections to the behavior beyond the energy surface, but all the conclusions remain valid qualitatively.

The goal of this work is the construction of a consistent theory of the reactions of quasifree scattering of protons using the nonfactorized impulse approximation with distorted waves and the application of this theory to specific nuclei. It is expected that this approach will be appropriate for the incident proton energies higher than $\sim$ 100 MeV.

In [1, 2], the stable methods of unambiguous determination of parameters of the model distorted waves were proposed, which describe the motion of interactive particles that can be absorbed, focused and can participate in various reactions. In what follows, we use only central optical potentials to describe the effects of the wave distortions and do not take into account a spin-orbital component, which is important only for studying the effects of polarization. Studying the spatial behavior of the three-dimensional distorted waves, we derived the following analytic form for their central component:

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \exp(-\gamma k R + iD\mathbf{k}\cdot\mathbf{r})\times$$

$$\times \left\{1 + F\exp\left[-(\mathbf{r} - R\hat{\mathbf{k}})^2/a^2\right]\right\}, \Psi_{\mathbf{k}}^{(-)}(\mathbf{r}) = \left[\Psi_{-\mathbf{k}}^{(+)}(\mathbf{r})\right]^*, \quad (1)$$

where $D = \beta + i\gamma$ is the refractive index, βk is the modified wave number of the relative motion, γ characterizes the wave decay, and F, R, and a are the focusing parameters that determine the value, position, and width of the diffraction maximum of the absolute value of the wave function in the shadow region of the nucleus.

The angular correlation cross-section (the differential cross-section with respect to the spectrum of a scattered nucleon and the angles of the scattered and knocked-out nucleons) is written in the form

$$\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dE_1} = \frac{(2\pi)^4}{2(2J_i+1)}\frac{p_1}{p_0}\frac{MM_{A-1}p_2^3}{|M_{A-1}p_2^2 - M\mathbf{p}_2\cdot\mathbf{p}_{A-1}|}\times$$

$$\times \sum_{\substack{\mu_0\mu_1\mu_2\\ M_iM_f}} \left|\sum_{\mu_3} t^{2c.m}(12;03) g_{fi}^{(\mu_3\nu_3)}(\mathbf{p})_3\right|^2, \quad (2)$$

where the distorted momentum distribution is related to the overlapping integral [3] in the following way:

$$g_{fi}^{(\mu_3\nu_3)}(\mathbf{p}_3) = (2\pi)^3 \int \psi_{\mathbf{k}_1}^{(-)*}(\mathbf{x}_1)\psi_{\mathbf{k}_2}^{(-)*}(\mathbf{x}_1)\psi_{\mathbf{k}_0}^{(+)}\times$$

$$\times\left(\frac{A-1}{A}\mathbf{x}_1\right) g_{fi}^{(\mu_3\nu_3)}(\mathbf{x}_1) d\mathbf{x}_1. \quad (3)$$

The factor $\frac{A-1}{A}$ in the wave function $\psi^{(+)}$ obviously has a kinematic origin.

The exact description of a three-particle final state in the knocking-out reaction leads to the appearance of the term, which is related to the motion of two nucleons, in the Hamiltonian. Depending on the choice of the kinematic variables that describe the system, this term can appear in the kinetic or potential energy of the Hamiltonian. Therefore, the wave function of the final state cannot be strictly represented as a product of functions. However, this term is of the order of $\frac{1}{A}$ and is usually neglected [27]. This so-called kinetic energy approximation leads to the wave function of a final state in the form of a product of two distorted wave functions for each escaping particle and the wave function of the residual nucleus. This approximation is valid in the static limit or for an infinitely heavy nucleus-remainder.

In the case of the symmetric geometry (no necessarily coplanar), one can use the bi-nucleon approximation [28], in which the wave function of a final state is presented as a product of the distorted wave function of the center-of-mass of two escaping nucleons, the plane wave function of their relative motion, and the wave function of the residual nucleus. This approximation is valid within the plane wave approximation and considerably simplifies calculations.

The characteristic feature of the T-matrix element of the reaction in (2) is the absence of the full factorization in the nucleon-nucleon and nucleus parts (this is the origin of the term "nonfactorized" in the title, in contrast to other works), which makes it impossible to represent

the reaction cross-section in the form of the product of the nonpolarized cross- section of NN-scattering and the distorted momentum distribution.

In the multiparticle shell model, the overlapping integral of the wave function of the initial nucleus φ_i and a certain final state φ_f of the residual nucleus is written down as a decomposition in one-particle wave functions that are characterized by the total angular momentum and the parity ($\lambda = nljm\nu$):

$$g_{fi}(\mathbf{x}_1) = \sqrt{A}\langle \mathbf{x}_1 f|i\rangle = \sum_{\lambda} \phi_\lambda(\mathbf{x}_i)\langle f|\mathbf{a}_\lambda|i\rangle =$$

$$= \sum_{\lambda} C^{J_i M_i}_{J_f M_f jm} C^{T_i N_i}_{T_f N_f \frac{1}{2}\nu} \sqrt{S_{fi}}\phi_\lambda(\mathbf{x}_1). \qquad (4)$$

The spectroscopic factor $S_{fi}(nlj)$ determines the fragmentation of a hole state λ into the final states of the residual nucleus $(A-1)$ and is proportional to the reduced matrix element of the annihilation operator of a nucleon $\mathbf{a}_\lambda$ in the state λ. The two-nucleon t-matrix in the center-of-mass system of two nucleons (2-c.m.s.) lies beyond the energy surface. It depends on the relative momenta of the nucleons $\mathbf{p} = (\mathbf{p}_0 + \mathbf{p}_{A-1})/2$ and $\mathbf{p}' = (\mathbf{p}_1 - \mathbf{p}_2)/2$ before and after a collision and on the collision energy, which is determined ambiguously. The corresponding relative collision energies are $\varepsilon = p^2/M$ and $\varepsilon' = p'^2/M$. From the energy conservation, it follows that $\varepsilon = \varepsilon' + E_S + \frac{A}{A-1}\frac{p^2_{A-1}}{2M}$, i.e. the descent from the energy surface (the deviation of ε' from ε) of the t-matrix increases with both the separation energy of a nucleon E_S and the momentum of the knocked-out nucleon which is equal to the momentum of the residual nucleus p_{A-1}. It is clear that, with the increase in the energy of the incident particle at fixed values of E_S and p_{A-1}, the effect of this descent will be reduced. The scattering angle in the NN-system is determined by the relation

$$\cos\bar{\theta} = \hat{\mathbf{p}}\cdot\hat{\mathbf{p}}' = \frac{1}{2pp'}\left[p_0(p_1\cos\vartheta_1 - p_2\cos\vartheta_2) - \frac{p_1^2 - p_2^2}{2}\right]$$

which gives the escape angle $\bar{\theta} = \pi/2$ in the case of the coplanar geometry.

In the channel spin representation, the t matrix of two nucleons in their center-of-mass system takes the form

$$t^{\text{2c.m.s.}} = \sum_{SM_S M'_S}\sum_{TM_T} C^{SM_S}_{\frac{1}{2}\mu_1\frac{1}{2}\mu_2} C^{SM'_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu_3} C^{TM_T}_{\frac{1}{2}\nu_1\frac{1}{2}\nu_2} \times$$

$$\times C^{TM_T}_{\frac{1}{2}\nu_0\frac{1}{2}\nu_3} t^{ST}_{M_S M'_S}(\mathbf{p},\mathbf{p}';\varepsilon). \qquad (5)$$

The elements of this t-matrix that correspond to the scattering in the state with the total spin S and the isotopic spin T are determined with the help of the partial decomposition that takes into account the identity of nucleons:

$$t^{ST}_{M'_S M_S} =$$

$$= \sum_{JML'M'_L LM_L}\left[1 - (-1)^{L'+S+T}\right] i^{L-L'} t^{JST}_{L'L}(p',p;\varepsilon)\times$$

$$\times C^{JM}_{L'M'_L SM'_S} C^{JM}_{LM_L SM_S} Y_{L'M'_L}(\hat{\mathbf{p}}') Y^*_{LM_L}(\hat{\mathbf{p}}). \qquad (6)$$

The partial components $t^{JST}_{L'L}(p',p;\varepsilon)$ were found in the explicit form for a nonlocal separable NN-potential [4] of rank 2 with the Gaussian form-factors as a solution of the Lippmann—Schwinger equation.

In the case of the (p,2p) reaction, we put $\nu_0 = \nu_3 = \nu_1 = \nu_2 = -\frac{1}{2}$ in (5). Then

$$t_{pp}(12;03) = \sum_{SM_S M'_S} C^{SM_S}_{\frac{1}{2}\mu_1\frac{1}{2}\mu_2} C^{SM'_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu_3} t^{S1}_{M_S M'_S}.$$

In the case of the (p,pn) reaction, we set $\nu_3 = \nu_2 = \frac{1}{2}$ and $T = 0, 1$. Then

$$t_{pn}(12;03) = \frac{1}{2}\sum_{SM_S M'_S} C^{SM_S}_{\frac{1}{2}\mu_1\frac{1}{2}\mu_2} C^{SM'_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu_3}(t^{S0}_{M_S M'_S} + t^{S1}_{M_S M'_S}).$$

Thus, all formulas obtained below for the (p,2p) reaction will be also valid for the (p,pn) reaction after the substitution $t^{S1}_{M_S M'_S} \Rightarrow \frac{1}{2}(t^{S0}_{M_S M'_S} + t^{S1}_{M_S M'_S})$. As a result, the cross-section of reaction (2) contains the interference term involving the isospin states of two nucleons with $T = 0$ and 1. This interference disappears only after the summation over the charged states ν_1 and ν_2 (due to the orthogonality of the coefficients of the sum of vectors). This corresponds to the registration of the contribution of coincidences of all the combinations of possible charge states of the escaping pair of nucleons in each shoulder of the spectrometer, which is not true in the case of the (p,pn) reaction. This assumption led the authors of [10,11], who used the analogy with the ordinary spin, to the application of the erroneous formulas to this reaction that did not contain this interference.

It is known that there are many potentials that are different by their form but are equivalent on the energy surface. This means that all these potentials are able to

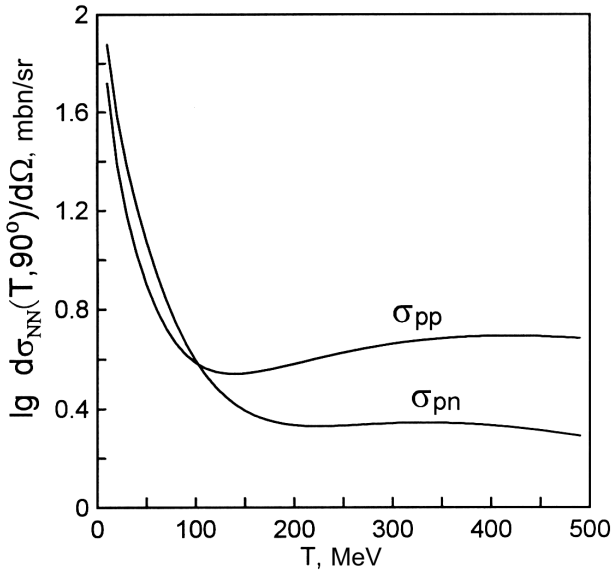

Fig. 1. Cross-section dependeces of the NN scattering in the 2-c.m.s. on the kinetic energy in the l. s., calculated for the nonlocal separable potential [4] at the angle $\bar{\theta} = 90°$

represent any incomplete collection of available two-nucleon data. Predictions of these different phase-equivalent potentials for multinucleon observable data, which are determined by the behavior of the NN-interaction beyond the energy surface, can differ significantly. In particular, a number of properties of the nuclear matter and the systems of three and four nucleons at low energies is especially sensitive to this behavior, which could help to select the most realistic potentials.

The reactions of quasifree scattering of protons by intra-nucleus nucleons also depend on the NN-interaction beyond the energy surface [13]. Unfortunately, theoretical uncertainties and experimental difficulties, which are not completely overcome yet, put obstacles in a way of obtaining the information on the effects beyond the energy surface from such experiments. In works [14,15], it was shown how, by choosing a special geometry of an experiment and taking into account the ratio of cross-sections, one can eliminate the most serious theoretical uncertainties in the distorted momentum distribution and carry out the independent investigation of the pN-collision (N = p,n) beyond the energy surface or of a distorted momentum distribution. Unfortunately, up to now, these proposals and the corresponding program of experimental investigations were not practically realized.

As an illustration of various energy dependences of the cross- sections of the NN-scattering, we present the results obtained with the use of a nonlocal separable NN-potential of rank 2 [4] which describes very well the scattering characteristics up to energies of 500 MeV in the laboratory system (l.s.).

The angular distribution of the scattering of unpolarized nucleons in the 2-c.m.s. is determined by the t-matrix which is given in the channel spin representation by formula (5). This cross-section (at the energy surface or beyond it) enters, as a factor, into the factorized momentum approximation and determines the angular correlation cross-section in the (p,pN) reaction. In the case of the coplanar and symmetric geometries, this cross-section, as was mentioned above, is characterized by the scattering angle $\bar{\theta} = \pi/2$ in the 2-c.m.s.

Fig. 1 shows these theoretically calculated differential cross-sections $\sigma_{pp}(T, \pi/2)$ and $\sigma_{pn}(T, \pi/2)$ as functions of the kinetic energy $T = E - Mc^2$ in the l.s.

From the dependences of cross-sections presented in Fig. 1, it is seen that, in the energy range that exceeds $\sim$ 100—150 MeV, the effects beyond the energy surface are not very large, because the "plateau-like" cross-section in that region is slightly sensitive to which value of the energy (the initial or final one) of the relative motion of two colliding nucleons in reaction (p,pN) is used in calculations of the two-nucleon t-matrix. In the energy range near 50 MeV, the picture is essentially different. Indeed, the exact T-matrix approach [16] and the nonfactorized momentum approximation developed in this paper, which takes approximately a descent from the mass surface into account, produce the results that are noticeably different even at the same collision energy.

The momentum distribution (3) that corresponds to the overlapping integral (4) takes the form

$$g_{fi}^{(\mu_3\nu_3)}(\mathbf{p}_3) =$$

$$= \sqrt{S_{fi}(nlj)} C^{T_i N_i}_{T_f N_f \frac{1}{2}\nu_3} \sum_{m m_l} C^{J_i M_i}_{J_f M_f jm} C^{jm}_{l m_l \frac{1}{2}\mu_3} g^{m_l}_{nlj}(\mathbf{p}_3), \tag{7}$$

where

$$g^{m_l}_{nlj}(\mathbf{p}_3) =$$

$$= (2\pi)^3 \int d\mathbf{x} \psi^{(-)*}_{\mathbf{k}_1}(\mathbf{x}) \psi^{(-)*}_{\mathbf{k}_2}(\mathbf{x}) \psi^{(+)}_{\mathbf{k}_0}(\tfrac{A-1}{A}\mathbf{x}) \phi_{nlj}(x) Y_{l m_l}(\hat{\mathbf{x}}). \tag{8}$$

From (2), it is seen that this reaction can be considered as the scattering by a nucleus' nucleon which is polarized and moves with momentum $\mathbf{p}_3 = -\mathbf{p}_{A-1}$. In this case, the spin function of the nucleus' nucleon

$$\chi = \sum_{\mu_3} a_{\mu_3}\chi_{\mu_3} =$$

$$= \left(\sum_{\mu_3} g_{fi}^{(\mu_3\nu_3)}(\mathbf{p}_3)\chi_{\mu_3}\right) / \left(\sum_{\mu_3}\left|g_{fi}^{(\mu_3\nu_3)}(\mathbf{p}_3)\right|^2\right)^{\frac{1}{2}} \qquad (9)$$

turns out to be dependent on magnetic quantum numbers of the initial and final nuclei through the overlapping integrals of their wave functions. In the case of unpolarized nuclei, when the orientations of the initial and final nuclei are not registered, the nucleon of the target has the effective polarization ($\mu = 0, \pm 1$)

$$P^{(\mu)} = (-1)^l\sqrt{\frac{2l+1}{3l(l+1)}}\left[j(j+1) - l(l+1) - \frac{3}{4}\right]\times$$

$$\times\frac{\sum\limits_{m_l m_l'}(-1)^{m_l}C^{1\mu}_{lm_l'l-m_l}g^{m_l'}_{nlj}(\mathbf{p}_3)g^{m_l\,*}_{nlj}(\mathbf{p}_3)}{\sum\limits_{m_l}\left|g^{m_l}_{nlj}(\mathbf{p}_3)\right|^2} \qquad (10)$$

which depends strongly on the total angular momentum j of the intra-nucleus nucleon. It can be found from (10) that, within the plane wave approximation and in the case $l = 0$, we have $P^{(\mu)} \equiv 0$ for any geometry of escaping protons. With reghard for distortions, the zero value will be obtained only for symmetric coplanar geometry. Due to the fact that, according to (8), the cross-section depends on the projection m_l, the distortion effects act as a filter that selects magnetic substates of the active nucleon and assigns them different weights. Because of this, the nucleon is perceived by the incident proton as a polarized particle.

The characteristic feature of the symmetric coplanar case of the (p,2p) reaction is the fact that, under the rotation by the π angle around the z axis, the system is transformed into the other system physically indistinguishable from the initial system. In this case,

$$e^{-i\pi J_z}g^{m_l}e^{i\pi J_z} = e^{-im_l\pi}g^{m_l} = (-1)^{m_l}g^{m_l} = g^{m_l}, \qquad (11)$$

i.e. $g^{m_l} = 0$, if m_l is odd.

In the plane wave approximation, it follows from (8) that

$$g^{m_l}_{nlj}(\mathbf{p}_3) = (-i)^l g_{nlj}(p_3)Y_{lm_l}(\hat{\mathbf{p}}_3),$$

$$g_{nlj}(p_3) = \sqrt{\frac{2}{\pi}}\int\limits_0^\infty dx\, x^2 j_l(p_3\, x)\phi_{nlj}(x). \qquad (12)$$

Therefore, $g^{m_l}(\mathbf{p}_3) \neq 0$ only at $m_l = 0$, because the momentum $\mathbf{p}_3$ is collinear with the z axis. However, with the distorted waves, we obtain additional contributions from other even m_l-components. This difference is related to the introduction of additional transverse momentum components at the expense of the distorted waves. If the escaping nucleons move with different energies and under different angles or if the incident proton and the escaping nucleons are non-coplanar, the above-mentioned cylindrical symmetry breaks, and the momentum of a recoil nucleus $\mathbf{p}_{A-1} \equiv -\mathbf{p}_3 = \mathbf{p}_0 - (\mathbf{p}_1 + \mathbf{p}_2)$ is not coplanar with the direction $\mathbf{p}_0$. This momentum has the transverse components and thus, even in the plane wave approximation, all m_l-components contribute.

Since the experiments on the knocking-out of nucleons allow us to measure the angular correlation, these experiments give the information on the population of nuclear substates. This information can be obtained by comparison of the symmetric coplanar (p,2p) reaction with a non-symmetric non-coplanar one. Indeed, let us consider the knocking-out of a $1p_{3/2}$ proton from the ground state 0^+ of a ^{12}C nucleus. Then, as follows from (7) and (11), only the substates with $M_f = \pm\frac{1}{2}$ will be occupied in the symmetric coplanar reaction. But, in the case of a broken symmetry, the occupation of the substates with $M_f = \pm\frac{3}{2}$ also begins.

The analysis of the momentum distribution (8) shows that the cross-section of the (p,2p) reaction has a maximum at $p_3 = 0$ if the overlapping integral contains the s-component, and a minimum of the s-component is absent. In the plane wave approximation, this minimum is exactly equal to zero. Small longitudinal momentum components which are introduced by the distorted waves partially fill the minimum, though the distortion effect is usually not sufficient to explain the degree of population that was observed in the majority of experiments performed in the symmetric coplanar geometry. The additional population of the minimum [12] can be explained by the finite experimental energetic and angular resolutions, because this accommodate the cases of a deviation from

the symmetric coplanar geometry and the appearance of non-zero transversal momenta of the recoil nucleus even in the case where the longitudinal component vanishes. The angle ϑ_m, at which the recoil nucleus momentum vanishes (the position of the minimum), is determined in the symmetric coplanar geometry by the expression $\cos\vartheta_m = 1/\sqrt{2}[T_0/(T_0 - E_S)]^{1/2}$. At the zero energy of separation the angle, $\vartheta_m = 45°$. But, for the bound nucleon, $E_S > 0$ and $\vartheta_m < 45°$. The angle ϑ_m shifts slightly with energy also due to the distortion and the relativistic effects. For the energy of incident protons $\sim$ 100 MeV in the l.s., the cross-section of the free NN-scattering at the angle $\bar{\theta} = 90°$ increases with decrease in the relative energy of collision. Therefore, the value and direction of the momentum of an intra-nucleus nucleon $\mathbf{p}_3$ make an essential effect on the NN-scattering cross-section. Namely, this cross-section increases, when $|\mathbf{p}_0 - \mathbf{p}_3| < p_0$, and decreases, when $|\mathbf{p}_0 - \mathbf{p}_3| > p_0$. In the first case, the incident proton flies after the nucleus' nucleon, and, in the second case, the proton and the nucleon move toward each other. This explains, to a considerable degree, the difference of the values of the maxima in the angular correlation cross-sections during the knocking-out of a proton from the p-state. The analogous effect on the peak values is made by the energy dependence of the optical potentials for emitted nucleons, because their energy depends of the escape angle. This means that, in the calculations of the angular distributions, the optical potentials for the final state should change together with a change in the scattering angle. This energy dependence leads to the increase in the distortion and, consequently, to the decrease in the cross-section at the scattering angles $\vartheta > 45°$ in comparison to the scattering angles $\vartheta < 45°$.

The averaged square of the absolute value of the T-matrix element of the transition for the (p,2p) reaction is

$$M_{fi} = \frac{1}{2}\sum_{\mu_0\mu_1\mu_2}\frac{1}{2J_i+1}\sum_{M_iM_f}|T_{fi}|^2 \equiv A_{fi} + B_{fi}, \qquad (13)$$

where A_{fi} and B_{fi} denote the contributions from the singlet ($S = 0$) and triplet ($S = 1$) states of two protons. After the summation over $\mu_1\mu_2M_iM_f$, we obtain

$$M_{fi} = \frac{C^2S_{lj}}{2(2j+1)}\times$$

$$\times \sum_{SM_Sm\mu_0}\left|\sum_{m_lM'_S\mu_3} C^{jm}_{lm_l\frac{1}{2}\mu_3}C^{SM'_S}_{\frac{1}{2}\mu_0\frac{1}{2}\mu_3}t^{S1}_{M_SM'_S}g^{m_l}\right|^2. \qquad (14)$$

From this, we find the contribution of the singlet state,

$$A_{fi} = \frac{C^2S_{lj}}{4(2l+1)}\sum_{m_l}|H_1^{m_l}|^2, \quad H_T^{m_l} = t_{00}^{0T}g^{m_l}, \qquad (15)$$

and that of the triplet state:

$$B_{fi} = \frac{C^2S_{lj}}{2(2j+1)}\times$$

$$\times \sum_{mM_S}\left\{\left|C^{jm}_{lm-\frac{1}{2}\frac{1}{2}\frac{1}{2}}O^{m-\frac{1}{2}}_{1M_S} + \frac{1}{\sqrt{2}}C^{jm}_{lm+\frac{1}{2}\frac{1}{2}-\frac{1}{2}}P^{m+\frac{1}{2}}_{1M_S}\right|^2 +\right.$$

$$\left. + \left|\frac{1}{\sqrt{2}}C^{jm}_{lm-\frac{1}{2}\frac{1}{2}\frac{1}{2}}P^{m-\frac{1}{2}}_{1M_S} + C^{jm}_{lm+\frac{1}{2}\frac{1}{2}-\frac{1}{2}}Q^{m+\frac{1}{2}}_{1M_S}\right|^2\right\}, \qquad (16)$$

where we introduced the notation

$$O^{m_l}_{TM_S} = t^{1T}_{M_S1}g^{m_l}, \quad P^{m_l}_{TM_S} = t^{1T}_{M_S0}g^{m_l},$$

$$Q^{m_l}_{TM_S} = t^{1T}_{M_S,-1}g^{m_l}. \qquad (17)$$

Formulas (15) and (16) obtained above are new and generalize the known ones [16] to the general geometries of the momenta of escaping protons. In this connection, we note that the use of expressions from [16] for the description of non-symmetric experiments in a number of works of the Saint—Petersburg group is erroneous.

As was mentioned above, the formulas obtained for the (p,2p) reaction are valid also for the (p,pn) reaction, if we replace (p,pn)$t^{S1} \Rightarrow (t^{S0} + t^{S1})/2$. Such a substitution is usually carried out also during the formal transformation of the formulas for the free pp-scattering to the case of the pn-scattering.

On the basis of the nonfactorized momentum approximation constructed in this work, we carry out the theoretical analysis of the ^{12}C(p,2p)^{11}B reaction at an energy of 156 MeV.

Fig. 2 shows our results of the DWIA-calculations of the angular correlation cross section (2) in the symmetric coplanar geometry ($\vartheta_1 = \vartheta_2 = \vartheta$) and their comparison with the data at 156 MeV for p-protons [5] and at 160 MeV for s-protons [6]. The contribution from the indirect mechanism is found to be insignificant. The distorted waves were taken for the input channel at 156 MeV and for the output channel at 72 MeV in the case of the optical potential with the volume and surface absorption [1]. Because we failed to find the data on the elastic (p—^{11}B) scattering in the literature for the description of the output channel at necessary energies,

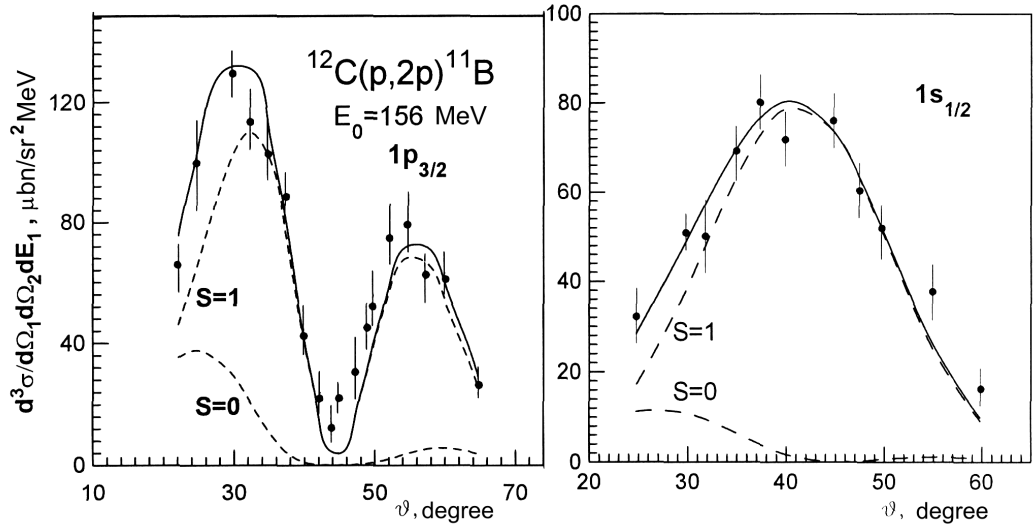

Fig. 2. Dependences of the differential cross-section of the ^{12}C(p,2p)^{11}B reaction on the escape angle ϑ of nucleons for the knocking-out of protons from the $1p_{3/2}$ and $1s_{1/2}$ states of a ^{12}C nucleus. The solid lines correspond to the theoretical calculations in DWIA. The dashed lines show the contribution from the singlet ($S = 0$) and the triplet ($S = 1$) states of the two protons

we used the data on the (p — ^{12}C) scattering. This should not introduce a great error to the explanation of the interaction in the final state. The use of the distorted waves for the potentials with a different type of absorption changes, mainly, only the general normalization of the cross section, which influences the value of the obtained spectroscopic factor C^2S and, to a less degree, affects the shape of the correlation curves. In figures, we used $C^2S(1p_{3/2}) = 3.68$ and $C^2S(1s_{1/2}) = 2.0$. In the case of s-protons, this value of C^2S completely exhausts the value given by the sum rule [3] for spectroscopic factors:

$$\sum_{\alpha_f J_f T_f} \left\{ C^{T_i N_i}_{T_f N_i - \nu \frac{1}{2} \nu} \right\}^2 S_{fi}(nlj) =$$

$$= \sum_{fm} |\langle f|\mathbf{a}_\lambda|i\rangle|^2 = \langle i| \sum_m \mathbf{a}^+_\lambda \mathbf{a}_\lambda |i\rangle = N_{nlj\nu}, \qquad (18)$$

where $N_{nlj\nu}$ is interpreted as the number of active nucleons (protons with $\nu = -1/2$ or neutrons with $\nu = 1/2$) at the nlj-subshell of the nucleus A which is in the state $\alpha_i J_i T_i N_i$.

In the case of p-protons, the obtained value corresponds to the contribution from the ground $\frac{3}{2}^-$ and first excited $\frac{1}{2}^-$ (2.125 MeV) states of a nucleus ^{11}B, which follows from the calculated values of the spectroscopic factors [8] for these states and agrees with the energy resolution ($\sim$ 3.5 MeV) in the analyzed experiment. Generally speaking, the $p_{3/2}$ distribution can contain the contributions from three states of the residual ^{11}B nucleus: the ground state $\frac{3}{2}^-$ and two excited states $\frac{1}{2}^-$(2.125 MeV) and $\frac{3}{2}^-$(5.020 MeV). According to the data on the (e,e′p) reaction [17], their relative contributions constitute 79, 12, and 9%.

The elements of the two-nucleon t-matrix that enter (17) were calculated by half beyond the energy surface from the left, that is, at the collision energy of two protons that corresponds to the input channel. For these energies, the ways of the descent from the energy surface have no basic importance and lead to the cross-sections that do not differ much from each other. The method of the descent used in our calculations gives a better representation of the shapes of the correlation curves.

For the description of the one-particle $1p_{3/2}$ and $1s_{1/2}$ bound proton states in a ^{12}C nucleus, we used a local shell Woods—Saxon potential [18]. The parameters of the potential were chosen from the condition of fitting the data on the elastic scattering of electrons and the separation energy eigenvalues obtained from the data on the (p,2p) and (e,e′p) reactions with a poor energy resolution. The latter provides automatically the averaging over all final excited states of the residual nucleus. In this way, we obtain the Hartree—Fock energy

$$E_S = -(E^i_A - \sum_p [\Gamma^i_p(lj)]^2 E^p_{A-1}) = |Q| + \sum_p (\Gamma^i_p)^2 E^*_p,$$

where the second term on the right-hand side determines the average value of the binding energy of final states and the average excitation energy of the residual nucleus. For the ^{12}C nucleus, we take $E_S(1p_{3/2}) = 16.86$ MeV and $E_S(1s_{1/2}) = 33.89$ MeV [19].

The integral (8) is evaluated explicitly if the radial wave function of the bound state is presented by the Gaussian functions $\phi_l(r) = r^l \sum_{n=1}^{3} \alpha_n \exp(-\beta_n r^2)$.

In the case of the high energy resolution of the experiment, when the final states of the residual nucleus are clearly separated and the wave distortion can be reliably calculated, the analysis of the experimental data with the use of the theory reduces to obtaining the information on the structure of a nucleus: the spectroscopic factor C^2S (for the orbital with quantum numbers nlj) and the mean square radius r_{rsm} (relative to the $A-1$ system) of the wave function of the bound state. In this case, the depth of the well, for which we calculate the bound state, is chosen from the condition that the separation energy of the nucleon corresponds to the studied final state in the $A-1$ nucleus.

As a rule, with increase in the excitation energy E_p^* of the residual nucleus, we observe the increase in r_{rsm} [17], which corresponds to the increase in the parameter r_0 in the potential of the bound state and imitates the effects of the discarded surface terms in the inhomogeneous differential equation for the exact overlapping integral [20, 21]. This is, in fact, the essence of one of the methods of taking the effects of the inhomogeneity of the equation into account, which consists in a modification of the geometry of the Woods—Saxon well in the homogeneous equation for a one-particle state. As shown by our calculations, the increase in the parameter r_0 of the binding potential leads to the increase in r_{rsm} of the radial function. This strongly decreases, in turn, the value of C^2S obtained from the comparison with the experiment. This is explained by the fact that, with increase in r_0, the effective region of space that gives a noticeable contribution to integral (8) for the components of the distorted momentum distribution also increases.

It is seen that, in all cases, we obtain a good agreement with the experimental data, which is a result of the thorough calculation of the distorting interactions [1] in channels and the reliable description of the properties of bound states. The contribution of the triplet state ($S = 1$) of two colliding protons to the cross section of the reaction is found to be dominant (the dashed curve), and the way of the descent from the energy surface in the two-nucleon t-matrix in the considered energy range is found to be of minor relevance.

In the first (p,2p) experiments on nuclei of the $1p$ shell, the total energy spectra demonstrated two broad peaks for $A \leq 12$ and three peaks for $A > 12$ which were explained by the knocking-out from the states $s_{1/2}$, $p_{3/2}$, and $p_{1/2}$. The angular distributions obtained from these experiments were analyzed, as a rule, using the sum rule (18). This approach assumes that every peak in the energy spectrum and the corresponding angular distribution contain the contributions from all allowed final states of the residual nucleus, and that the measured separation energy is equal to the mean separation energy. In the experiments with a better energy resolution, it was revealed that these peaks have a fine structure. At first, it was assumed that this structure is due to the mixing of configurations, i.e. to the introduction of additional one-particle states in the ground state of the nucleus-target. Later on, it was found that these additional peaks (fine structure) are due to the splitting of the one-hole force (fragmentation) over the possible excited states of the residual nucleus, and the peak positions correspond to the energies of these states. The matter is that the hole state, which arises as a result of the removal of a nucleon from the shell, is not, generally speaking, an eigenstate of the residual nucleus, but it is fragmented over these states according to values of the spectroscopic factor. Therefore, the study of the energy spectra of escaping nucleons allows us to know the states of the residual nucleus which are excited in the course of the reaction and their relative shares. This allows us to identify the states that are not observable in other reactions. The experimental relative probabilities of the excitation of various states in the same reaction can serve as a criterion of the validity of sum rules. In the cases where the observed states do not exhaust the sum rule, the estimates of the mean energy of removal can be unreliable due to the impossibility to verify the assumption that the peak position determined in experiments with the poor energy resolution corresponds to the sought value of the mean energy of removal.

The measurement of quantities of the Q-reactions provides the information on the nucleon binding energies in various one-particle states in the nucleus-target. The study of the angular distribution, which corresponds to each excited state, allows us to determine the experimental spectroscopic factor and the transferred angular momentum and also provides the information on the one-particle wave function of a nucleon in the nucleus-target.

Based on the occupation number formalism, the analysis of the experimental data [22], in which the separate final states were determined, showed the necessity of the thorough spectroscopic analysis. Such an analysis was first performed for nuclei of the $1p$ shell [23] with the use of the spectroscopic factors obtained from the genealogical coefficients [8] and one-particle wave functions of the bound protons, which were found from the electron scattering description. These calculations contained no fitting parameters. The same reactions on the nuclei of the same shell were studied in [24, 25] with the use of the diagram technique. From the comparison with the experimental cross sections, the reduced width and the nucleus radius were determined for momentum distributions of the Butler's type. In both cases, the quality of the description was inadequate.

Earlier, the cross-section of this reaction was calculated [23] using the diproton model [28] at 160 MeV in the factorized DWIA for the symmetric coplanar geometry. The cross-section of the free pp-scattering $(d\sigma/d\bar{\Omega})_0$ lay totally on the energy surface at the kinetic energy $T_0 = E_0\sqrt{\frac{p_{A-1}^2}{M^2}+1}+\frac{\mathbf{p}_0\cdot\mathbf{p}_{A-1}}{M}-Mc^2$ in the l.s. and for the scattering angle of 90° in the 2-c.m.s. Moreover, at that time, the distorting potential for the pair of escaping protons (diproton) was not well known. This led to the quite poor description of data [6]. In particular, the right maximum of the p-correlation cross-section was found to be twice smaller than the experimental one. In earlier calculations [29], the agreement of the theoretical values and the experimental data was even worse because the functions of bound states were calculated using the non-adequate oscillating and exponential functions, and the distorting potentials were not known at all and were calculated theoretically on the basis of the Gaussian distribution of the nucleon density in a nucleus and the amplitude of the $NN-$ scattering by the method described in [30]. The later calculations [30] suffered from the same drawbacks.

Unfortunately, the high-energy data for the (p,2p) reactions are not well suitable for a detailed spectroscopic analysis. The situation is better at intermediate energies, where the experiment resolution is higher. For example, the data on ^{12}C [26] at 50 MeV reveal five clearly visible peaks in the total energy spectrum. These peaks correspond to the ground state $\frac{3}{2}^-$ (2.14 MeV) of ^{11}B and to its excited states $\frac{3}{2}^-$ (5.04 MeV), $\frac{5}{2}^-$ (4.46 MeV), and $\frac{7}{2}^-$ (6.76 MeV). Among all these states, only the states with $\frac{1}{2}^-$ and $\frac{3}{2}^-$ can be described within the shell model with the intermediate bond [8]. The states $\frac{5}{2}^-$ and $\frac{7}{2}^-$ are forbidden within the one-step mechanism, and their explanation requires a two-stage mechanism with the excitation of ^{12}C at the level 2^+ (4.43 MeV). This state is strongly collectivized and is easily excited on the inelastic scattering. Therefore, such states should possess the structure of a $1p_{3/2}-$hole related to the state 2^+ of a ^{12}C nucleus.

Our calculations show that a consistent theory, which takes into account the distortion effects and the calculated spectroscopic characteristics of residual nuclei, allows us to obtain a good agreement with the experimental data without using additional fitting parameters. Our results support the assumption about the validity of the proposed approach for the proton energies higher than 100 MeV and demonstrate the suitability of the developed formalism for obtaining the detailed information on the structural specific features of nuclei related to the one-particle degrees of freedom.

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НЕФАКТОРИЗОВАНЕ ІМПУЛЬСНЕ НАБЛИЖЕННЯ ЗІ СПОТВОРЕНИМИ ХВИЛЯМИ В ТЕОРІЇ КВАЗИВІЛЬНОГО РОЗСІЯННЯ. РЕАКЦІЯ ^{12}C(p, 2p)^{11}B ПРИ ЕНЕРГІЇ 156 МеВ

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Р е з ю м е

Запропоновано новий простий формалізм нефакторизованого імпульсного наближення із спотвореними хвилями (DWIA), в якому проведено теоретичний аналіз реакції ^{12}C(p,2p)^{11}B при енергії 156 МеВ в симетричній компланарній геометрії з урахуванням тільки прямого механізму. Одночастинкові $1p_{3/2}$ і $1s_{1/2}$ зв'язані стани протонів в ^{12}C генерувалися оболонковим потенціалом Вудса — Саксона. Як спотворені хвилі використовували їхні тривимірні аналітичні представлення. В результаті ретельного врахування спотворюючих взаємодій у вхідному і вихідному каналах і надійного опису протонних зв'язаних станів одержано хороший кількісний опис кутових кореляційних спектрів вилітаючих протонів. Внесок триплетного стану двох протонів, що зіштовхуються, в переріз реакції виявився домінуючим, а спосіб сходу з енергетичної поверхні в двонуклонній t-матриці — неістотним.